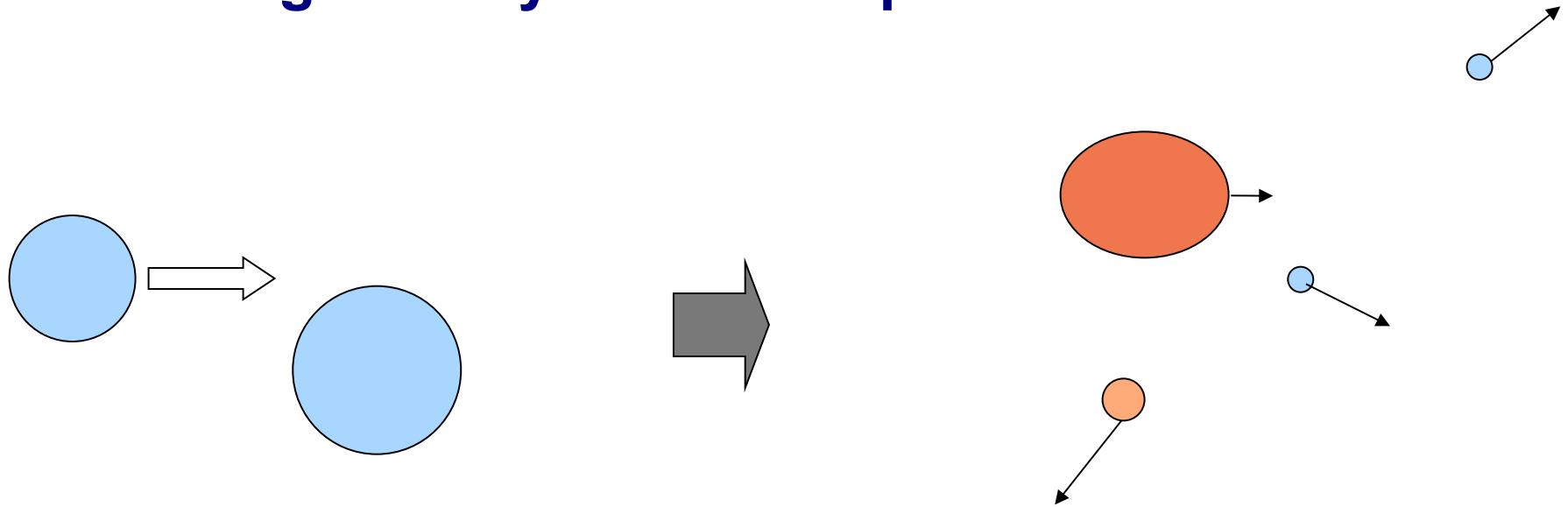


# The geometry of hot compound nuclei



T. J. Santos, B.V. Carlson – ITA, Brazil

L.C. Chamon – IF – USP, Brazil

S. R. Souza – IF - UFRGS and IF - UFRJ, Brazil

R. Donangelo – IF - UFRJ, Brazil and IF – Universidad de la República, Uruguay

NuSYM14 - Liverpool

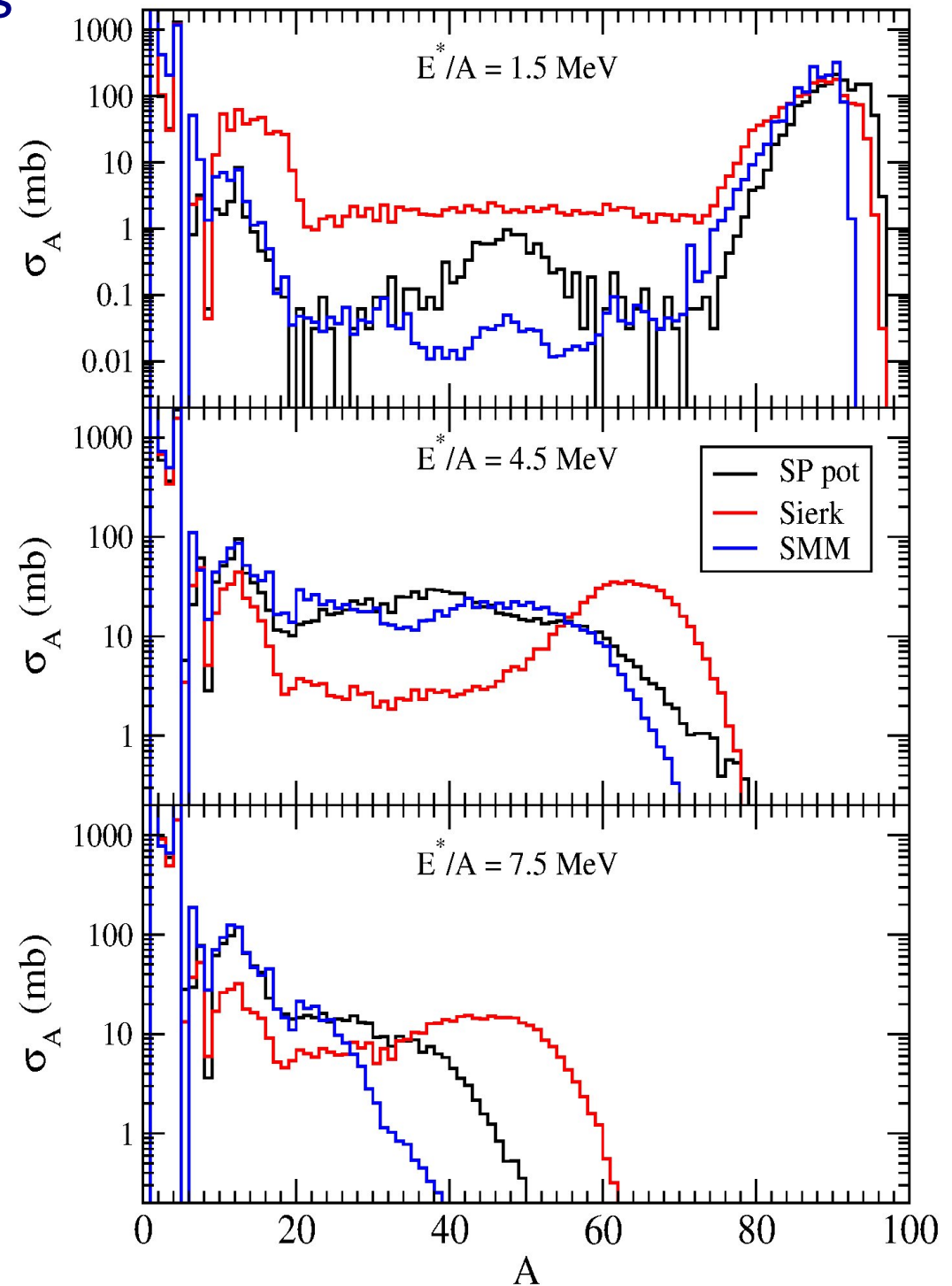
July 7 - 9, 2014

# Decay of the compound nucleus – residual nuclei

The GEMINI code with Sierk barriers has less emission of IMF fragments at intermediate and high energies (red) but describes spallation experiments fairly well.

