(Heavy) dense QCD and nuclear matter from a (3d) effective lattice theory

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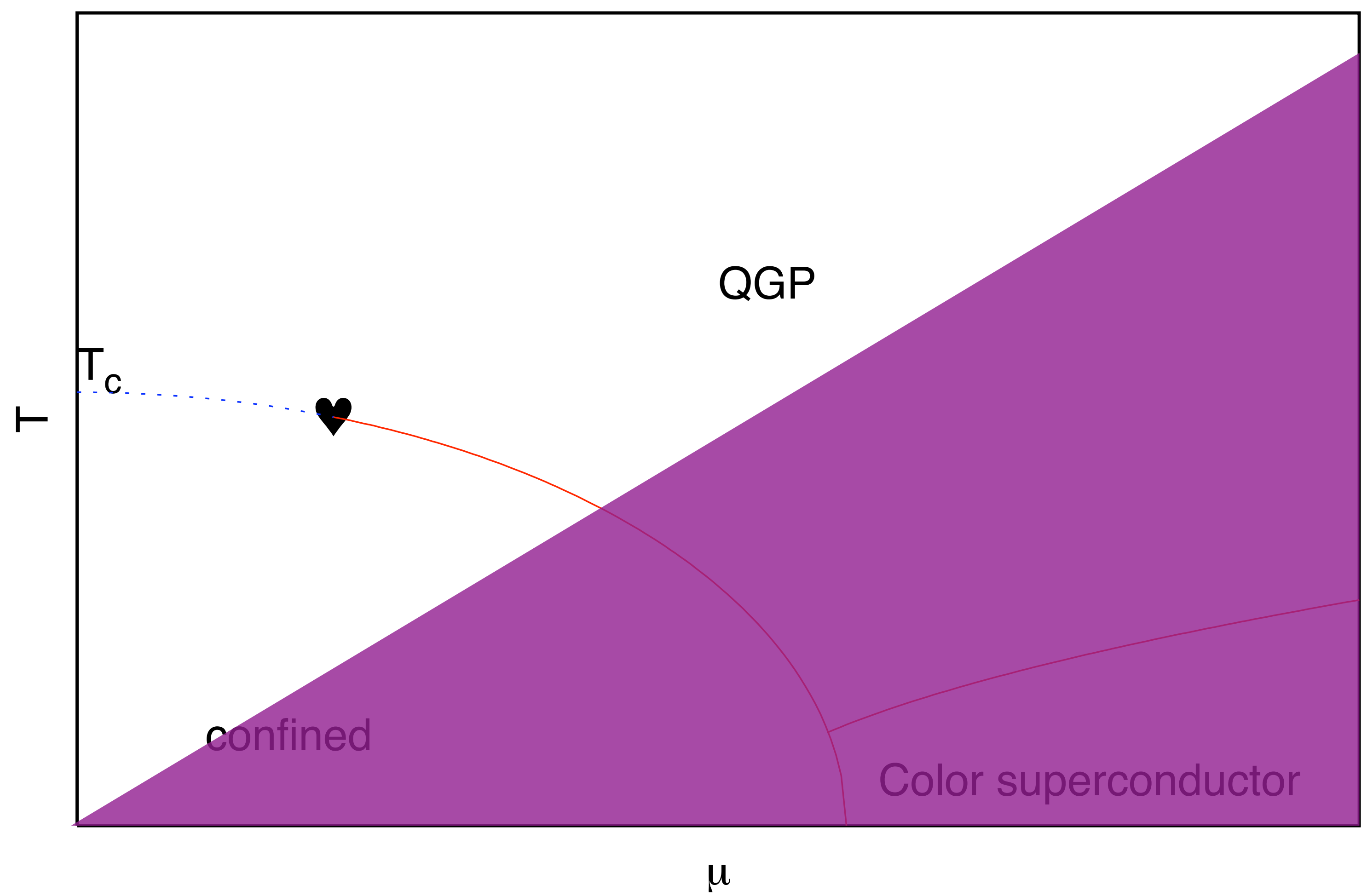
- 3d effective lattice theories derived by strong coupling methods
- The deconfinement transition in QCD with heavy dynamical quarks JHEP 1201 (2012)
- Cold and dense QCD: transition to nuclear matter JHEP 1409 (2014)
- Cold and dense chiral QCD on (very) coarse lattices PRL 113 (2014)

The (lattice) calculable region of the QCD phase diagram



- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \;\; (\mu = \mu_B/3)$
- No critical point in the controllable region
- Complex Langevin: lots of progress, but not in all parameter space, no "guarantees"

The effective lattice theory approach I

- Two-step treatment:

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: split temporal and spatial link integrations:

$$Z = \int DU_0 DU_i \; \det Q \; e^{S_g[U]} \equiv \int DU_0 e^{-S_{eff}[U_0]} = \int DL \; e^{-S_{eff}[L]}$$

  Spatial integration after analytic strong coupling and hopping expansion

- Result: 3d spin model of QCD, infinitely many couplings, ordered parametrically
- Truncation: valid for heavy quarks, sufficiently close to the continuum
- Step II: sign problem milder: Monte Carlo, complex Langevin
- Numerical simulations in 3d without fermion matrix inversion, very cheap!

Starting point: Wilson's lattice Yang-Mills action

Partition function; link variables as degrees of freedom

$$Z = \int \prod_{x,\mu} dU(x;\mu) \exp(-S_{YM}) \equiv \int DU \exp(-S_{YM})$$

Wilson's gauge action

$$S_W = -\frac{\beta}{N} \sum_p \mathrm{ReTr}(U_p) = \sum_p S_p \qquad \beta = \frac{2N}{g^2}$$

Plaquette:

$$\square \to 1 + ia^2 g F_{\mu\nu} - \frac{a^4 g^2}{2} F_{\mu\nu} F^{\mu\nu} + O(a^6) + \ldots$$

$U_\mu(x) = \mathrm{e}^{-iagA_\mu(x)}$

$$T = \frac{1}{aN_t}$$ continuum limit $a \to 0, N_t \to \infty$

Small $\beta(a) \Rightarrow$ small T

- Leading order graph in case of $N_\tau = 4$:

Figure: 4 plaquettes in fundamental representation lead to a 2 Polyakov loop interaction term

- Integration of spatial link variables leads to

$$-S_1 = u^{N_\tau} \sum_{\langle ij \rangle} \mathrm{tr}\, W_i\, \mathrm{tr}\, W_j$$

Expansion parameter: $u = a_f(\beta) = \beta/18 + \cdots$

- Possible generalizations: larger distance, higher dimensional representations, larger number of loops involved, ...
- *Here*: Decorate LO graph with additional spatial and temporal plaquettes

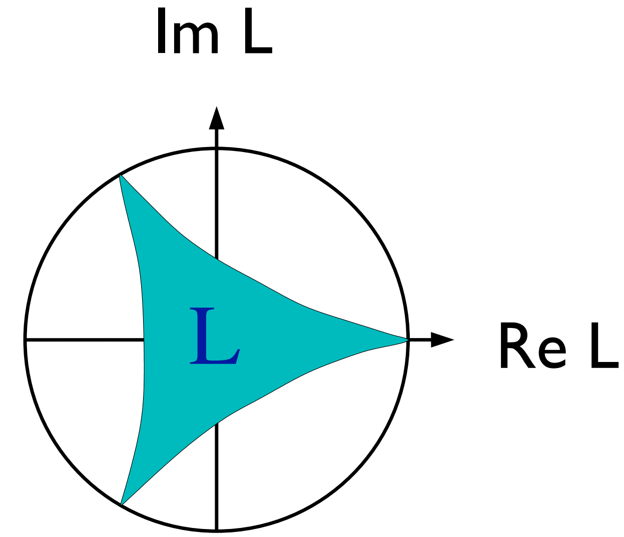
Effective one-coupling theory for SU(3) YM

(L= Tr W)

$$Z = \int [dL] \exp\left[-S_1 + V_{SU(3)}\right]$$

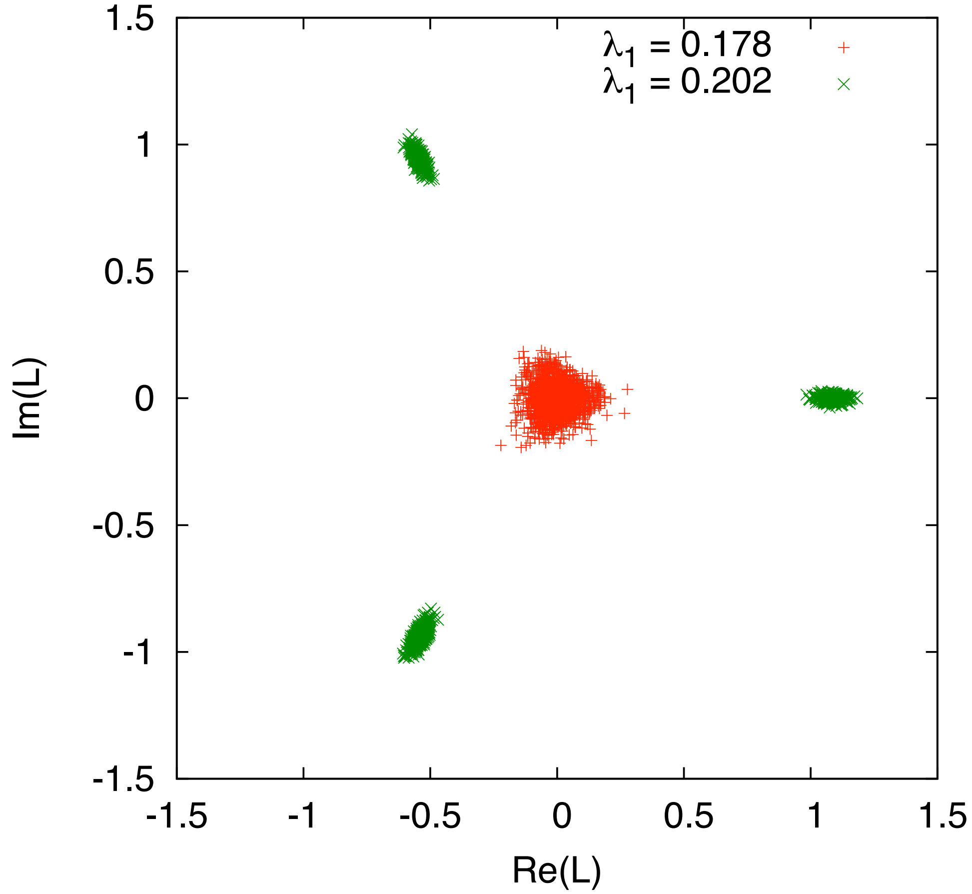
$$= \int [dL] \prod_{\langle ij \rangle} \left[1 + 2\lambda_1 \mathrm{Re}\left(L_i L_j^*\right)\right] *$$

$$* \prod_i \sqrt{27 - 18|L_i|^2 + 8\mathrm{Re}L_i^3 - |L_i|^4}$$

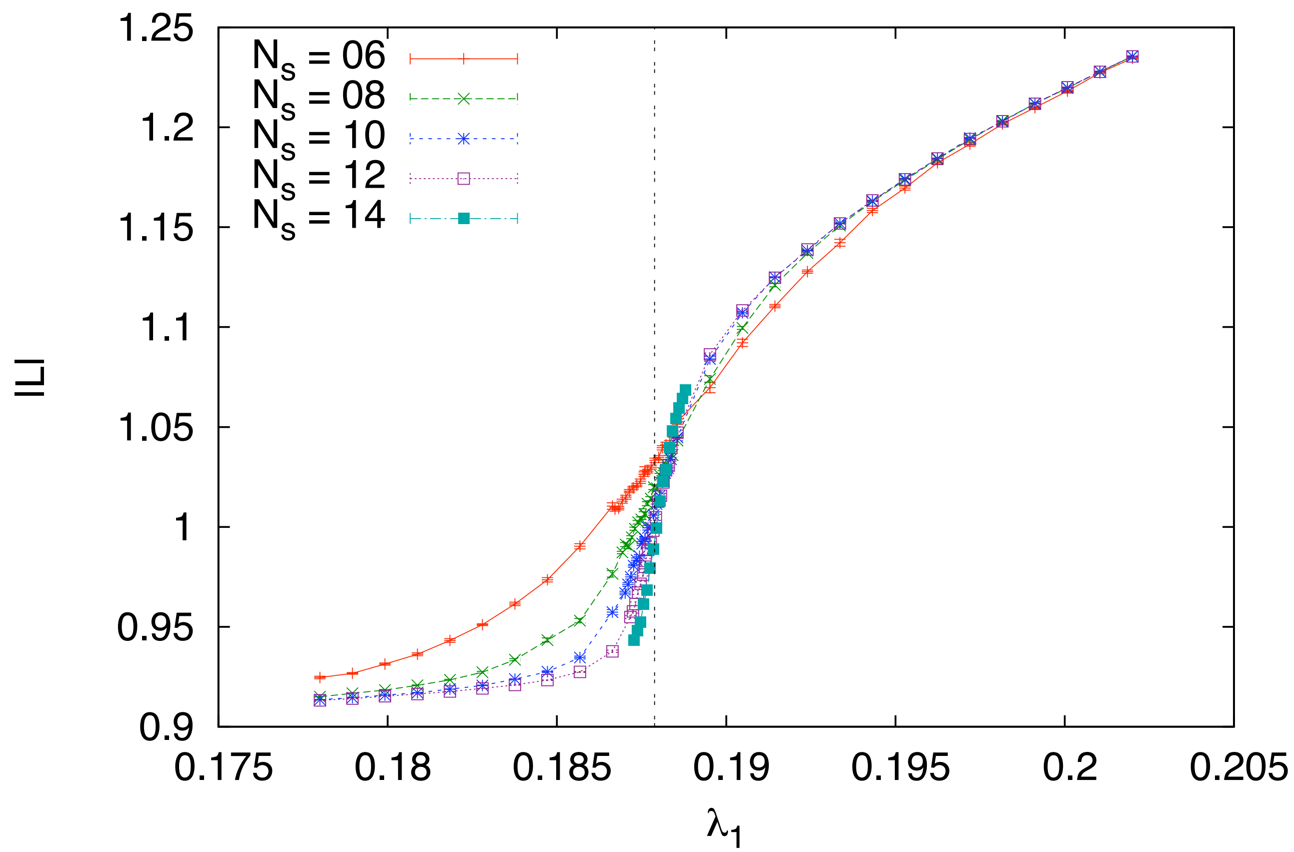


$$\lambda(u, N_\tau \geq 5) = u^{N_\tau} \exp\left[N_\tau\left(4u^4 + 12u^5 - 14u^6 - 36u^7 + \frac{295}{2}u^8 + \frac{1851}{10}u^9 + \frac{1055797}{5120}u^{10}\right)\right]$$

