### Constraining the nuclear matter equation of state using the elliptic flow of light clusters

by <u>A. Le Fèvre<sup>1</sup></u>, Y. Leifels<sup>1</sup>, W. Reisdorf<sup>1</sup>, J. Aichelin<sup>2</sup>, Ch. Hartnack<sup>2</sup>, and N. Herrmann<sup>3</sup>

<sup>1</sup>GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany

<sup>2</sup>SUBATECH, UMR 6457, Ecole des Mines de Nantes - IN2P3/CNRS - Université de Nantes, France

<sup>3</sup>Physikalisches Institut der Universität Heidelberg, Heidelberg, Germany



### Constraining the nuclear matter equation of state using the elliptic flow of light clusters

by <u>A. Le Fèvre<sup>1</sup></u>, Y. Leifels<sup>1</sup>, W. Reisdorf<sup>1</sup>, J. Aichelin<sup>2</sup>, Ch. Hartnack<sup>2</sup>, and N. Herrmann<sup>3</sup>

<sup>1</sup>GSI Helmholtzzentrum für Schwerionenforschung GmbH, Darmstadt, Germany

<sup>2</sup>SUBATECH, UMR 6457, Ecole des Mines de Nantes - IN2P3/CNRS - Université de Nantes, France

<sup>3</sup>Physikalisches Institut der Universität Heidelberg, Heidelberg, Germany

Constraining the stiffness of the EOS with the elliptic flow.A clusterisation approach...

Towards the determination of the stiffness of the asymmetry energy.



Soft 200 MeV

----- Hard 380 MeV

2

з

- The equation of state (EOS) of nuclear matter:
  - of fundamental interest
  - object of intense theoretic
  - an important ingredient in n
    - compact stars<sup>[1]</sup>
    - core collapse supernov
- The calculation of the nuclear E [3], is a very complex task.
- ρ/ρ Nuclear physics based on empir nuclear forces requires a confrontation with empirical tacts.
- 1st method, from astrophysicists: from 'neutron' star masses and radii. But missing:

MeV)

E<sup>®/A</sup>

100

50

0

- precise model-independent radii,
- composition of the matter in the center of the stars.
  - [1] J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. 62 (2012) 485.
  - [2] A. Burrows, Rev. Mod. Phys. 85 (2013) 245.
  - [3] A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. 111 (2013) 032501 ASSOCIATION

ides



NGC 1952. Crab Nebula pulsar neutron star imaged by the NASA/ESA Hubble Space Telescope

ch as very recently attempted in

vsical phenomena such as:

he most 'fundamental' theory of



- The equation of state (EOS) of nuclear matter:
  - of fundamental interest
  - object of intense theoretical efforts since several decades



NGC 1952, Crab Nebula pulsar neutron star imaged by the NASA/ESA Hubble Space Telescope

- an important ingredient in modeling fascinating astrophysical phenomena such as:
  - compact stars<sup>[1]</sup>
  - core collapse supernovae<sup>[2]</sup>
- The calculation of the nuclear EOS from first principles, such as very recently attempted in [3], is a very complex task.
- Nuclear physics based on empirical observations => even the most 'fundamental' theory of nuclear forces requires a confrontation with empirical facts.
- 1st method, from astrophysicists: from 'neutron' star masses and radii. But missing:
  - precise model-independent radii,
  - composition of the matter in the center of the stars.



[2] A. Burrows, Rev. Mod. Phys. 85 (2013) 245.

ASSOCIATION [3] A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. 111 (2013) 032501



- Alternative method: in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.











- Alternative method: in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.





### Flows at high density in heavy-ion collisions

$$\frac{dN}{d(\phi - \phi_R)}(y, p_t) = \frac{N_0}{2\pi} \left(1 + 2\sum_{n \ge 1} v_n \cos n(\phi - \phi_R)\right) \frac{1}{j}$$

Y = rapidity p<sub>t</sub> = transverse momentum

- $\Phi_{\text{R}}$  = reaction plane azimuthal angle
- $V_1$  = 'side/directed flow',  $\langle p_x/p_t^2 \rangle$

$$V_2(y, p_t) = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle$$

ELMHOLTZ ASSOCIATION

'Elliptic flow':  $cos(2(\Phi-\Phi_R))$  mode, competition between 'in-plane' (V<sub>2</sub>>0) and 'out-of-plane' ejection (V<sub>2</sub><0).