This is mostly due to the neglect of the angular momentum effects of IMF emission, taken into account using the classical Ericson-Strutinsky formalism and SP potential barriers (black).

The SMM uses a volume 2-3 times larger than the ground state volume but describes much HI data fairly well.



# Formation of the compound nucleus - Equilibration

The compound nucleus is assumed to be in equilibrium. We estimate the local equilibration time in terms of the typical energy width of a shell-model state,

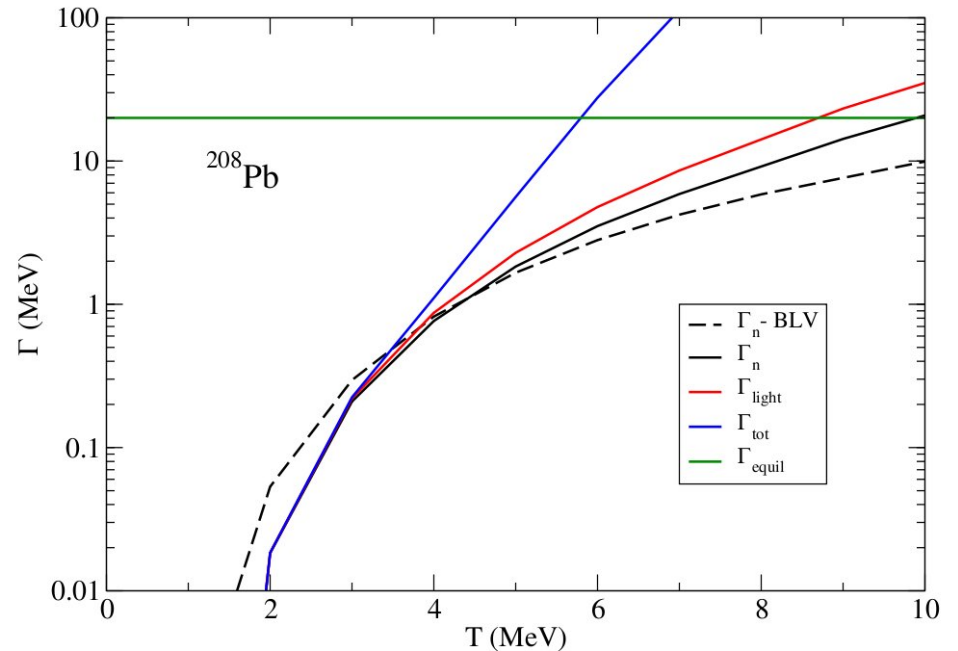
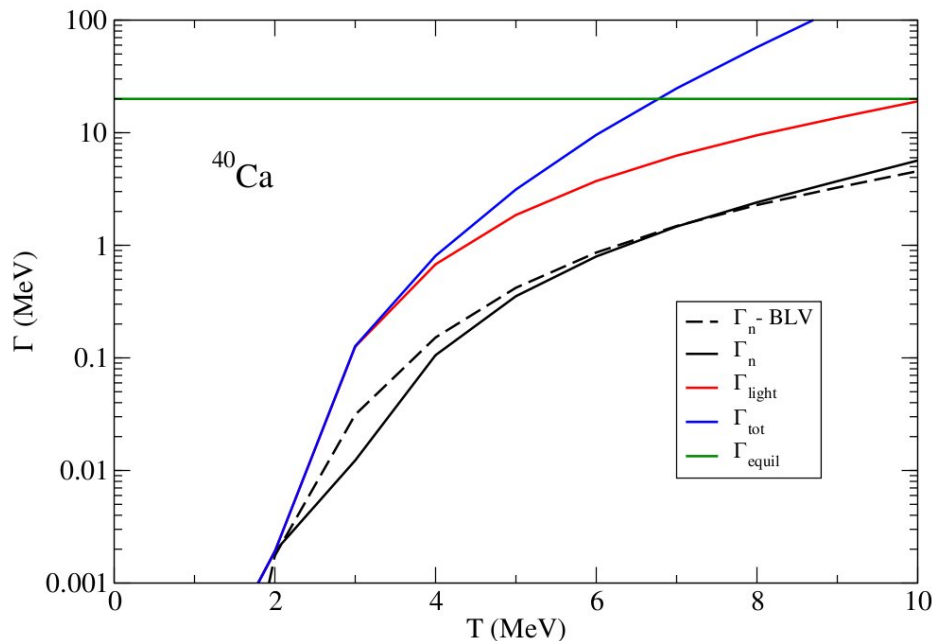
$$\hbar/\tau_{eq} = \Gamma_{eq} \approx 20 \text{ MeV} \quad \text{N. Frazier, B.A. Brown, V. Zelevinsky, Phys. Rev. C54 (1996) 1665.}$$