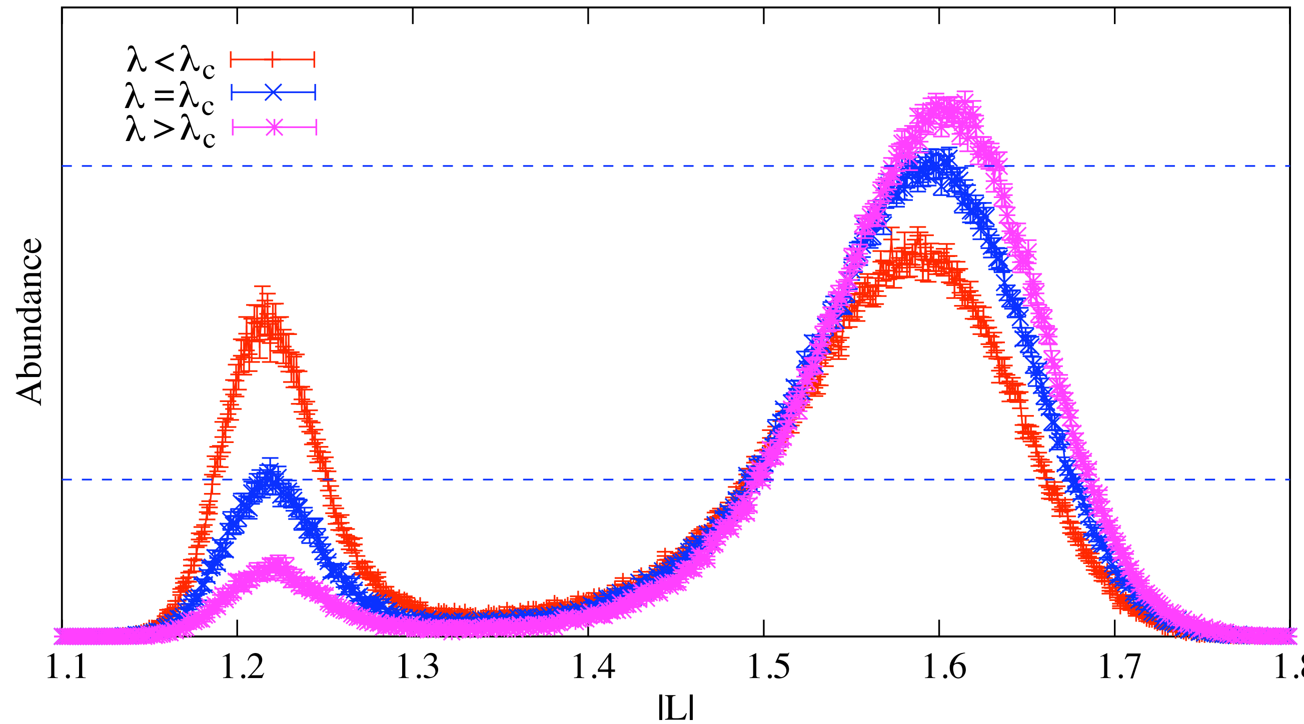
Numerical results for SU(3), one coupling



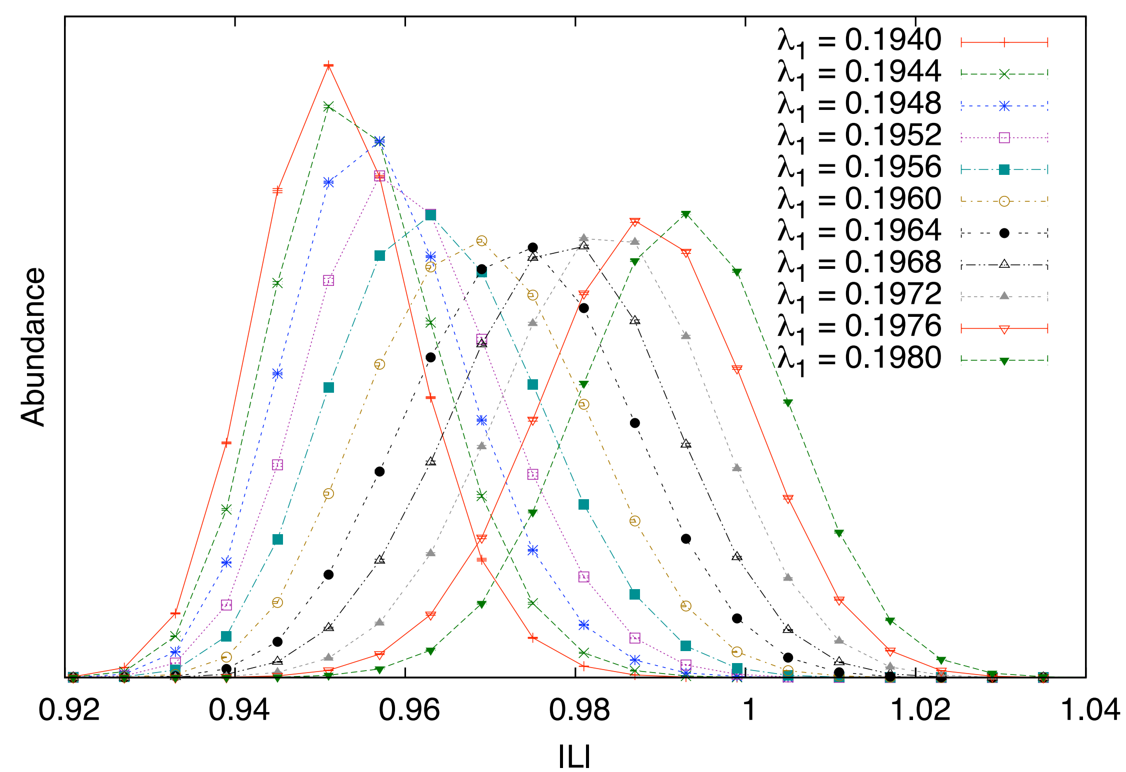
Order-disorder transition
=Z(3) breaking



First order phase transition for SU(3):



Second order (3d Ising) phase transition for SU(2):

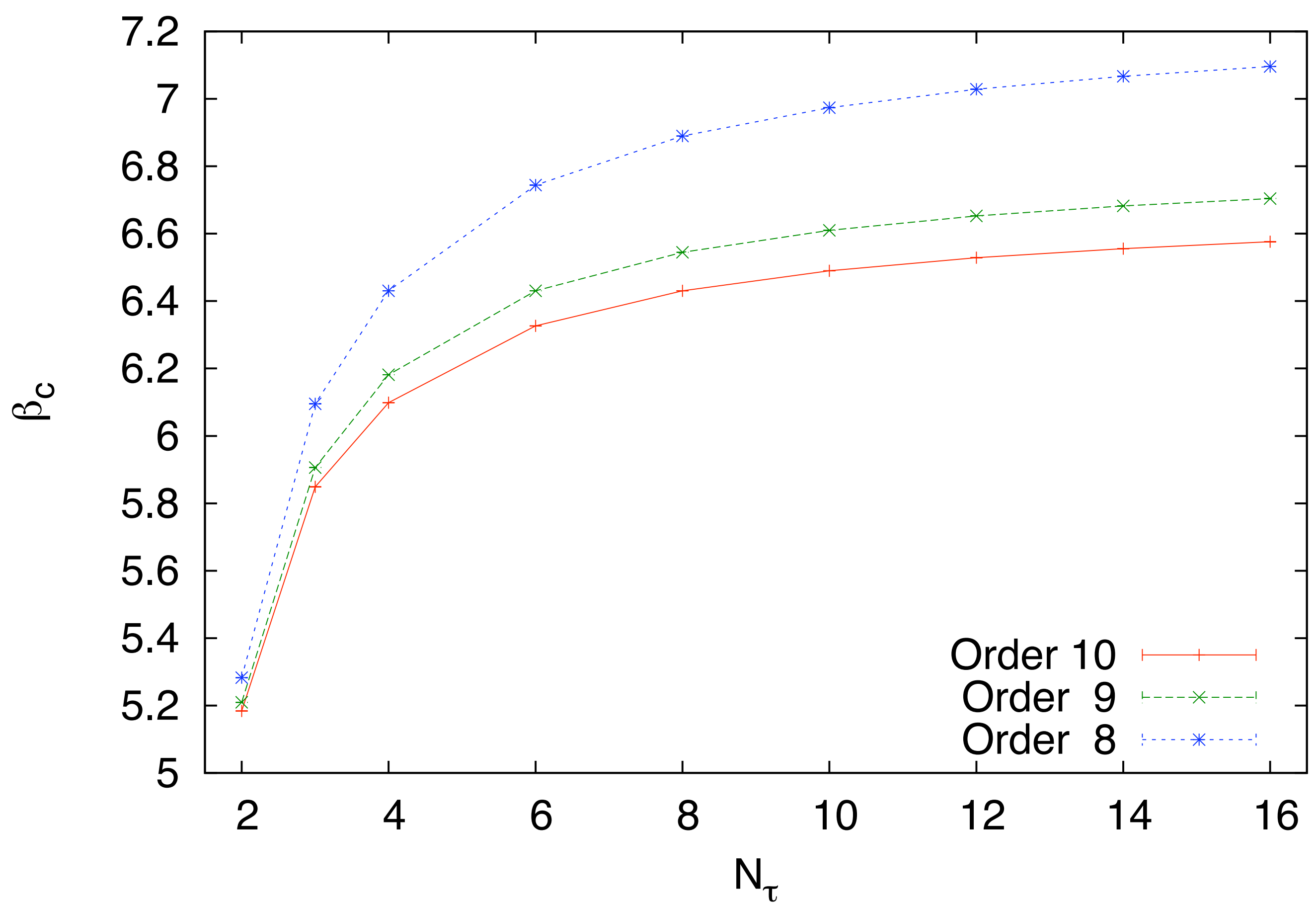


Mapping back to 4d finite T Yang-Mills

Inverting

$$\lambda_1(N_\tau, \beta) \rightarrow \beta_c(\lambda_{1,c}, N_\tau)$$

...points at reasonable convergence

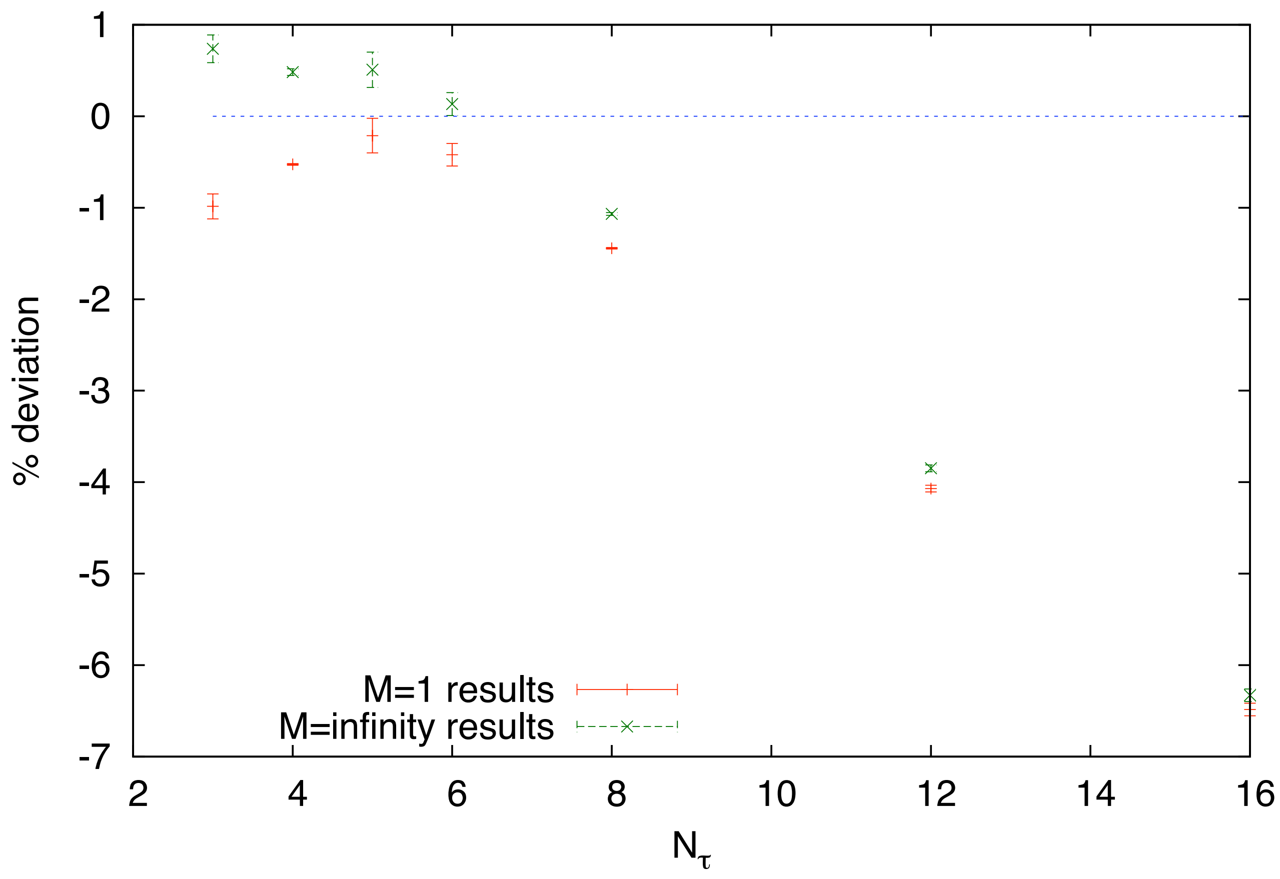


SU(3)