### Flows at high density in heavy-ion collisions

$$\frac{dN}{d(\phi - \phi_R)}(y, p_t) = \frac{N_0}{2\pi} \left(1 + 2\sum_{n \ge 1} v_n \cos n(\phi - \phi_R)\right)$$

Y = rapidity p<sub>t</sub> = transverse momentum

- $\Phi_{\text{R}}$  = reaction plane azimuthal angle
- V<sub>1</sub> = 'side/directed flow', <p<sub>x</sub>/p<sub>t</sub><sup>2</sup>>

$$V_2(y, p_t) = \left\langle \frac{p_x^2 - p_y^2}{p_t^2} \right\rangle$$

ELMHOLTZ ASSOCIATION

'Elliptic flow':  $cos(2(\Phi-\Phi_R))$  mode, competition between 'in-plane' (V<sub>2</sub>>0) and 'out-of-plane' ejection (V<sub>2</sub><0).





- Alternative method: in earth laboratories, heavy ion collisions over a wide range of incident energies, system sizes and compositions.



- Present work: improve the situation in the 1 A.GeV regime, from extensive flow data published recently by the FOPI Collaboration (Au+Au @ 0.4-1.5 A.GeV)<sup>[4]</sup>
  - → close look at the elliptic flow data with improvements:
    - 1) not only protons: d, t, <sup>3</sup>He <sup>4</sup>He having larger flow signals than single nucleons.
    - 2) not only mid-rapidity data: 80% of the target- projectile rapidity gap.



[4] W. Reisdorf, et al. (FOPI Collaboration), Nucl. Phys. A 876 (2012) 1.

#### Elliptic flow







Arnaud Le Fèvre - NuSYM14 – 7-9 July 2014 – Liverpool

Complete shape of  $v_2(y_0)$ : a new observable:  $v_{2n} = |v_{20}| + |v_{22}|$ , from fit  $v_2(y_0) = v_{20} + v_{22} \cdot y_0$ 

→ v<sub>2n</sub>(E<sub>beam</sub>) varies by a factor
 ≈1.6, >> measured uncertainty
 (≈1.1)
 → clearly favors a 'soft' EOS.



FOPI Collaboration / NPA 876 (2012) 1-60

HELMHOLTZ

Complete shape of  $v_2(y_0)$ : a new observable:  $v_{2n} = |v_{20}| + |v_{22}|$ , from fit  $v_2(y_0) = v_{20} + v_{22} \cdot y_0$ 

→ v<sub>2n</sub>(E<sub>beam</sub>) varies by a factor
 ≈1.6, » measured uncertainty
 (≈1.1)
 → clearly favors a 'soft' EOS.

FOPI Collaboration / NPA 876 (2012) 1-60

HELMHOLTZ

ASSOCIATION



1.6

1.6

- Phenomenological EOS
   HM and SM include the saturation point at p/p<sub>0</sub> = 1,
   E/A = -16 MeV by construction.
- fixes the absolute position of the curves:
- the heavy ion data are only sensitive to the shape, i.e. the pressure (derivative).
- → a stiff EOS, characterised by K<sub>0</sub> = 380 MeV is not in agreement with the flow data in the incident energy range 0.4 -1.5 A.GeV.





/

- Phenomenological EOS
   HM and SM include the saturation point at p/p<sub>0</sub> = 1,
   E/A = -16 MeV by construction.
- fixes the absolute position of the curves:
- the heavy ion data are only sensitive to the shape, i.e. the pressure (derivative).
- → a stiff EOS, characterised by K<sub>0</sub> = 380 MeV is not in agreement with the flow data in the incident energy range 0.4 -1.5 A.GeV.





/

- Phenomenological EOS
   HM and SM include the saturation point at p/p<sub>0</sub> = 1,
   E/A = -16 MeV by construction.
- fixes the absolute position of the curves:
- the heavy ion data are only sensitive to the shape, i.e. the pressure (derivative).
- → a stiff EOS, characterised by K<sub>0</sub> = 380 MeV is not in agreement with the flow data in the incident energy range 0.4 -1.5 A.GeV.





1

### Which density has been probed?

Purpose = characterise which 'typical' densities where probed in the FOPI experiments => at which time V<sub>2</sub> develops, and which conditions influence it the most.

