Methods in Amplitude Analysis

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PANDA Collaboration meeting
12 September 2014, Frascati (Italy)

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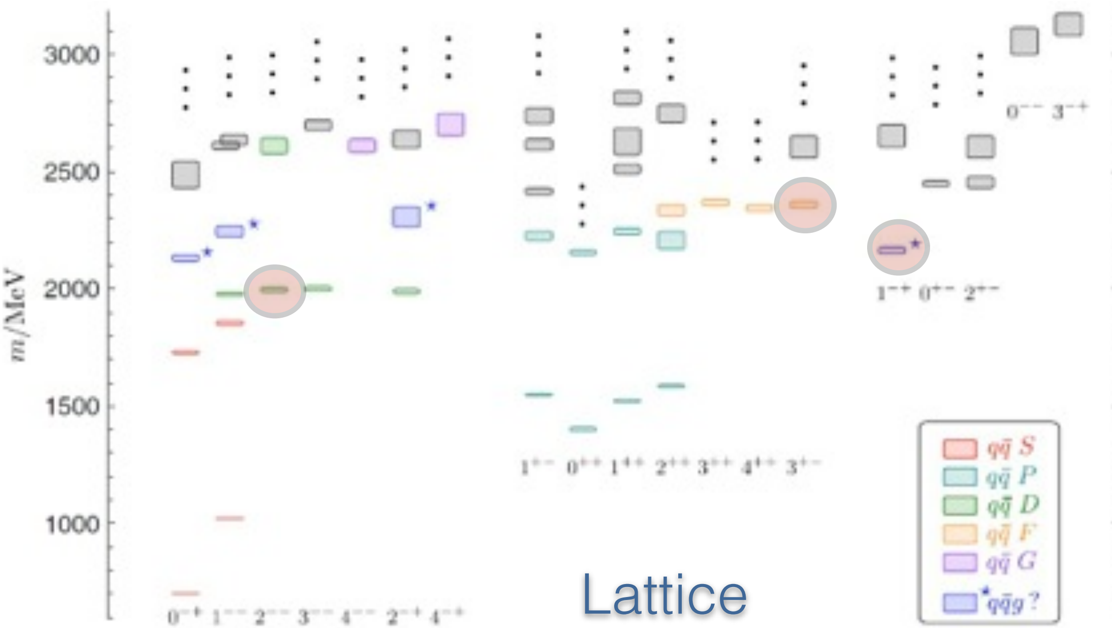
- Summary

Motivation

Aim to:

Complete understanding of the hadron spectrum and discover new resonances



Jozef J. Dudek
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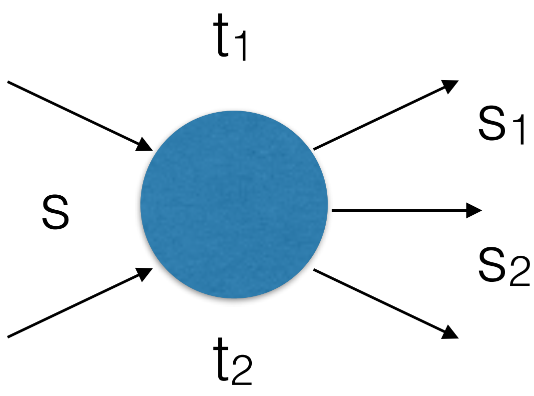
$n^{2s+1}\ell_J$	J^{PC}	$I=1$	$I=1/2$	$I=0$	$I=0$	EXD
1^1S_0	0^{-+}	π	K	η	η'	R2
1^3S_0	1^{--}	$\rho(770)$	$K^*(982)$	$\omega(782)$	$\phi(1020)$	R1
1^1P_1	1^{+-}	$b_1(1235)$	$K_1(1400)$	$h_1(1170)$	$h_1(1380)$	R2
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1370)$	$f_0(1710)$	R4
1^3P_1	1^{++}	$a_1(1260)$	$K_1(1270)$	$f_1(1285)$	$f_1(1420)$	R3
1^3P_2	2^{++}	$a_2(1320)$	$K_2^{**}(1430)$	$f_2(1270)$	$f_2'(1525)$	R1
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	R2
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$	$\omega(1650)$		R4
1^3D_2	2^{--}	★	$K_2^*(1820)$			R3
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	R1
1^1F_3	3^{+-}	★				R2
1^3F_2	2^{++}		$K_2^*(1980)$	$f_2(1910)$	$f_2(2010)$	R4
1^3F_3	3^{++}		$K_3(2320)$			R3
1^3F_4	4^{++}	$a_4(2040)$	$K_4^{**}(2045)$		$f_4(2050)$	R1

JPAC:

Provide theoretical support needed to analyze the data

Motivation



$$d^5\sigma \sim |A(s, s_1, s_2, t_1, t_2, \{\lambda\}|^2$$

- Physics of interest resides in A evaluated at values of kinematical variables outside the experimentally accessible region

In Amplitude analysis a model of A is constructed, fitted to data and continued to regions of interest

Collaboration strategy

theory

experiment

```
      double complex function A(gamma,target,recoil,pip,pim,
                               ,lambda_g,lambda_t,lambda_r,
                                params)
      implicit double precision (a-h,o-z)
      dimension gamma(4)
      dimension target(4)
      dimension recoil(4)
      dimension pip(4),pim(4)
      dimension params(100)

       double complex Ampl

      s = (gamma(4)+target(4))**2 - (gamma(1)+target(1))**2
     $    - (gamma(2)+target(2))**2 - (gamma(3)+target(3))**2

      s1 = (pip(4)+pim(4))**2 - (pip(1)+pim(1))**2
     $     - (pip(2)+pim(2))**2 - (pip(3)+pim(3))**2

      s2 = (pip(4)+recoil(4))**2 - (pip(1)+recoil(1))**2
     $     - (pip(2)+recoil(2))**2 - (pip(3)+recoil(3))**2

      t1 = (gamma(4)-pim(4))**2 - (gamma(1)-pim(1))**2
     $     - (gamma(2)-pim(2))**2 - (gamma(3)-pim(3))**2

      t1 = (target(4)-recoil(4))**2 - (target(1)-recoil(1))**2
     $     - (target(2)-recoil(2))**2 - (target(3)-recoil(3))**2

      call Ath(s,s1,s2,t1,t2,lambda_g,lambda_t,lambda_r,params,Ampl)

      A = Ampl

      return
      end
```

