

Cluster Correlations in Dense Matter and Equation of State

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NuSYM14
Fourth International Symposium
on the Nuclear Symmetry Energy
University of Liverpool

Outline

- **Introduction**

Dense Matter in Nature, Nuclear and Stellar Matter, Correlations

- **Generalized Relativistic Density Functional**

Details of gRDF Model, Effective Interaction, Mass Shifts, Chemical Composition of Matter

- **Symmetry Energy and Neutron Skins**

Density Dependence, Neutron Skins with α -Cluster Correlations

- **Conclusions**

Details:

S. Typel, G. Röpke, T. Klähn, D. Blaschke, H.H. Wolter, Phys. Rev. C 81 (2010) 015803

M.D. Voskresenskaya, S. Typel, Nucl. Phys. A 887 (2012) 42

G. Röpke, N.-U. Bastian, D. Blaschke, T. Klähn, S. Typel, H.H. Wolter, Nucl. Phys. A 897 (2013) 70

S. Typel, H.H. Wolter, G. Röpke, D. Blaschke, Eur. Phys. J. A 50 (2014) 17

S. Typel, Phys. Rev. C 89 (2014) 064321

Introduction

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- **astrophysical objects**
 - neutron stars and neutron star mergers
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 - length scales: macroscopic/microscopic
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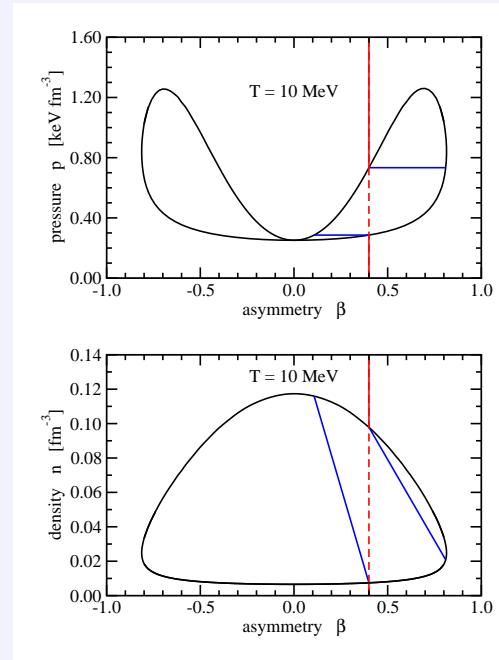
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- **very different systems**
 - length scales: macroscopic/microscopic
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- **interacting many-body systems**
 - correlations essential
 - assuming equilibrium conditions ⇒ **equation of state (EoS)**
⇒ thermodynamic properties and chemical composition
 - subsaturation densities ⇒ **clustering**

Nuclear Matter & Stellar Matter

nuclear matter

- only strongly interacting particles
- no electromagnetic interaction, no charge neutrality
- densities below nuclear saturation density
⇒ ‘non-congruent’ liquid-gas phase transition:
coexistence of low-density and high-density phases
with different isospin asymmetries



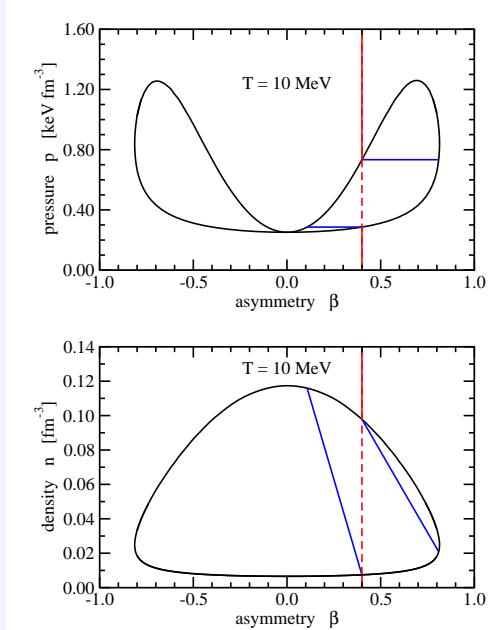
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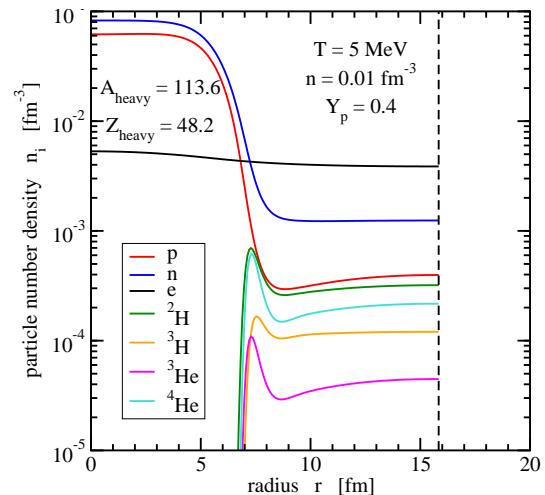
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stellar matter

