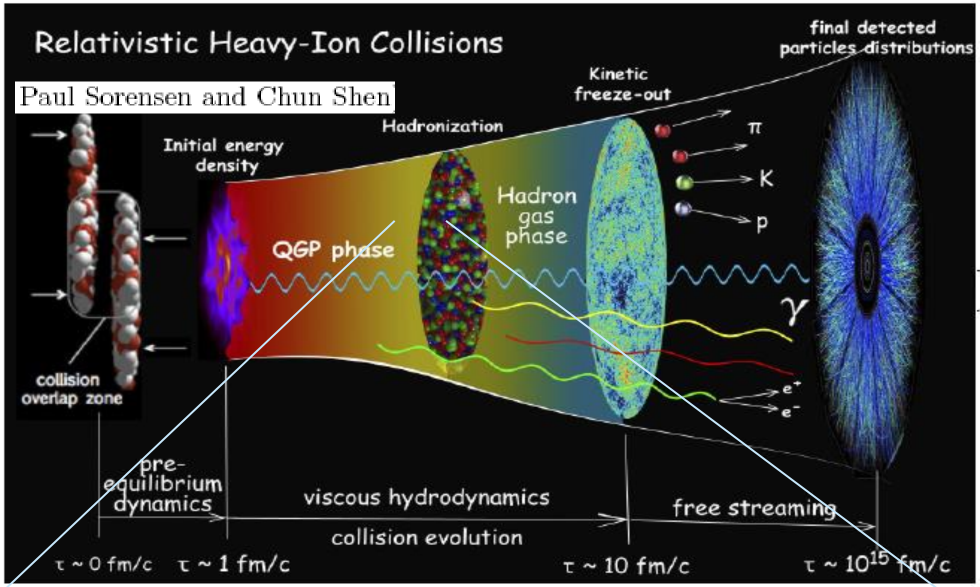
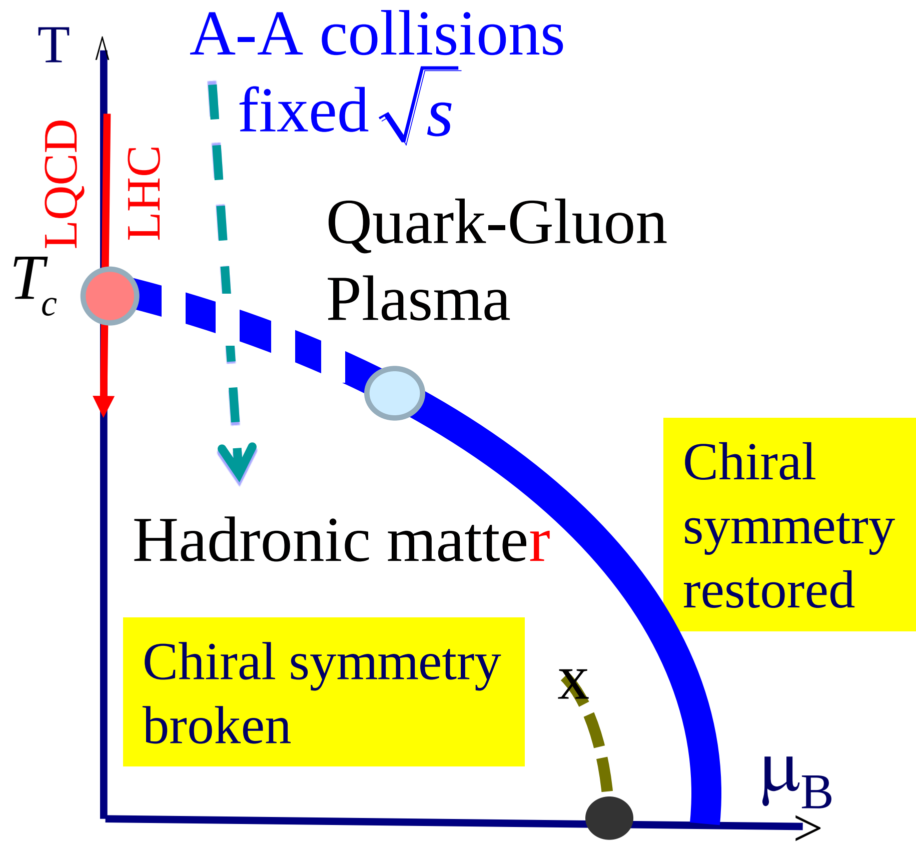
Fluctuations in the QCD Phase Diagram





- Fluctuations of conserved charges at the LHC and LQCD results
- The influence of critical fluctuations on the probability distribution of net baryon number

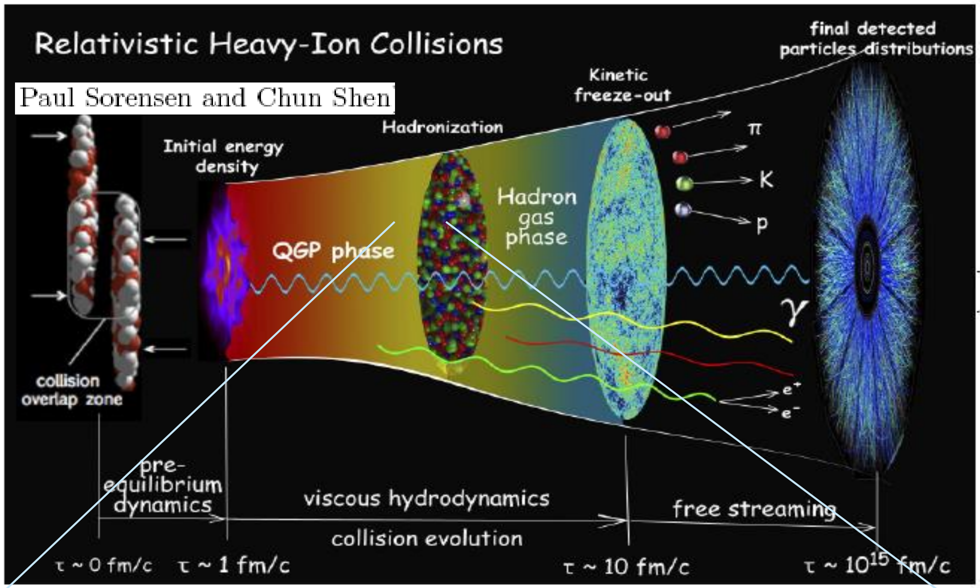
1st principle calculations:

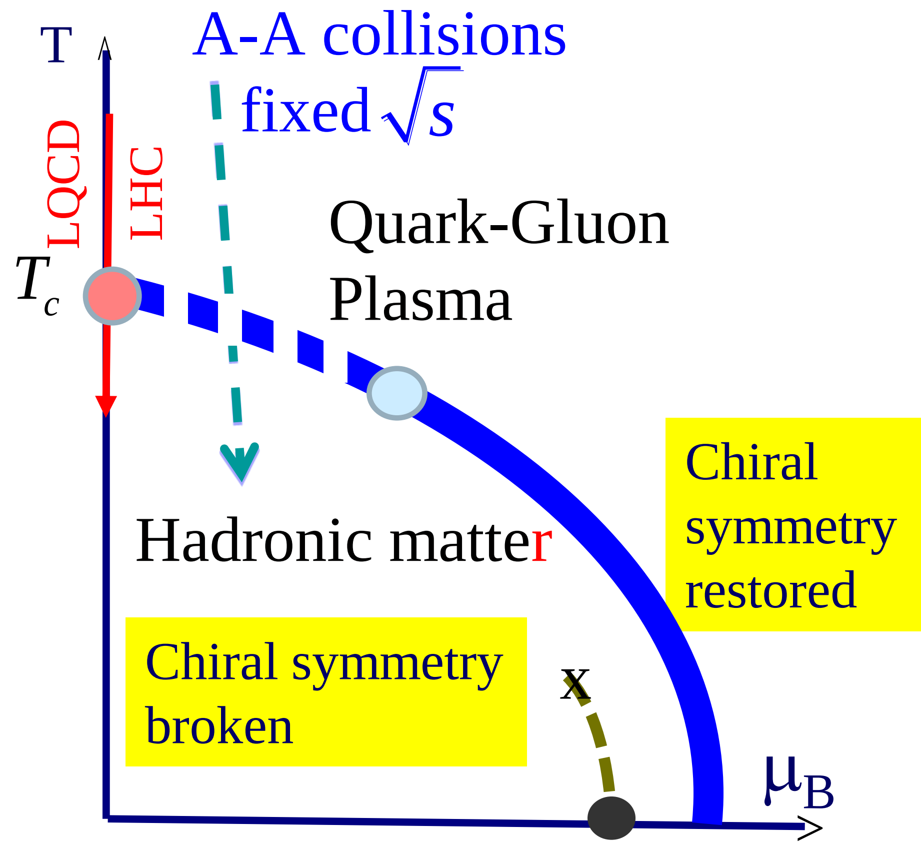
$\mu, T << \Lambda_{QCD}$: χ − perturbation theory

$\mu, T >> \Lambda_{QCD}$: pQCD >

$\mu_q < T$: LGT

Fluctuations in the QCD Phase Diagram





- P. Braun-Munzinger, A. Kalweit and J. Stachel
- B. Friman & K. Morita

within EMMI collaboration

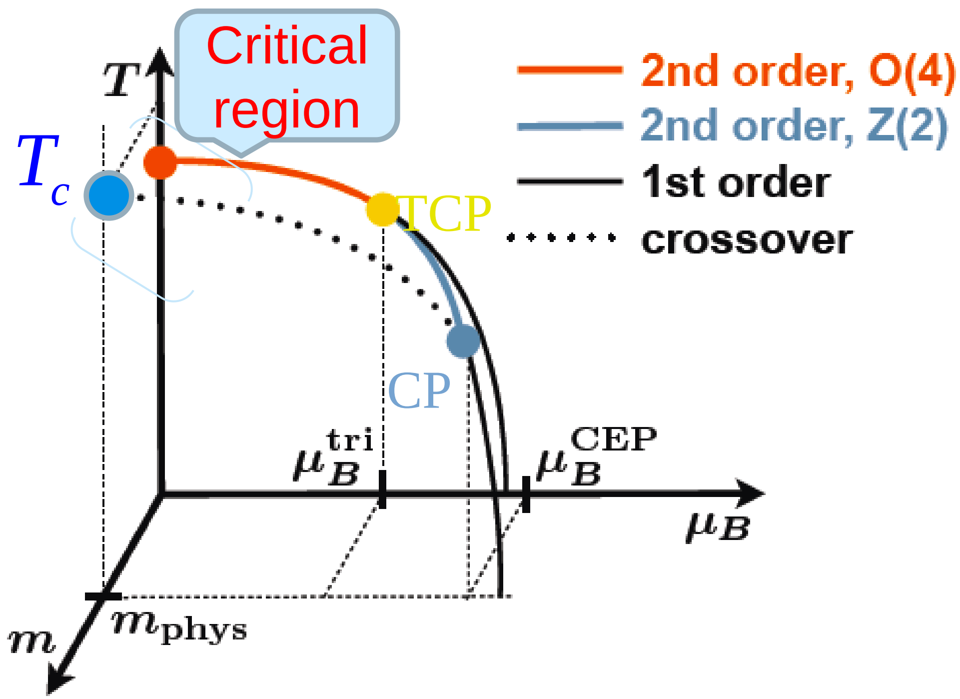
1^{st} principle calculations:

$\mu, T << \Lambda_{QCD}$: $\chi -$perturbation theory

$\mu, T >> \Lambda_{QCD}$: pQCD >

$\mu_q < T$: LGT

Deconfinement and chiral symmetry restoration in QCD



- The QCD chiral transition is crossover   Y.Aoki, et al Nature (2006)
  and appears in the O(4) critical region