Name	Last modified	Size	Description
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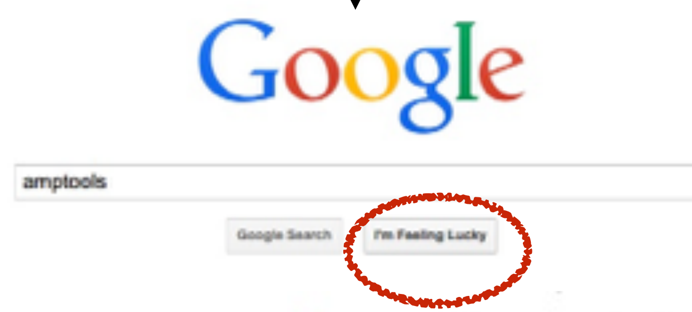
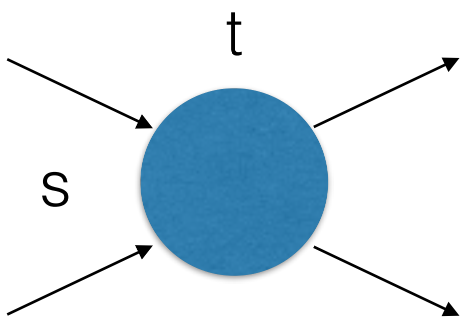
g12_mc_gen.txt.gz	05-Aug-2014 12:13	385M	
g12_mc_rec.txt.gz	05-Aug-2014 12:13	82M	



AmpTools
Brought to you by: mashephe

Amplitude analysis vs p.w. Amplitude analysis



$$A(s,t,\{\lambda\}) = \sum_J^{\infty} (2J+1)\, d^J_{\mu,\nu}(\theta_s)\, f^J(s,\{\lambda\})$$

$$\mu = \lambda_1 - \lambda_2, \quad \nu = \bar{\lambda}_1 - \bar{\lambda}_2$$

- A(s,t,{λ}): amplitude expressed in terms of kinematical variables

↓

Enter comparison with data

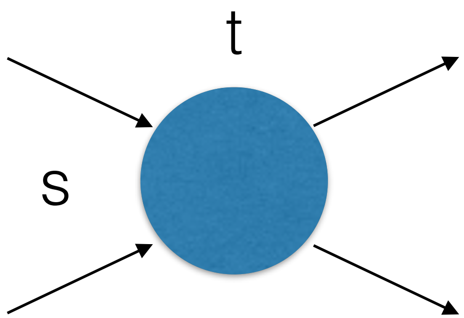
- Partial Wave Amplitudes: decomposition in terms of rotational functions

↓

These "diagonalize unitarity" and contain resonance information

Entire dynamical information that does not depend on the underlying theory (e.g. QCD) comes from **unitarity**

Unitarity defines singularities of partial waves



$$A(s,t) = \sum_J^{\infty} (2J+1)\, P_J(z)\, f_J(s)$$

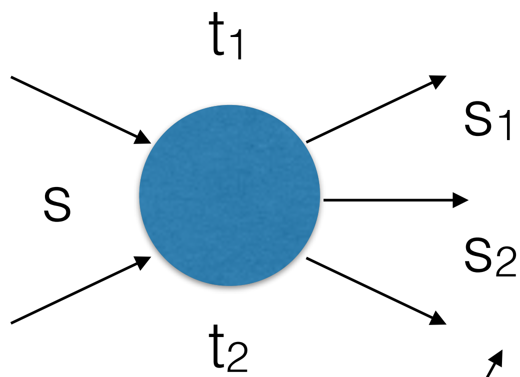
$$\text{Disc}\, f_J(s) = \rho(s)\, f_J(s+)\, f_J(s-)$$

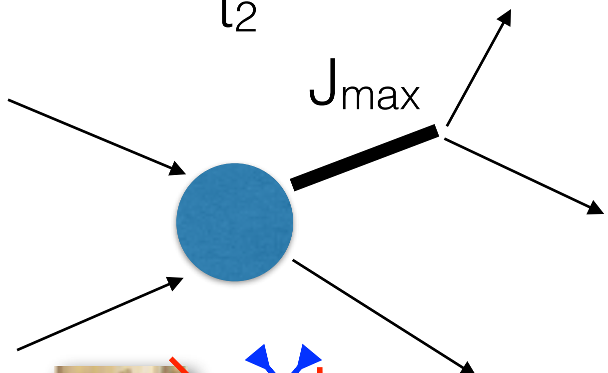
- For small s, s-channel unitarily is “simple”
- Isobar model = truncate the partial waves: $\sum_J^{\infty} \rightarrow \sum_J^{J_{max}}$
  Isobars = partial waves

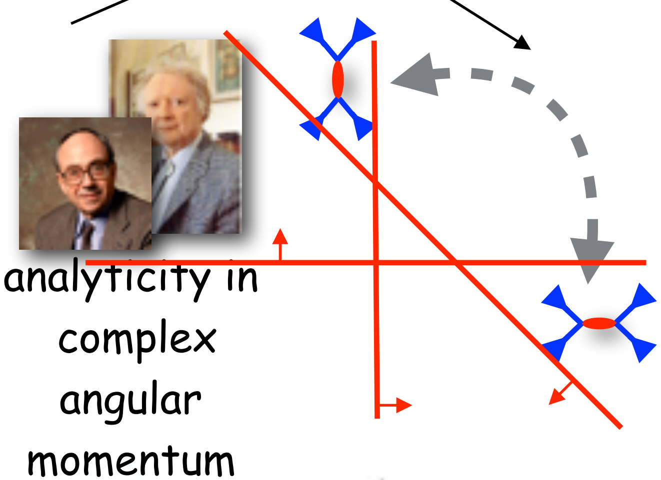
When is this a bad thing to do?