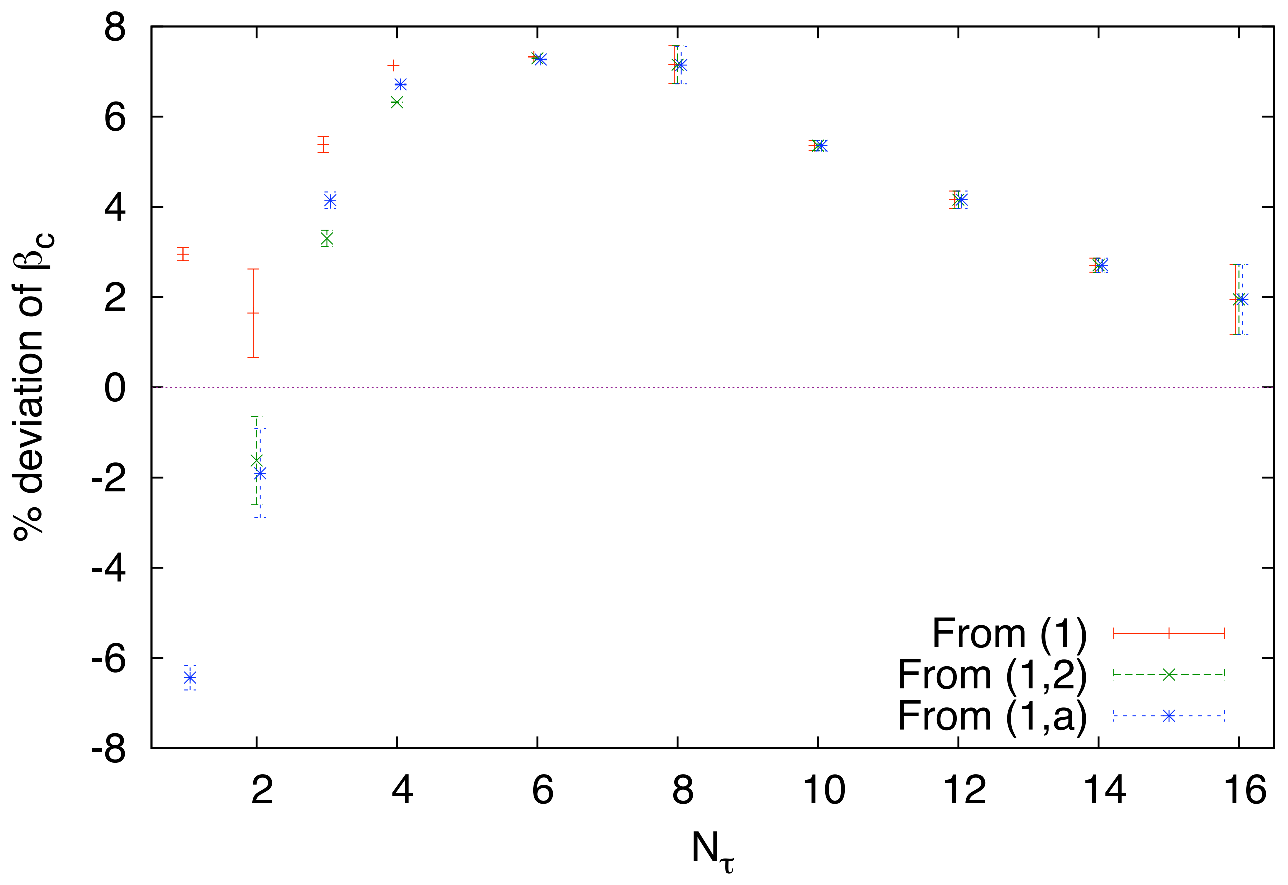
Comparison with 4d Monte Carlo

Relative accuracy for β_c compared to the full theory

SU(2)

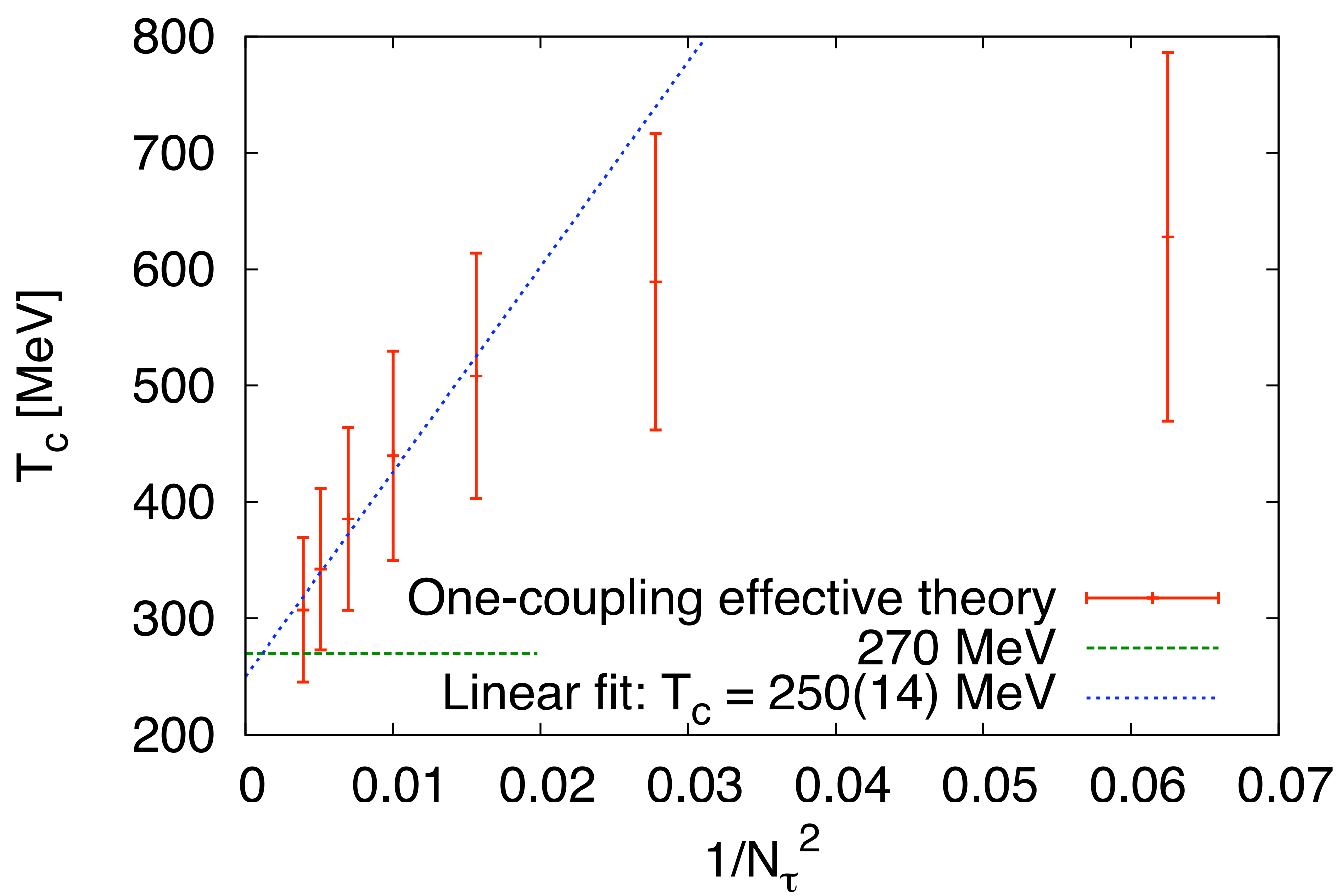


SU(3)



Note: influence of additional couplings checked explicitly!

Continuum limit feasible!



-error bars: difference between last two orders in strong coupling exp.

-using non-perturbative beta-function (4d T=0 lattice)

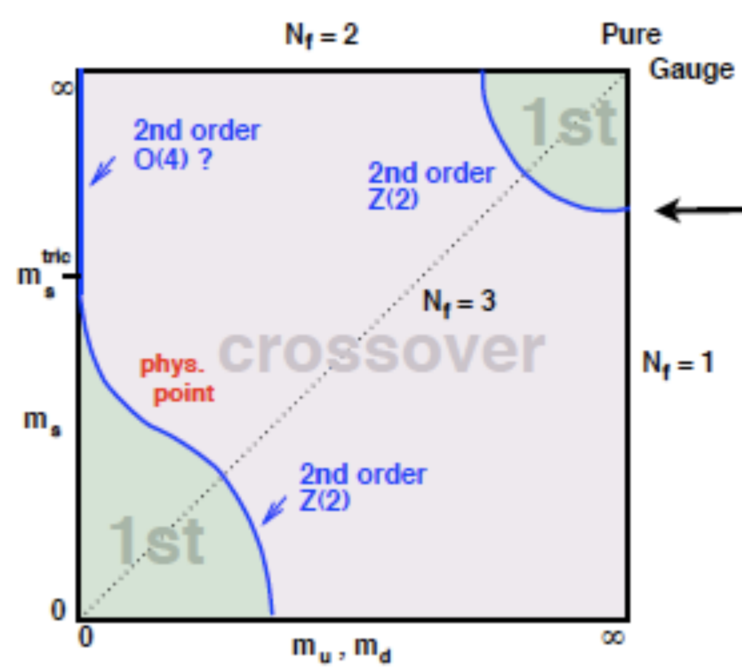
-all data points from one single 3d MC simulation!

Including heavy, dynamical Wilson fermions

Expand in the *hopping parameter* $\kappa = 1/(2aM+8)$:

$$-S_{\mathrm{eff}} = \sum_i \lambda_i(u,\kappa,N_\tau) S_i^{\mathrm{S}} - 2N_f \sum_i \left[h_i(u,\kappa,\mu,N_\tau) S_i^{\mathrm{A}} + \overline{h}_i(u,\kappa,\mu,N_\tau) S_i^{\dagger\mathrm{A}} \right]$$

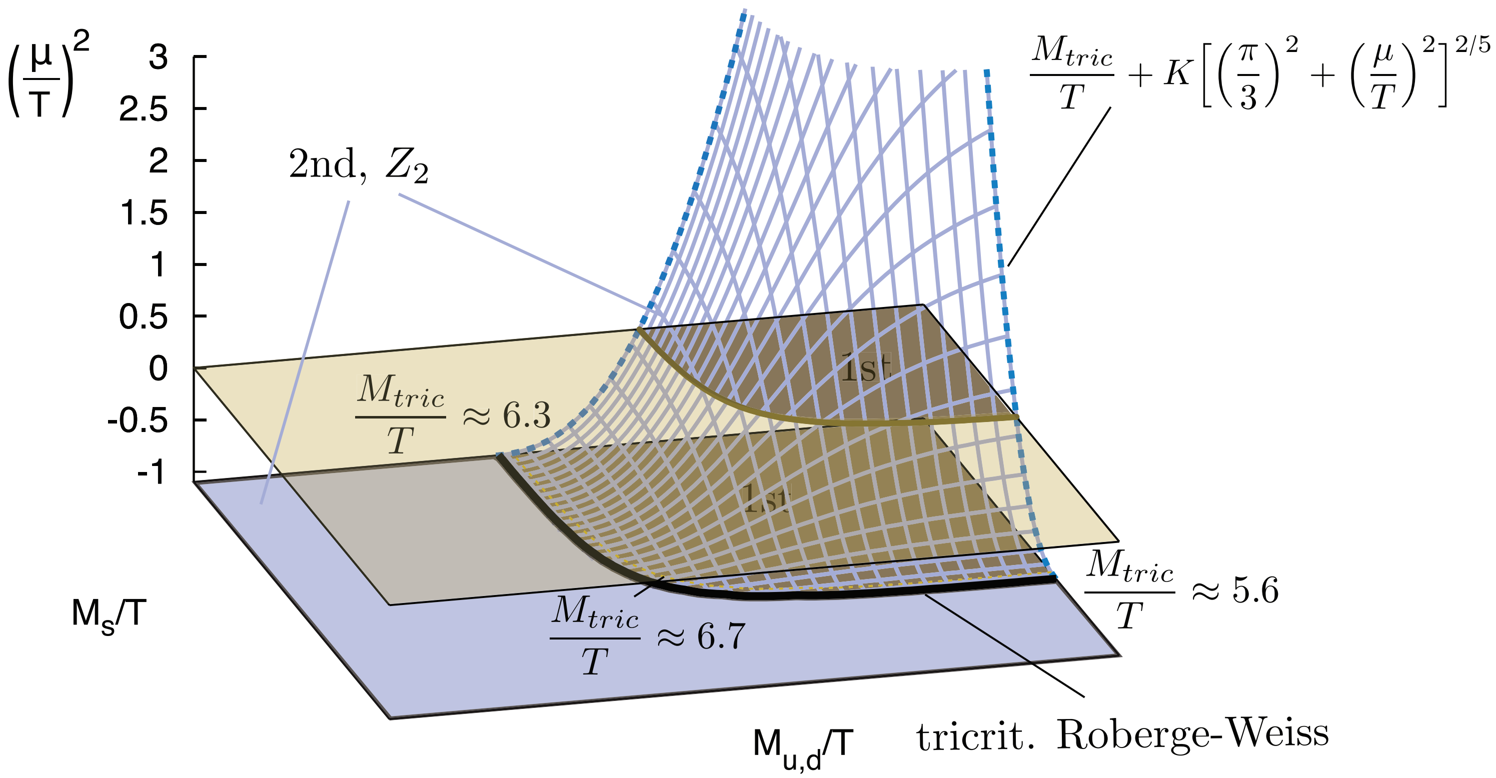
Now, keep only $\lambda_1 S_1^{\mathrm{S}}$ and $h_1 S_1^{\mathrm{A}} + \overline{h}_1 S_1^{\dagger\mathrm{A}}$ NLO: $\sim \kappa^2$



	eff. theory		4d MC, WHOT	4d MC, de Forcrand et al
N_f	M_c/T	$\kappa_c(N_\tau = 4)$	$\kappa_c(4)$, Ref. [23]	$\kappa_c(4)$, Ref. [22]
1	7.22(5)	0.0822(11)	0.0783(4)	~ 0.08
2	7.91(5)	0.0691(9)	0.0658(3)	–
3	8.32(5)	0.0625(9)	0.0595(3)	–