  O. Kaczmarek et.al. Phys.Rev. D83, 014504 (2011)

- Chiral transition temperature

  $$T_c = 155(1)(8) \text{ MeV}$$

  T. Bhattacharya et.al.
  Phys. Rev. Lett. 113, 082001 (2014)

- Deconfinement of quarks sets in at the chiral crossover

  A.Bazavov, Phys.Rev. D85 (2012) 054503

- The shift of T_c with chemical potential

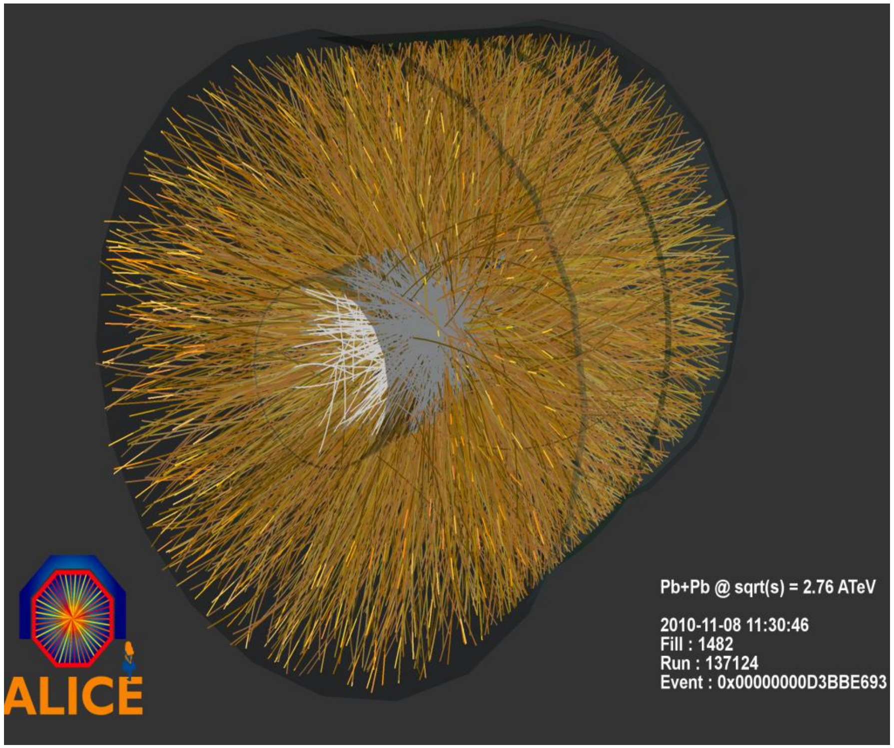
$$T_c(\mu_B) = T_c(0)[1 - 0.0066 \cdot (\mu_B / T_c)^2]$$

Ch. Schmidt Phys.Rev. D83 (2011) 014504

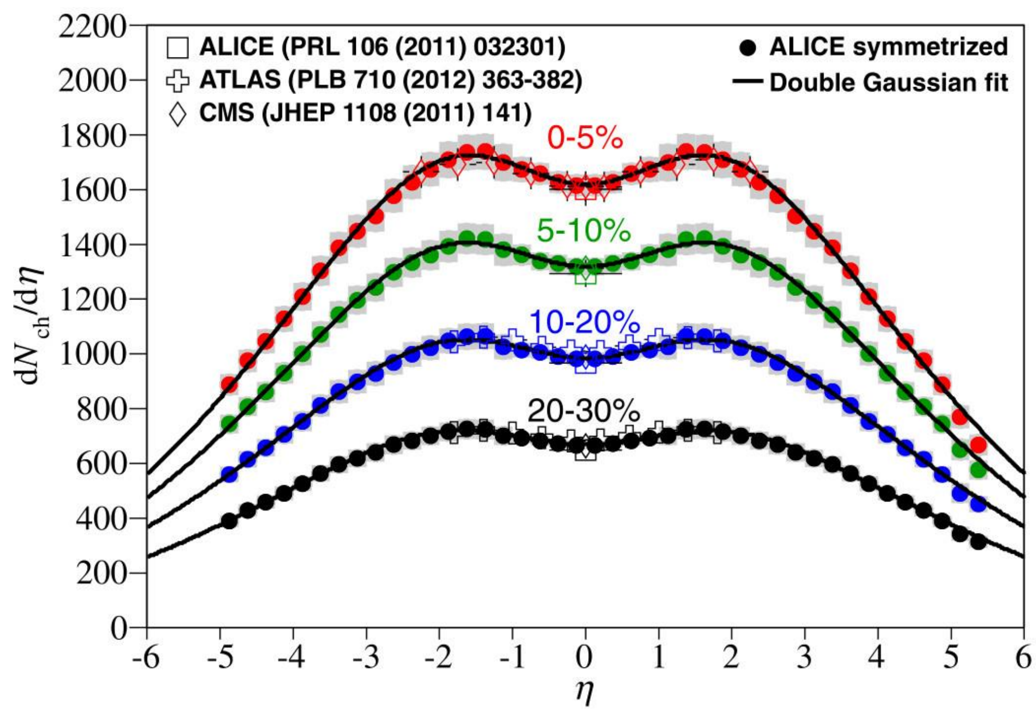
See also:
Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, *et al.*
JHEP, 0906 (2009)

Can the fireball created in central A-A collisions be considered a matter in equilibrium?

ALICE charged particles event display



Excellent data of LHC experiments on charged particles pseudo-rapidity density

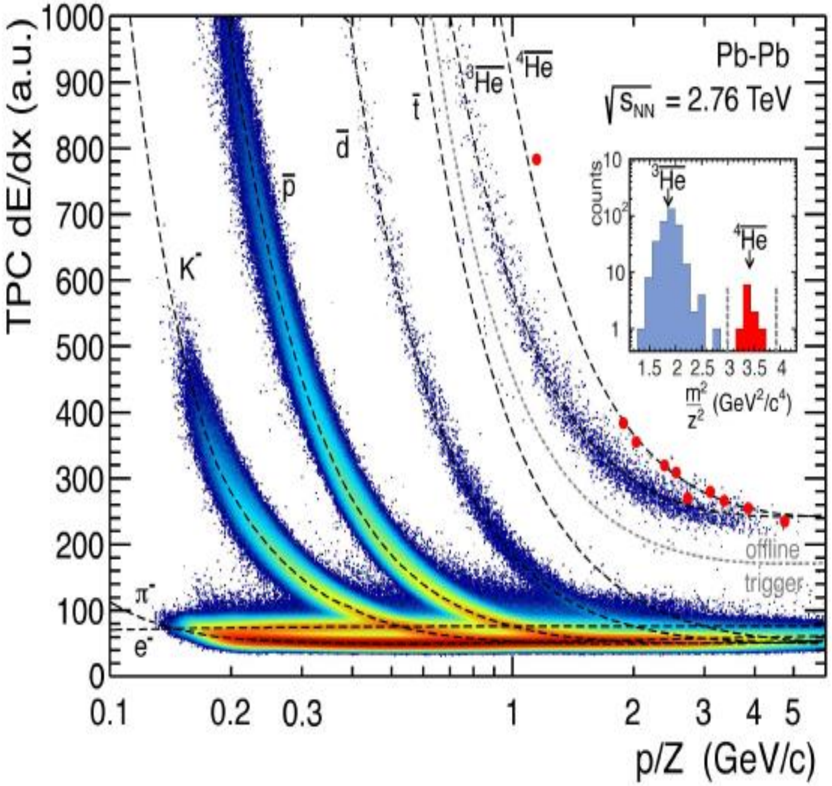


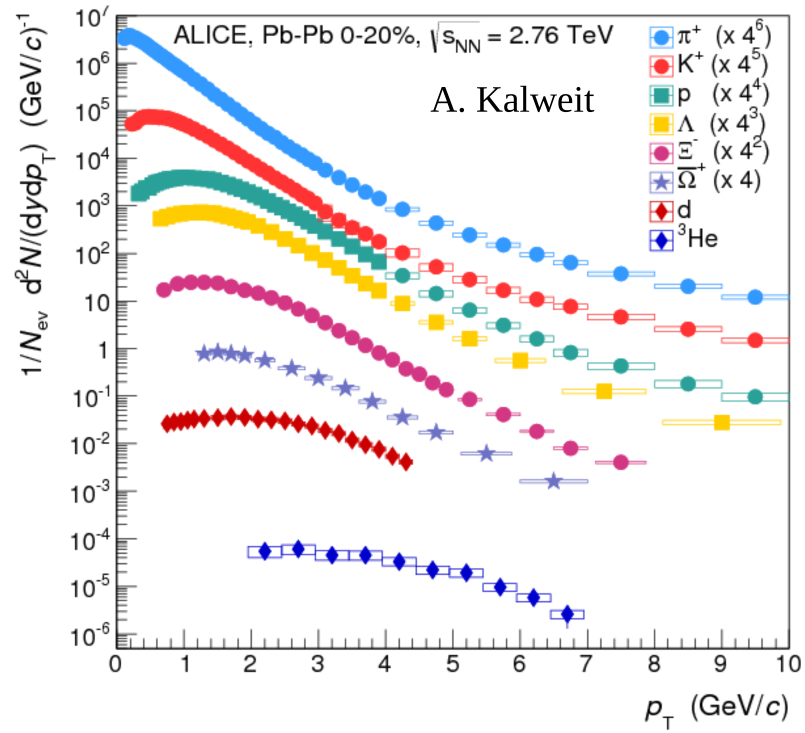
P. Braun-Munzinger, J. Stachel & Ch. Wetterich (2004)

Multi-hadron production near phase boundary brings hadrons towards equilibrium

Excellent performance of ALICE detectors for particles identification

Paolo Giubellino & Jürgen Schukraft for ALICE Collaboration





ALICE Time Projection Chamber (TPC), Time of Flight Detector (TOF), High Momentum Particle Identification Detector (HMPID) together with the Transition Radiation Detector (TRD) and the Inner Tracking System (ITS) provide information on the flavour composition of the collision fireball, vector meson resonances, as well as charm and beauty production through the measurement of leptonic observables.

Test of thermalization in HIC:

- With respect to what statistical operator?

- We will use the Statistical QCD partition function

  i.e. LQCD data as the solution of QCD

  at finite temperature,

and confront them

with ALICE data taken in central Pb-Pb

collisions at $\sqrt{s} = 2.75\ \text{TeV}$

Consider fluctuations and correlations of conserved charges

- They are quantified by susceptibilities:

  If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2} = \frac{\partial^2 (P)}{\partial(\mu_N)^2} \qquad \frac{\chi_{NM}}{T^2} = \frac{\partial^2 (P)}{\partial\mu_N \partial\mu_M}$$

$$N = N_q - N_{-q}, \;\; N,M = (B,S,Q), \quad \mu = \mu / T, \quad P = P / T^4$$

- Susceptibility is connected with variance

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N^2 > - < N >^2)$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$< N^n >= \sum_N N^n \mathrm{P}(N)$$

Consider special case:

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

$$< N_q > \equiv \overline{N}_q \quad =>$$

- Charge and anti-charge uncorrelated and Poisson distributed, then
- $P(N)$ the Skellam distribution

$$P(N) = \left(\frac{\overline{N}_q}{\overline{N}_{-q}} \right)^{N/2} I_N(2\sqrt{\overline{N}_{-q}\overline{N}_q}) \exp[-(\overline{N}_{-q} + \overline{N}_q)]$$

- Then the susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

Consider special case: particles carrying $q = \pm 1, \pm 2, \pm 3$

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

- The probability distribution

$$< S_{-q} > \equiv \bar{S}_{-q}$$

$$q = \pm 1, \pm 2, \pm 3$$

$$P(S) = (\frac{\bar{S}_1}{\bar{S}_{\bar{1}}})^{\frac{S}{2}} \exp[\sum_{n=1}^{3}(\bar{S}_n + \bar{S}_{\bar{n}})] \sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} (\frac{\bar{S}_3}{\bar{S}_{\bar{3}}})^{k/2} I_k(2\sqrt{\bar{S}_3 \bar{S}_{\bar{3}}}) (\frac{\bar{S}_2}{\bar{S}_{\bar{2}}})^{i/2} I_i(2\sqrt{\bar{S}_2 \bar{S}_{\bar{2}}}) (\frac{\bar{S}_1}{\bar{S}_{\bar{1}}})^{-i-3k/2} I_{2i+3k-S}(2\sqrt{\bar{S}_1 \bar{S}_{\bar{1}}})$$

Fluctuations

$$\frac{\chi_S}{T^2} = \frac{1}{VT^3} \sum_{n=1}^{|q|} n^2 (\langle S_n \rangle + \langle S_{-n} \rangle)$$

Correlations

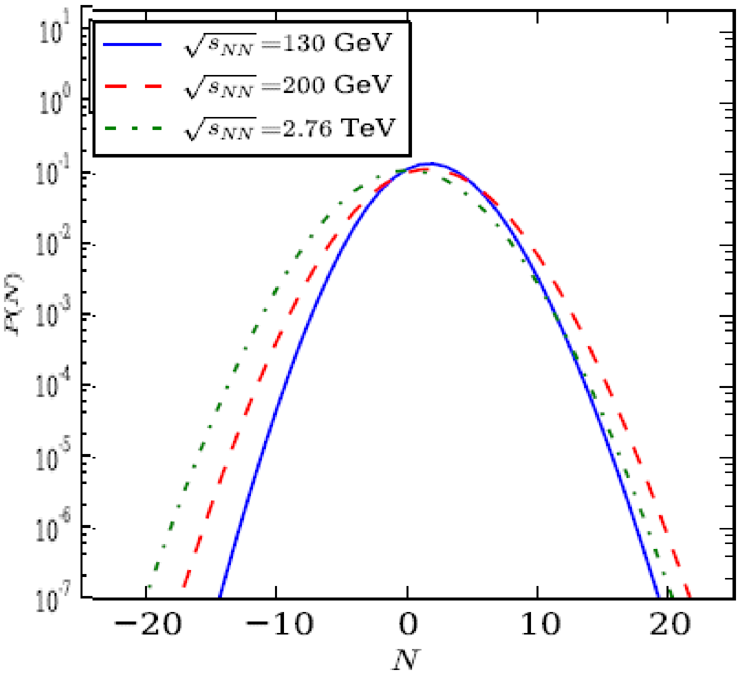
$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} \sum_{n=-q_N}^{q_N} \sum_{m=-q_M}^{q_M} nm \langle N_{n,m} \rangle$$

$\langle N_{n,m} \rangle$, is the mean number of particles
; carrying charge $N = n$ and $M = m$.