- For large-s, s-channel unitarity is hopeless. It is the low-l t-channel p.w. which become relevant (Regge physics).

Truncated partial wave series



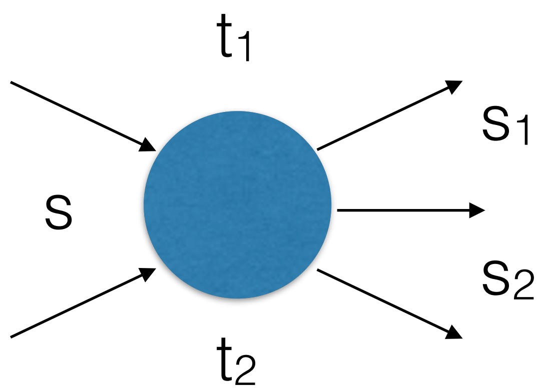




$$A = \sum_{J_1, J_2, \lambda} d^{J_2}_{\lambda_b - \lambda_t, \lambda - \lambda_r}(\theta_2)\, d^{J_1}_{\lambda 0}(\theta_1)\, e^{i\lambda\phi_1} f_{J_1, J_2, \lambda}(s_1, s)$$

- Suppose the s_1 series is truncated $\sum_{J_1}^{\infty} \to \sum_{J_1}^{J_{max}}$
- Then $A \sim s_2^{J_{max}}$ becomes “wild” for high energies
- The correct behavior $A \sim s_2^{\alpha} < s_2$ can only emerge if $J_{max} = \infty$
- The “machinery” to account for the contribution to infinite number of terms from cross-channel exchanges is due to Regge and Mandelstam

Isobar model

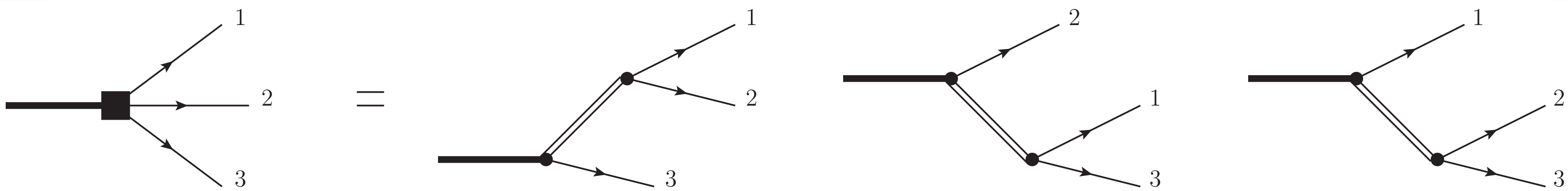


$$A = \sum_{J_1, J_2, \lambda} d^{J_2}_{\lambda_b - \lambda_t, \lambda - \lambda_r}(\theta_2)\, d^{J_1}_{\lambda 0}(\theta_1)\, e^{i\lambda\phi_1}\, f_{J_1, J_2, \lambda}(s_1, s)$$

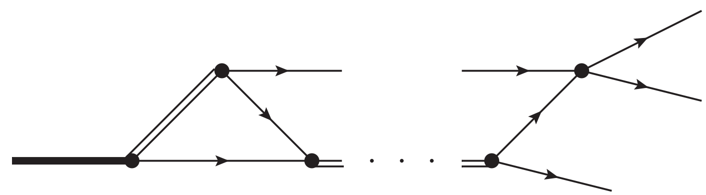
If all s_1,s_2,s are small it is OK to truncate

ω/ϕ→3π

$$A(s,t) = \sum_J^{J_{max}} (2J+1)\, d^J_{1,0}(\theta_s)\, f_J(s) + \sum_J^{J_{max}} (2J+1)\, d^J_{1,0}(\theta_t)\, f_J(t) + \sum_J^{J_{max}} (2J+1)\, d^J_{1,0}(\theta_u)\, f_J(u)$$



+ unitarity



Unitarity relation for the p-wave F(s):

$$\text{Disc}\, F(s) = \rho(s)\, \overset{\pi\pi \to \pi\pi}{t^*(s)} \Big(F(s) + \hat{F}(s)\Big)$$

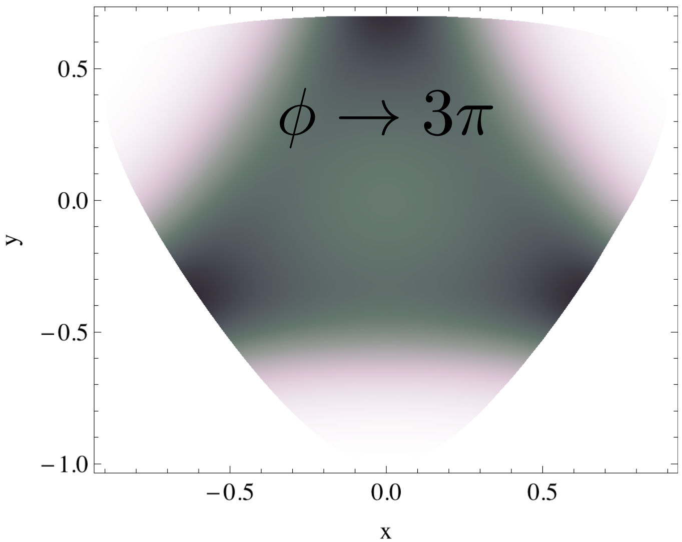
$$\hat{F}(s) = 3 \int_{-1}^{+1} \frac{dz_s}{2}\, (1 - z_s^2)\, F(t)$$