- **hadrons** and **leptons**
- **strong** and **electromagnetic interaction**
- specific condition: **charge neutrality**
- formation of **inhomogeneous matter**
⇒ new particle species (nuclei)
⇒ ‘**pasta phases**’
- lattice formation at low temperatures
⇒ **phase transition**: liquid/gas ↔ solid



gRDF, spherical Wigner-Seitz cell



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⇒ in general complicated structure

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(see, e.g., C. J. Horowitz, A. Schwenk, Nucl. Phys. A 776 (2006) 55)

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⇒ construction of generalized relativistic density functional
with correct limits and explicit cluster degrees of freedom

Generalized Relativistic Density Functional

Generalized Relativistic Density Functional I

- **grand canonical approach**
 - extension of relativistic mean-field models with density-dependent meson-nucleon couplings
⇒ grand canonical potential density $\omega(T, \{\mu_i\})$

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- medium modifications of composite particles (mass shifts, internal excitations)
- scattering correlations considered (essential for correct low-density limit)
- particles and antiparticles included
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- **stellar matter**
 - charge neutrality condition
 - Coulomb correlations with correct limits \Rightarrow phase transition to crystal

Generalized Relativistic Density Functional II

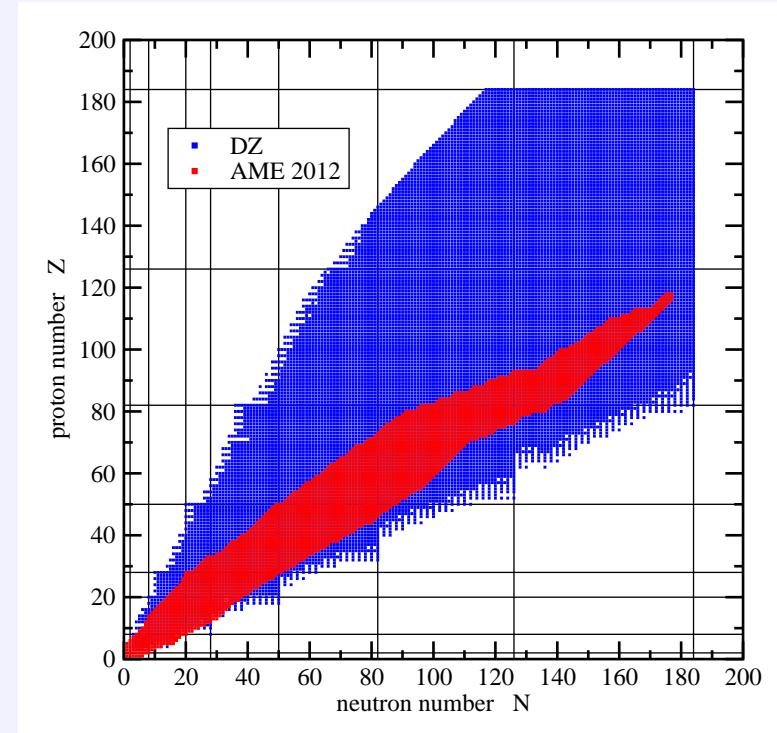
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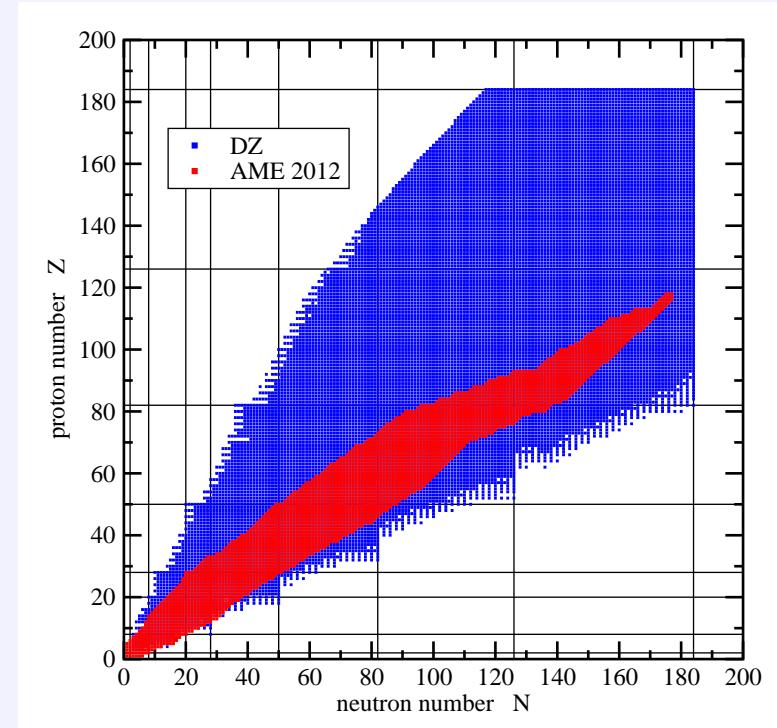
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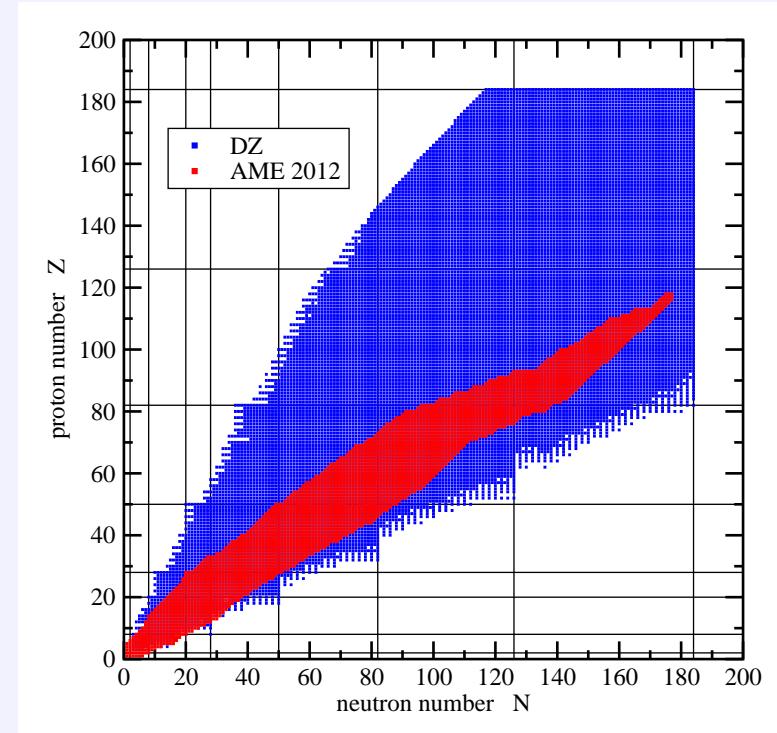
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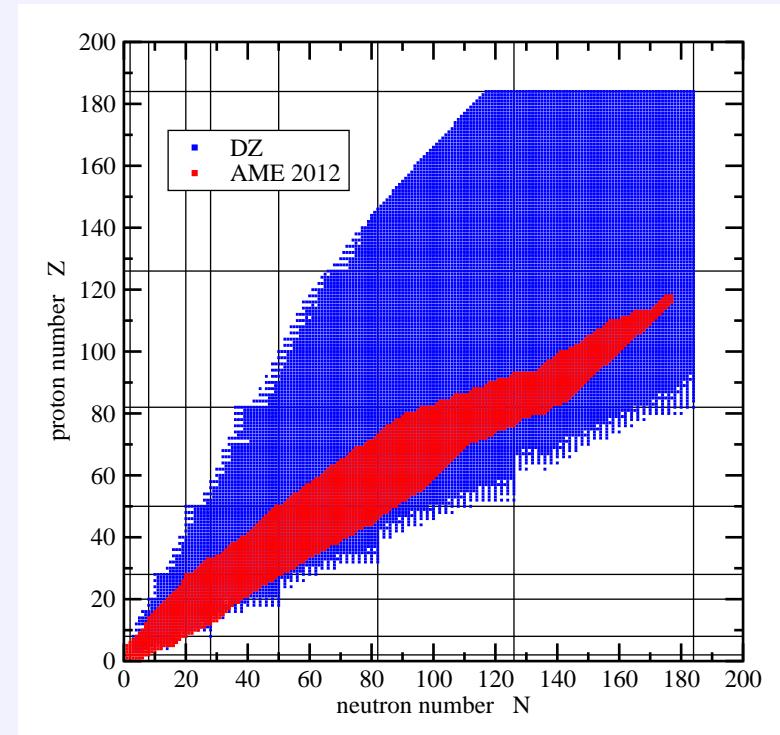
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- **quasiparticles** with scalar potential S_i and vector potential V_i