IQMD transport model<sup>[5,6]</sup> various phenomenological EOS's:

- » 'stiff' = H & HM (+ momentum dependent), K<sub>0</sub> = 380 MeV
- » 'soft' = S & SM (+momentum dependent), K<sub>0</sub> = 200 MeV.
  Here: protons in Au+Au at 1.5 A.GeV, b=3 fm

3 53 X

HELMHOLTZ

ASSOCIATION



full target-projectile overlap

### Which density has been probed?



- The elliptic flow in his final dependence with rapidity develops fast: during the passing time.
- The elliptic flow in strength and shape is mostly influenced by the force of the mean field
- The 'typical' density of the 'measured' EOS can be built from the mean value weighted by this force up to the passing time.



#### **Simulations: the scenario**



 The density range, relevant to the EOS evidenced by the FOPI Collaboration, spans in the range ρ = (1.25 - 2.0) ρ<sub>0</sub>.



- \* Simulated Annealing Procedure: PLB301:328,1993; later called SACA.
- \* P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390
- **2 steps:** \* 2010 version: publication in progress...
- 1) Pre-select good «candidates» for fragments according to proximity criteria: real space coalescence = Minimum Spanning Tree (MST) procedure.





- \* Simulated Annealing Procedure: PLB301:328,1993; later called SACA.
- \* P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390
- **2 steps:** \* 2010 version: publication in progress...
- 1) Pre-select good «candidates» for fragments according to proximity criteria: real space coalescence = Minimum Spanning Tree (MST) procedure.
- 2) Take randomly 1 nucleon
- out of one fragment







- \* Simulated Annealing Procedure: PLB301:328,1993; later called SACA.
- \* P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390
- **2 steps:** \* 2010 version: publication in progress...
- 1) Pre-select good «candidates» for fragments according to proximity criteria: real space coalescence = Minimum Spanning Tree (MST) procedure.
- 2) Take randomly 1 nucleon out of one fragment



3) Add it randomly to another fragment





11

- \* Simulated Annealing Procedure: PLB301:328,1993; later called SACA.
- \* P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390
- **2 steps:** \* 2010 version: publication in progress...
- 1) Pre-select good «candidates» for fragments according to proximity criteria: real space coalescence = Minimum Spanning Tree (MST) procedure.
- 2) Take randomly 1 nucleon out of one fragment



3) Add it randomly to another fragment



 $\underline{\text{If } E' < E}$  take the new configuration



- \* Simulated Annealing Procedure: PLB301:328,1993; later called SACA.
- \* P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390
- \* 2010 version: publication in progress...
- 1) Pre-select good «candidates» for fragments according to proximity criteria: real space coalescence = Minimum Spanning Tree (MST) procedure.
- 2) Take randomly 1 nucleon out of one fragment

2 steps:



3) Add it randomly to another fragment



<u>If E' < E</u> take the new configuration <u>If E' > E</u> take the old with a probability depending on E'-E



- \* Simulated Annealing Procedure: PLB301:328,1993; later called SACA.
- \* P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390
- \* 2010 version: publication in progress...
- 1) Pre-select good «candidates» for fragments according to proximity criteria: real space coalescence = Minimum Spanning Tree (MST) procedure.
- 2) Take randomly 1 nucleon out of one fragment

2 steps:



3) Add it randomly to another fragment



<u>If E' < E</u> take the new configuration <u>If E' > E</u> take the old with a probability depending on E'-E Repeat this procedure very many times...

- \* Simulated Annealing Procedure: PLB301:328,1993; later called SACA.
- \* P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390
- \* 2010 version: publication in progress...
- 1) Pre-select good «candidates» for fragments according to proximity criteria: real space coalescence = Minimum Spanning Tree (MST) procedure.
- 2) Take randomly 1 nucleon out of one fragment

2 steps:



3) Add it randomly to another fragment



<u>If E' < E</u> take the new configuration <u>If E' > E</u> take the old with a probability depending on E'-E Repeat this procedure very many times...



Ingredients of the binding energy of the clusters :

Volume component: mean field (Skyrme, dominant), for NN, NΛ (hypernuclei)
 Surface effect correction: Yukawa term.

(3) Asymmetry energy : 23.3 MeV.  $(<\rho'_B>)^{(\gamma_{ASY-1})} \cdot (<\rho'_n> - <\rho'_p>)^2/<\rho'_B>$ 

(4) Extra « structure » energy  $(N,Z,\rho) = B_{MF}(\rho).((B_{exp}-B_{BW})/(B_{BW}-B_{Coul}-B_{asy}))(\rho_0)$ 

(5) <sup>3</sup>He+n recombination.