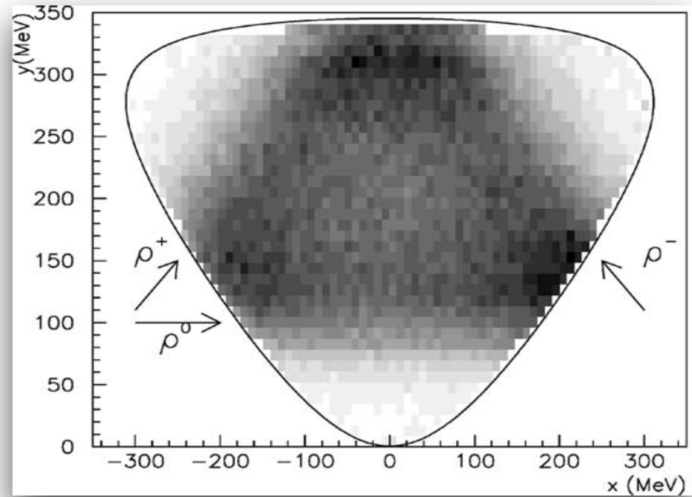
↓

dispersive relation

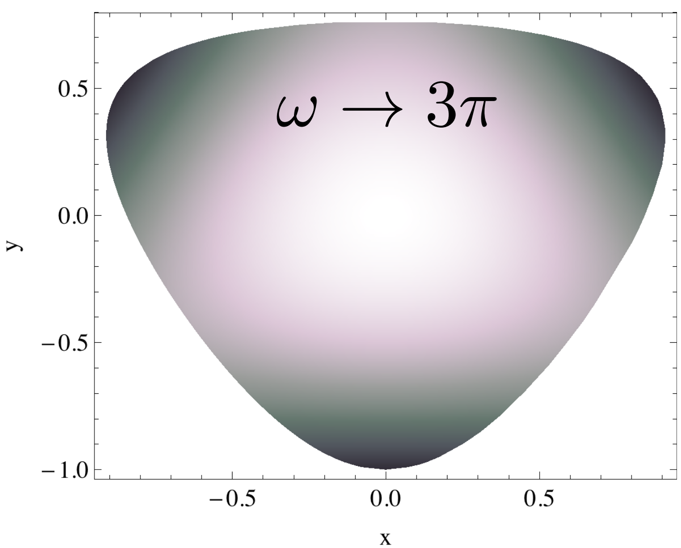
Khuri, Treiman 1960
Aitchison 1977

Dalitz plots





KLOE (2003)



- Only one parameter (overall normalization) → fixed from $\Gamma_{exp}(\omega/\phi \to 3\pi)$
- **ϕ→3π**: distribution clearly shows ρ-meson resonances
- **ω→3π**: distribution is relatively flat
- Upcoming data from CLAS g12, KLOE, WASA, etc.

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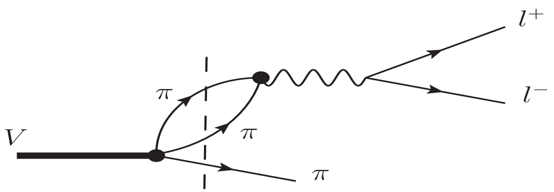
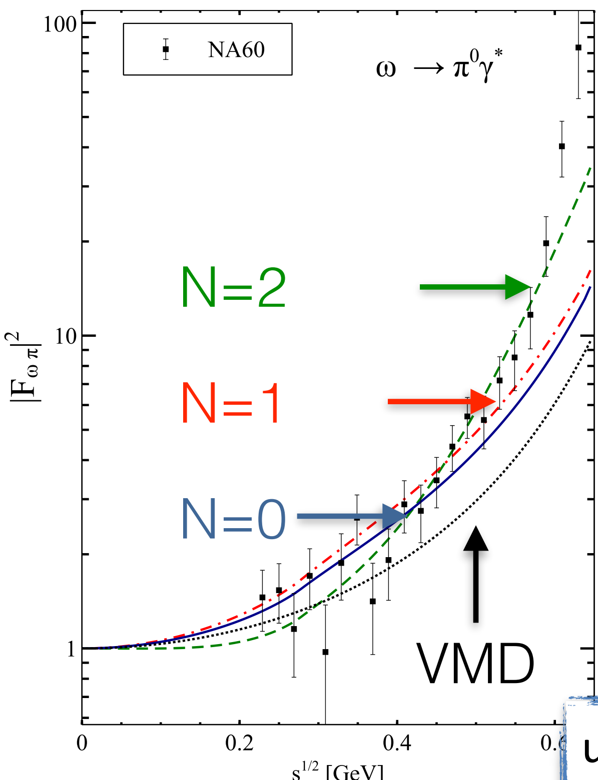
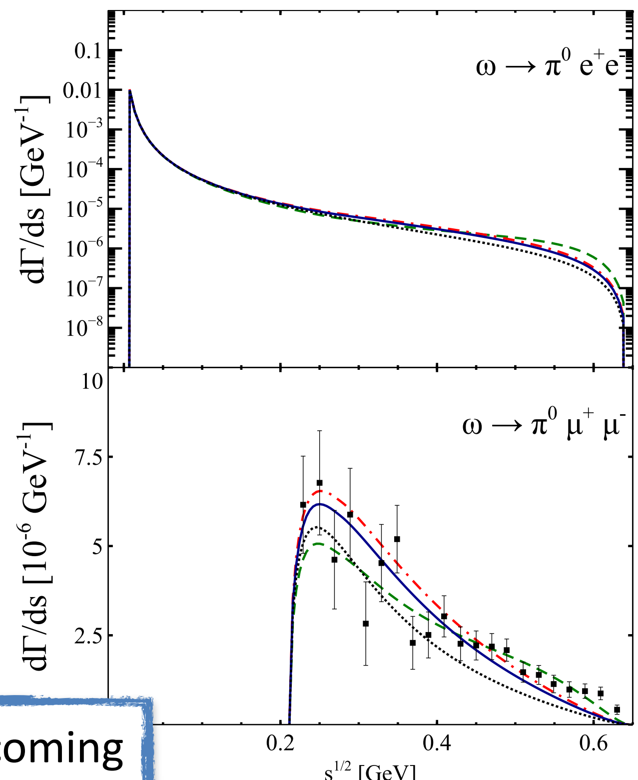
ω→π⁰γ*