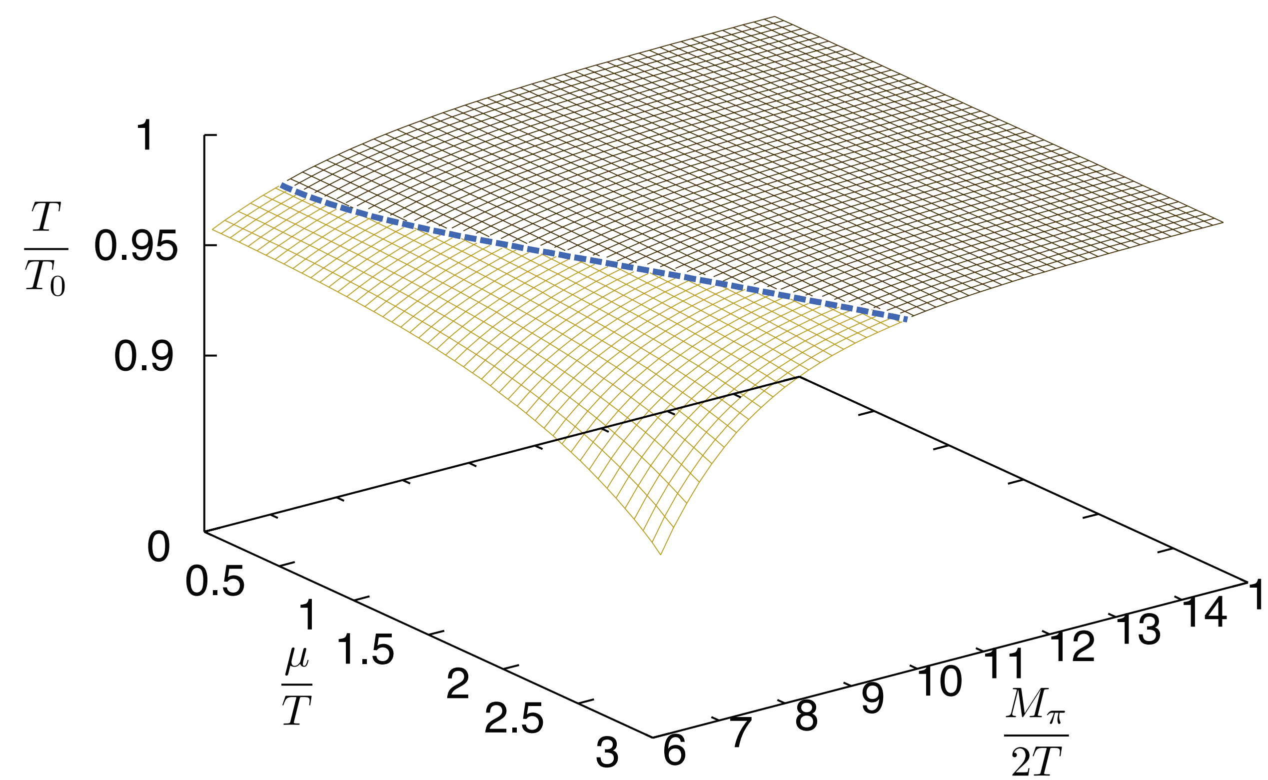
Accuracy ~5%, predictions for Nt=6,8,... available!

The fully calculated deconfinement transition

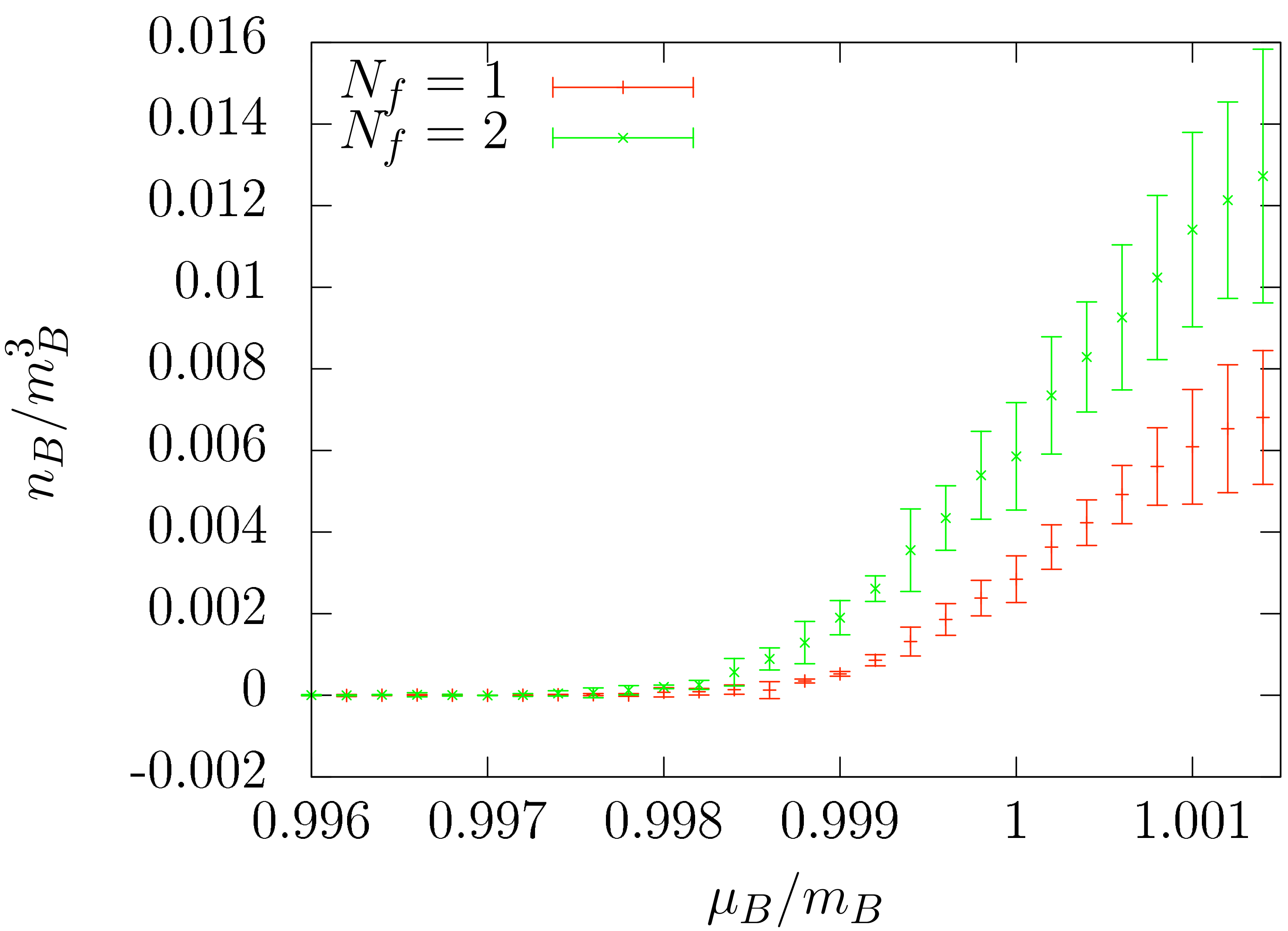


deconfinement critical surface

phase diagram for Nf=2, Nt=6



The equation of state for nuclear matter



$$S_{eff} \sim \kappa^n u^m, \quad n + m = 4$$

$$m_\pi = 20 \text{ GeV}, T = 10 \text{ MeV}$$

$$\frac{\mu}{T} \sim 4000$$

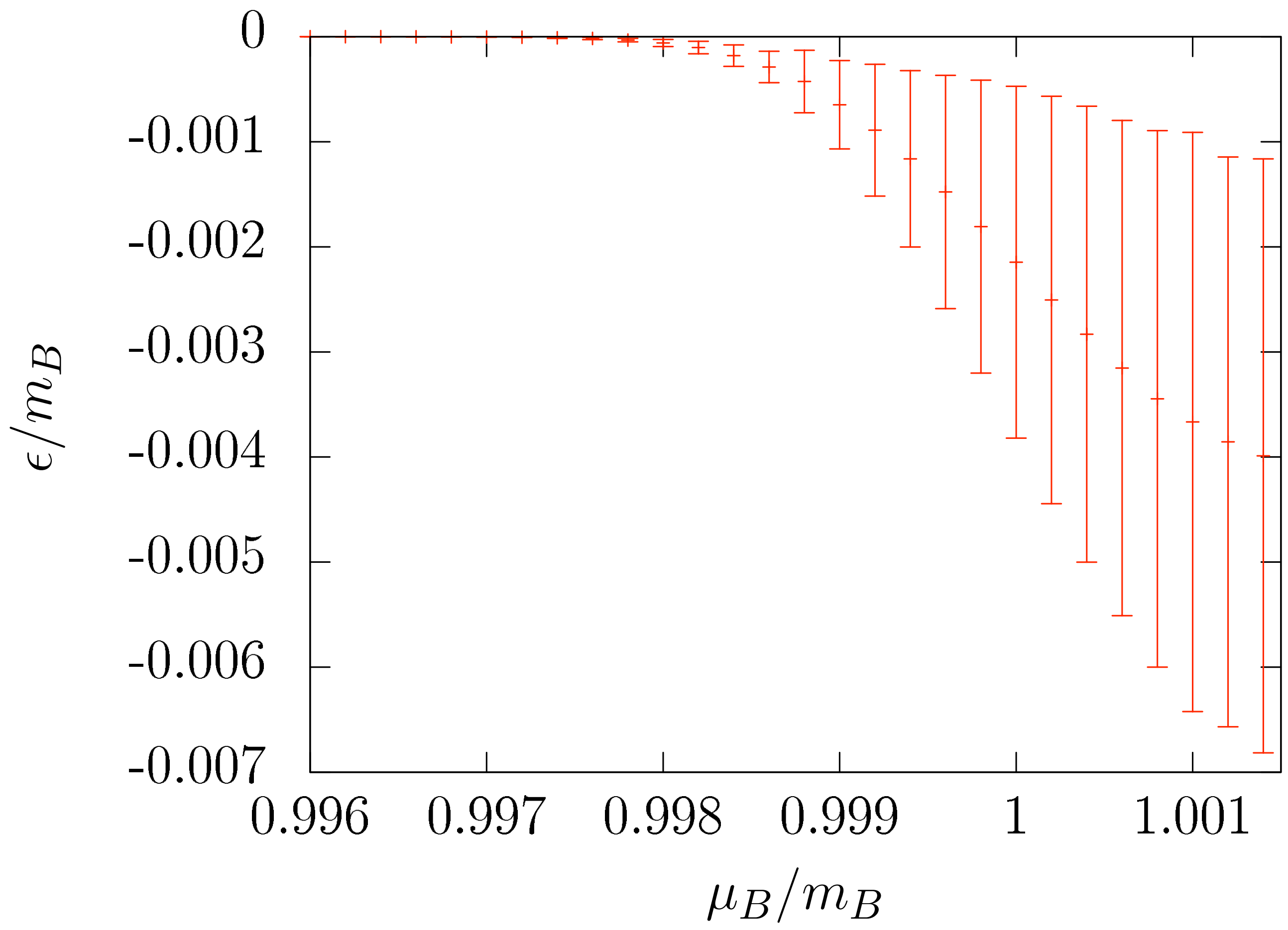
Effect of binding between baryons: $\mu_c < m_B$

