Sub-saturation matter in Compact Stars : nuclear modelling in the framework of the Extended Thomas-Fermi theory

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ENSICAEN
ÉCOLE NATIONALE SUPÉRIEURE D'INGÉNIEURS DE CAEN & CENTRE DE RECHERCHE

- Sub-saturation matter: Wigner-Seitz cells $\rightarrow$ nucleus + gas
- Stellar matter EoS: wide range of (ρ_b, y_p, T)
  $\Rightarrow$ wide variety of nuclei (A, δ_r) in any nucleon gas (ρ_g,δ_g)
  $\rightarrow$ Self-consistent mean-field theory
- Standard approach for $\rho_b < \rho_0$: Single Nucleus Approximation
  (*J.M. Lattimer and F. Douglas Swesty*, NPA 535, 331 (1991),
  H. Shen et al., NPA, 435 (1998))
- Improvement: extended NSE
  (*Ad. R. Raduta and F. Gulminelli*, PRC 82, 8065801 (2010),
  M. Hempel and J. Schaffner-Bielich, NPA 837, 210-254 (2010))
  + beyond SNA: statistical distribution of nuclei
    - $T = 0$: $\min[E_{WS}/V_{WS}]$
    - Finite temperature: $P_{WS} \propto \exp\left[\beta(E_{WS} - TS_{WS})/V_{WS}\right]$
  – no in-medium effects: ideal clusters with vacuum energies

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  $\Rightarrow$ Extended Thomas-Fermi approach

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2. **Nuclei without gas: symmetry energy from ETF**
3. **In-medium modifications**

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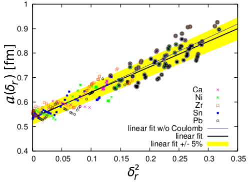
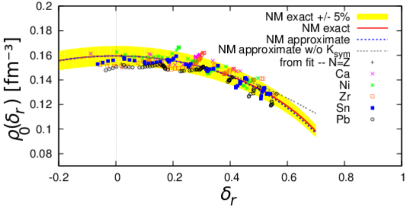
Model

- Energy density: Skyrme + ETF at $o(\hbar^4) \rightarrow \mathcal{E}^{Sky}_{ETF}(\rho_q, \nabla\rho_q)$
- Matter density profiles:
  - $\rho_q(r) = \dfrac{\rho_{0q}}{1+\mathrm{e}^{(r-R_q)/a_q}} + \dfrac{\rho_{gq}}{1+\mathrm{e}^{(R_q-r)/a_q}}$
    - $\rho_0(\delta_r) = \rho_0^{\delta_r=0}\left[1 - \frac{3L}{K_\infty + K_{sym}\delta_r^2}\delta_r^2\right]$
    - $a_q(\delta_r) = \alpha_q\delta_r^2 + \beta_q$
    - $R_q(\delta_r)$: ensure the conservation of the particle numbers

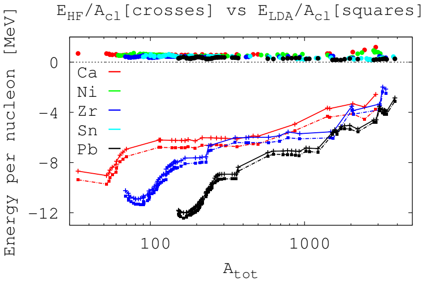


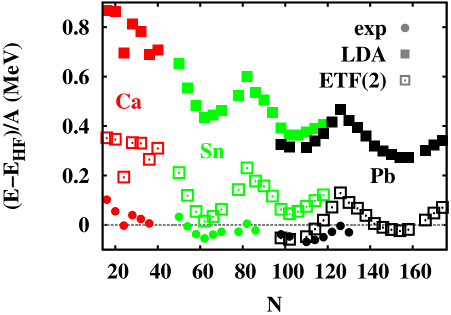


P. Papakonstantinou et al., Phys. Rev. **C 88** 045805 (2013)

Comparison with Hartree-Fock calculations

$$E^{WS}_{ETF} = \int_{V_{WS}} \mathcal{E}^{Sky}_{ETF}(\rho_q, \nabla\rho_q)\mathrm{d}\mathbf{r}$$