This is about the time needed for light to cross a nucleus of mass 64,

$$\hbar c/\Gamma_{eq} \approx 10 \text{ fm} \approx 2 * 1.25 * 64^{1/3} \text{ fm}$$

We estimate the decay width of a nucleus as the sum of its partial statistical decay widths.

$$\Gamma_W = \frac{g\mu}{\pi^2 \hbar^2} \int e \sigma_{inv}(e) \frac{\rho_f(\epsilon_0 - Q - e, 0)}{\rho_{cn}(\epsilon_0, 0)} de$$



# Caloric curves

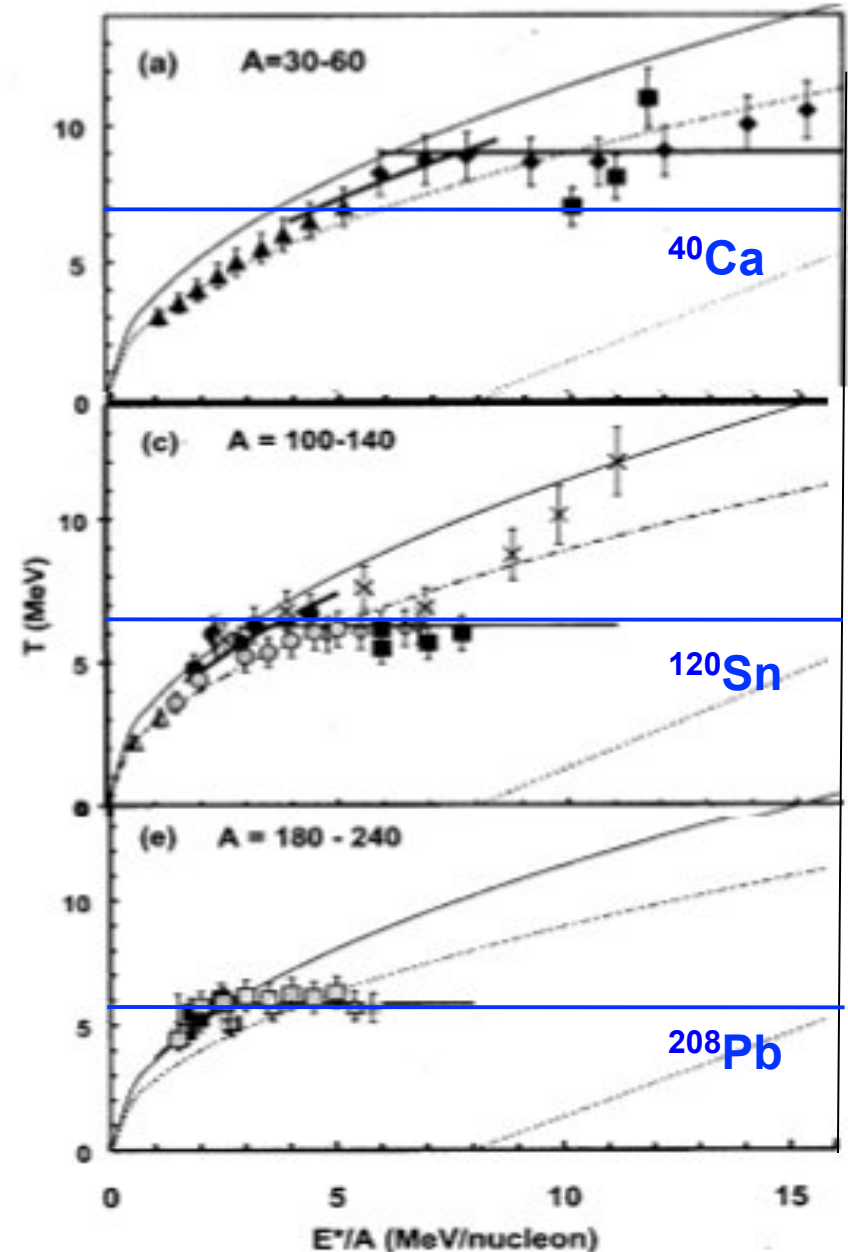
The caloric curve – the relation between

- the effective temperature of emitted particles and
- the energy deposited in the composite system.

It is reconstructed from measurements of the emitted particles.

Except for the lowest mass range, the saturation is consistent with the limiting temperature for equilibrium – the limit for formation of the compound nucleus.

Are these then non-equilibrium reactions?



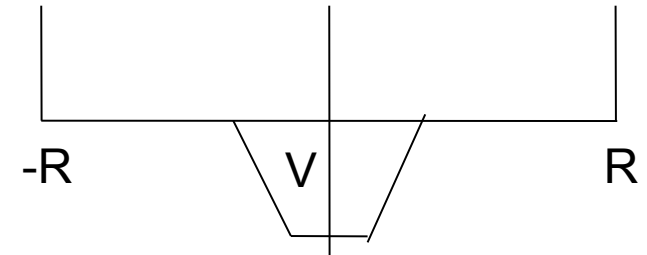
## Bound and continuum states

Consider scattering from a potential  $V(x)$  in a 1-D box  $[-R, R]$  with the condition that the wave function is zero on the edges of the box.

For a positive energy continuum state, this implies that

$$2kR + \delta(E) = n\pi$$

where  $\delta(E)$  is the phase shift due to scattering from the potential.