(6) Secondary decay: GEMINI.



12

Ingredients of the binding energy of the clusters :

Volume component: mean field (Skyrme, dominant), for NN, NΛ (hypernuclei)
 Surface effect correction: Yukawa term.

(3) Asymmetry energy : 23.3 MeV.  $(<\rho'_B>)^{(\gamma_{ASY-1})} . (<\rho'_n> - <\rho'_p>)^{2/<\rho'_B>}$ 

(4) Extra « structure » energy  $(N,Z,\rho) = B_{MF}(\rho).((B_{exp}-B_{BW})/(B_{BW}-B_{Coul}-B_{asy}))(\rho_0)$ 

(5) <sup>3</sup>He+n recombination.

A HELMHOLTZ

(6) Secondary decay: GEMINI.

#### **C** Remarks:

- Advantage of SACA : the fragment partitions can reflect the early dynamical conditions (Coulomb, density, flow details, strangeness...). Fragment partitions already determined at the passing time of the colliding system.
- In the framework of QMD, HSD, <ρ<sub>clusters</sub> < 0.5. ρ<sub>0</sub> ⇒ isotope yields of SACA with E<sub>asy</sub> probe it at sub-saturation densities

ELMHOLTZ

ASSOCIATION

2 52 X



FOPI

multiplicity 01

10<sup>0</sup>

10<sup>-1</sup>

HELMHOLTZ ASSOCIATION



#### Another application of SACA : hypernuclei production



#### Directed flow



FOPI Collaboration / NPA 876 (2012) 1-60



#### Directed flow



FOPI Collaboration / NPA 876 (2012) 1-60

#### Directed flow



FOPI Collaboration / NPA 876 (2012) 1-60

#### Elliptic flow

ASSOCIATION



Au+Au ut0>0.4 FOPI Collaboration / NPA 876 (2012) 1-60

#### Elliptic flow



Arnaud Le Fèvre - NuSYM14 – 7-9 July 2014 – Liverpool

Au+Au ut0>0.4 FOPI Collaboration / NPA 876 (2012) 1-60







The higher the bombarding energy, the stronger the sensitivity.

**ELMHOLTZ** ASSOCIATION





19

Arnaud Le Fèvre - NuSYM14 - 7-9 July 2014 - Liverpool

 A single parameter v<sub>2n</sub>, characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.



- A single parameter v<sub>2n</sub>, characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- Relevant density range: estimated from the simulations to span  $\rho = (1.25 2.0)\rho_0$ .



- A single parameter v<sub>2n</sub>, characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- Relevant density range: estimated from the simulations to span  $\rho = (1.25 2.0)\rho_0$ .
- The spectator clock can presumably be used to try to extend improved EOS constraints to densities  $(3-4 \rho_0)$  in future accelerator systems such as FAIR.



- A single parameter v<sub>2n</sub>, characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- Relevant density range: estimated from the simulations to span  $\rho = (1.25 2.0)\rho_0$ .
- The spectator clock can presumably be used to try to extend improved EOS constraints to densities  $(3-4 \rho_0)$  in future accelerator systems such as FAIR.
- Beyond 4 A.GeV, other ideas are needed to extract EOS information from heavy ion data.



- A single parameter v<sub>2n</sub>, characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- Relevant density range: estimated from the simulations to span  $\rho = (1.25 2.0)\rho_0$ .
- The spectator clock can presumably be used to try to extend improved EOS constraints to densities (3-4  $\rho_0$ ) in future accelerator systems such as FAIR.
- Beyond 4 A.GeV, other ideas are needed to extract EOS information from heavy ion data.
- A realistic treatment of the clusterisation is needed to account for e.g. Easymmetry effects.



- A single parameter v<sub>2n</sub>, characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- Relevant density range: estimated from the simulations to span  $\rho = (1.25 2.0)\rho_0$ .
- The spectator clock can presumably be used to try to extend improved EOS constraints to densities  $(3-4 \rho_0)$  in future accelerator systems such as FAIR.
- Beyond 4 A.GeV, other ideas are needed to extract EOS information from heavy ion data.
- A realistic treatment of the clusterisation is needed to account for e.g. Easymmetry effects.
- The stiffness of the asymmetry energy can be discriminated by the shape (v<sub>2n</sub>) the elliptic flow over a large range of rapidity (not only mid-rapidity) of <sup>3</sup>He and tritons. -> Prelimininary indication of 0.5<=  $\gamma_{asy}$  < 1 by confronting IQMD-SACA to FOPI data.