Variance at 200 GeV AA central col. at RHIC

STAR Collaboration

P. Braun-Munzinger, et al.
Nucl. Phys. A880 (2012) 48)



- Consistent with Skellam distribution

$$\frac{<p>+<\bar{p}>}{\sigma^2} = 1.022 \pm 0.016 \qquad \frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$

- The maxima of $P(N)$ have a very

$$P_{max} \approx \frac{1}{\sqrt{2\pi(\bar{N}_p + \bar{N}_{\bar{p}})}}$$

similar values at RHIC and LHC

thus $N_p + N_{\bar{p}} \approx$ const., indeed

Momentum integrated:

$RHIC \quad \sigma^2 \approx <p>+<\bar{p}> = 61.4 \pm 5.7$

$LHC \quad \sigma^2 \approx <p>+<\bar{p}> = 61.04 \pm 3.5$

$$\frac{\sigma^2}{p-\bar{p}} = 6.18 \pm 0.14 \ \ in \ \ 0.4 < p_t < 0.8 GeV$$

$$\frac{p+\bar{p}}{p-\bar{p}} = 7.67 \pm 1.86 \ \ in \ \ 0.0 < p_t < \infty \ GeV$$

Probing O(4) chiral criticality with charge fluctuations

- Due to expected O(4) scaling in QCD the free energy:

$$P = P_R(T,\mu_q,\mu_I) + b^{-1} P_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta\delta/\nu} h)$$

- Generalized susceptibilities of net baryon number

$$c_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = c_R^{(n)} + c_S^{(n)} \quad \text{with} \quad \begin{aligned} c_s^{(n)}\big|_{\mu=0} &= d\ h^{(2-\alpha-n/2)/\beta\delta}\ f_\pm^{(n)}(z) \\ c_s^{(n)}\big|_{\mu\neq 0} &= d\ h^{(2-\alpha-n)/\beta\delta}\ f_\pm^{(n)}(z) \end{aligned}$$

- At $\mu = 0$ only $c_B^{(n)}$ with $n \geq 6$ receive contribution from $c_S^{(n)}$
- At $\mu \neq 0$ only $c_B^{(n)}$ with $n \geq 3$ receive contribution from $c_S^{(n)}$

- $c_B^{n=2} = \chi_B / T^2$ Generalized susceptibilities of the net baryon number never critical with respect to ch. sym.

Constructing net charge fluctuations and correlation from ALICE data

- Net baryon number susceptibility

$$\frac{\chi_B}{T^2} \approx \frac{1}{VT^3}\left(\langle p\rangle + \langle N\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + \langle \Xi^-\rangle + \langle \Xi^0\rangle + \langle \Omega^-\rangle + \overline{par}\ \right)$$

- Net strangeness

$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}\left(\langle K^+\rangle + \langle K_S^0\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + 4\langle \Xi^-\rangle + 4\langle \Xi^0\rangle + 9\langle \Omega^-\rangle + \overline{par} - (\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-} + \Gamma_{\varphi\to K_S^0} + \Gamma_{\varphi\to K_L^0})\langle \varphi\rangle\ \right)$$

- Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}\left(\langle K^+\rangle + 2\langle \Xi^-\rangle + 3\langle \Omega^-\rangle + \overline{par} - (\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-})\langle \varphi\rangle - (\Gamma_{K_0^*\to K^+} + \Gamma_{K_0^*\to K^-})\langle K_0^*\rangle\ \right)$$

χ_B , χ_S , χ_{QS} from ALICE mid-rapidity yield data

- use also $\Sigma^0 / \Lambda = 0.278$ from pBe at $\sqrt{s} = 25\ GeV$

- Net baryon fluctuations $\frac{\chi_B}{T^2} = \frac{1}{VT^3}(203.7 \pm 11.4)$

- Net strangeness fluctuations $\frac{\chi_S}{T^2} = \frac{1}{VT^3}(504.2 \pm 16.8)$

- Charge-Strangeness corr. $\frac{\chi_{QS}}{T^2} = \frac{1}{VT^3}(191 \pm 12)$

- Ratios is volume independent

$$\frac{\chi_B}{\chi_S} = 0.404 \pm 0.026$$

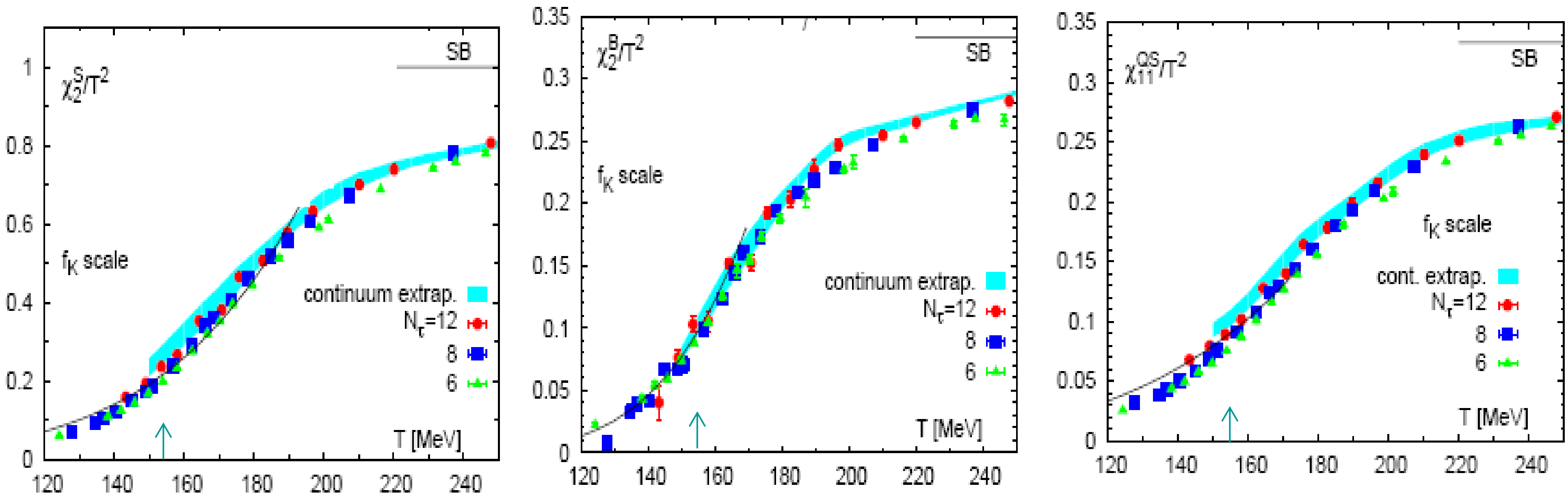
and

$$\frac{\chi_B}{\chi_{QS}} = 1.066 \pm 0.09$$

Compare the ratio with LQCD data:

A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee

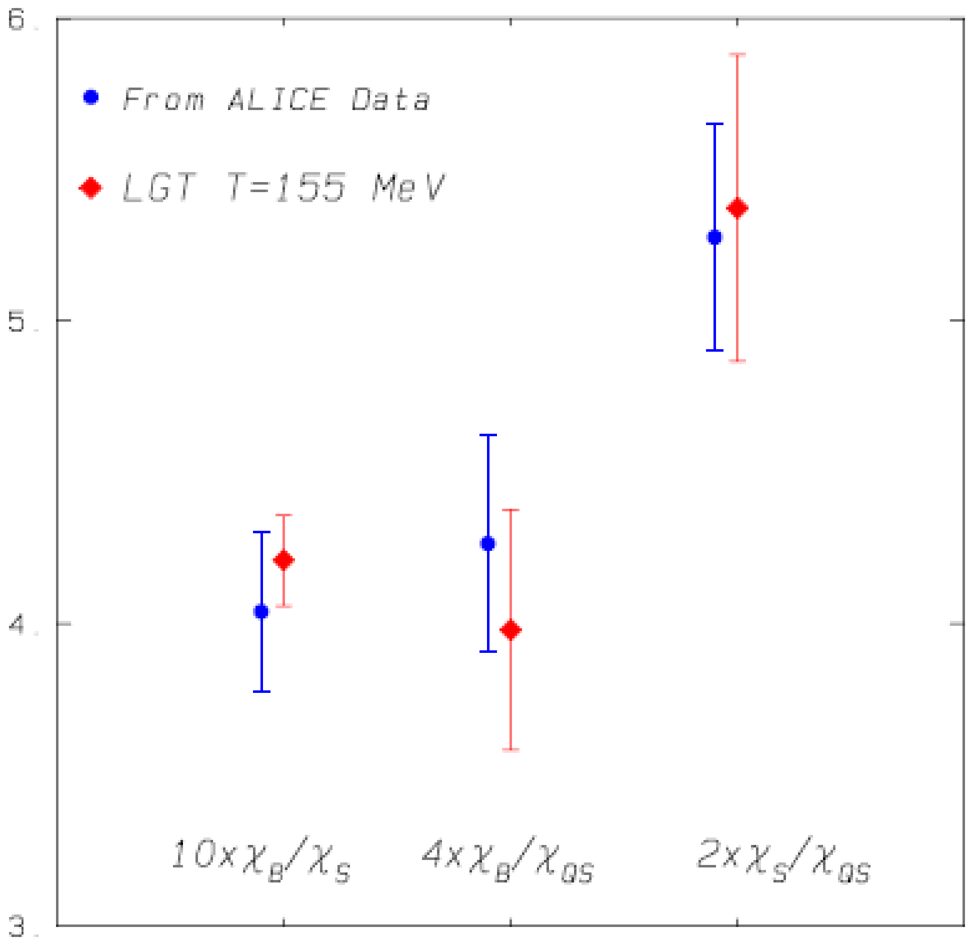
Phys.Rev.Lett. 113 (2014) and HotQCD Coll. A. Bazavov et al. Phys.Rev. D86 (2012) 034509



- Is there a temperature where calculated ratios from ALICE data agree with LQCD?

Baryon number strangeness and Q-S correlations

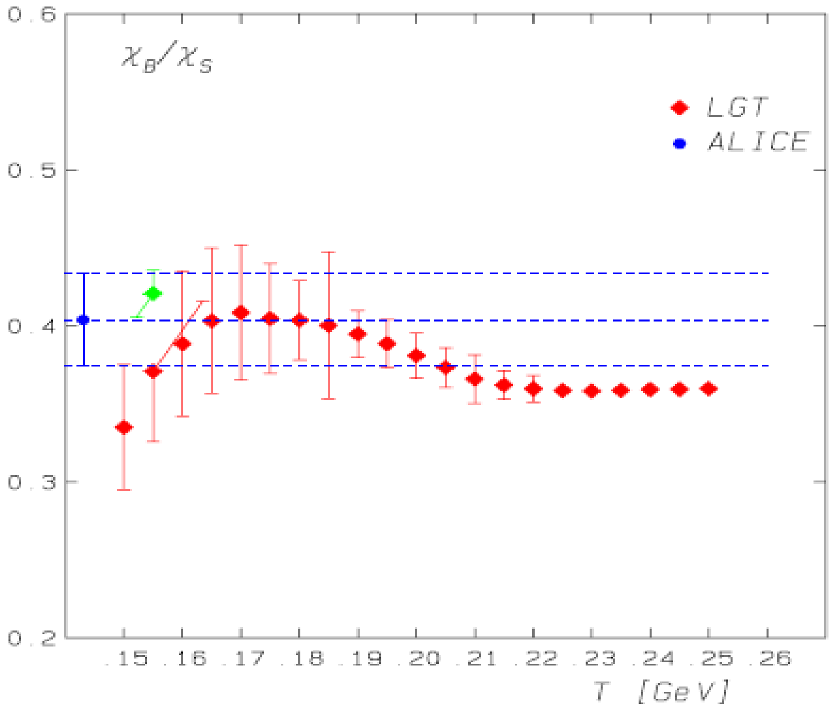
Compare at chiral crossover

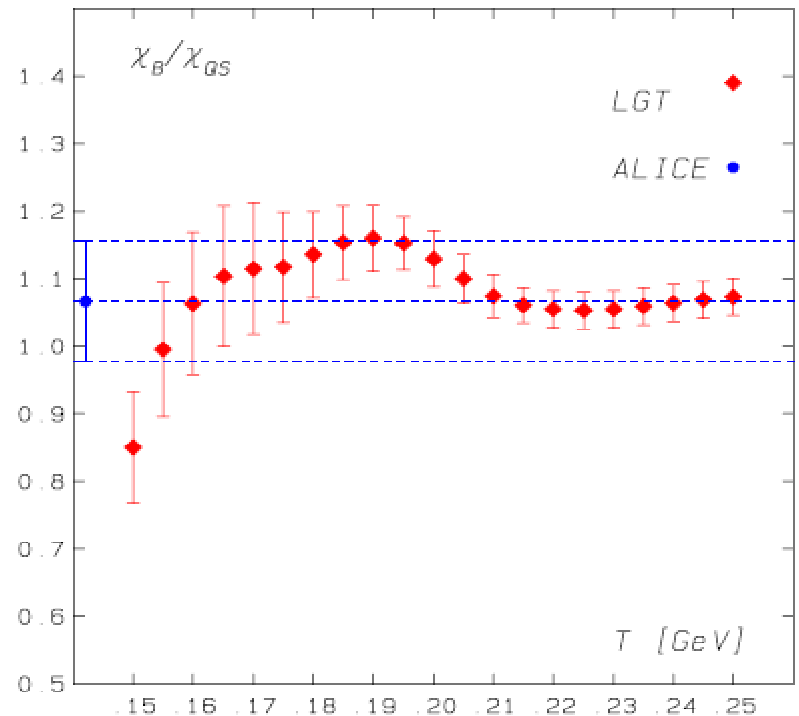


- There is a very good agreement, within systematic uncertainties, between extracted susceptibilities from ALICE data and LQCD at the chiral crossover

- How unique is the determination of the temperature at which such agreement holds?

