Non-Equilibrium Dynamics in a Holographic Superfluid

Andreas Samberg
with M. Karl, T. Gasenzer, C. Ewerz

arXiv: 1410.3472

Institut für Theoretische Physik
Universität Heidelberg

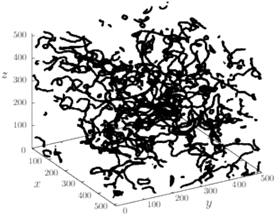
EMMI Physics Days, November 11, 2014

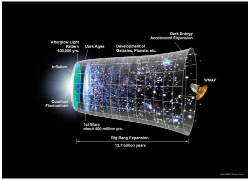
HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

HGSFP

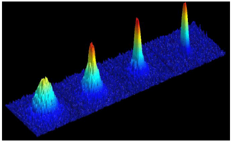
EMMI

Non-Equilibrium Dynamics

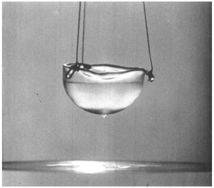




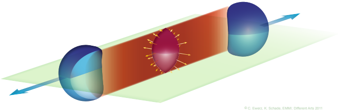
Early universe

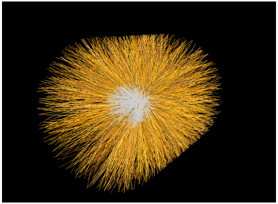


Quantum gases/
Superfluids

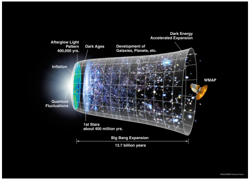


Heavy-ion collisions

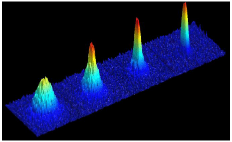


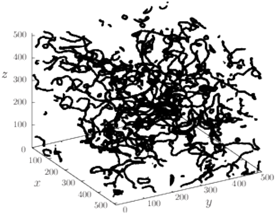


Non-Equilibrium Dynamics

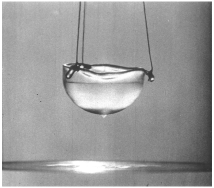


Early universe

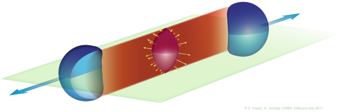


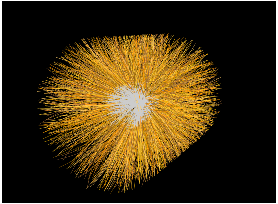


Quantum gases/ Superfluids



Heavy-ion collisions

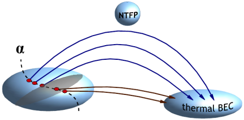




Non-Thermal Fixed Points

Far-from-equilibrium Relaxation Thermal equilibrium

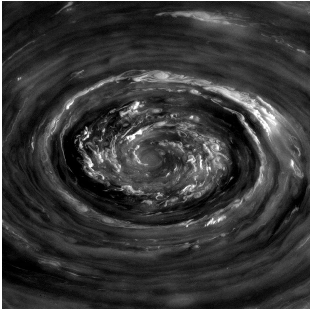
transient regime
quasi-stationary



Universality?

Turbulence

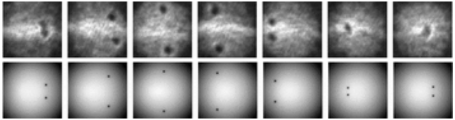
Classical



[NASA/JPL-Caltech/Space Sci. Inst., 2012]

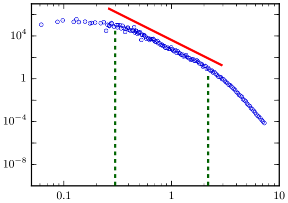
- A. N. Kolmogorov (1941): Kinetic energy spectrum
  - inertial range
  - power-law scaling, $E(k) \sim k^{-5/3}$

Quantum



Neely *et al.*, PRL 104 (2010)

$E(k) \sim k^{-\zeta}$



Many Methods

- Gross–Pitaevskii equation
  - Classical statistical simulations

Nowak *et al.*, PRA 85 (2012)
Schole *et al.*, PRA 86 (2012)
Nowak *et al.*, New J.Phys. 16 (2014)

- Non-perturbative QFT techniques
  - 2PI effective action

Berges *et al.*, PRL 101 (2008)
Berges *et al.*, Nucl.Phys.B 813 (2009)
Scheppach *et al.*, PRA 81 (2010)

- Holography

Adams *et al.*, Science 341 (2013)
Ewerz, Gasenzer, Karl, AS, `arXiv:1410.3472`

- ...