The density of continuum single-particle states is

$$\rho(E) = \frac{dn}{dE} = \frac{2R}{\pi} \frac{dk}{dE} + \frac{1}{\pi} \frac{d\delta}{dE}$$

so that the thermodynamic potential can be written as the sum of three contributions

$$\begin{aligned} \Omega &= T \sum_i \ln(1 - n_i) \\ &\rightarrow T \sum_{i \in b} \ln(1 - n_i) + \frac{T}{\pi} \int_0^\infty \ln(1 - n(E)) \frac{d\delta}{dE} dE + 2R \frac{T}{\pi} \int_0^\infty \ln(1 - n(E)) \frac{dk}{dE} dE \end{aligned}$$

Subtracting the infinite continuum contribution, we take

$$\Delta\Omega(T, \mu) = \Omega(T, \mu, V) - \Omega(T, \mu, V = 0)$$

# Nuclear densities

Two self-consistent calculations are performed  
– for the nucleus + gas and for the gas.

The chemical potentials are such that  $Z$  and  $A$  correspond to  $\rho_{\text{NG}} - \rho_{\text{G}}$ .

RMF calculations w/pairing

- Harmonic oscillator basis – 30 major shells;
- NL3 and DDME1 parameter sets;

Skyrme Thomas-Fermi calculations

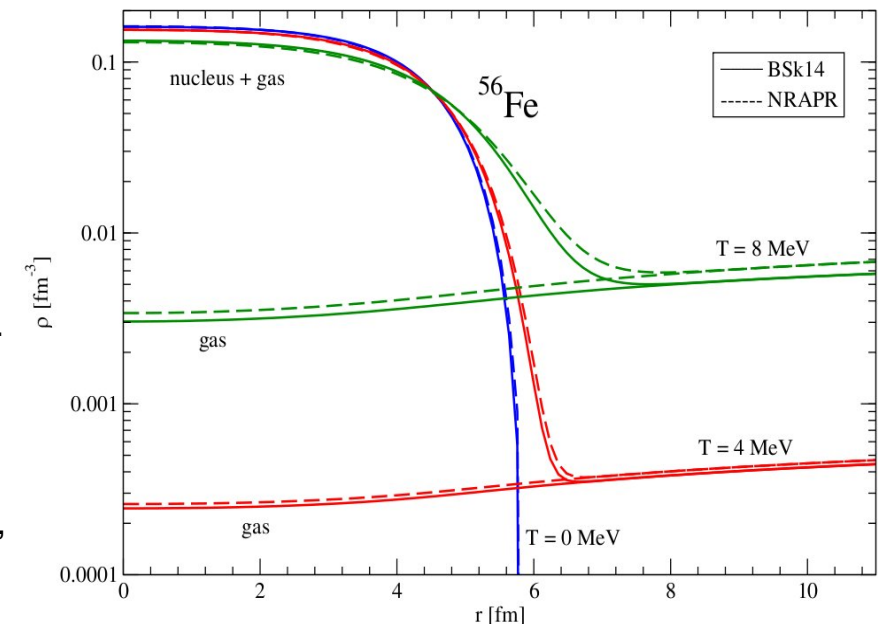
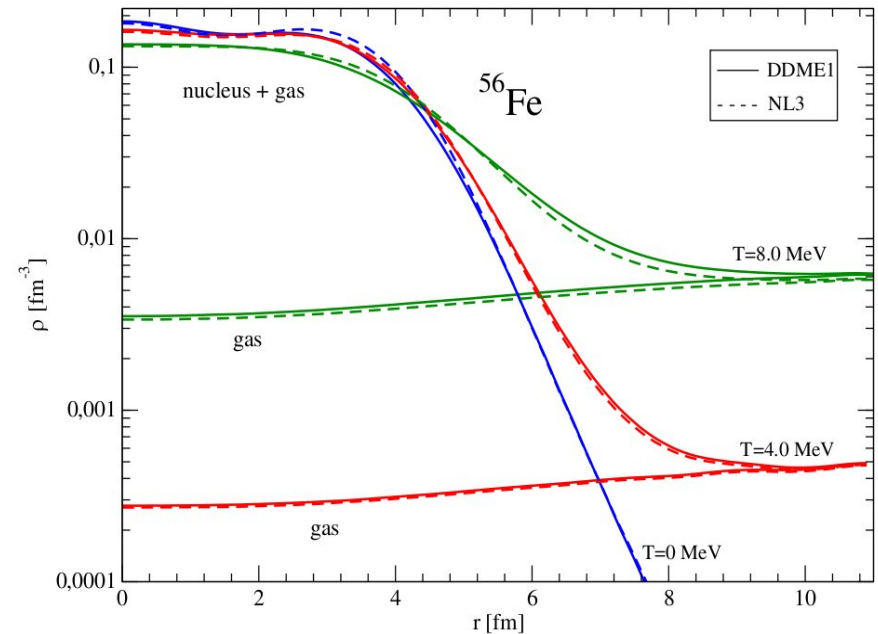
- Regular grid in a 1-D box;
- BSk14 and NPAPR parameter sets

NL3 – G.A. Lalazissis, J. König and P. Ring, Phys.Rev. C 55, 540 (1997).

DD-ME1 – T. Niksic, D. Vretenar, P. Finelli, and P. Ring, Phys. Rev. C 66, 024306 (2002).

BSk14 - S. Goriely, M. Samyn, J. Pearson, Phys. Rev. C 75, 064312 (2007).

NPAPR - A. W. Steiner, M. Prakash, J. M. Lattimer, P. J. Ellis, Phys. Rep. 411, 325 (2005)



## Rms radii

The BLV radii diverge between 9 and 11 MeV, depending on the interaction.