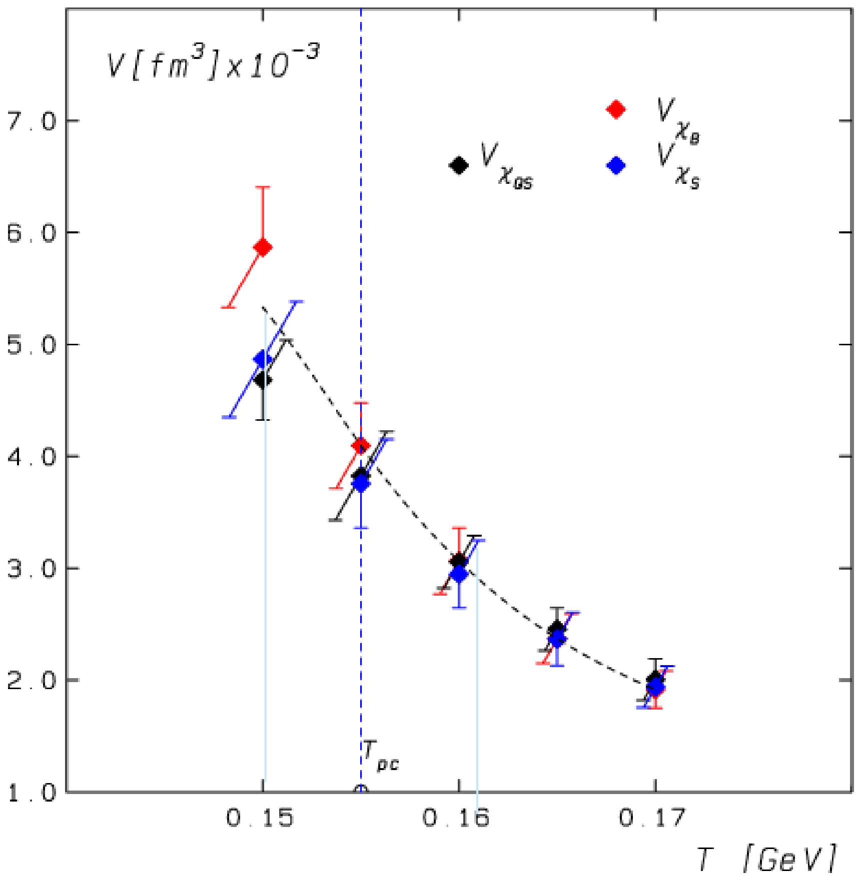
Consider T-dependent LQCD ratios and compare with ALICE data





- The LQCD susceptibilities ratios are weakly T-dependent for $T \geq T_c$
- We can reject $T \leq 0.15\ GeV$ for saturation of χ_B, χ_S *and* χ_{QS} at LHC and fixed to be in the range $0.15 < T \leq 0.21\ GeV$, however
- LQCD => for $T > 0.163\ GeV$ thermodynamics cannot be anymore described by the hadronic degrees of freedom

Extract the volume by comparing data with LQCD



- Since

$$(\chi_N / T^2)_{LQCD} = \frac{(<N^2> - <N>^2)_{LHC}}{V_N T^3}$$

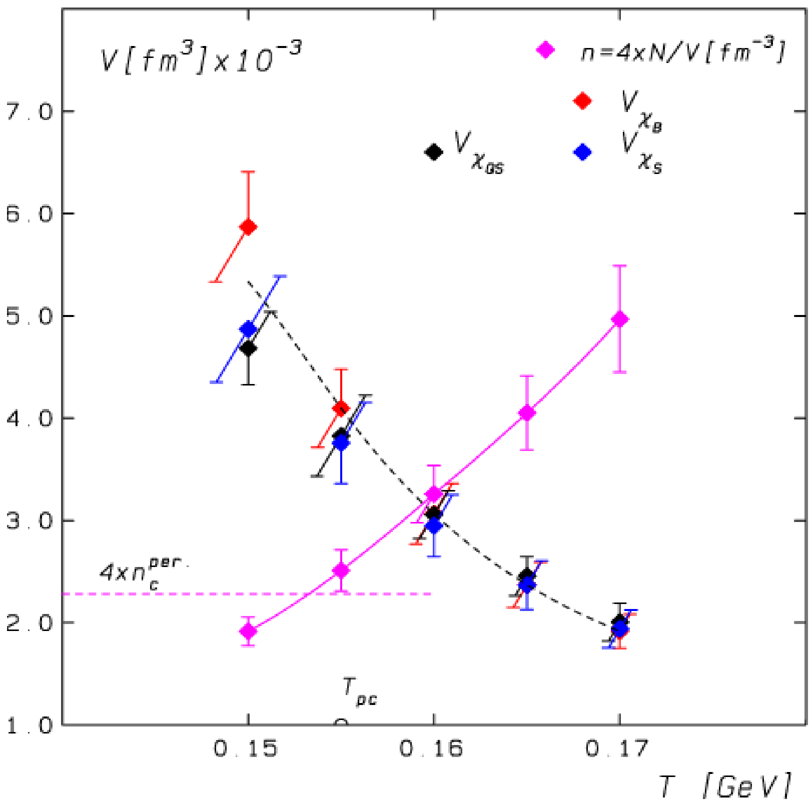
thus

$$V_{\chi_B}(T) = \frac{203.7 \pm 11.4}{T^3 (\chi_B / T^2)_{LQCD}} \qquad V_{\chi_S}(T) = \frac{504.2 \pm 24.2}{T^3 (\chi_B / T^2)_{LQCD}}$$

$$V_{\chi_{QS}}(T) = \frac{191 \pm 12}{T^3 (\chi_B / T^2)_{LQCD}}$$

- All volumes, should be equal at a given temperature if originating from the same source

Particle density and percolation theory



- Density of particles at a given volume $n(T) = \frac{N^{exp}_{total}}{V(T)}$

- Total number of particles in HIC at LHC, ALICE

$$\langle N_t \rangle = 3\langle \pi \rangle + 4\langle p \rangle + 4\langle K \rangle + (2 + 4 \times 0.2175)\langle \Lambda_\Sigma \rangle + 4\langle \bar{\Xi} \rangle + 2\langle \bar{\Omega} \rangle,$$

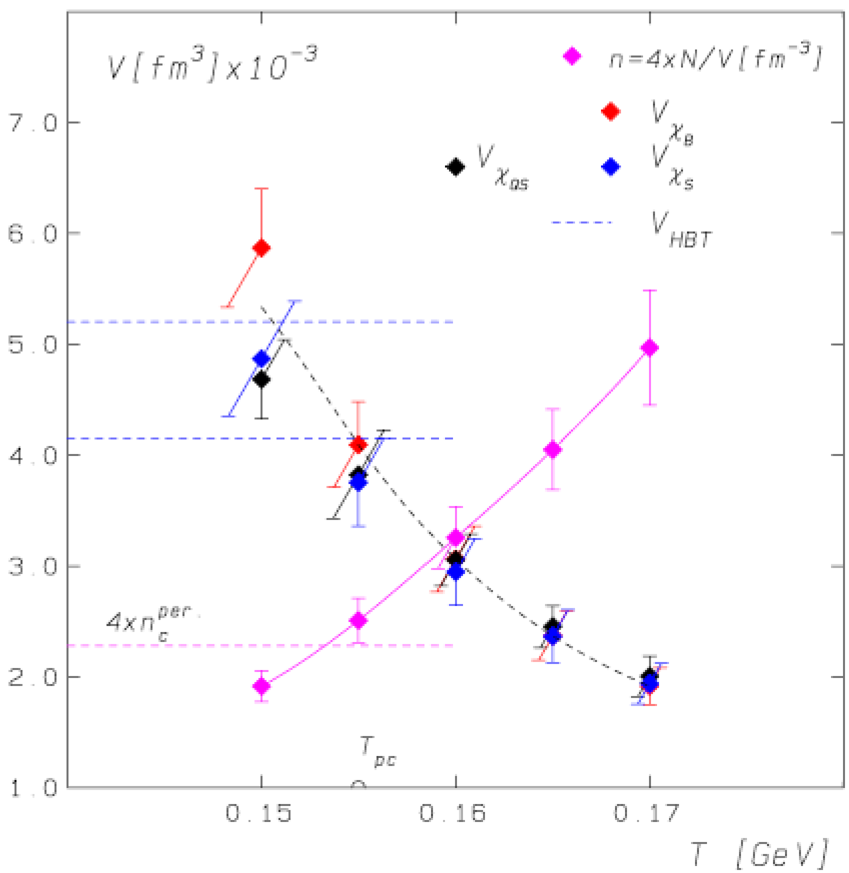
$$\langle N_t \rangle = 2486 \pm 146$$

- Percolation theory: 3-dim system of objects of volume $V_0 = 4/3\pi R_0^3$

$n_c = \frac{1.22}{V_0}$ take $R_0 \approx 0.8\, fm$ => $n_c \approx 0.57\ [fm^{-3}]$ => $T_c^p \approx 153.5\ [MeV]$

P. Castorina, H. Satz &K.R. Eur.Phys.J. C59 (2009)

Constraining the volume from HBT and percolation theory



- Some limitation on volume from Hanbury-Brown–Twiss: HBT volume $V_{HBT} = (2\pi)^{3/2} R_l R_o R_s.$

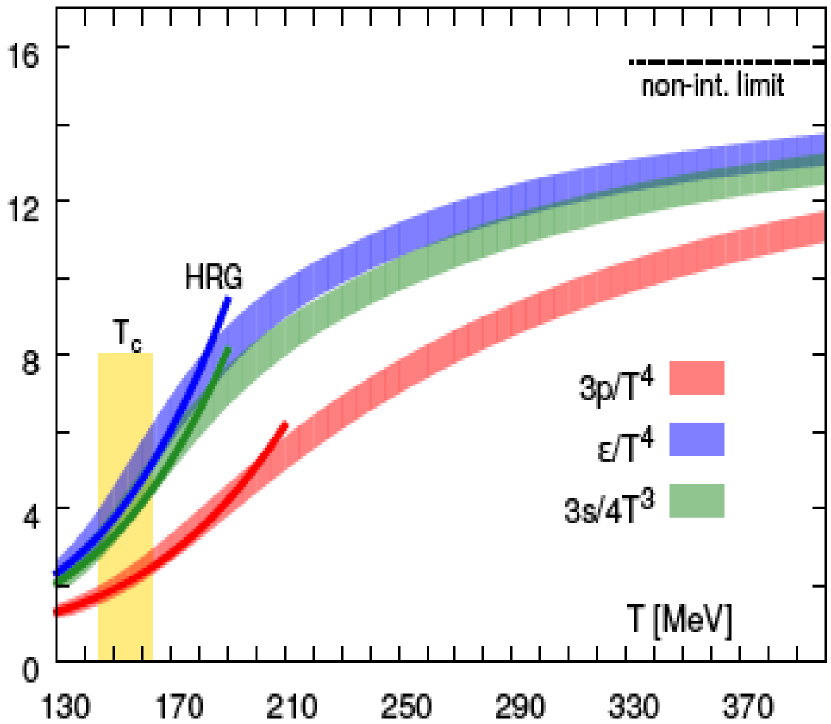
  Take ALICE data from pion interferometry $V_{HBT} = 4800 \pm 640\, fm^{-3}$