F. Aymard,

Proceeding JRJC (2014)

F. Aymard, J. Margueron, F. Gulminelli,

Phys. Rev. C 89, 065807 (2014)

Surface energy

- Surface energy: $E_s = E_{ETF} - \frac{A}{\rho_0}\mathcal{E}^{Sky}_{ETF}\left(\rho_{0q}, \nabla\rho_q = 0\right)$
  - $\rightarrow$ Saturation density ρ_0 depends on nucleus isospin asymmetry
- 2 isospin asymmetries for a nucleus:
  - $\rightarrow$ global: $I = \frac{N-Z}{A}$
  - $\rightarrow$ bulk: $\delta_r = \frac{\rho_{0n} - \rho_{0p}}{\rho_0} = \frac{I + \frac{3a_c}{8Q}\frac{Z^2}{A^{5/3}}}{1 + \frac{9J}{4Q}A^{-1/3}} \simeq I\left(1 - \frac{9J}{4Q}A^{-1/3} + \cdots\right)$

$\Rightarrow$ 2 surface energy definitions for a nucleus:

- $\rightarrow$ $E_s(\delta) = E_{ETF} - \frac{A}{\rho_0(\delta)}\mathcal{E}^{Sky}_{ETF}\left(\rho_{0q}(\delta)\right)$
- $\rightarrow$ $E_s(I) = E_{ETF} - \frac{A}{\rho_0(I)}\mathcal{E}^{Sky}_{ETF}\left(\rho_{0q}(I)\right) \simeq E_s(\delta) - \left(\frac{9J^2}{2Q}A^{2/3} - \cdots\right)I^2$

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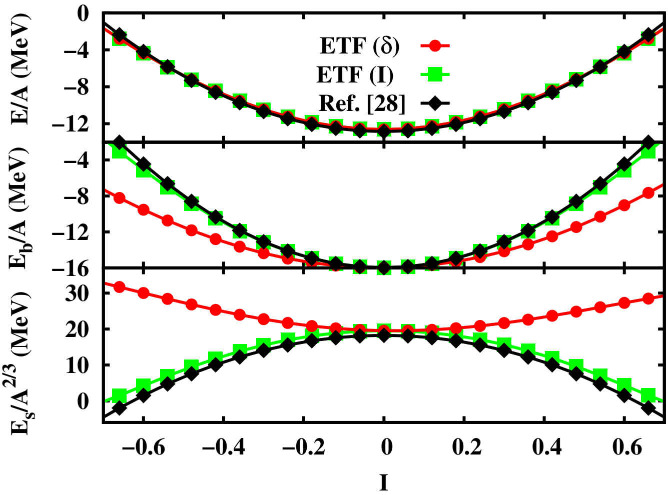
Symmetry energy from ETF



Ref. [28]:

$$E_{LDM} = -a_v A + a_s A^{2/3} + \frac{a_v^a}{1+a_v^a/(a_s^a A^{1/3})} A I^2$$

P. Danielewicz and J. Lee, Nucl. Phys. A 818 (2009)

F. Aymard, J. Margueron, F. Gulminelli, Phys. Rev. C 89, 065807 (2014)

Global or bulk isospin asymmetry

- Euler-Lagrange equations:

$$\lambda_q = \frac{\partial \mathcal{E}}{\partial \rho_q}(r) \Rightarrow \rho_q(r) = cst = \rho_{0q} \text{ at local saturation} \Rightarrow \delta_r$$

- Comparison with HF calculations (Pb isotopic chain):

E_{HF}/A	$\%(I)$	$\%(\delta)$	$\bar{r}^p_{HF}$	$\%(I)$	$\%(\delta)$
-11.17	+1.4	+0.1	5.53	+2.3	-0.1

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Wigner-Seitz cell energetics