Without Coulomb, the BLV radii diverge at about 12 MeV.

The BLV matter radii are well fit at  $T < 6$  MeV by

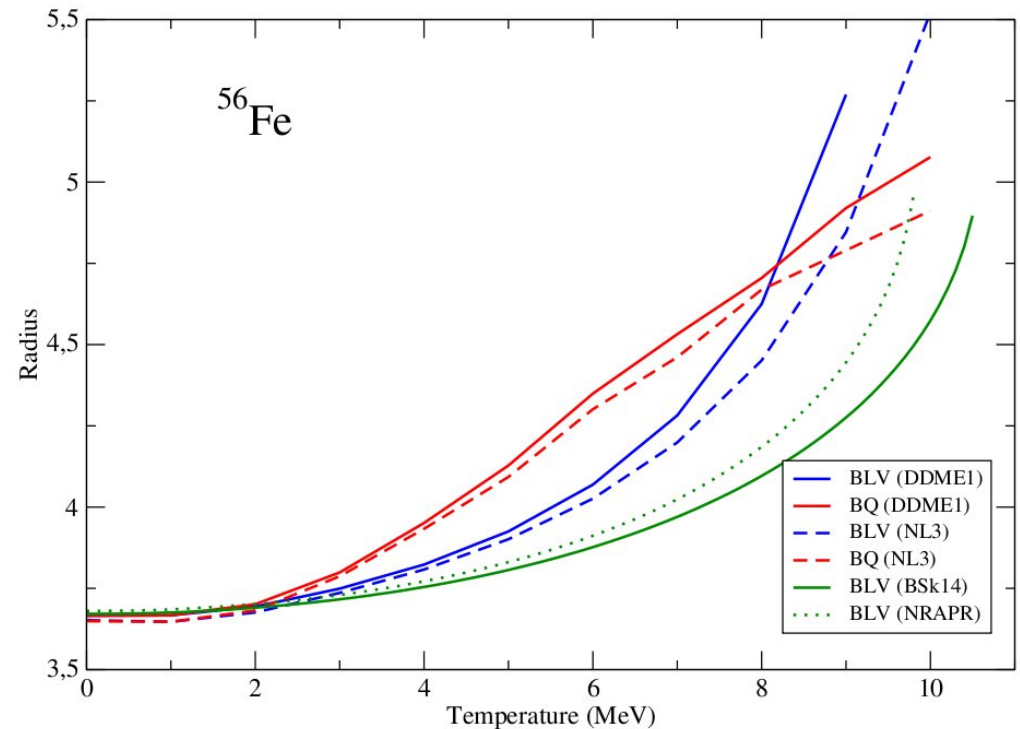
$$\langle r_m^2 \rangle = r_{m0}^2 A^{2/3} (1 + c_m T^2)$$

with

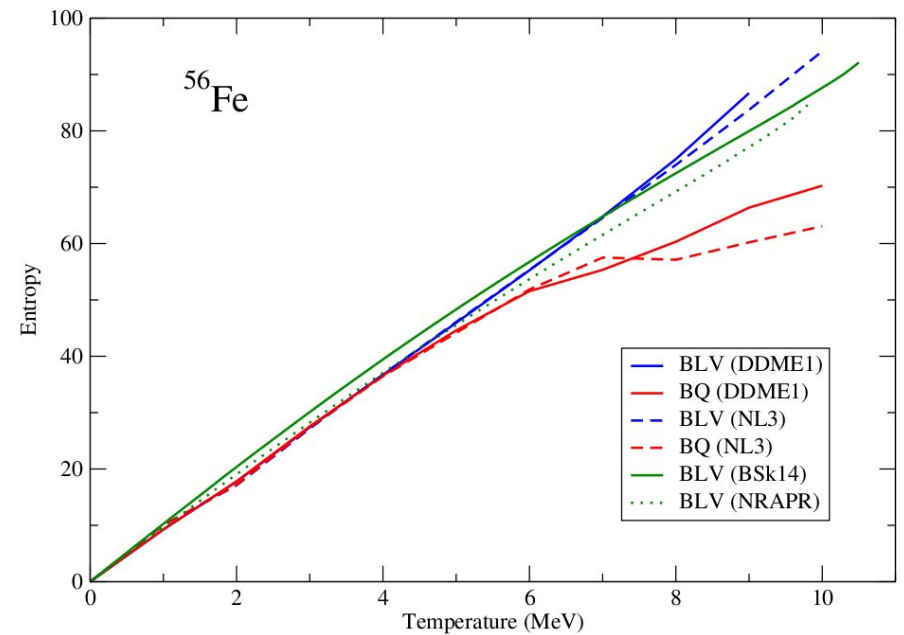
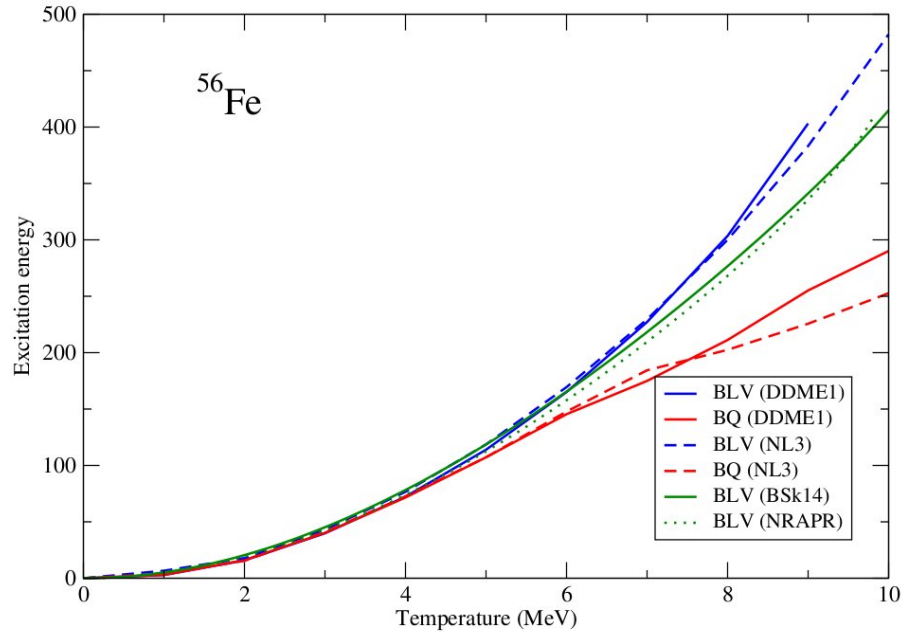
$$r_{m0} = 0.95 \pm 0.05 \text{ fm}$$