If the system would decouple at the chiral crossover, then $V \geq V_{HBT}$

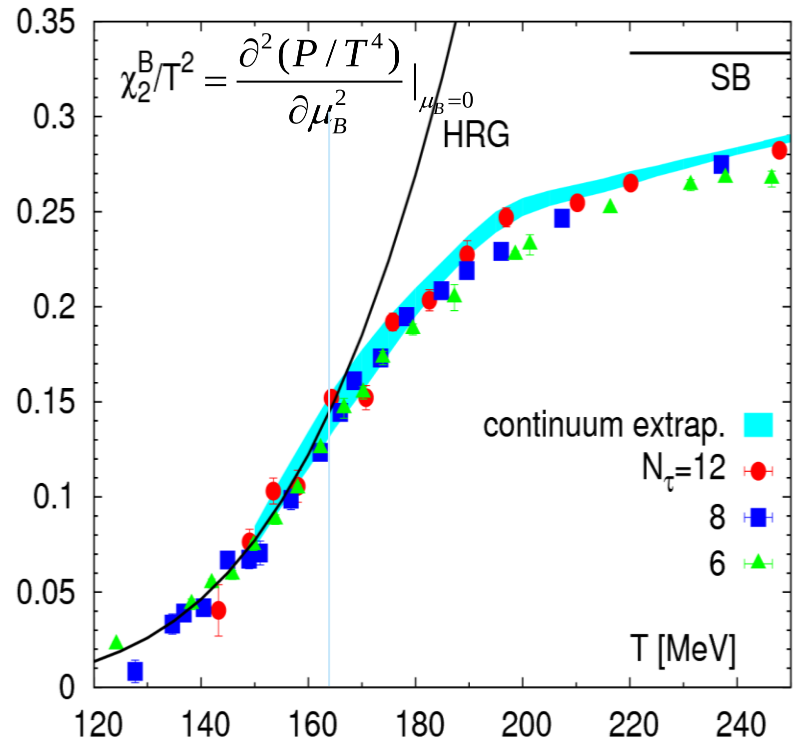
From these results: variance extracted from LHC data consistent with LQCD at $T \approx 154 \pm 2\ MeV$ where the fireball volume $V \approx 4200\ fm^3$

Excellent description of the QCD Equation of States by Hadron Resonance Gas

A. Bazavov et al. HotQCD Coll. July 2014



F. Karsch et al. HotQCD Coll.



- "Uncorrelated" Hadron Gas provides an excellent description of the QCD equation of states in confined phase

- "Uncorrelated" Hadron Gas provides also an excellent description of net baryon number fluctuations

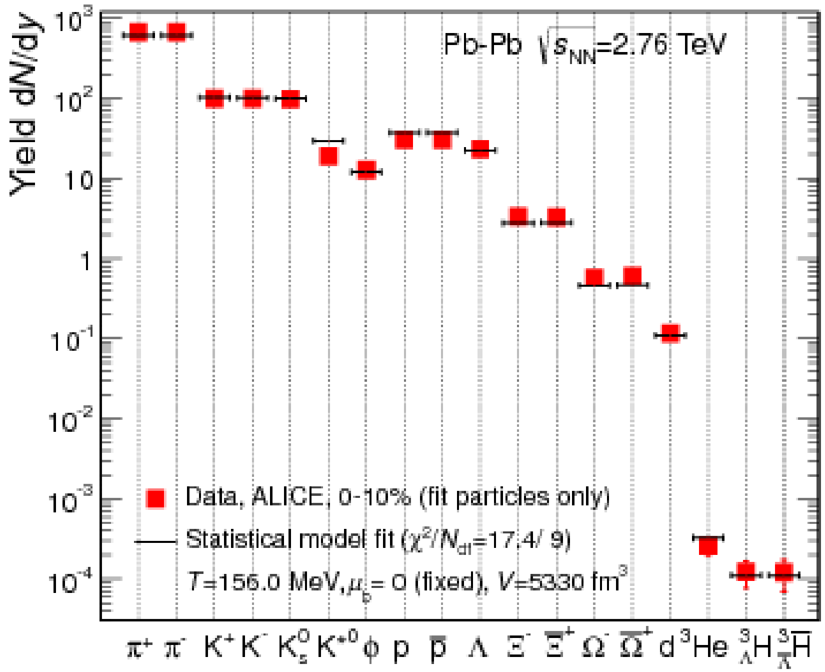
Thermal origin of particle yields with respect to HRG

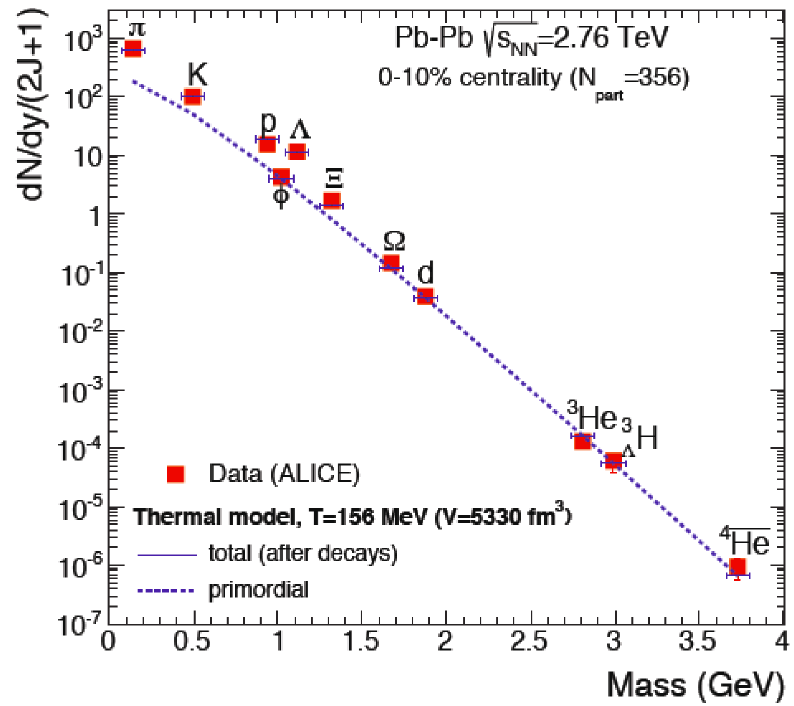
Rolf Hagedorn => the Hadron Resonace Gas (HRG):

"uncorrelated" gas of hadrons and resonances

$$< N_i > = V \, [n_i^{th}(T, \vec{\mu}) + \sum_K \Gamma_{K \to i} n_i^{th-Res.}(T, \vec{\mu})]$$

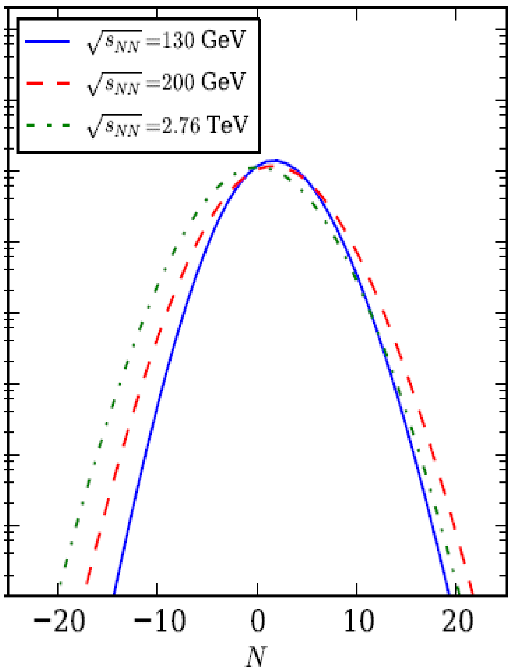
A. Andronic, Peter Braun-Munzinger, & Johanna Stachel,





- Measured yields are reproduced with HRG at $T = 156$ MeV

What is the influence of O(4) criticality on P(N)?



- For the net baryon number use the Skellam distribution (HRG baseline)

$$P(N) = \left(\frac{B}{\bar{B}}\right)^{N/2} I_N(2\sqrt{B\bar{B}}) \exp[-(B+\bar{B})]$$

  as the reference for the non-critical behavior

- Calculate P(N) in an effective chiral model which exhibits O(4) scaling and compare to the Skellam distribtuion

Modelling O(4) transtion: effective Lagrangian and FRG

$$\mathscr{L}_{\rm QM} = \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U(\sigma,\vec{\pi})$$

Effective potential is obtained by solving *the exact flow equation* (Wetterich eq.) with the approximations resulting in the O(4) critical exponents

B.J. Schaefer & J. Wambach,; B. Stokic, B. Friman & K.R.

$$\partial_k\Omega_k(\sigma) = \frac{Vk^4}{12\pi^2}\left[\sum_{i=\pi,\sigma}\frac{d_i}{E_{i,k}}[1+2n_B(E_{i,k})] - \frac{2\nu_q}{E_{q,k}}[1-n_F(E^+_{q,k})-n_F(E^-_{q,k})]\right]$$

$$E_{\pi,k} = \sqrt{k^2+\Omega'_k}$$

$$E_{\sigma,k} = \sqrt{k^2+\Omega'_k+2\rho\Omega''_k}$$

$$E_{q,k} = \sqrt{k^2+2g^2\rho}$$

$$\Omega'_k \equiv \frac{\partial\Omega_k}{\partial(\sigma^2/2)}$$

Full propagators with $k < q < \Lambda$

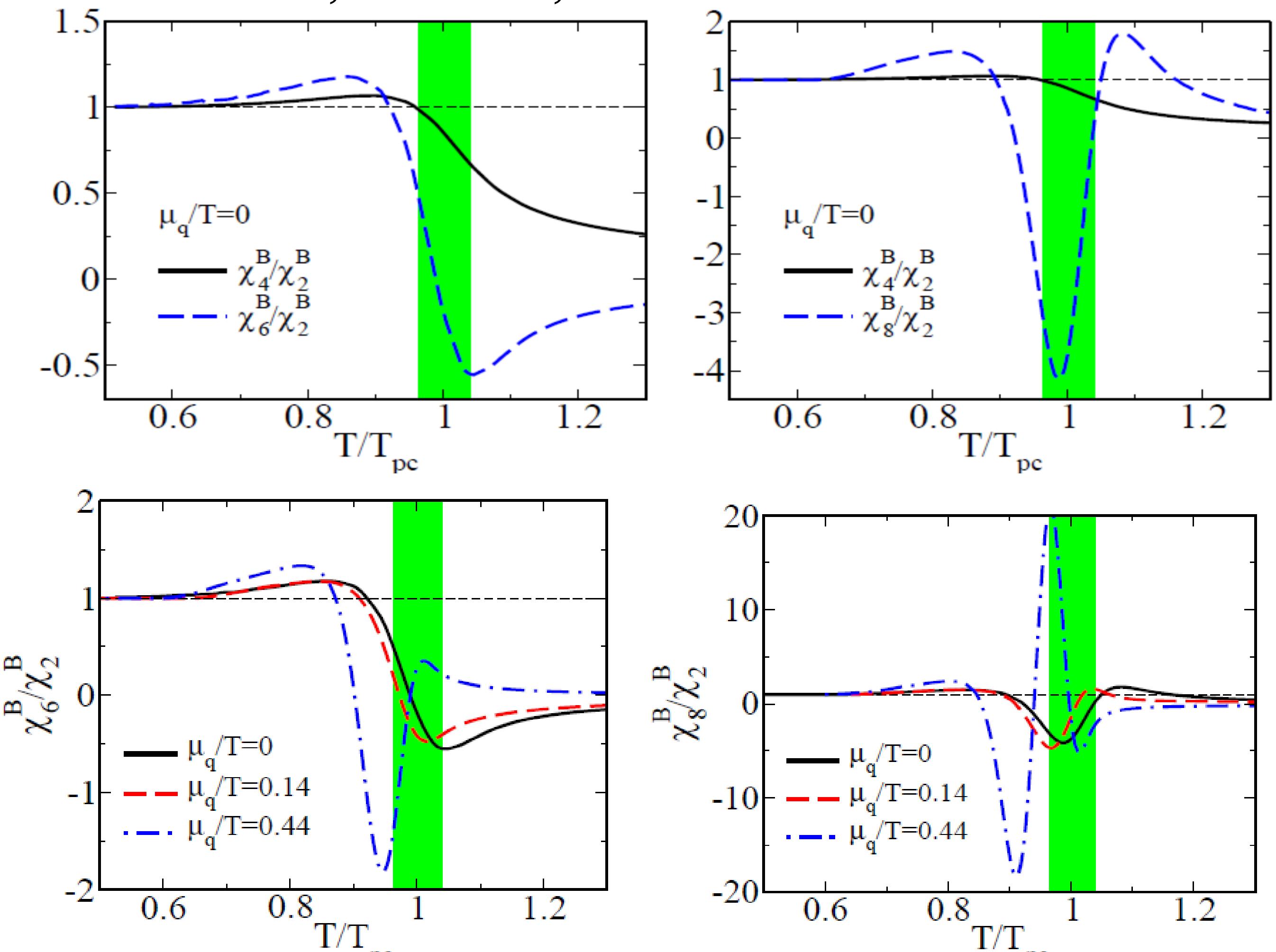
$\Gamma_\Lambda = S_{\text{classical}}$

Integrating from $k=\Lambda$ to $k=0$ gives a full quantum effective potential

Put $\Omega_{k=0}(\sigma_{\min})$ into the integral formula for P(N)

Higher moments of baryon number fluctuations

B. Friman, K. Morita, V. Skokov & K.R.



- If freeze-out in heavy ion collisions occurs from a thermalized system close to the chiral crossover temperature, this will lead to a negative sixth and eighth order moments of net baryon number fluctuations.