- A single parameter v<sub>2n</sub>, characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- Relevant density range: estimated from the simulations to span  $\rho = (1.25 2.0)\rho_0$ .
- The spectator clock can presumably be used to try to extend improved EOS constraints to densities (3-4  $\rho_0$ ) in future accelerator systems such as FAIR.
- Beyond 4 A.GeV, other ideas are needed to extract EOS information from heavy ion data.
- A realistic treatment of the clusterisation is needed to account for e.g. Easymmetry effects.
- The stiffness of the asymmetry energy can be discriminated by the shape (v<sub>2n</sub>) the elliptic flow over a large range of rapidity (not only mid-rapidity) of <sup>3</sup>He and tritons. -> Prelimininary indication of 0.5<=  $\gamma_{asy}$  < 1 by confronting IQMD-SACA to FOPI data.
- The same can be obtained via the ratio of the elliptic flow values of neutrons and protons.



- A single parameter v<sub>2n</sub>, characterising the elliptic flow over a large rapidity interval, for protons and other light isotopes -> clear discrimination for soft EOS.
- Relevant density range: estimated from the simulations to span  $\rho = (1.25 2.0)\rho_0$ .
- The spectator clock can presumably be used to try to extend improved EOS constraints to densities  $(3-4 \rho_0)$  in future accelerator systems such as FAIR.
- Beyond 4 A.GeV, other ideas are needed to extract EOS information from heavy ion data.
- A realistic treatment of the clusterisation is needed to account for e.g. Easymmetry effects.
- The stiffness of the asymmetry energy can be discriminated by the shape (v<sub>2n</sub>) the elliptic flow over a large range of rapidity (not only mid-rapidity) of <sup>3</sup>He and tritons. -> Prelimininary indication of 0.5<=  $\gamma_{asy}$  < 1 by confronting IQMD-SACA to FOPI data.
- The same can be obtained via the ratio of the elliptic flow values of neutrons and protons.

See AsyEOS experiment: ongoing analysis, forthcoming talks by P. Russotto and J. Brzychczyk.



### **Comparison to microscopic calculations**

(three representative microscopic calculations compared with our new constraints)



Dirac-Brueckner-Hatree-Fock (DBHF) calculation<sup>[10]</sup> using the Bonn A<sup>[11]</sup> nucleon-nucleon potential

[10] R. Brockmann, R. Machleidt, Phys. Rev. C 42 (1990) 1965.

[11] T. Katayama, K. Saito, Phys. Rev. C 88 (2013) 035805.

Arnaud Le Fèvre - NuSYM14 – 7-9 July 2014 – Liverpool

### **Comparison to microscopic calculations**

(three representative microscopic calculations compared with our new constraints)



### 2 symmetric nuclear matter EOS's from [12]:

 'DBHF' = meson theoretic potential together with the DBHF method
 'Chiral'= use of effective field theory (EFT) with density dependent interactions derived from leading order chiral three-nucleon forces.

[12] P. Danielewicz, G. Odyniec, Phys. Lett. B 157 (1985) 168.

#### **Comparison to microscopic calculations**

(three representative microscopic calculations compared with our new constraints)



Using the chiral approach<sup>[13]</sup>: 2 rather different EOS's including or not virtual  $\Delta$  excitations.

- » the virtual  $\Delta$ -excitations help locate the EOS at the right horizontal place around  $\rho = 0.16$  fm-3.
- » the ∆ leads to a rather marked stiffening of the EOS (KO = 304 MeV)
- » because 'cold' EOS ?
- » finite temperature in the reaction => the  $\Delta$  are real rather than virtual. The theoretical ' $\Delta$  stiffness' could then be a dispersion effect rapidly changing with temperature.

[13] S. Fritsch, N. Kaiser, W. Weise, Nucl. Phys. A 750 (2005) 259.

#### Beam energy dependence of elliptic flow



elliptic flow

- pressure gradient of compression zone
- shadowing of spectators
- $\succ$  at low energies
  - attraction due to mean field of nucleons
- $\succ$  at high energies
  - lacking shadowing of spectators

#### Elliptic flow and the nuclear matter EOS



P. Danielewicz et al. Science 298, 1592 (2002)

#### Elliptic flow and the nuclear matter EOS



P. Danielewicz et al. Science 298, 1592 (2002)