$$f_{V\pi}(s) = \int_{s_\pi}^{s_i} \frac{ds'}{\pi} \frac{\text{Disc}\, f_{V\pi}(s')}{s'-s} + \sum_{i=0}^{N} C_i\, \omega(s)^i$$



upcoming CLAS g12



- C_0 fixed from $\Gamma_{exp}(\omega\to\pi\gamma)$
- Nature of the steep rise?

  1. Upcoming data from CLAS g12

  2. Exp. analysis of φ→πγ is very important

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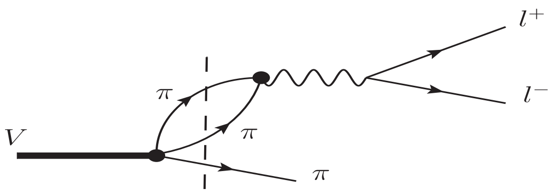
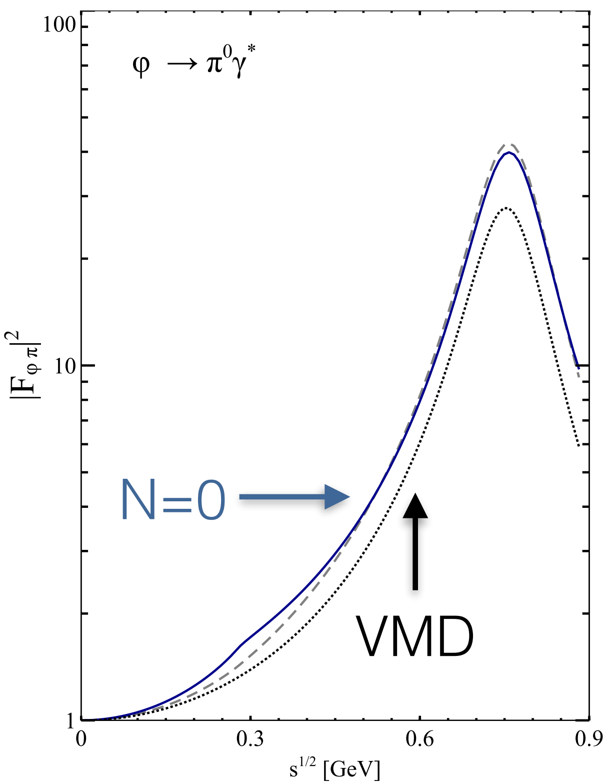
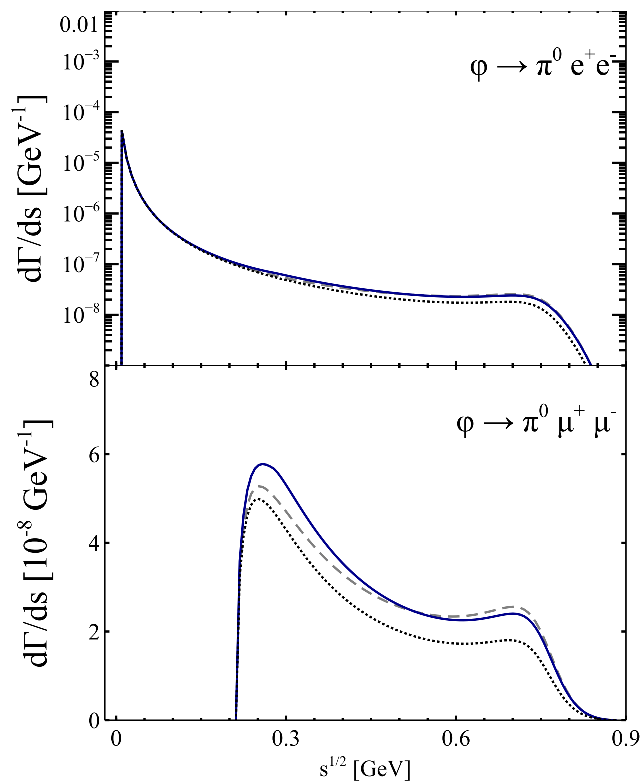
$\phi \to \pi^0 \gamma^*$



$$f_{V\pi}(s) = \int_{s_\pi}^{s_i} \frac{ds'}{\pi} \frac{\text{Disc}\, f_{V\pi}(s')}{s' - s} + \sum_{i=0}^{N} C_i\, \omega(s)^i$$