Binding energy per nucleon: $\epsilon = \dfrac{\mu_c - m_B}{m_B} \sim 10^{-3}$

Transition is smooth crossover: $T > T_c \sim \epsilon\, m_B$

Binding energy per nucleon

$$\epsilon = \frac{e - n_B m_B}{n_B m_B} = \frac{e}{n_B m_B} - 1$$



Binding energy per nucleon

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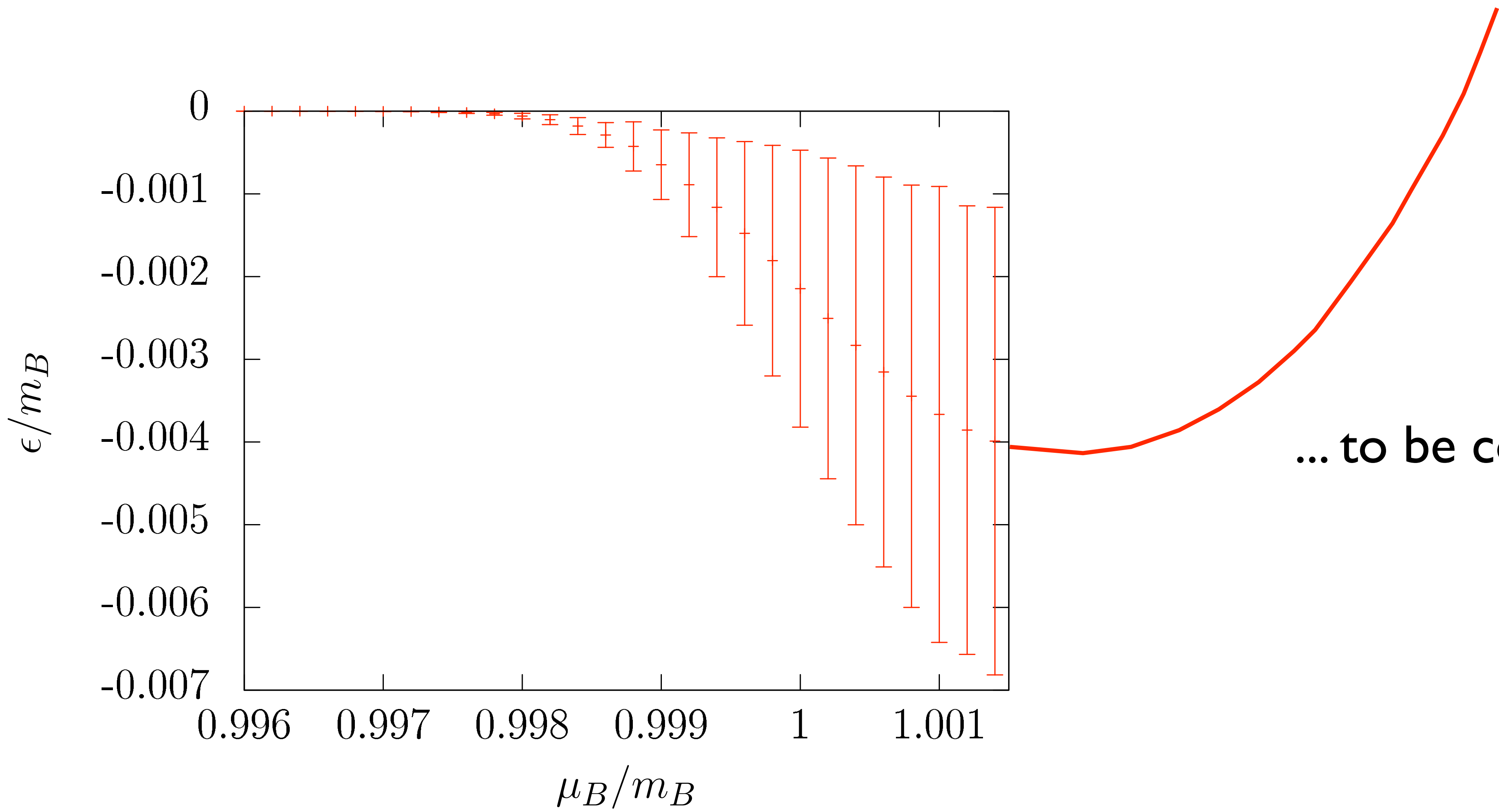
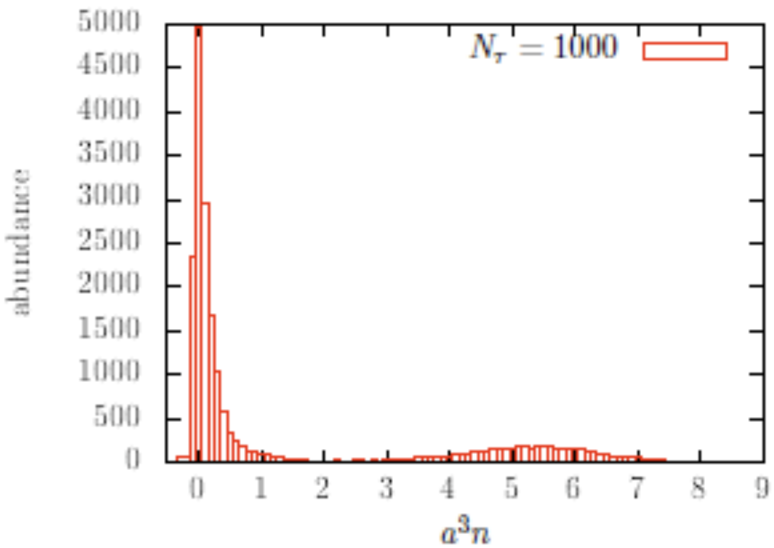


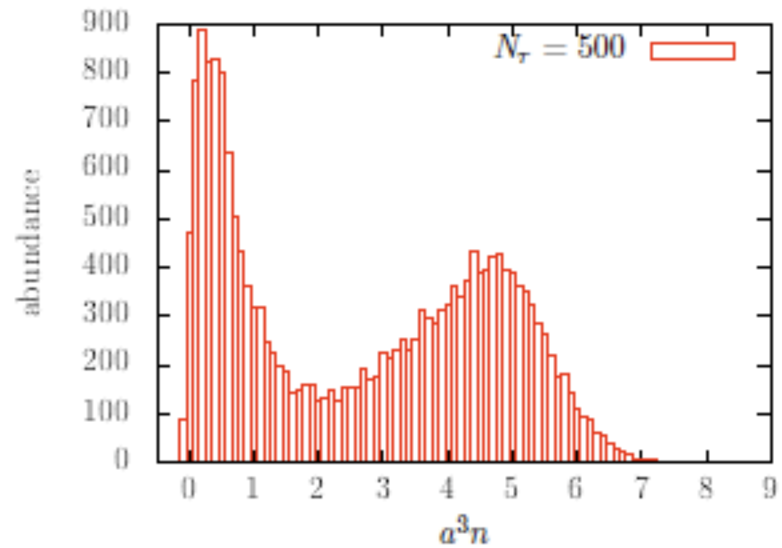
... to be continued...

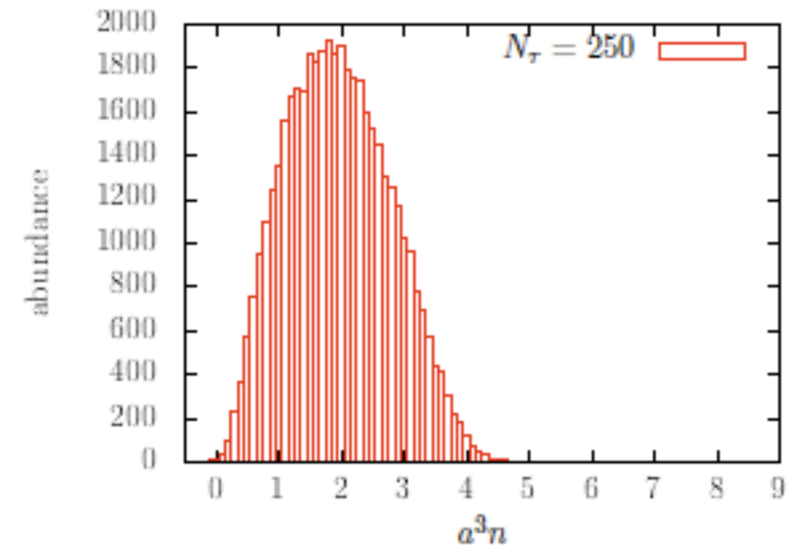
Minimum: access to nucl. binding energy, nucl. saturation density!

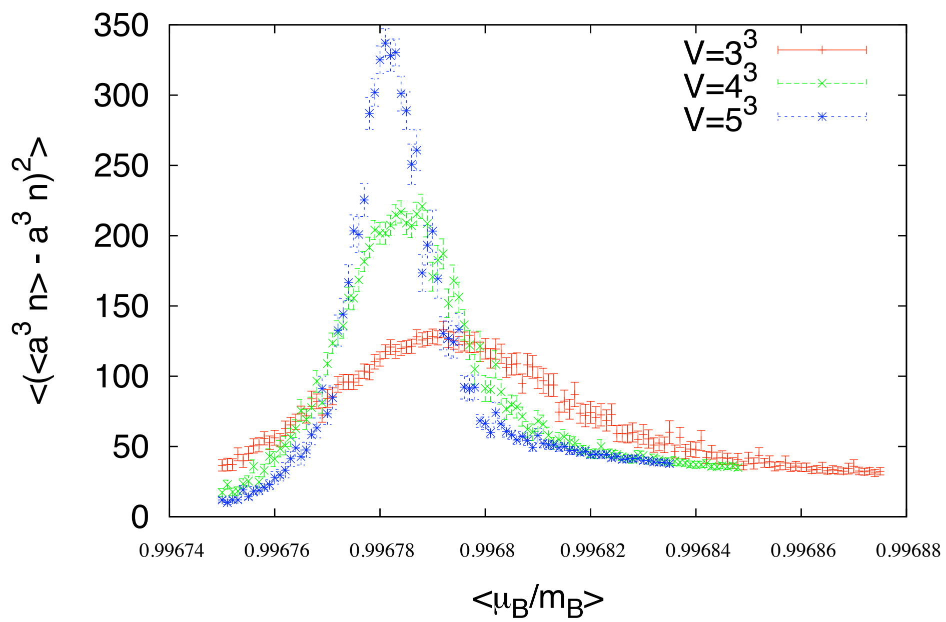
$\epsilon \sim 10^{-3}$ consistent with the location of the onset transition

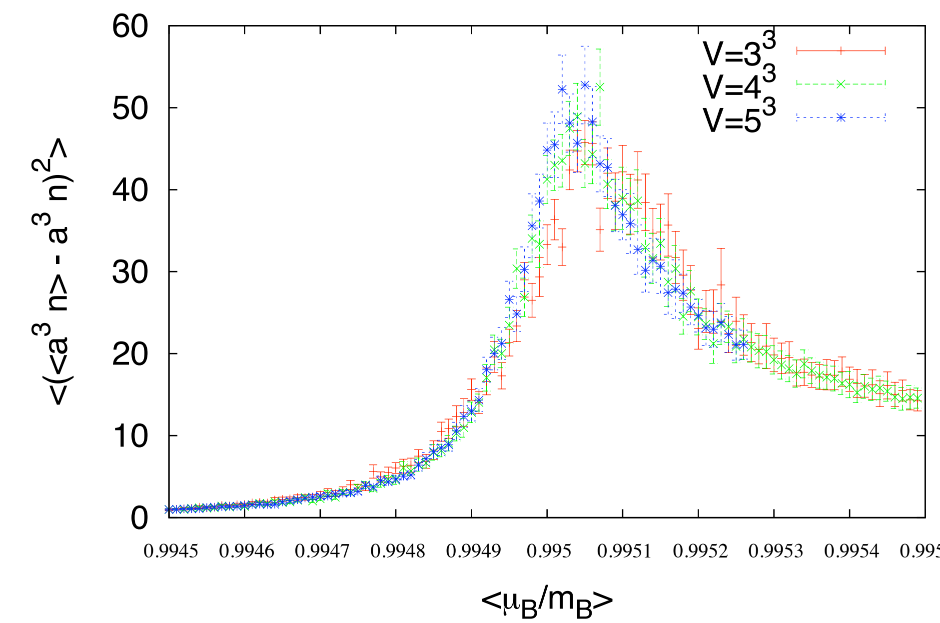
Lighter quarks: first order + endpoint!