Many Methods

- Gross–Pitaevskii equation
  - Classical statistical simulations

Nowak *et al.*, PRA 85 (2012)
Schole *et al.*, PRA 86 (2012)
Nowak *et al.*, New J.Phys. 16 (2014)

- Non-perturbative QFT techniques
  - 2PI effective action

Berges *et al.*, PRL 101 (2008)
Berges *et al.*, Nucl.Phys.B 813 (2009)
Scheppach *et al.*, PRA 81 (2010)

- Holography

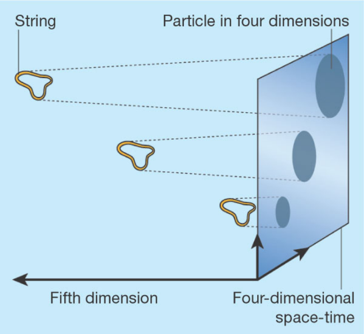
Adams *et al.*, Science 341 (2013)
Ewerz, Gasenzer, Karl, AS, arXiv:1410.3472

- . . .

Holography

Holographic principle: ['t Hooft, Susskind]

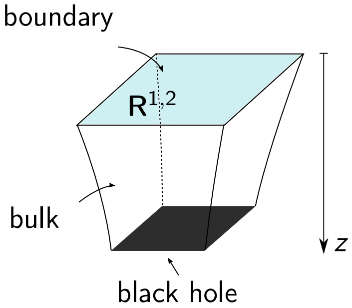
Quantum gravity	$\longleftrightarrow$	QFT
in $D+1$ dimensions		in D dimensions



Maldacena, 2003

- Concrete realization: [Maldacena, Gubser&Klebanov&Polyakov, Witten, 1997/98]
  Anti de Sitter (AdS) / Conformal Field Theory (CFT) Duality

Gauge/Gravity Duality



Bulk:

- Weakly coupled
- Fields: $\Phi, \ldots$
- Black hole mass m_{BH}

/

Boundary:

- Strongly coupled
- Operators: $\hat{\psi}, \ldots$
- Temperature $T \sim m_{\text{BH}}$

Classical gravity / *Quantum* field theory
(in certain limits)

Holographic Superfluid

Apply holography to study the non-equilibrium regime of a strongly correlated $(2+1)$D superfluid. [Adams *et al.*, Science 341 (2013)]

- Ingredients

$(2+1)$D	$(3+1)$D
thermal background @ μ, T	black hole
complex scalar field operator ψ	complex scalar field Φ
$U(1)$ conserved current j^{μ}	$U(1)$ gauge field A_M

Holographic Superfluid

Apply holography to study the non-equilibrium regime of a strongly correlated $(2+1)$D superfluid. [Adams *et al.*, Science 341 (2013)]

- Ingredients

$(2+1)$D	$(3+1)$D
thermal background @ μ, T	black hole
complex scalar field operator ψ	complex scalar field Φ
$U(1)$ conserved current j^{μ}	$U(1)$ gauge field A_M

- $(3+1)$D gravitational action

Gubser, PRD 78 (2008)
Hartnoll *et al.*, PRL 101 (2008)

$$S = \frac{1}{2\kappa}\int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{R} + \Lambda + \frac{1}{q^2}\mathcal{L}_{\text{matter}}\right)$$

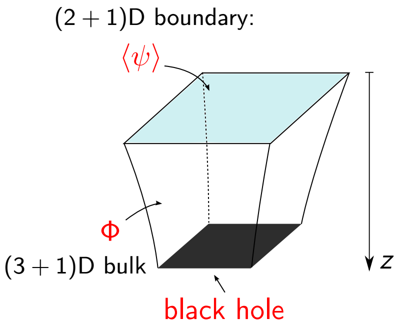
$$\mathcal{L}_{\text{matter}} = -\frac{1}{4}F_{MN}F^{MN} - |(\nabla_M - \mathrm{i}A_M)\Phi|^2 - m^2|\Phi|^2$$

Setup

Classical field equations for Φ and F^{MN} in bulk:

$$(D^2 - m^2)\,\Phi = 0$$
$$\nabla_M F^{MN} = J^N(\Phi)$$

encode *quantum* behavior on boundary.