- C_0 fixed from $\Gamma_{exp}(\phi \to \pi\gamma)$
- **Grey:** no 3b effects

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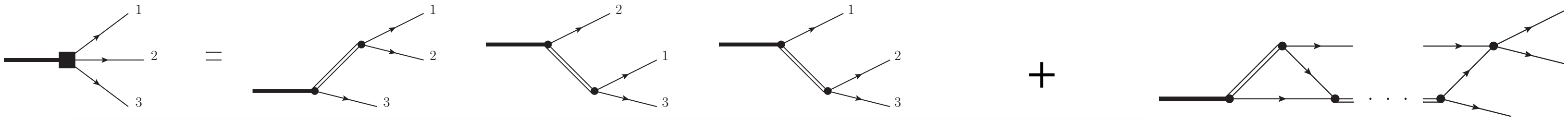
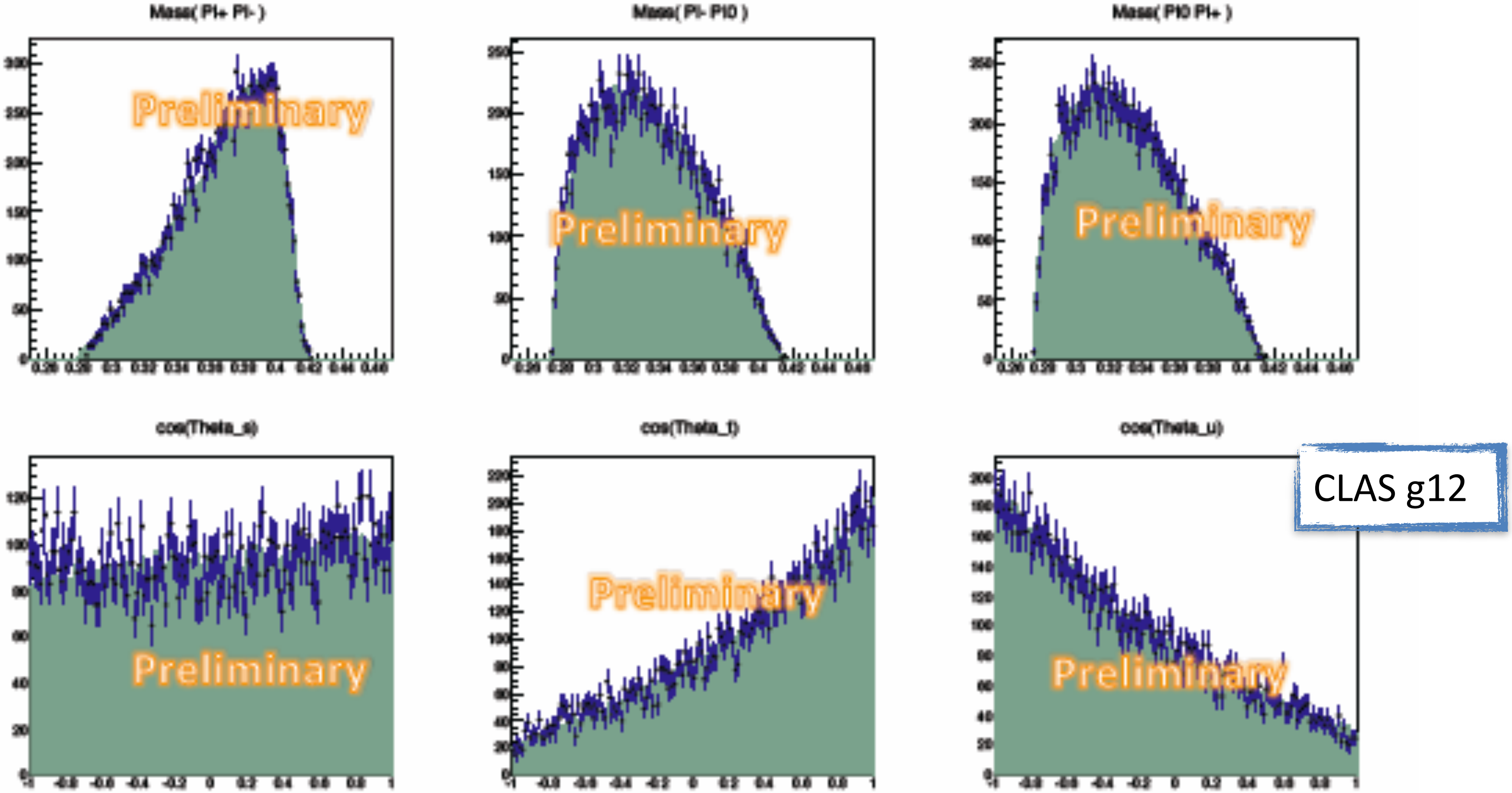
A. Szczepaniak

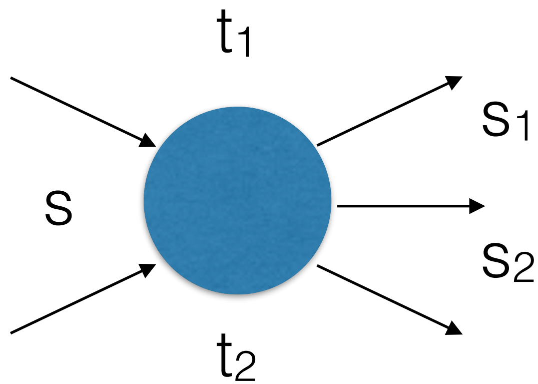
η→3π





P. Guo, D. Schott
A. Szczepaniak

Regge physics

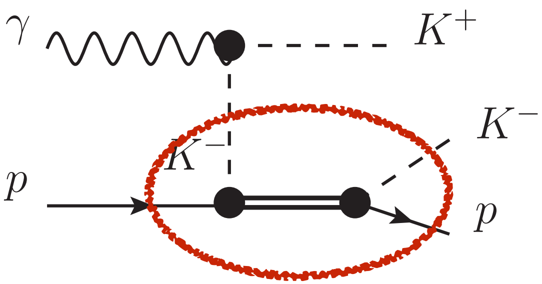


$$A = \sum_{J_1, J_2, \lambda} d^{J_2}_{\lambda_b - \lambda_t, \lambda - \lambda_r}(\theta_2)\, d^{J_1}_{\lambda 0}(\theta_1)\, e^{i\lambda\phi_1}\, f_{J_1, J_2, \lambda}(s_1, s)$$