$$c_m = 0.005 \pm 0.001 \text{ MeV}^{-2}$$

The rms radius is thus expected to increase by about 9% from  $T=0$  to 6 MeV.



# Excitation energy and entropy



On the scale shown here,

- the excitation energy appears to vary quadratically and the entropy linearly with the temperature, in all cases, up to about 5 MeV (Fermi gas behavior);
- Pairing and shell effects enter at low temperatures.



## Liquid-drop model fit to the energy

- Due to the effects of pairing and shell closures, both the constant and temperature dependent terms must be fit. We take

$$E = c_1 A + c_2 A^{2/3} + c_4 A d^2 + c_5 A^{1/3} + c_6 \frac{Z(Z-1)}{A^{1/3}} + (c_7 A + c_8 A^{2/3} + c_9 A d^2) T^2$$

where

$$d = \frac{1}{(1 + c_3 A^{-1/3})} \frac{N - Z}{A}$$

- 180 nuclei with  $8 \leq Z \leq 82$  and  $12 \leq A \leq 250$ ,  $2 \text{ MeV} \leq T \leq 6 \text{ MeV}$ .

Modelos	$c_1$	$c_2$	$c_3$	$c_4$	$c_5$	$c_6$	$c_7$	$c_8$	$c_9$	$\chi^2/N$
BSk14	-14.71	11.73	0.655	27.29	6.94	0.664	0.064	0.077	-0.091	3.5
NRAPR	-14.35	9.98	0.718	28.24	9.17	0.649	0.057	0.087	-0.095	3.6
DD-ME1	-15.83	21.16	1.042	32.79	-7.79	0.675	0.062	0.093	-0.112	7.0
NL3	-15.27	17.68	1.145	32.47	-1.88	0.650	0.059	0.090	-0.085	6.6
G.S.	-15.8	18.3	0.0	23.7	0.0	0.714	0.0625	0.139	0.0	—

# Symmetry energy

The symmetry energy is found to be

$$E_{sym} \approx A d^2 \left( 30 - 0.1 T^2 \right) \text{ MeV}$$

It is about 10% below its ground state value at a temperature of 6 MeV.

Why? The principal effect is the volume expansion.

With

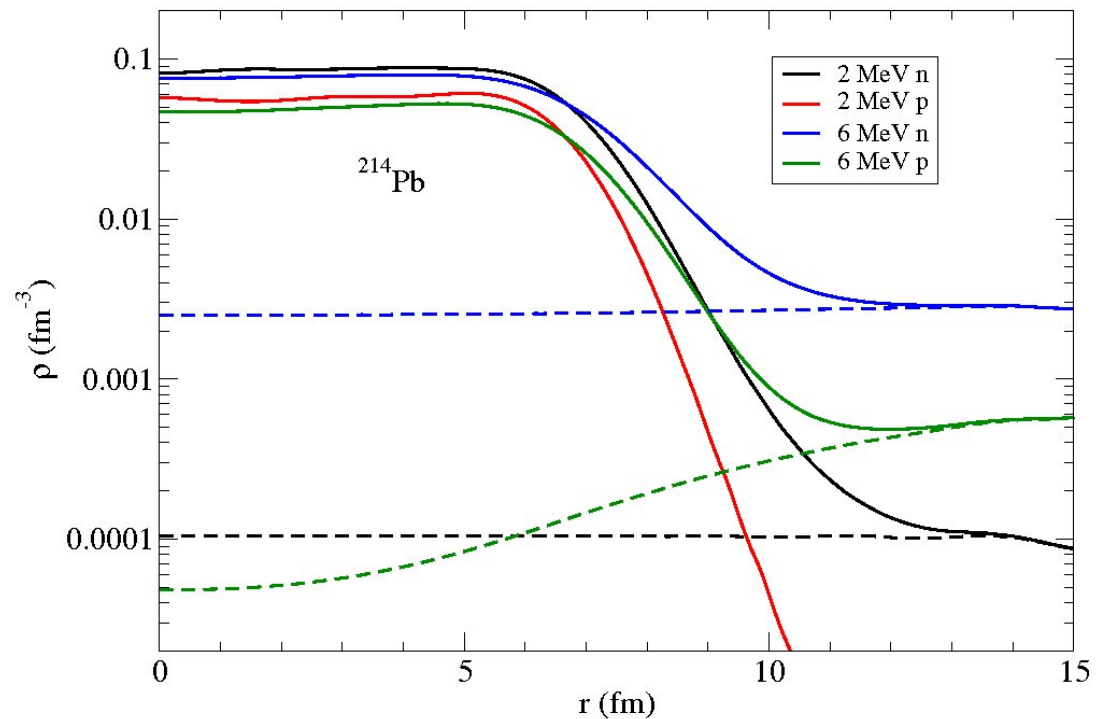
$$\langle r_m^2 \rangle = r_{m0}^2 A^{2/3} \left( 1 + c_m T^2 \right)$$