- $$E_{ETF}^{WS} = \int_0^{R_{WS}} \mathcal{E}_{ETF}^{Sky}(\rho_q, \nabla\rho_q,)\mathrm{d}^3 r$$
$$= \int_0^{R_{cl}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3 r + \int_{R_{cl}}^{R_{WS}} \mathcal{E}_{Sky}^{ETF}(\rho_q, \nabla\rho_q,)\mathrm{d}^3 r$$
$$= \mathcal{E}_{ETF}^{Sky}(\rho_{0q})V_{cl} + \mathcal{E}_{ETF}^{Sky}(\rho_{gq})(V_{WS} - V_{cl}) + E_{S,m}$$

- $$E_{B,m} = \left[\mathcal{E}_{ETF}^{Sky}(\rho_{0q}) - \mathcal{E}_{ETF}^{Sky}(\rho_{gq})\right]\frac{A}{\rho_0}$$

- $$E_{S,m} = \int_0^{R_{WS}} \mathcal{E}_{ETF}^{Sky}(\rho_q, \nabla\rho_q,)\mathrm{d}r - E_{B,m} - \mathcal{E}_{ETF}^{Sky}(\rho_{gq})V_{WS}$$

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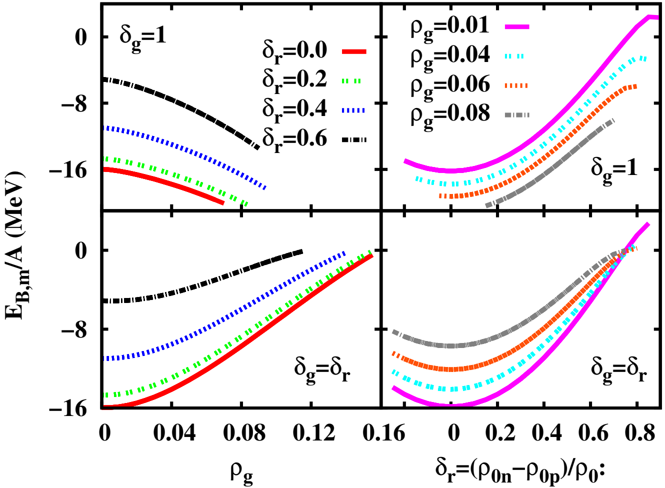
Bulk binding energy shift



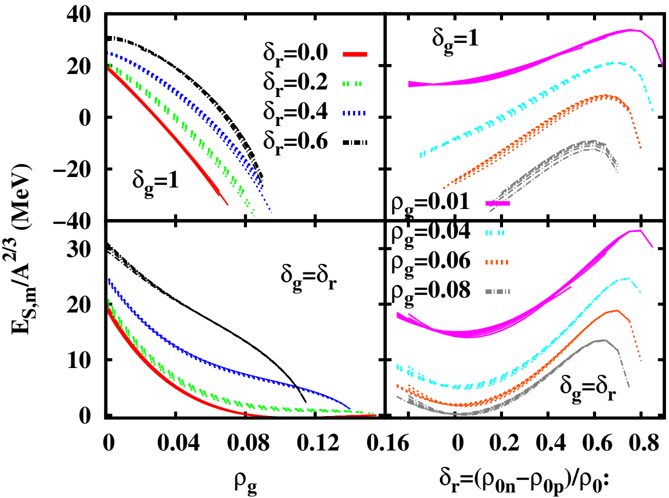
pure neutron gas

Homogeneous asymmetry

isospin asymmetry in the bulk cluster

F. Aymard, J. Margueron, F. Gulminelli, Phys. Rev. C 89, 065807 (2014)

Modification of the surface energy



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Summary and perspectives

- Semi-classical expansion (ETF): quasi-analytical model reproducing HF energy calculations with accuracy $\lesssim 200$ keV$/A$

$\Rightarrow$ Nuclei in vacuum:

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- Surface symmetry energy $\rightarrow$ positive

$\Rightarrow$ In-medium modifications to the cluster energies:

- Consistent treatment of nuclei and unbound nucleons with a non-artificial excluded volume
- Results:
  - Bulk effect: binding energy shift
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  - Very different effects depending on the proton fraction

$\star$ Tabulation of the surface in-medium energies as $\mathcal{T}(A, \delta_r, \rho_g, \delta_g)$

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