These properties are universal and should be observed in HIC experiments at LHC and RHIC

Figures: results of the PNJL model obtained within the Functional Renormalisation Group method

Moments obtained from probability distributions

- Moments obtained from probability distribution

$$< N^k >= \sum_N N^k P(N)$$

- Probability quantified by all cumulants

$$P(N) = \frac{1}{2\pi} \int_0^{2\pi} dy \exp[iyN - \chi(iy)]$$

Cumulants generating function: $\chi(y) = \beta V[p(T, y+\mu) - p(T,\mu)] = \sum_k \chi_k y^k$

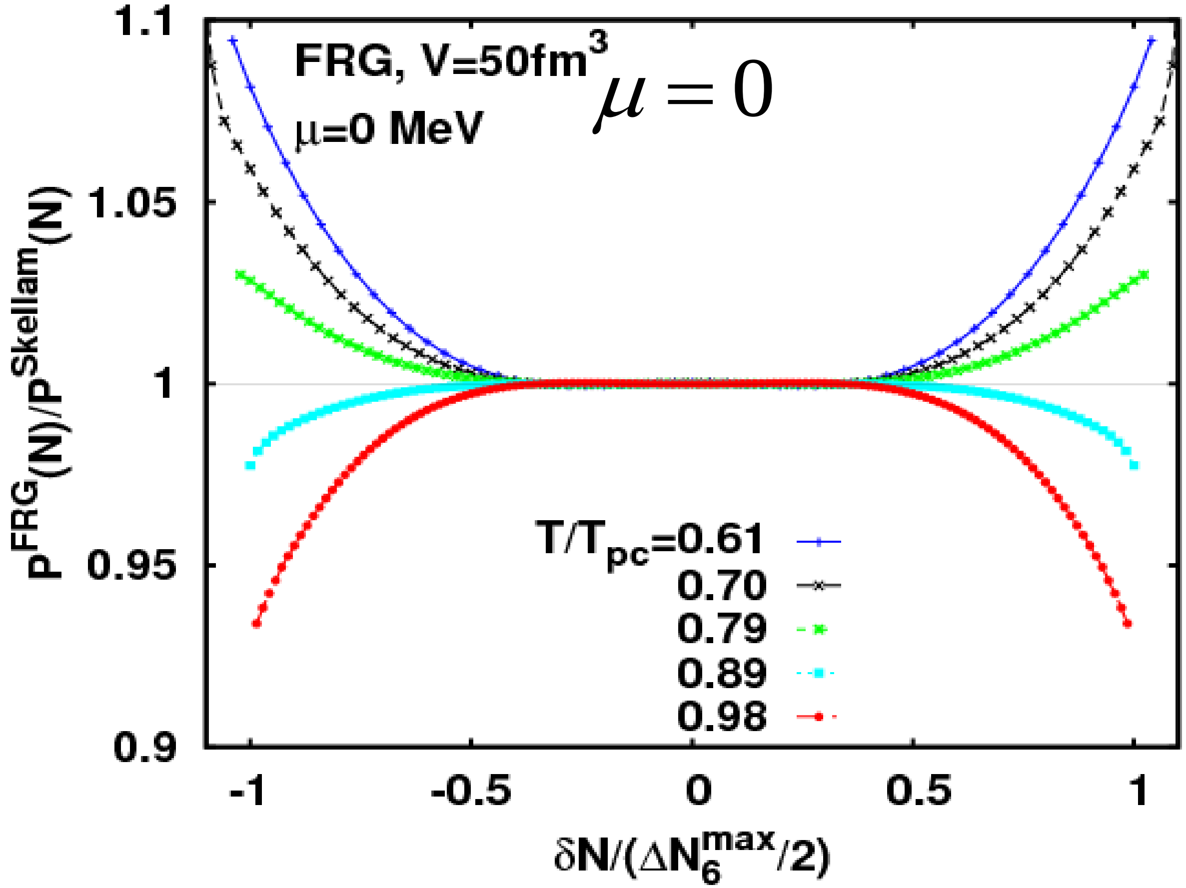
- In statistical physics

$$P(N) = \frac{Z_C(N)}{Z_{GC}} e^{\frac{\mu N}{T}} \quad Z(T,V,N) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta e^{-i\theta N} \mathscr{Z}(T,V,\theta)$$

The influence of O(4) criticality on P(N) for $\mu = 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance at different T/T_{pc}

K. Morita, B. Friman &K.R. (PQM model within renormalization group FRG)



- Ratios less than unity near the chiral crossover, indicating the contribution of the O(4) criticality to the thermodynamic pressure

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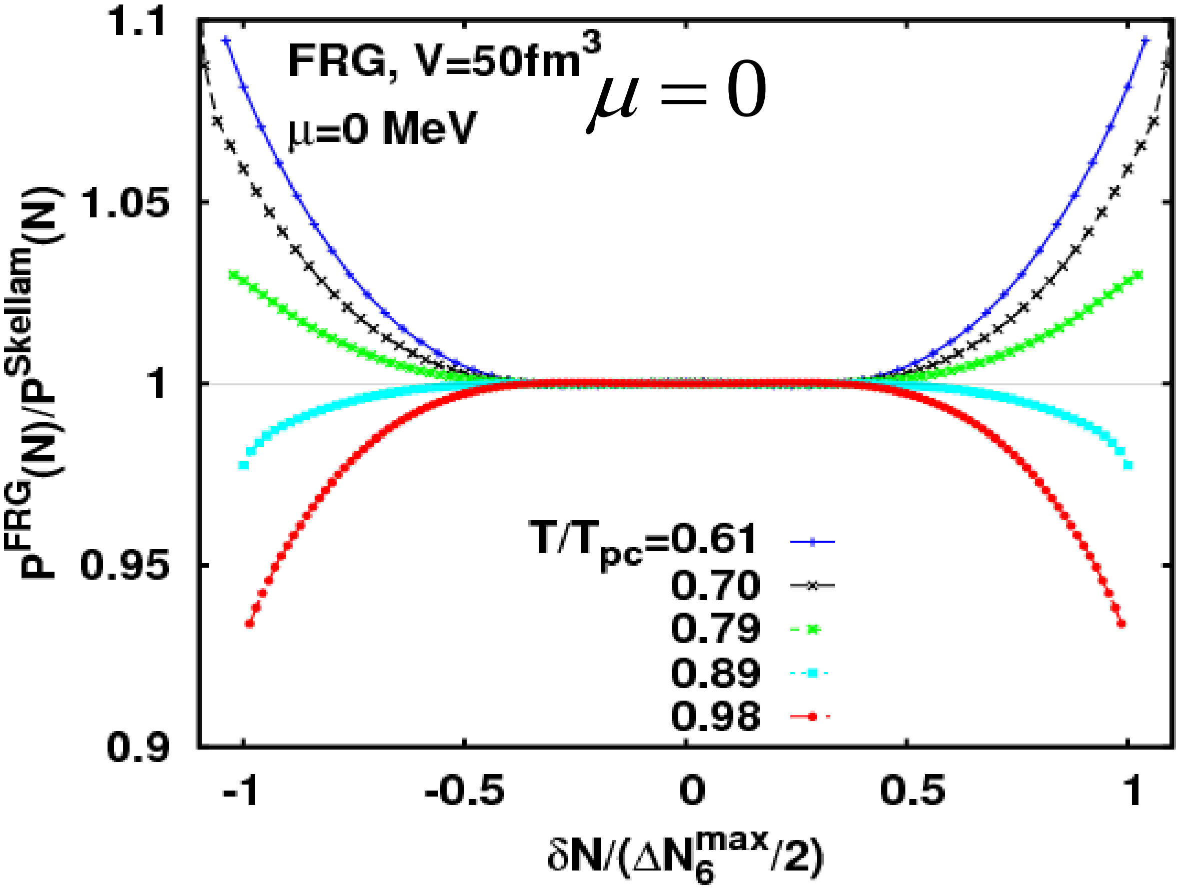

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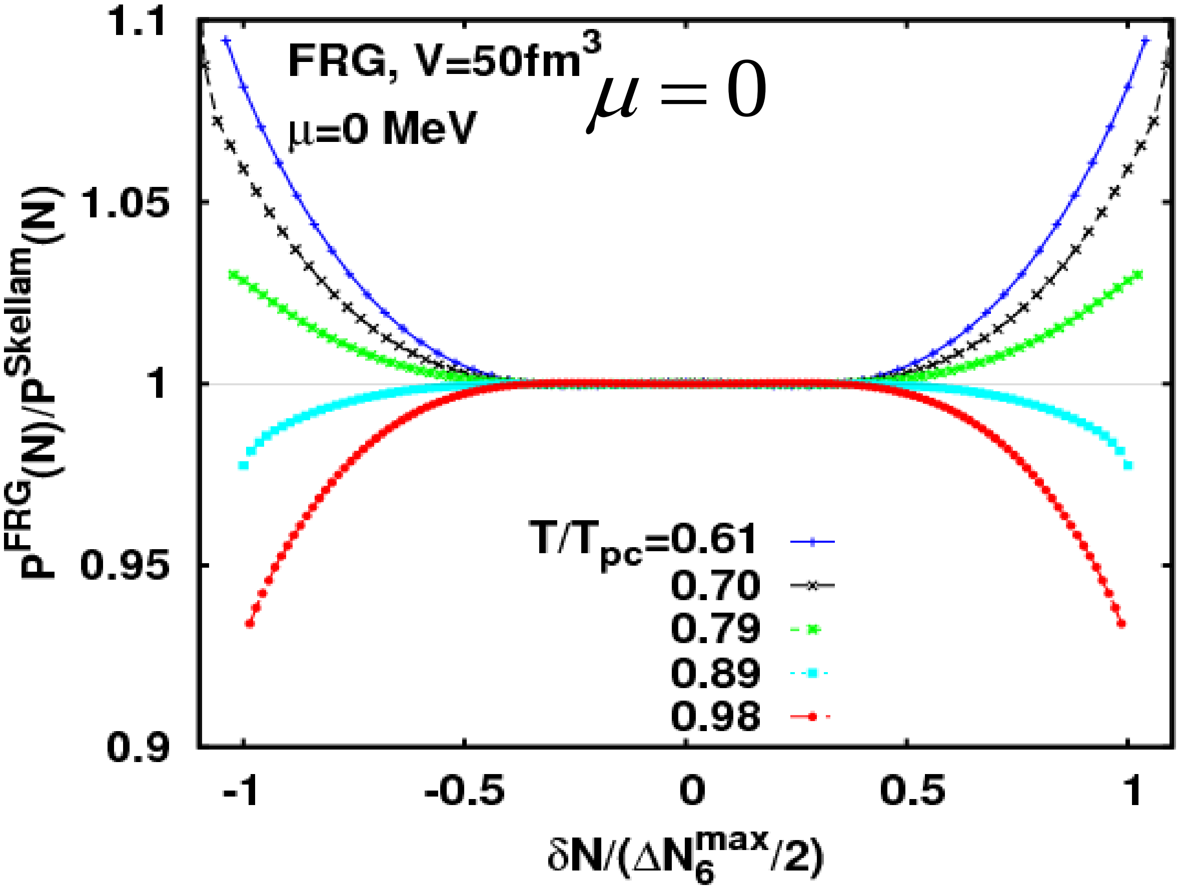
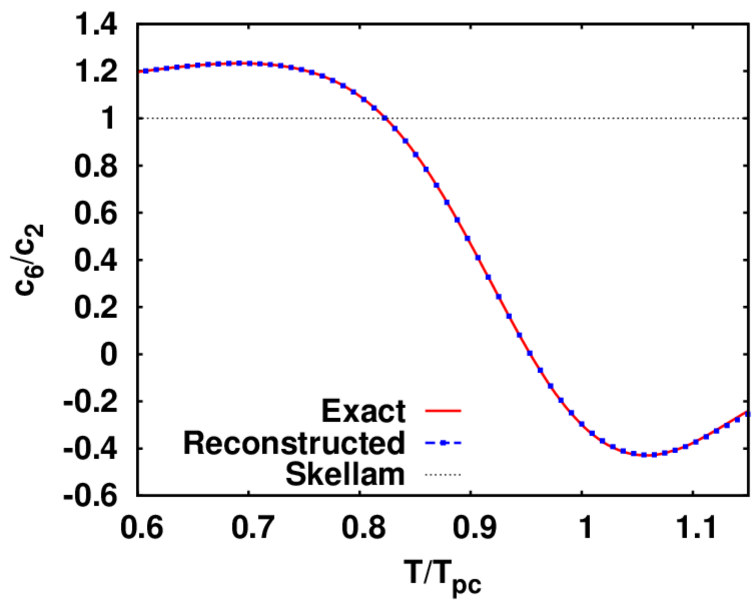