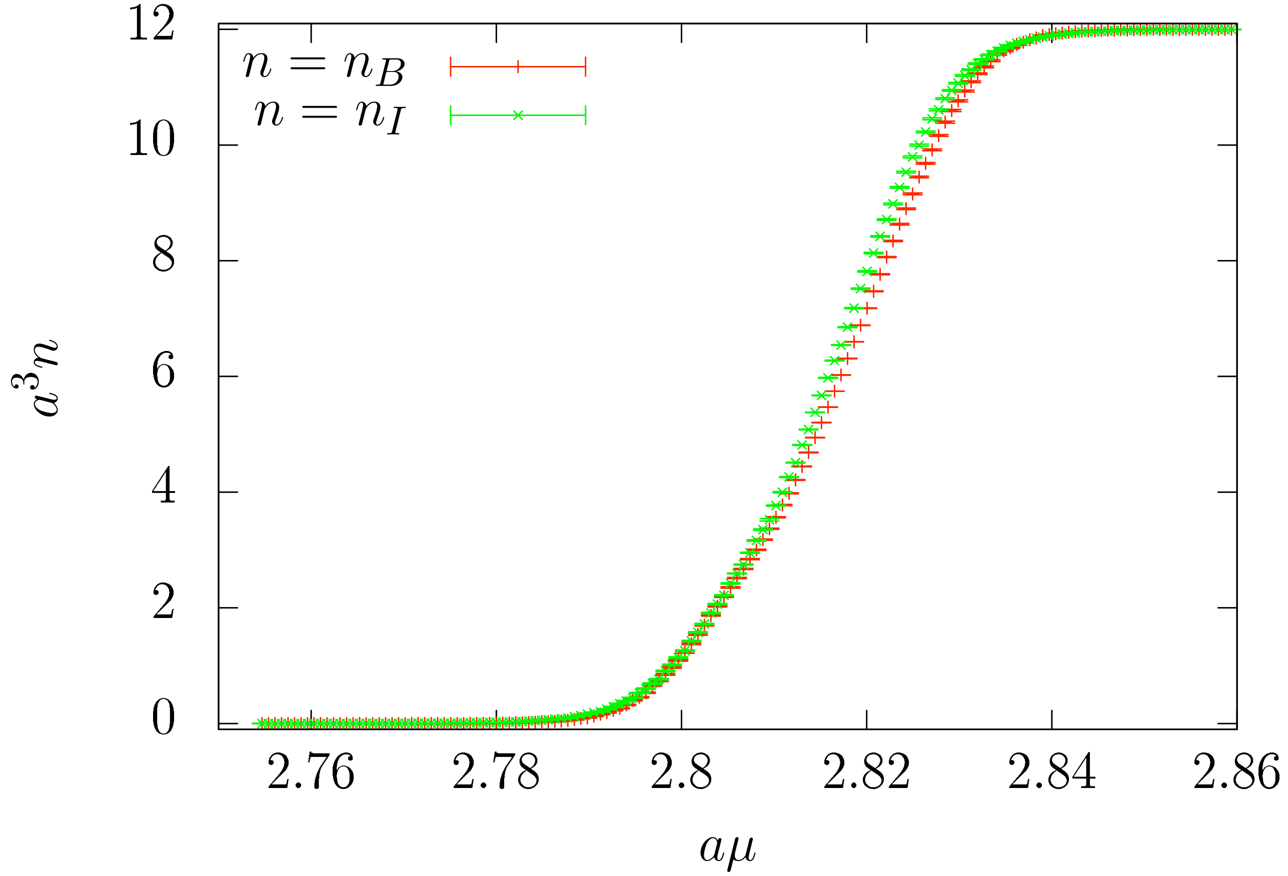


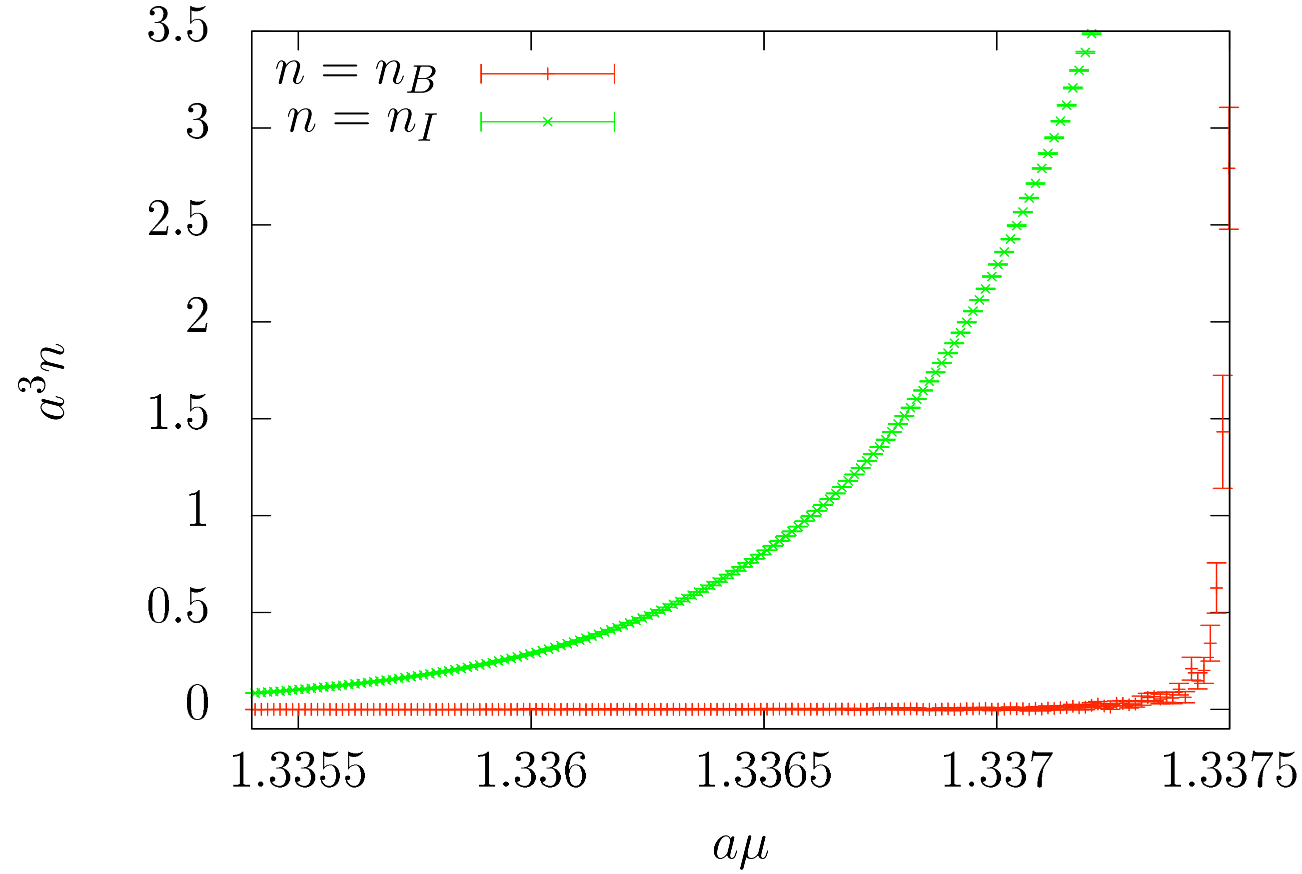
- $O(k^4)$:Stretching the hopping series, $\kappa = 0.12, \beta = 5.6$
- Coexistence of vacuum and finite density phase: 1st order
- If the temperature $T = \frac{1}{aN_\tau}$ or the quark mass is raised this changes to a crossover

nuclear liquid gas transition!!!

attn: no convergence yet!

Finite isospin vs baryon chemical potential





nearly static quarks

$$\frac{m_\pi}{2} \approx \frac{m_B}{3}$$

lighter quarks

$$\frac{m_\pi}{2} < \frac{m_B}{3}$$

onset at smaller chemical potential

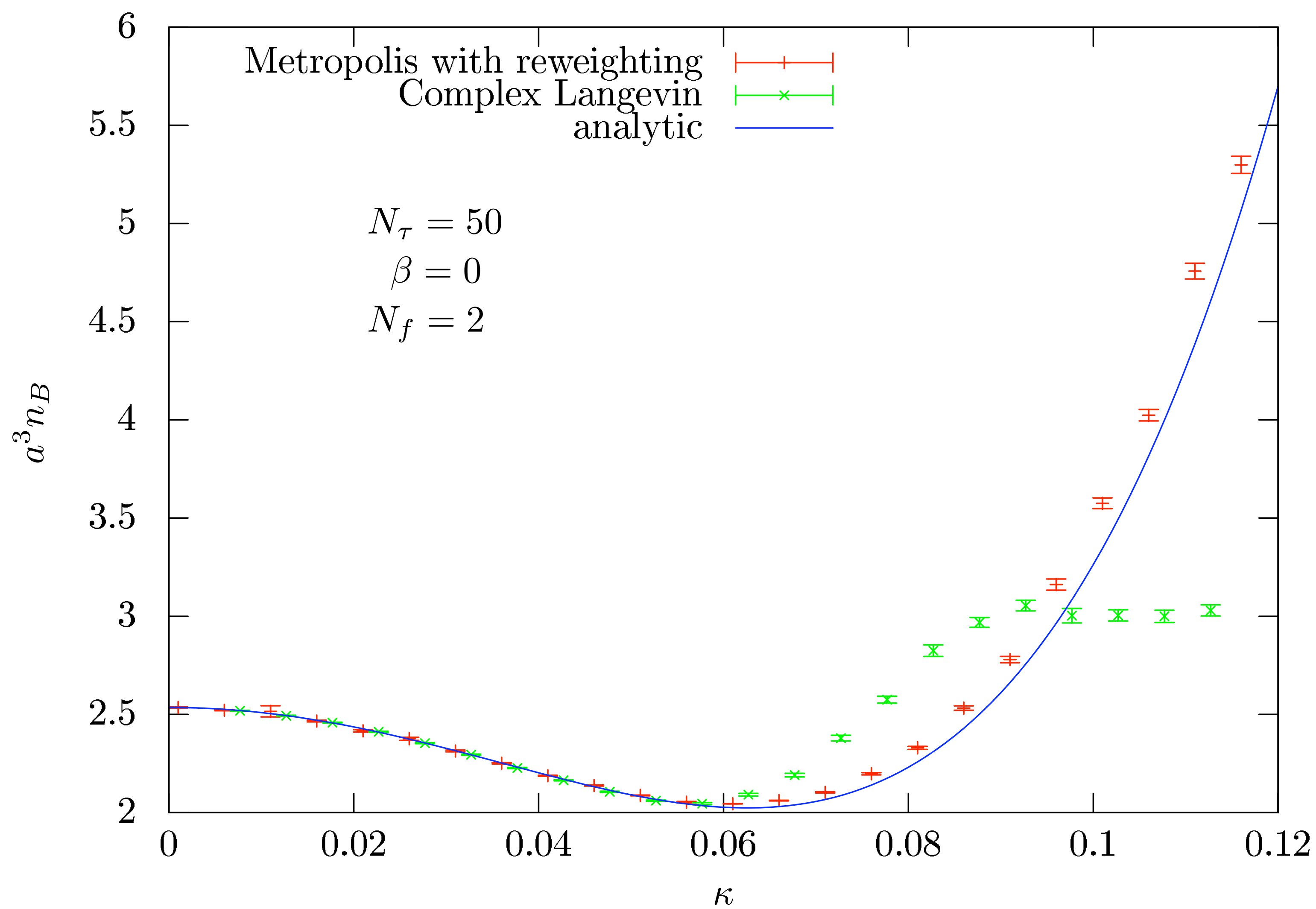
The problem with complex Langevin

No non-analytic phase transition seen yet

Aarts, Seiler, Sexty, Stamatescu

Does not work in all parameter regions!

In effective theory:



The effective lattice theory approach II

- Two-step treatment:

  de Forcrand, Langelage, O.P., Unger
  Phys.Rev.Lett. 113 (2014) 152002

  I. Calculate effective theory analytically
  II. Simulate effective theory

- Step I.: integrate over gauge links in strong coupling expansion, leave fermions

$$Z_{\mathrm{QCD}} = \int d\psi d\bar{\psi} dU \, e^{S_F + S_G} = \int d\psi d\bar{\psi} Z_F \, \langle e^{S_G} \rangle_{Z_F}$$

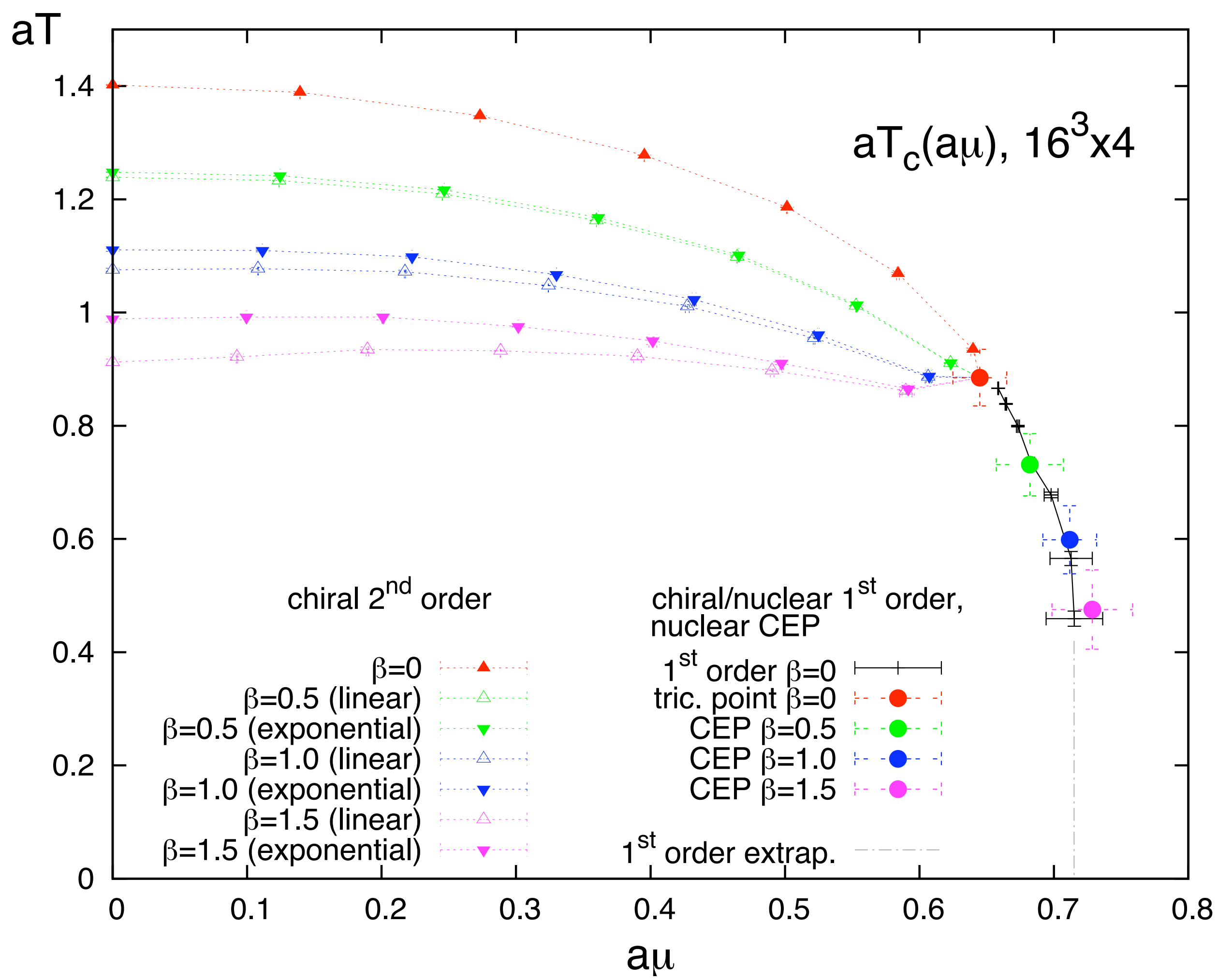
$$\langle e^{S_G} \rangle_{Z_F} \simeq 1 + \langle S_G \rangle_{Z_F} = 1 + \frac{\beta}{2N_c} \sum_P \left\langle \mathrm{tr}[U_P + U_P^\dagger] \right\rangle_{Z_F} \qquad Z_F(\psi, \bar{\psi}) = \int dU \, e^{S_F}$$

- Result: 4d "polymer" model of QCD (hadronic degrees of freedom!)
  Valid for all quark masses (also m=0!), at strong coupling (very coarse lattices)

- Step II: sign problem milder: Monte Carlo with worm algorithm

- Numerical simulations without fermion matrix inversion, very cheap!