- Near-boundary behavior of fields crucial

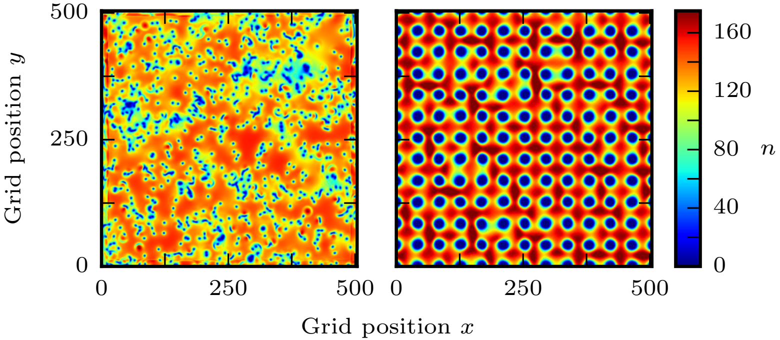
$$\Phi(t,\vec{x},z) = \eta(t,\vec{x})\,z + \langle\psi(t,\vec{x})\rangle\,z^2 + \mathcal{O}(z^3)$$

- Black hole
  $\longleftrightarrow$ Thermal normal component
  - Dissipation naturally built in
  - Use static black hole

Simulations

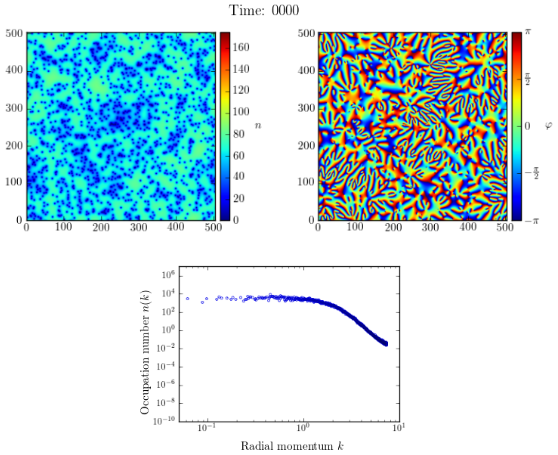
- Put on periodic grid, use spectral methods to solve
- Far-from-equilibrium initial conditions: vortices
- 6 initial conditions (10 runs each)

random distribution	2×144	2×432	2×720
vortex lattice (12×12)	± 2	± 6	± 10
# elementary vortices	288	864	1440



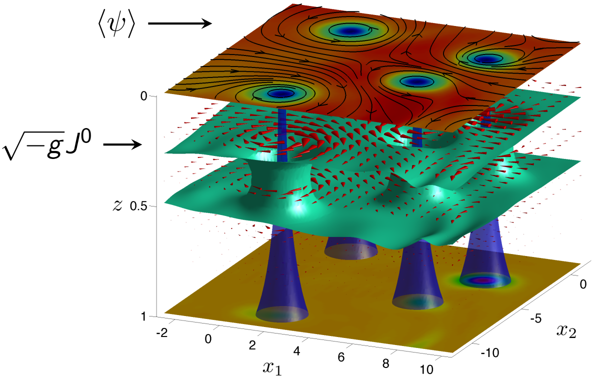
Sample Run

Movies and more on www.thphys.uni-heidelberg.de/holographic-superfluid



- Large grids: 504×504, 32 points in holographic direction
- Long propagation times: $t_{\text{final}} = 4000$

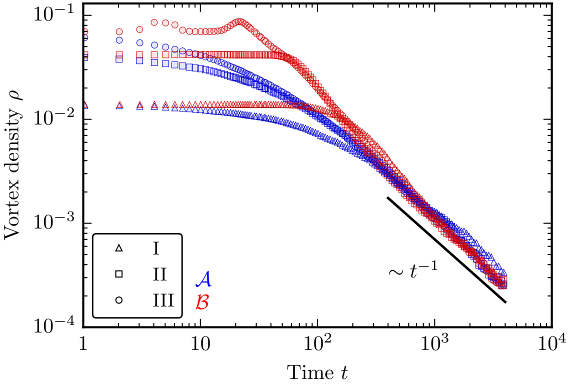
Bulk Realization of Superfluid



Adams *et al.*, Science 341 (2013)