we have

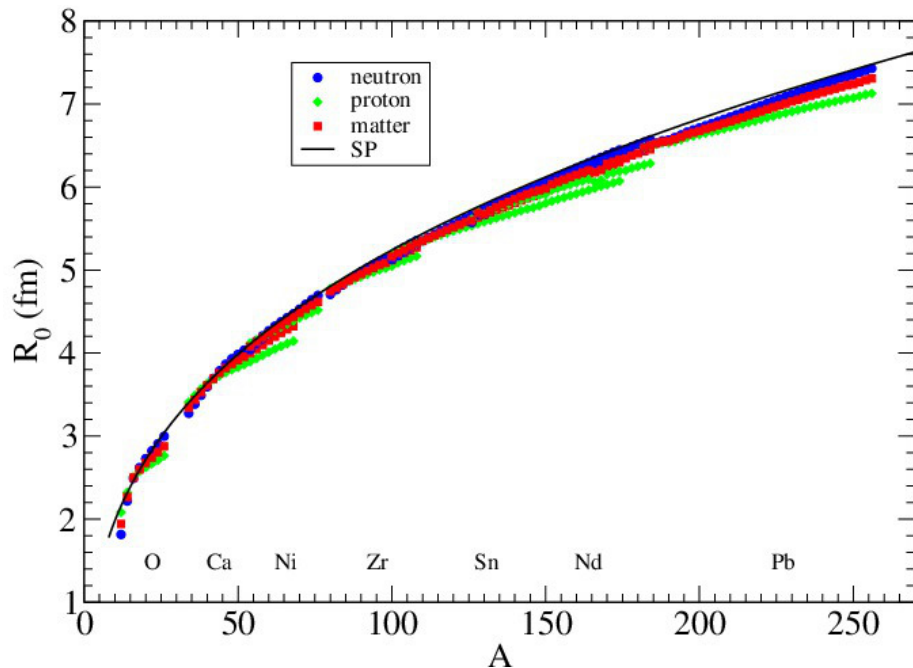
$$E_{sym} \approx E_{sym,0} - c_m \tilde{L} T^2 / 2$$

where

$$\tilde{L} = 3\rho_0 \left. \frac{dE_{sym}}{d\rho} \right|_{\rho_0} \approx 40 \text{ MeV}$$



## Geometry - radii



Global fits:

$$R_0 = 1.31A^{1/3} - 0.84 \text{ fm}$$

$$dR = 0.0033A^{1/3} - 0.014 \text{ fm/MeV}^2$$

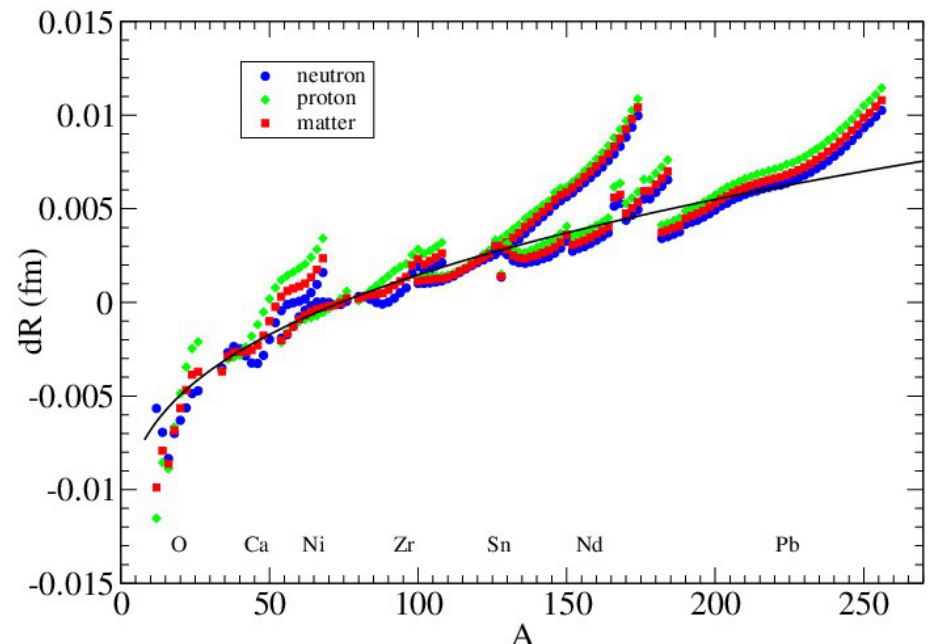
Note that

$$R(T) \approx R_0(1 + c_m T^2/2)$$

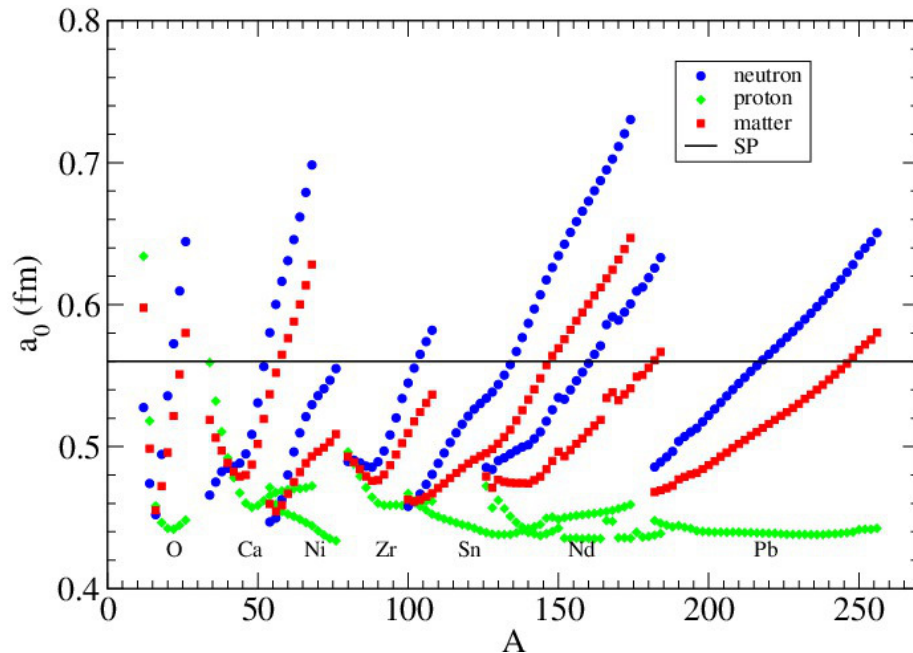
Fit from T=2 to 5 MeV:

$$\rho(r) = \frac{\rho_0(T)}{1 + \exp[(r - R(T))/a(T)]}$$

$$R(T) = R_0 + dRT^2$$



## Geometry – diffuseness parameter



Global fits:

$$a_0 = 0.56 \text{ fm}$$

$$da = 0.005 \text{ fm/MeV}^2$$

Here

$$da/a_0 \approx 4c_m$$

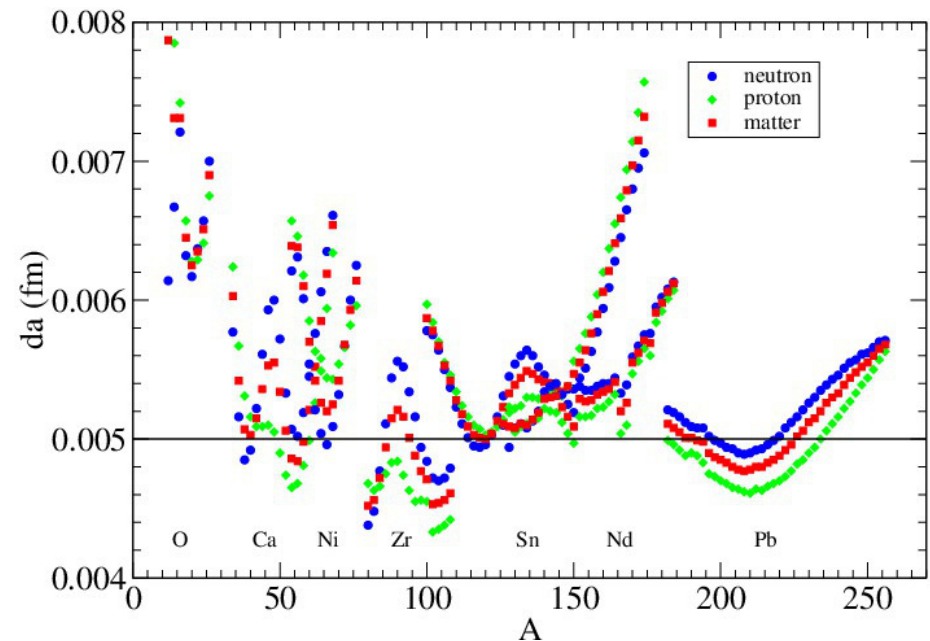
but

$$\langle r^2 \rangle \approx \frac{3}{5}R^2 + \frac{7}{5}\pi^2 a^2$$

Fit from  $T=2$  to 5 MeV:

$$\rho(r) = \frac{\rho_0(T)}{1 + \exp[(r - R(T))/a(T)]}$$

$$a(T) = a_0 + daT^2$$



## Global fit to potential barriers - $V_B$

The São Paulo folding potential was calculated with the parametrized temperature dependence of the densities and the maximum of the nuclear + Coulomb potential was calculated for about 5000 pairs of nuclei. These in turn were parametrized (to better than 1%) at  $T=0$  by

$$V_B(Z_1, A_1, Z_2, A_2) = Z_1 Z_2 e^2 / R_B(Z_1, A_1, Z_2, A_2)$$

with

$$\begin{aligned} R_B(Z_1, A_1, Z_2, A_2) = & 1.486 \left( A_1^{1/3} + A_2^{1/3} \right) - 0.996 \left( A_1^{-1/3} + A_2^{-1/3} \right) + 1.617 \\ & + 2.784 \left( Z_1^{1/3} + Z_2^{1/3} \right) - 0.515 \left( Z_1^{-1/3} + Z_2^{-1/3} \right) \text{ fm} \end{aligned}$$

When the fit is extended to include 5 temperature dependent terms of the same type, we find (to better than 2%) for  $T < 5$  MeV,

$$R_B(Z_1, A_1, Z_2, A_2, T) \approx R_B(Z_1, A_1, Z_2, A_2) (1 + c_V T^2) \quad c_V = 0.0031 \text{ MeV}^{-2}$$

Thus the barrier energies vary about 20% more than the matter density radii do ( $c_m/2 = 0.0025 \text{ MeV}^{-2}$ ). That is, they are expected to decrease by about 11% from  $T=0$  to 6 MeV.

# Summary

- We have studied the temperature dependence of the excitation energy, entropy, rms radii and the geometrical form of nuclear densities using the BLV prescription.
- Using a parametrization of the temperature dependence of the form of the densities, we have calculated the temperature dependence of the potential barrier of the São Paulo potential.
- We find the temperature dependence of the barrier to be only slightly larger than that of the rms radius of the densities.
- The modified barriers should change only slightly the limiting temperature of an equilibrated compound nucleus.