The QCD Phase diagram at strong coupling



- LO gauge correction included, simulation by worm algorithm
- Chiral phase transition with 2nd order and 1st order line meeting in tricritical point
- Nuclear liquid gas transition on top of first order chiral one at strong coupling

Conclusions

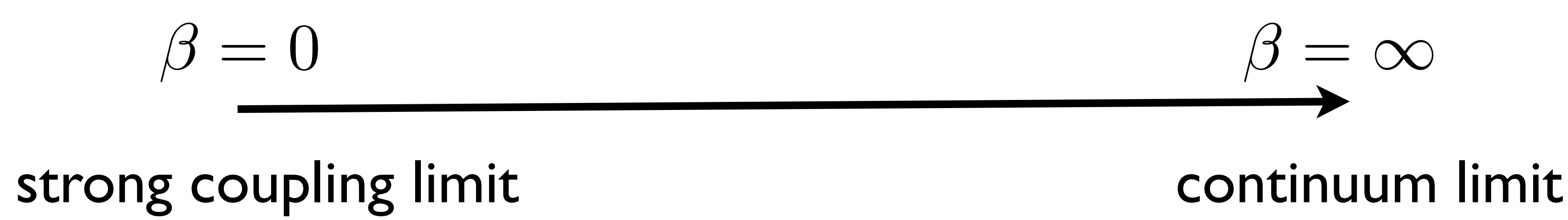
- Two-step treatment of QCD phase transitions:

  I. Derivation of effective action by strong coupling expansion
  II. Simulation of effective theory

- Finite T transition + nuclear liquid gas transition for:

  Heavy QCD on not so coarse or chiral QCD on very coarse lattices

How is this possible?



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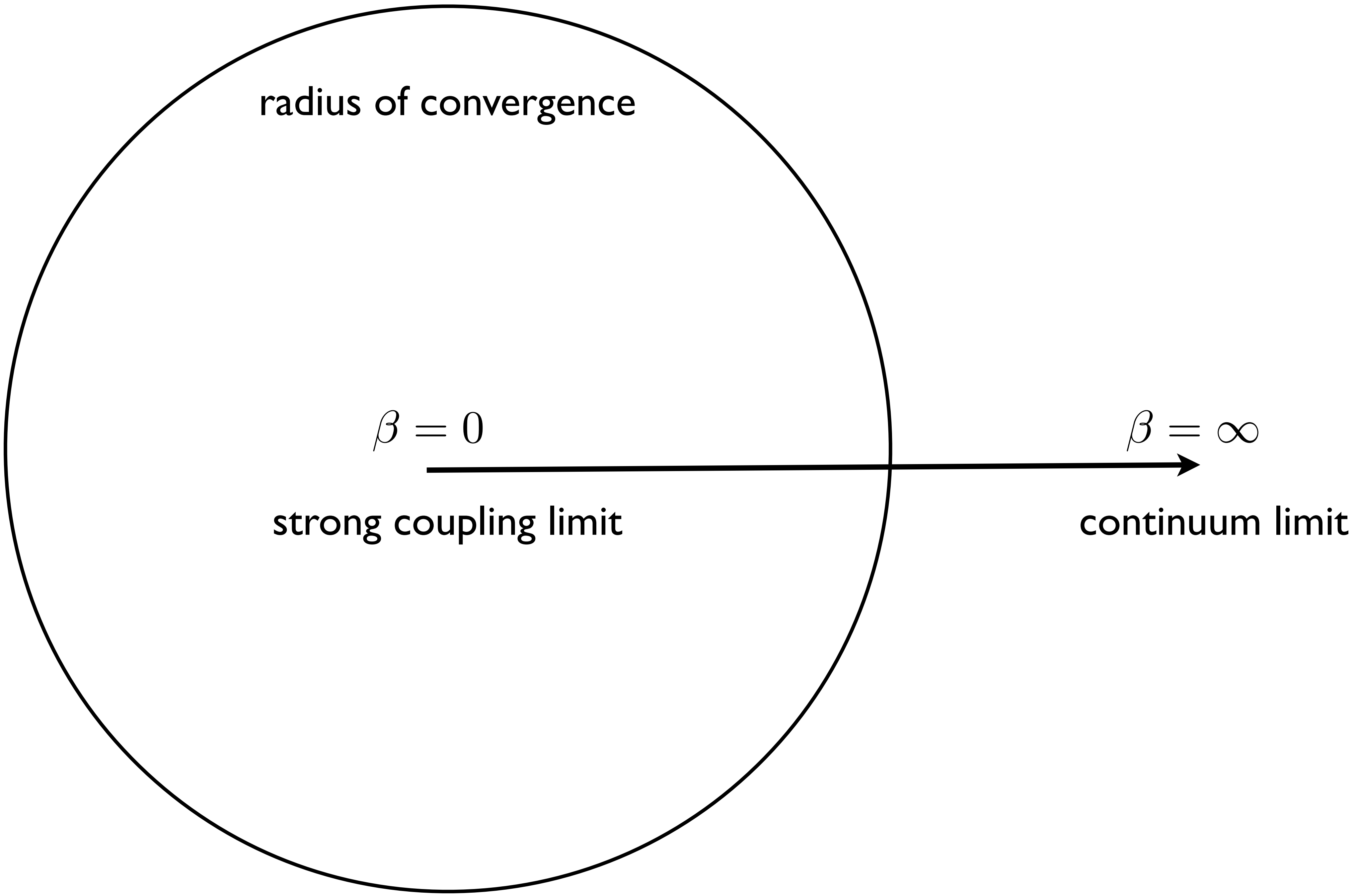

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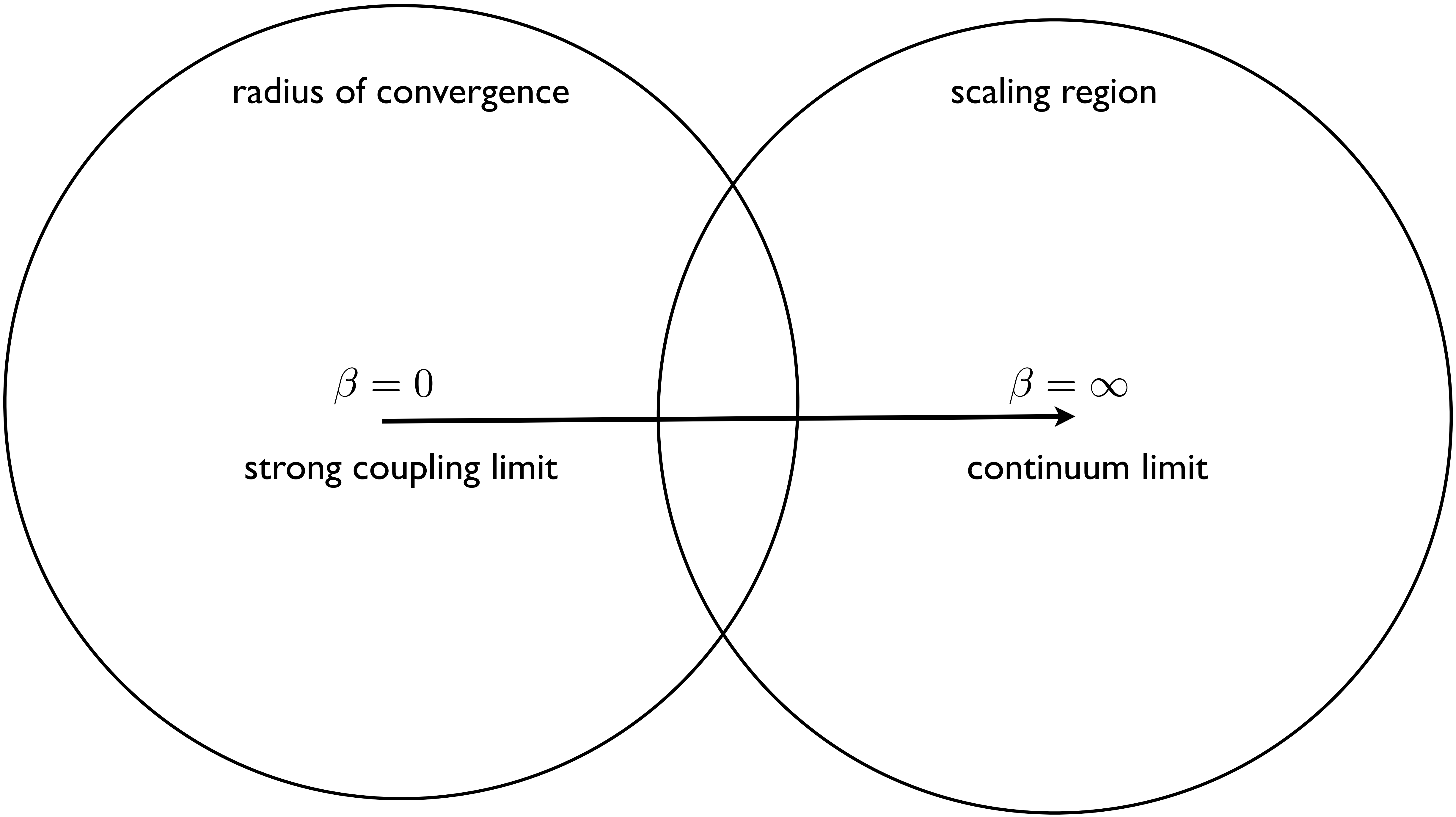

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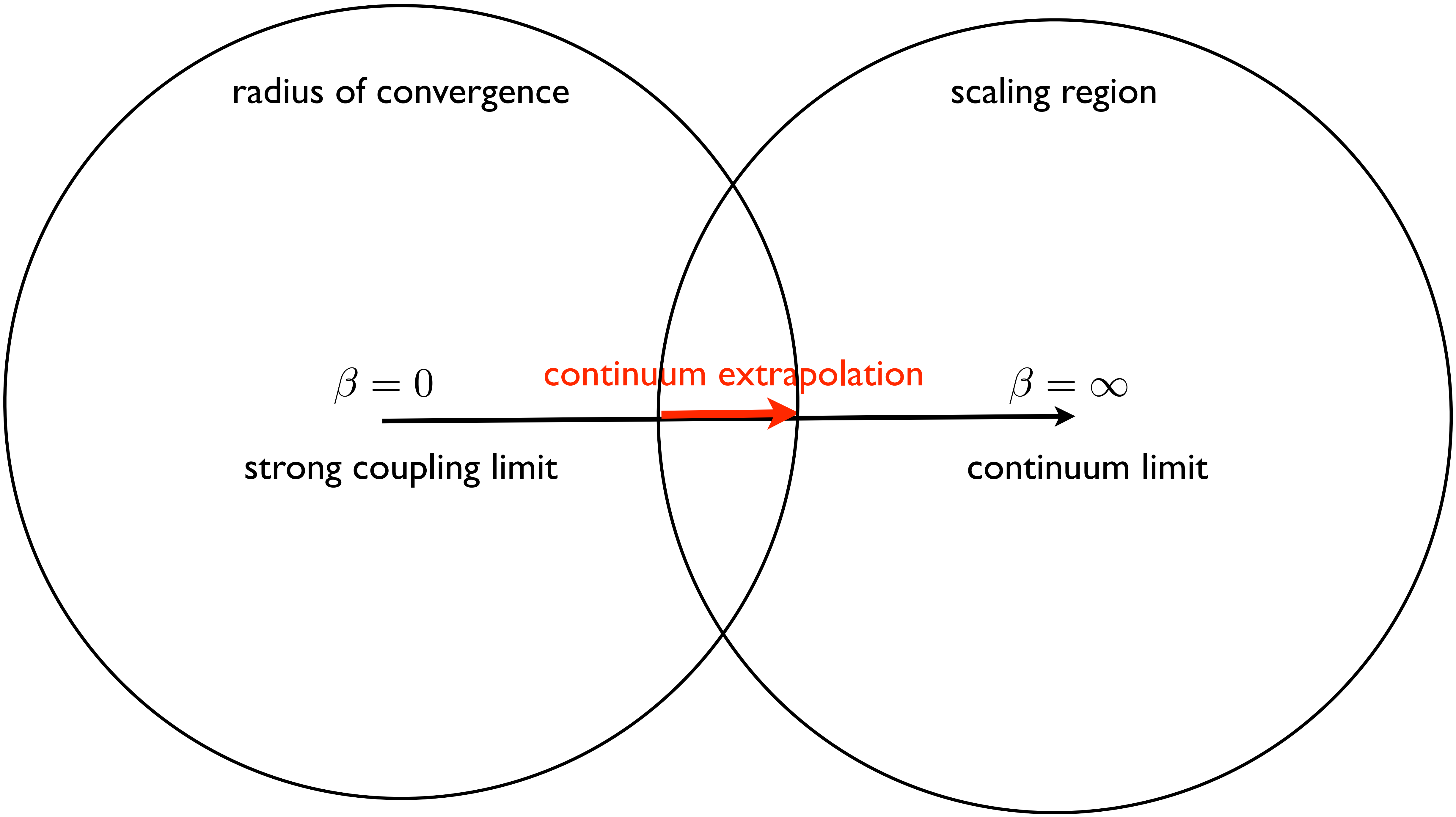
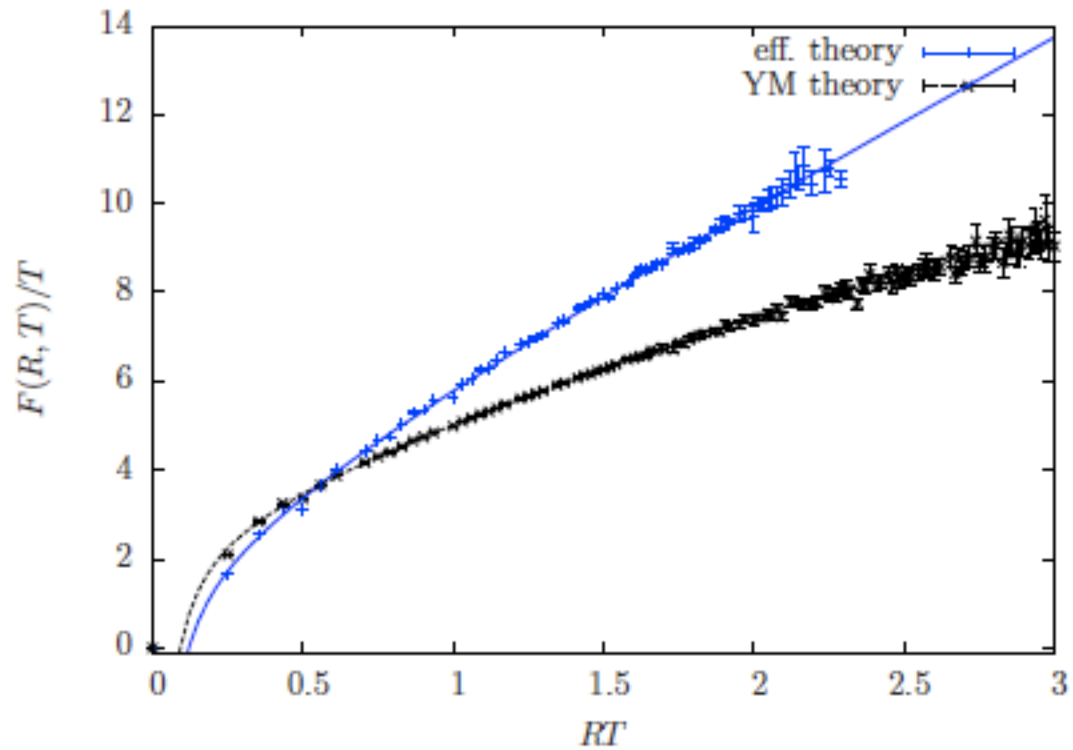
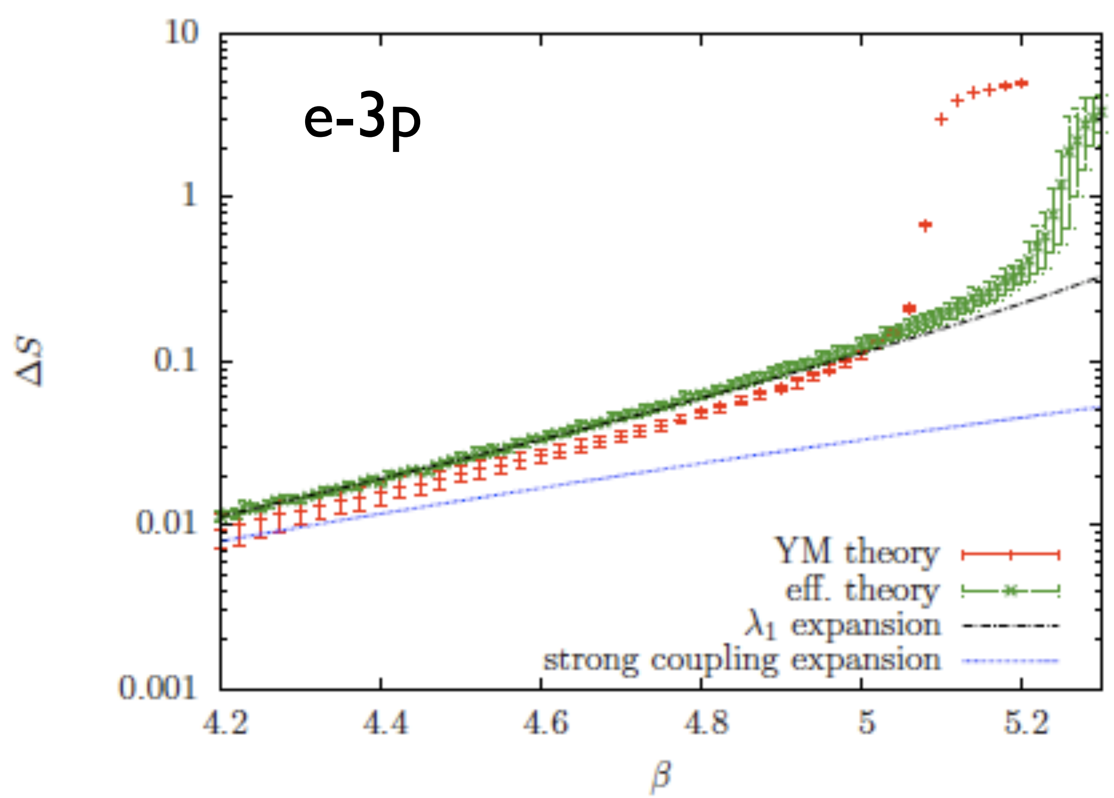