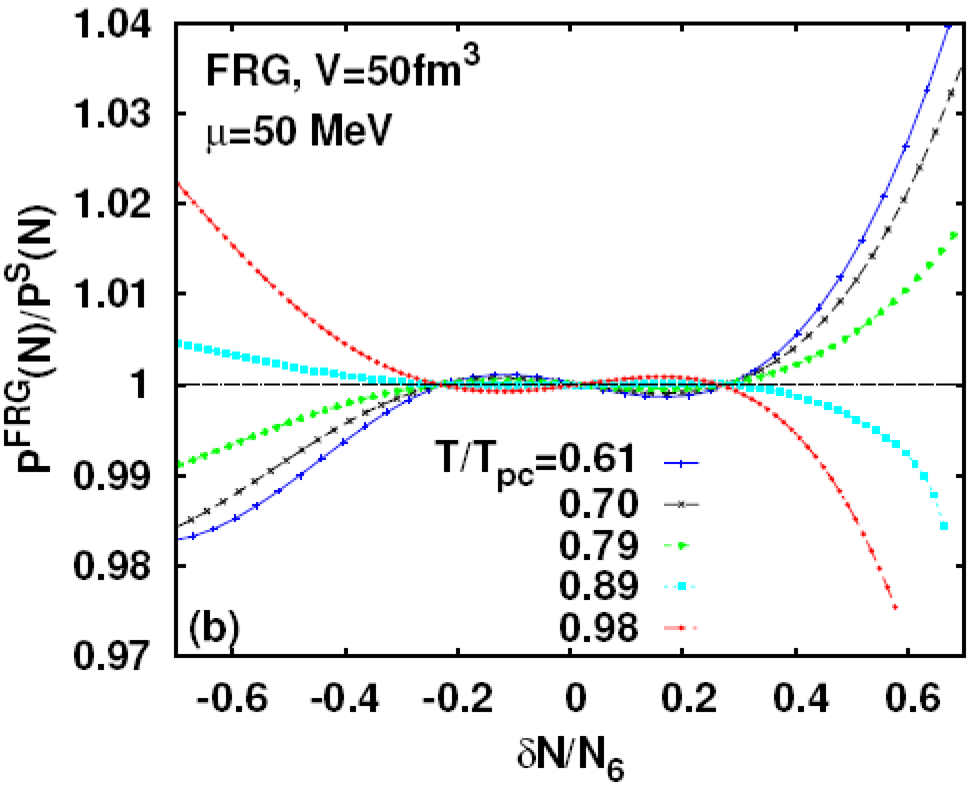
Ratio < 1 at larger |N| if $c_6/c_2 < 1$



The influence of O(4) criticality on P(N) at $\mu \neq 0$

- Take the ratio of $P^{FRG}(N)$ which contains O(4) dynamics to Skellam distribution with the same Mean and Variance near $T_{pc}(\mu)$

K. Morita, B. Friman et al.



- Asymmetric P(N) $N > < N >$
- Near $T_{pc}(\mu)$ the ratios less than unity for

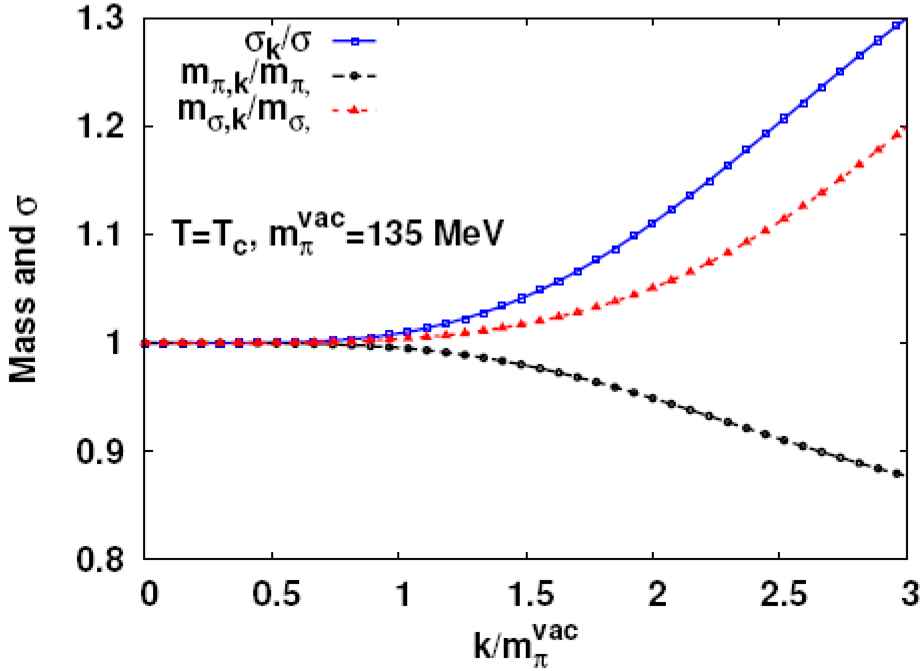
Conclusions:

From a direct comparison of ALICE data to LQCD:

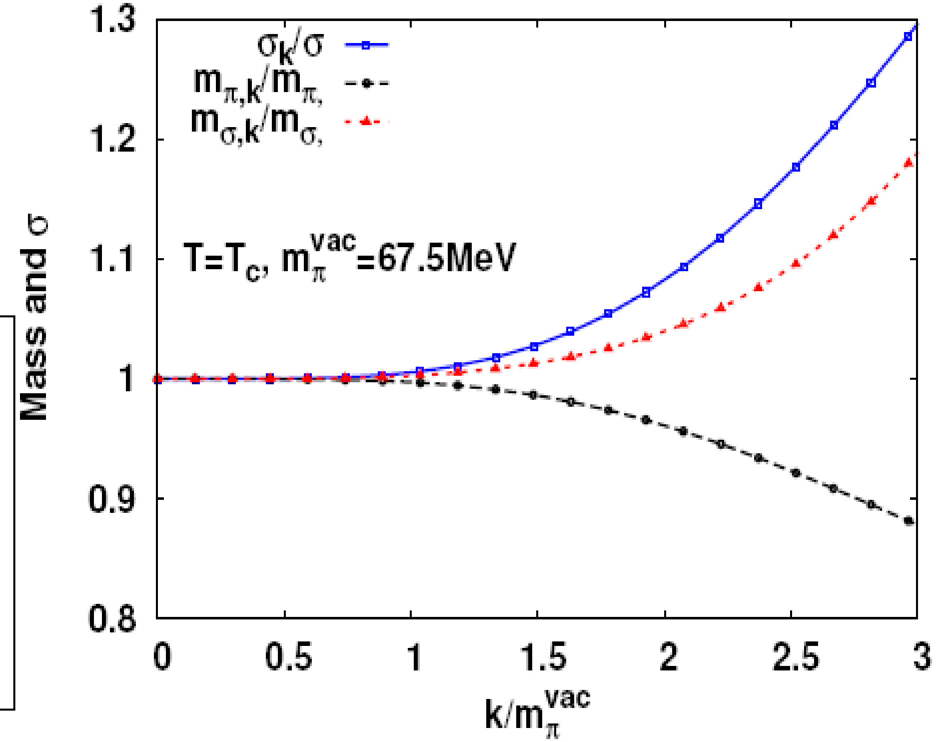
- there is thermalization in heavy ion collisions at the LHC and the charge fluctuations and correlations are saturated at the chiral crossover temperature

Skellam distribution, and its generalization, is a good approximation of the net charge probability distribution P(N) for small N. The criticality sets in at larger N and results in the shrinking of the Skellam function for positive N.

The influence of momentum cuts on sigma and pion mass

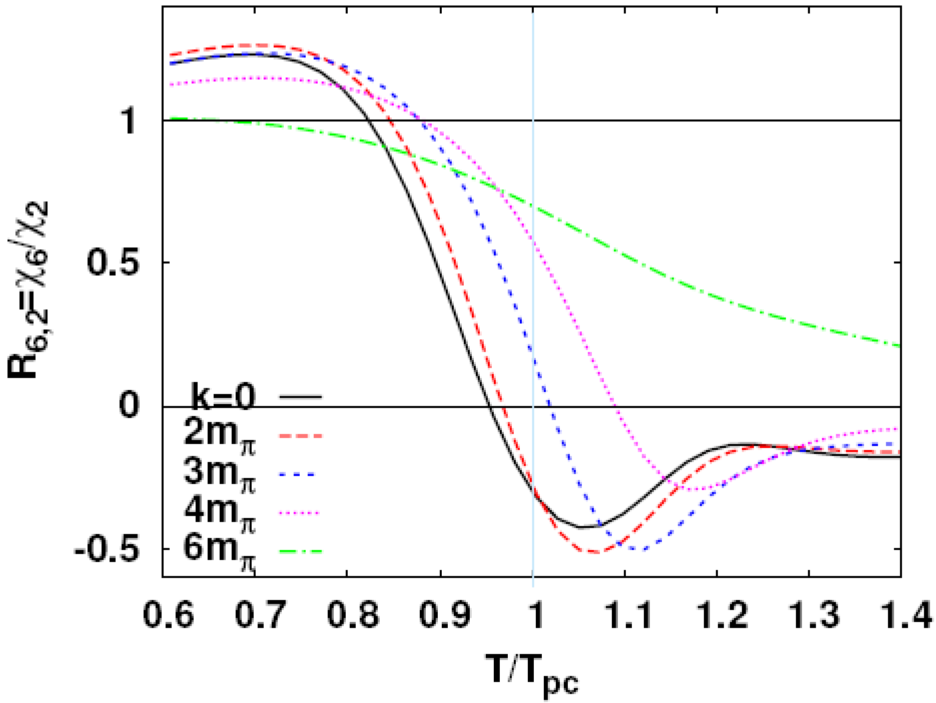


- Consider 1st the pion and sigma masses at T_{pc} and their dependence on the infrared momentum cut off



- Introducing soft momentum cut at $k \leq m_\pi$ will not modify relevant O(4) properties near chiral crossover of pion and sigma masses

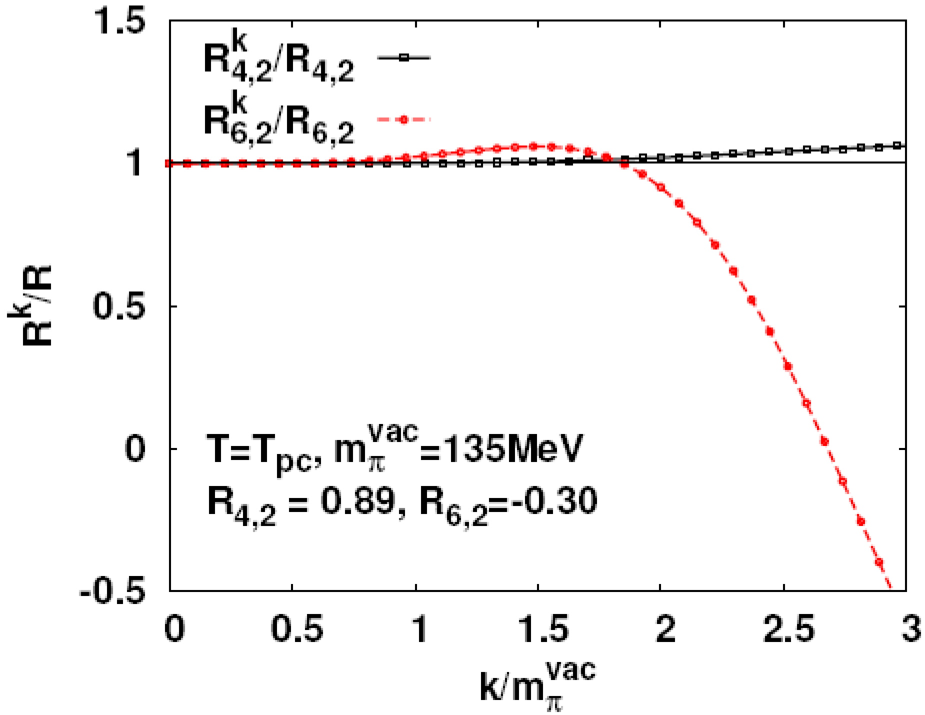
The influence of momentum cuts on critical fluctuations



- For $k > 5m_\pi$ the ratio shows a smooth change from unity to ideal quark gas value, thus there distribution is Skellam

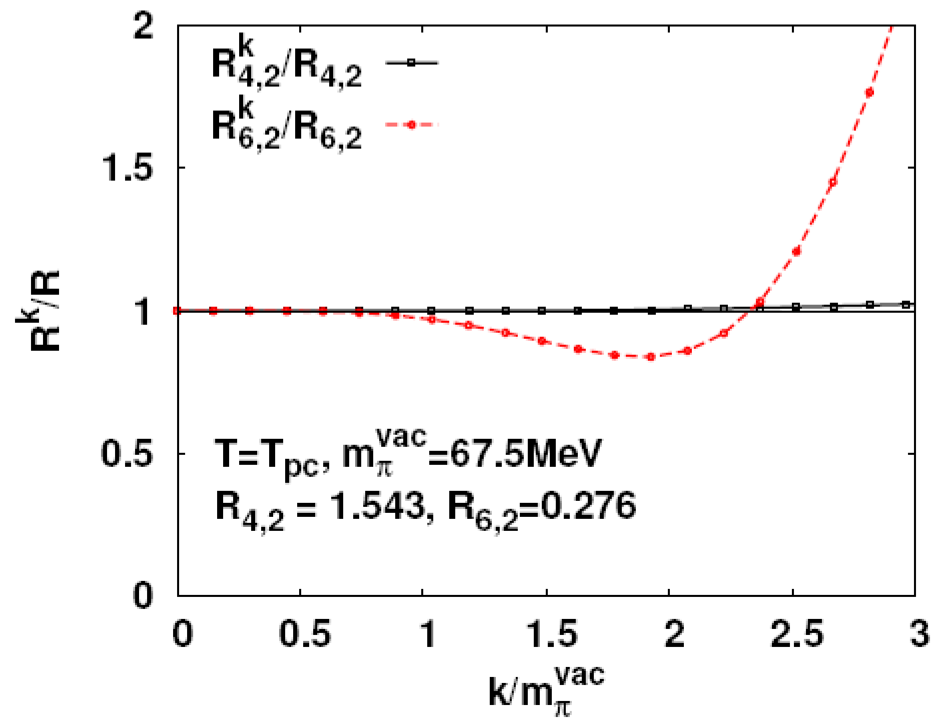
- With increasing infrared momentum cut off the suppression of R near T_{pc} due to O(4) criticality is weakened.
- For $k > 2m_\pi$, the characteristic negative structure of this fluctuation ratio, expected do to remnant of the O(4) criticality dissapears.