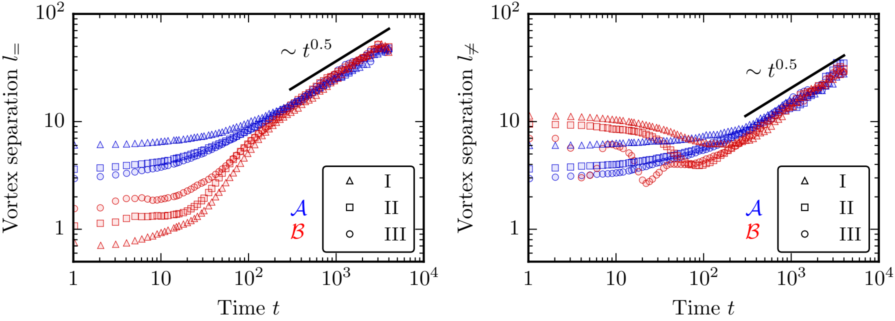
- Energy flux only significant in vortex cores and vortex annihilation events ⇒ UV dissipation only

Vortex Density



- Count vortices $\Rightarrow$ density
- Universal late-time, $t \gtrsim 400$, power law $\rho(t) \sim t^{-1}$

Vortex Separations



- Track (anti-)vortex positions
- Universal late-time power law, $\rho(t) \sim t^{1/2}$
  - Diffusive behavior
  - Comparison to Gross–Pitaevskii models?

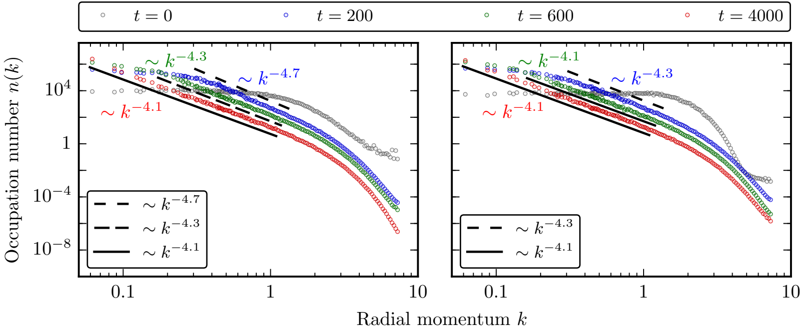
Turbulence: Kolmogorov Scaling

- Radial occupation number spectrum

$$n(k) = \int \frac{d\Omega_k}{2\pi} \langle \psi^*(\vec{k})\psi(\vec{k}) \rangle$$

- Radial kinetic energy spectrum

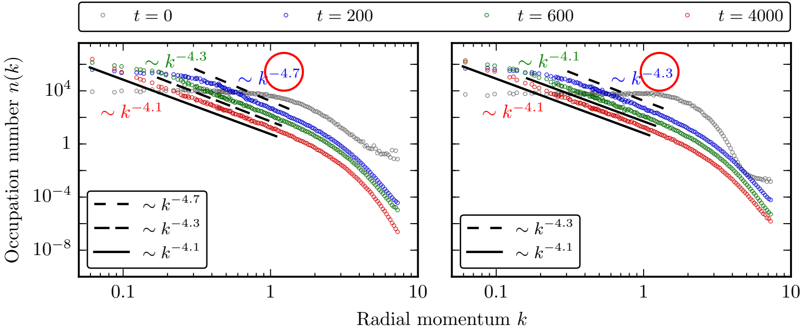
$$E(k) = k^3 n(k)$$



Turbulence: Kolmogorov Scaling

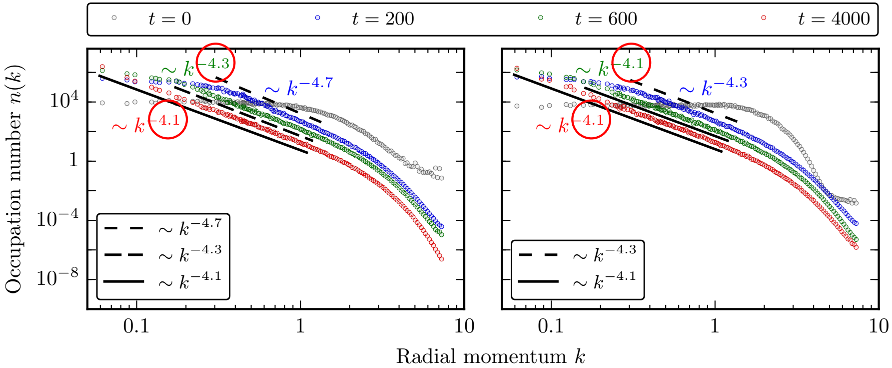
- Radial kinetic energy spectrum

$$E(k) = k^3 n(k)$$



- Early/intermediate times:
  Kolmogorov-type scaling $E(k) \sim k^{-5/3}$ [cf. Adams *et al.*, Science 341 (2013)]
- Observe only for random initial distributions of vortices; transient