What does and does not work?



Correlation functions and spectrum:
NO

couplings over large distances needed



Thermodynamics and critical coupling:
YES

partition function needed, ultra-local!

Subleading couplings

Subleading contributions for next-to-nearest neighbours:

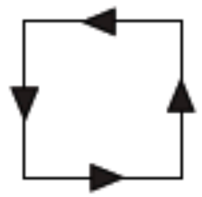
$$\lambda_2 S_2 \quad \propto \quad u^{2N_\tau+2} \sum_{[kl]}' 2\mathrm{Re}(L_k L_l^*) \quad \text{distance} \ = \sqrt{2}$$

$$\lambda_3 S_3 \quad \propto \quad u^{2N_\tau+6} \sum_{\{mn\}}'' 2\mathrm{Re}(L_m L_n^*) \quad \text{distance} \ = 2$$

as well as terms from loops in the *adjoint* representation:

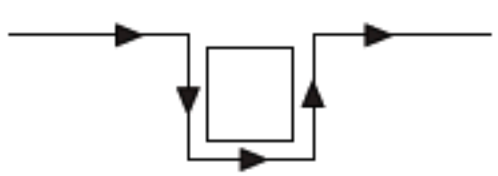
$$\lambda_a S_a \propto u^{2N_\tau} \sum_{<ij>} \mathrm{Tr}^{(a)} W_i \mathrm{Tr}^{(a)} W_j \quad ; \quad \mathrm{Tr}^{(a)} W = |L|^2 - 1$$

Links along imaginary time gain $\exp(\pm\mu a)$



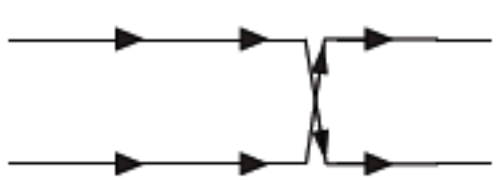
reabsorbed in gauge part: $\begin{cases} \beta \to \beta + \mathcal{O}(\kappa^4) \\ u(\beta) \to u(\beta,\kappa) \end{cases}$

LO Polyakov "magnetic" term $\sim \begin{cases} \underbrace{(2\kappa e^{+a\mu})^{N_\tau}}_{h_1} L \\ \underbrace{(2\kappa e^{-a\mu})^{N_\tau}}_{\bar{h}_1} L^* \end{cases}$



higher corrections to the above:

$$h_1 = (2\kappa e^{a\mu})^{N_\tau}\left[1 + \mathcal{O}(k^2) f(u) + \ldots\right]$$



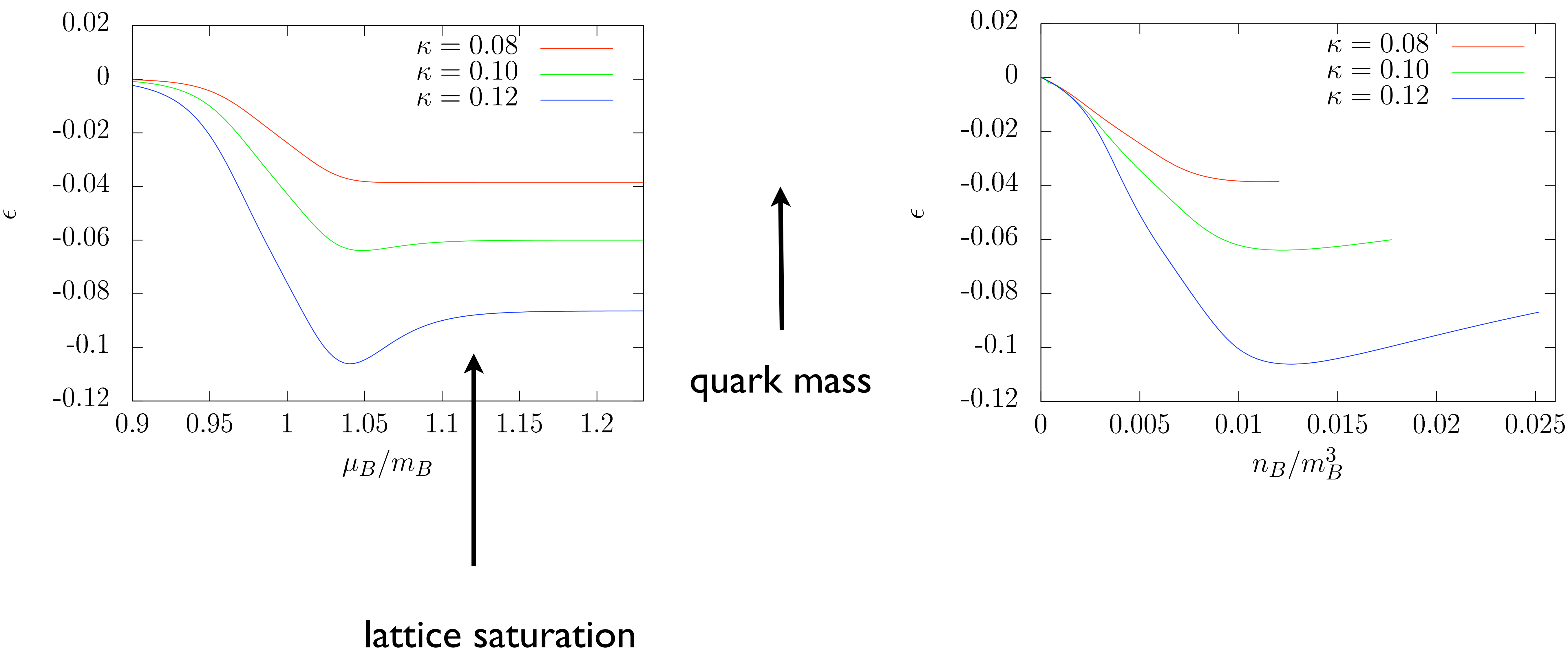
other (suppressed) terms, such as $h_2(L_x L_{x+\hat{i}})$,

$$h_2 \sim (2\kappa e^{a\mu})^{2N_\tau}\kappa^2 \ldots$$

Quark mass dependence of the binding energy:

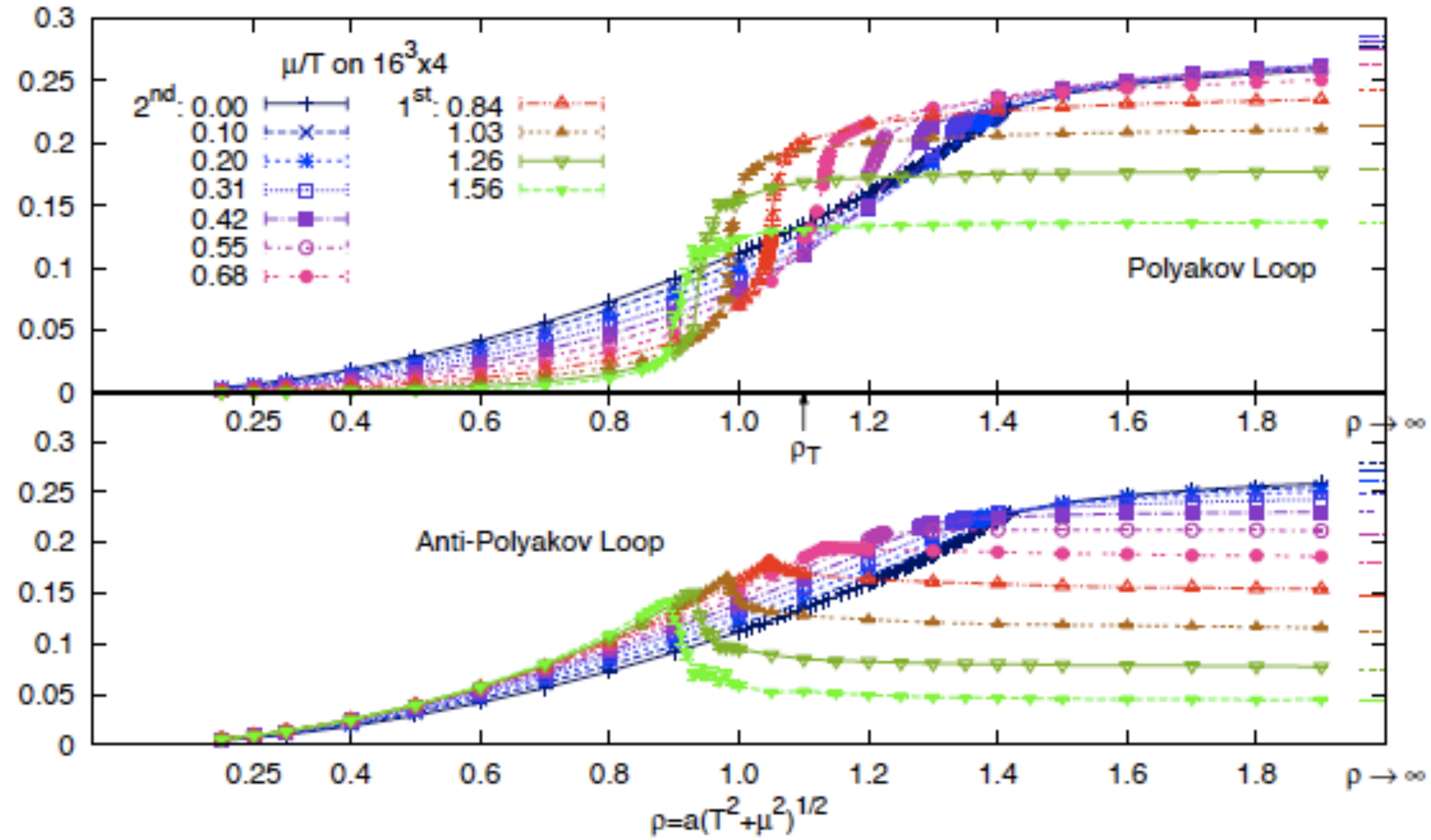
Expect short range nucl. potential for heavy pions, $V \sim \frac{e^{-m_\pi r}}{r}$

Analytic solution, finite lattice spacing:



Two transitions seen:

Chiral/deconfinement



Nuclear onset