If all s_1,s_2,s are large it is NOT OK to truncate

$\gamma p \to K^+K^-p$

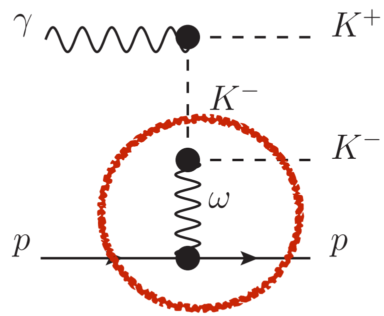
Deck model



$$K^- p \to K^- p$$

low energy fit

C. Fernandez-Ramirez,
A. Szczepaniak, M. Manley

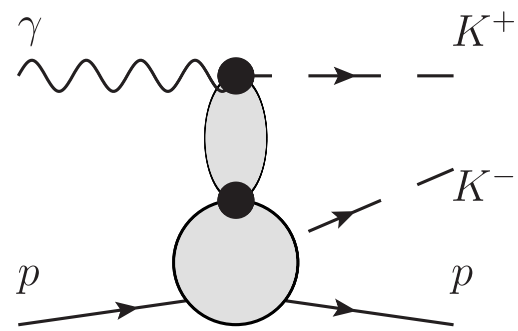


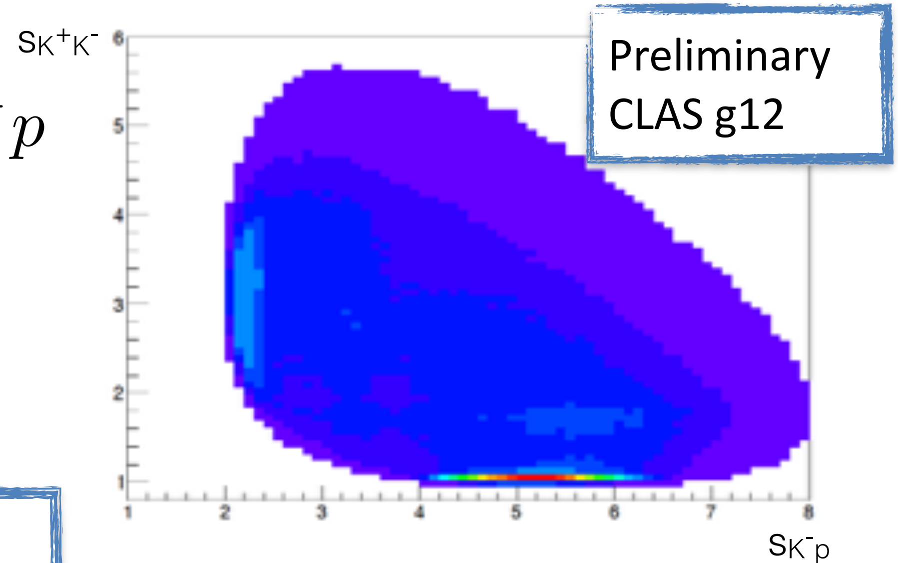
$$K^- p \to K^- p$$

High energy fit

V. Mathieu
A. Szczepaniak

B5 model

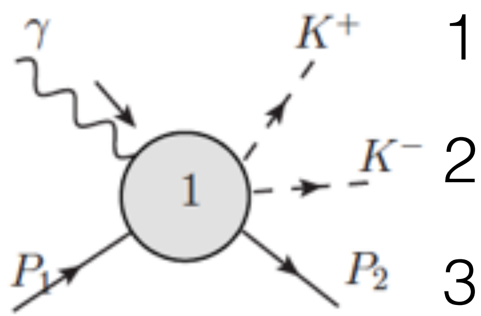


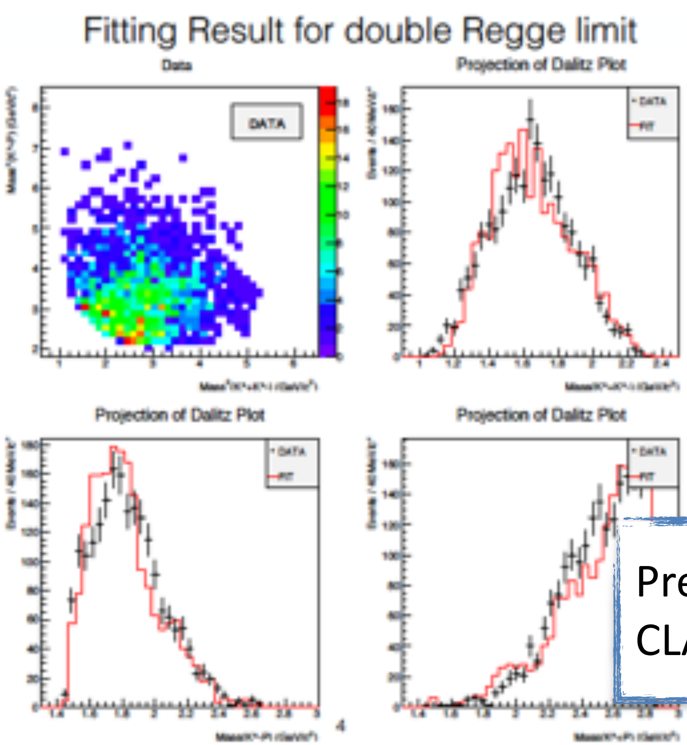


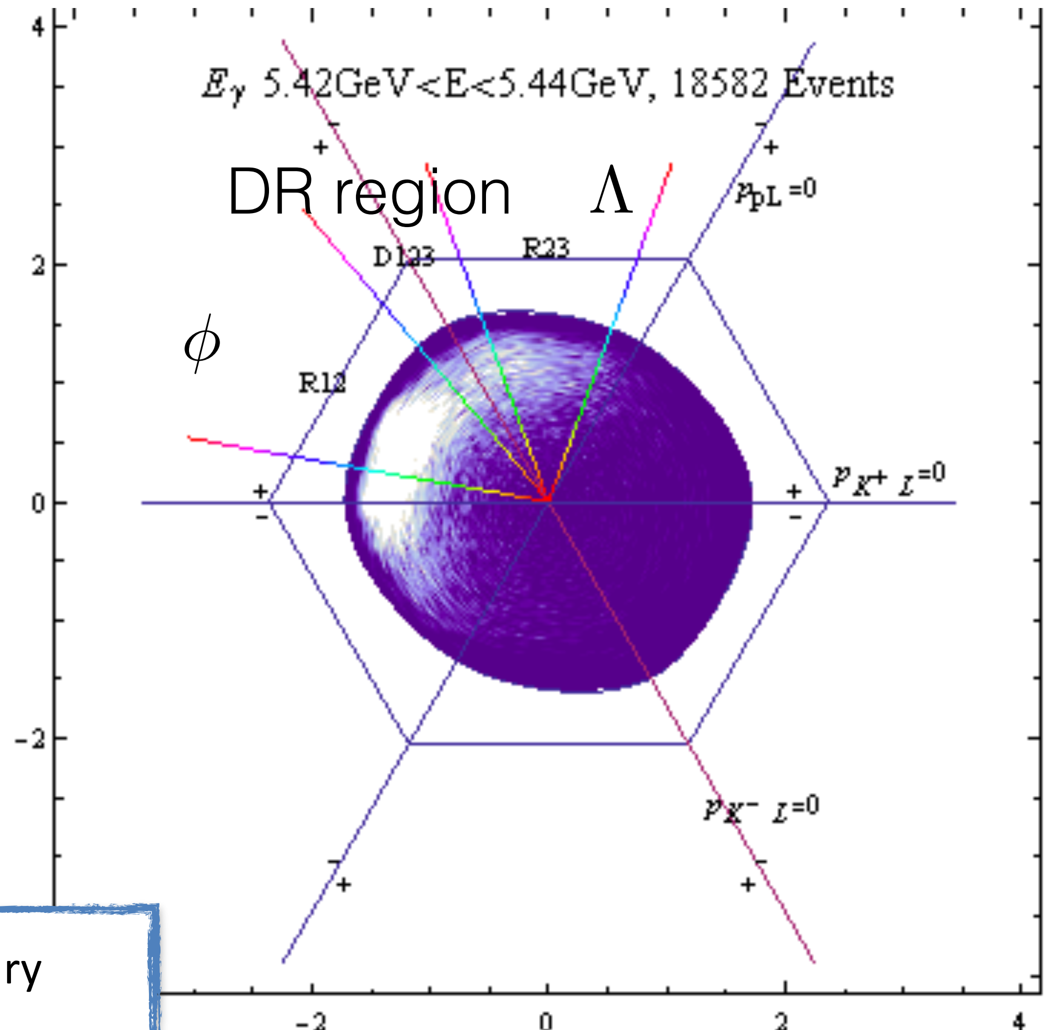
D. Schott

Analytical continuation between the two regions via dispersion relations (FESR)

$\gamma p \rightarrow K^+ K^- p$







Preliminary