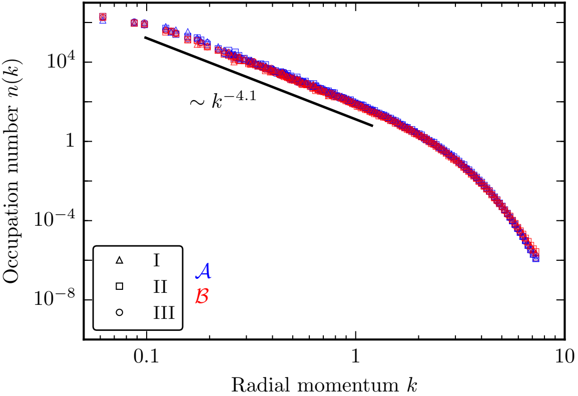
Turbulence: Late-Time Universal Behavior

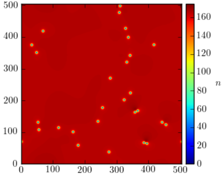


- Late times, $t \gtrsim 600$: power law $n(k) \sim k^{-\zeta}$ with $4.1 \lesssim \zeta \lesssim 4.3$ for *all initial conditions*
- Dilute gas of uncorrelated vortices, expect $n \sim k^{-4}$ in GPE

[Nowak *et al.*, PRA 85 (2012)]

Turbulence: Late-Time Universal Behavior





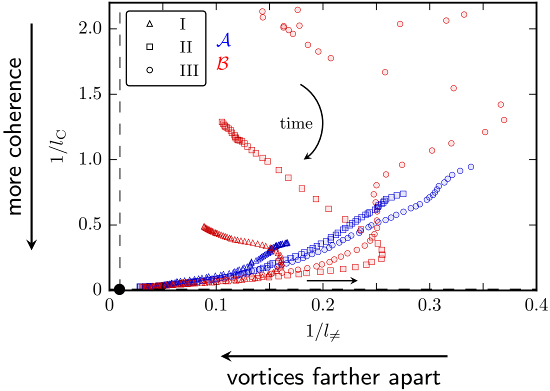
- Spectra for all 6 initial conditions ⇒ one universal spectrum

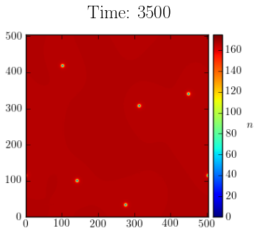
Fixed-Point Dynamics

Movies and more on www.thphys.uni-heidelberg.de/holographic-superfluid

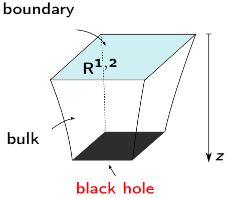
- Late-time scaling in *time* and *space*
- Non-thermal fixed point
  - Few vortices, far apart
  - On maximally coherent background
  - Evolution slowing down near NTFP

cf. ultracold Bose gas: Nowak *et al.*, PRB 84(R) (2011)
Nowak *et al.*, PRA 85 (2012)





Summary



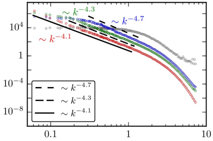
Non-perturbative access beyond mean-field

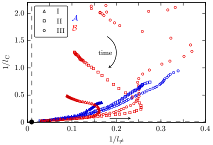
Strong correlations built in

Real-time dynamics

Kolmogorov scaling

Universal behavior



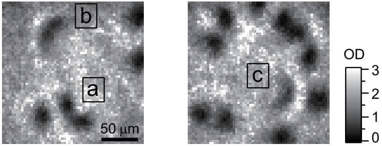


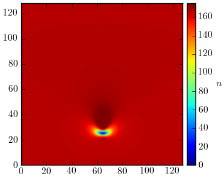
Late-time universal regime

Non-thermal fixed point

Outlook

[Kwon *et al.*, arXiv:1403.4658]





Comparison with experiments

Comparison with semi-classical methods (GPE)

Role of dissipation

Fluctuations

Dynamic normal component (dynamic metric)

Non-equilibrium in strongly coupled plasmas (similar methods)