The influence of momentum cuts and different pion masses



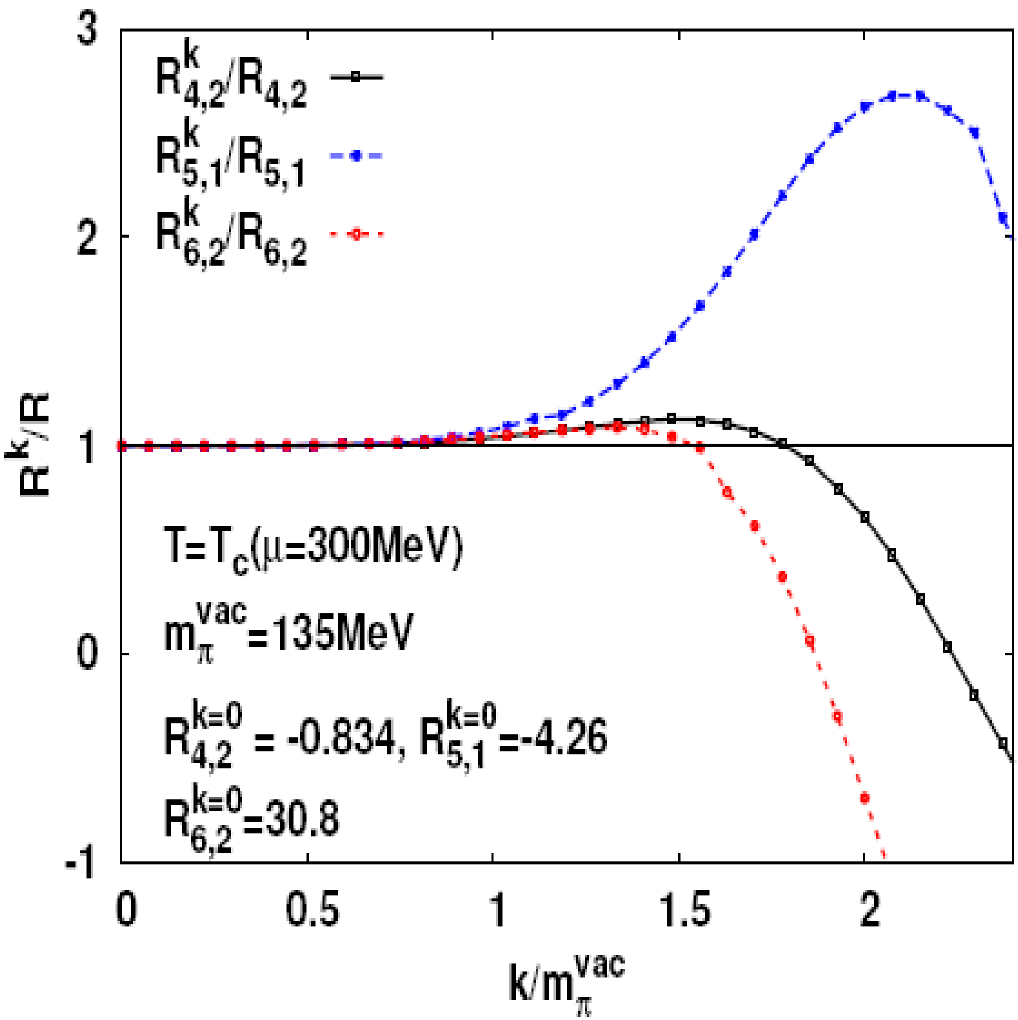
T_{pc}

- At lower pion mass the sensitivity to momentum cut is shifted to lower value. Also the sign is changed



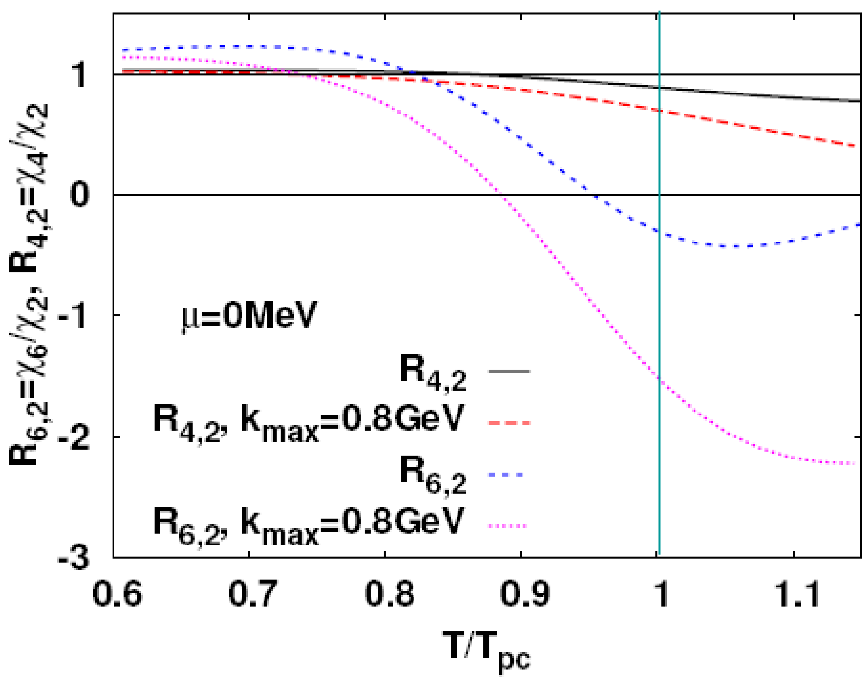
- At physical pion mass $R_{6,2}$ is weekly changing with cut off if $k < 2m_\pi$, for larger k, the $R^k{}_{6,2} > 0$
- $R_{4,2}$ is not O(4) critical, thus insensitive to any cut off change

Momentum cuts at finite chemical potential

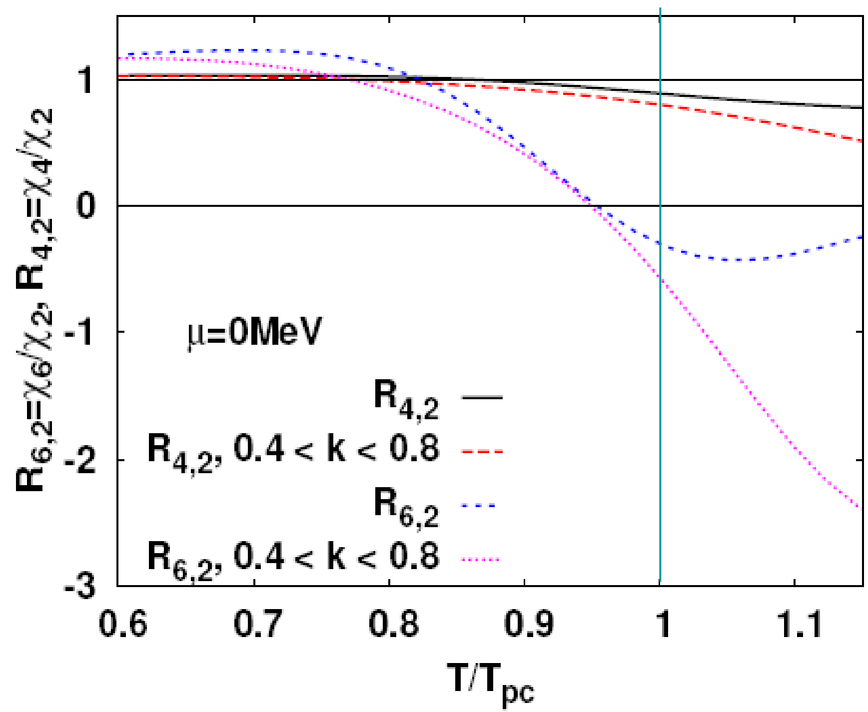


- At finite chemical potential all moments χ_n with $n \geq 3$ are influenced by O(4) criticality
- Consequently, the χ_4 also show a strong influence for infrared momentum cut off.
- Deviations from full results also are seen to deviate at lower cut off $k < 1.5 m_\pi$

The influence of ultraviolet and infrared momentum cuts

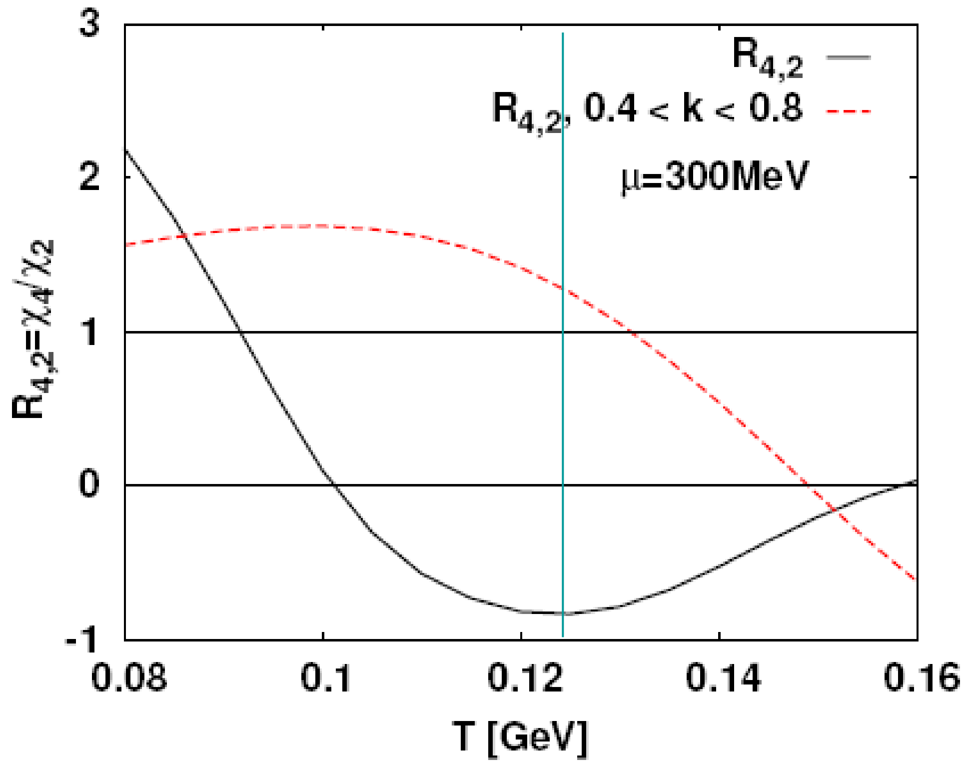


- Introducing simultanious cut $0.4 < k < 0.8\ GeV$ modifies χ_6 less at T_{pc}, since IR and UV cuts are working in an opposite directions



- Introducing ultraviolet momentum cut $k = 0.8 GeV$ suppresses χ_6 fluctuations at T_{pc}
- The suppression of χ_4 appears at high T due to quantum statistics

IR and UV momentum cuts at finite chemical potential



- At finite and large chemical potential, strongly modified χ_4 if $0.4 < k < 0.8\ GeV$ is imposed
- Characteristic negative O(4) structure of χ_4 is totally lost
- Here already the infrared cutoff $k > 2.2 m_\pi$ implies change of sign of χ_4 at T_{pc}

- Conclusions: Measuring fluctuations of the net proton number in HIC, to search for the O(4) chiral cross over or CP, a special care have to be made when introducing momentum cuts, as thy can falsify the physics.