CLAS g12

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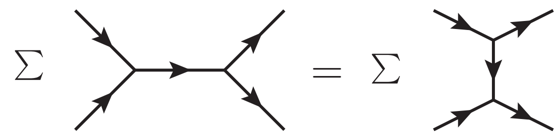
$J/\Psi \to 3\pi$

Dual model



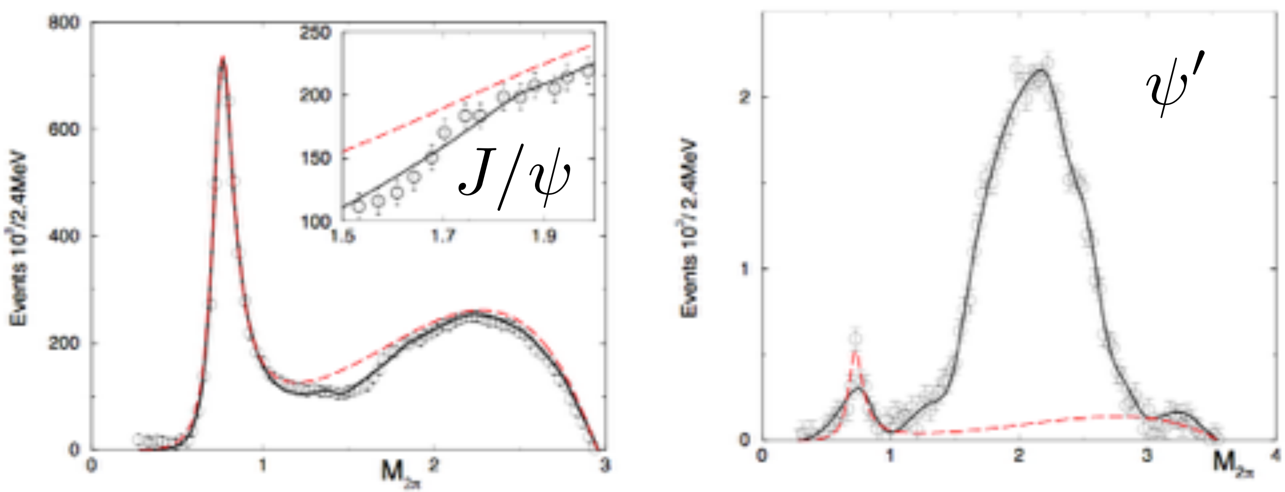
$$A(s,t) = \sum_{n,m} c_{n,m} \frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}$$

Parameters:

trajectory $\alpha(s)$

couplings $c_{n,m}$

$J/\psi, \psi' \to 3\pi$



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M. Pennington
arXiv:1403.5782

Summary

- Philosophy of JPAC (JLab Physics Analysis Center)
- Current projects

  $\omega/\phi \to 3\pi$, $\pi\gamma$; $\eta \to 3\pi$

  $\gamma p \to K^+ K^- p$; $J/\psi \to 3\pi$
- Wide range of applicability