Effective Interaction

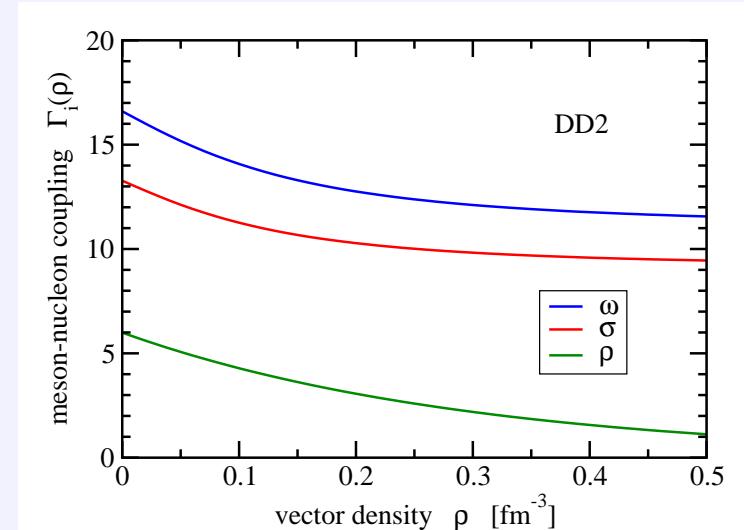
exchange of

- Lorentz scalar mesons $m \in \mathcal{S} = \{\sigma, \delta, \sigma_*, \dots\}$
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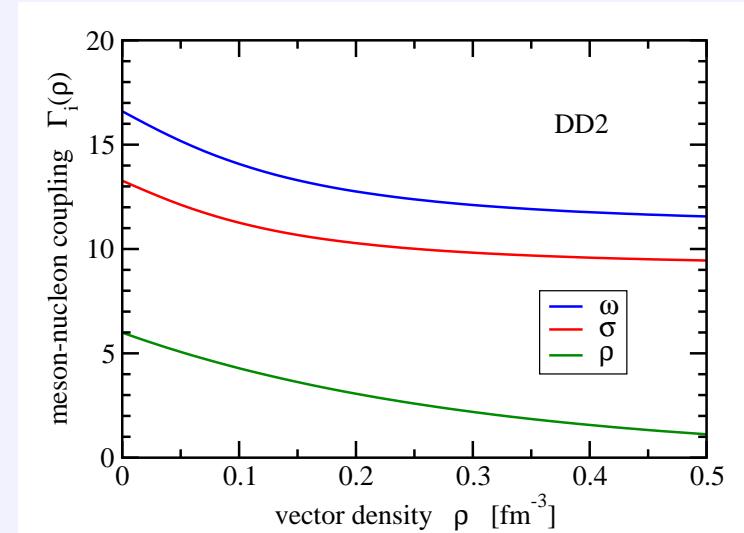
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- coupling to constituents: $\Gamma_{im} = g_{im}\Gamma_m$
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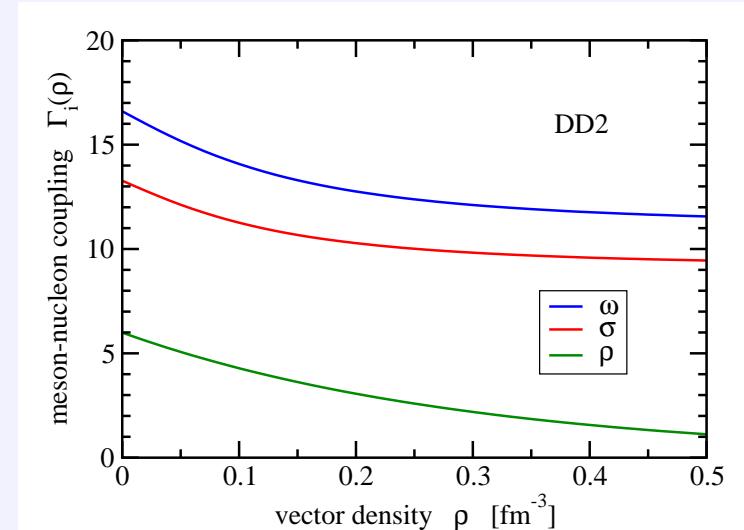
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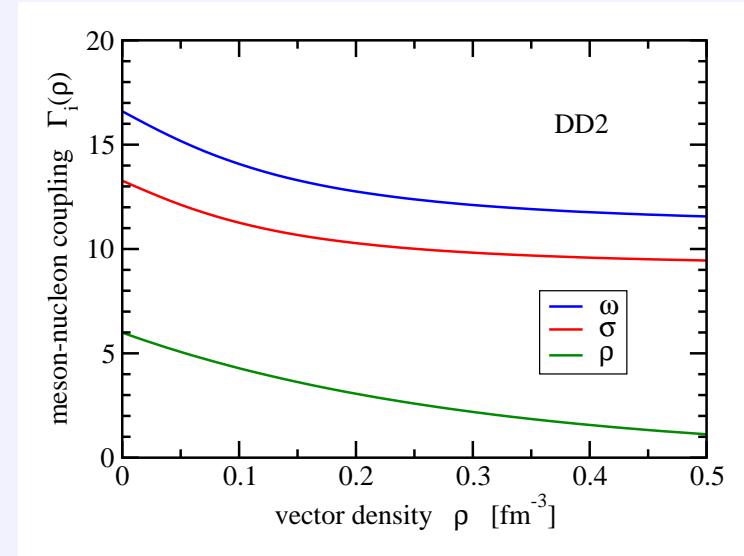
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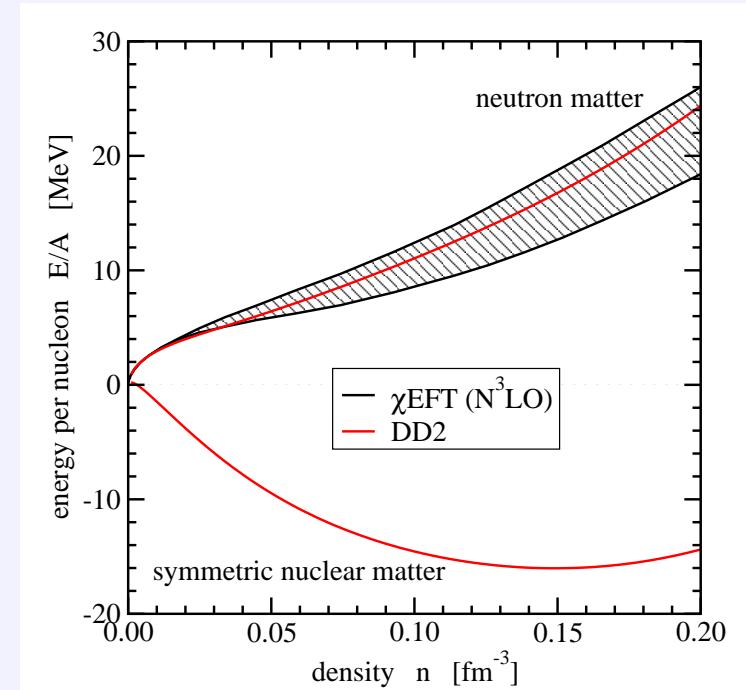
nuclear matter parameters

$$\begin{aligned} n_{\text{sat}} &= 0.149 \text{ fm}^{-3} \\ a_V &= 16.02 \text{ MeV} \\ K &= 242.7 \text{ MeV} \\ J &= 31.67 \text{ MeV} \\ L &= 55.04 \text{ MeV} \end{aligned}$$

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χ EFT(N^3LO):

- I. Tews et al., Phys. Rev. Lett 110 (2013) 032504
T. Krüger et al., Phys. Rev. C 88 (2013) 025802

Mass Shifts I

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- **electromagnetic shift** $\Delta E_i^{(\text{Coul})}$ (in stellar matter)
 - **electron screening** of Coulomb field
 - ⇒ increase of binding energies

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light nuclei

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with effective density

$$n_i^{(\text{eff})} = \frac{2}{A_i} (N_i n_n^{(\text{tot})} + Z_i n_p^{(\text{tot})})$$

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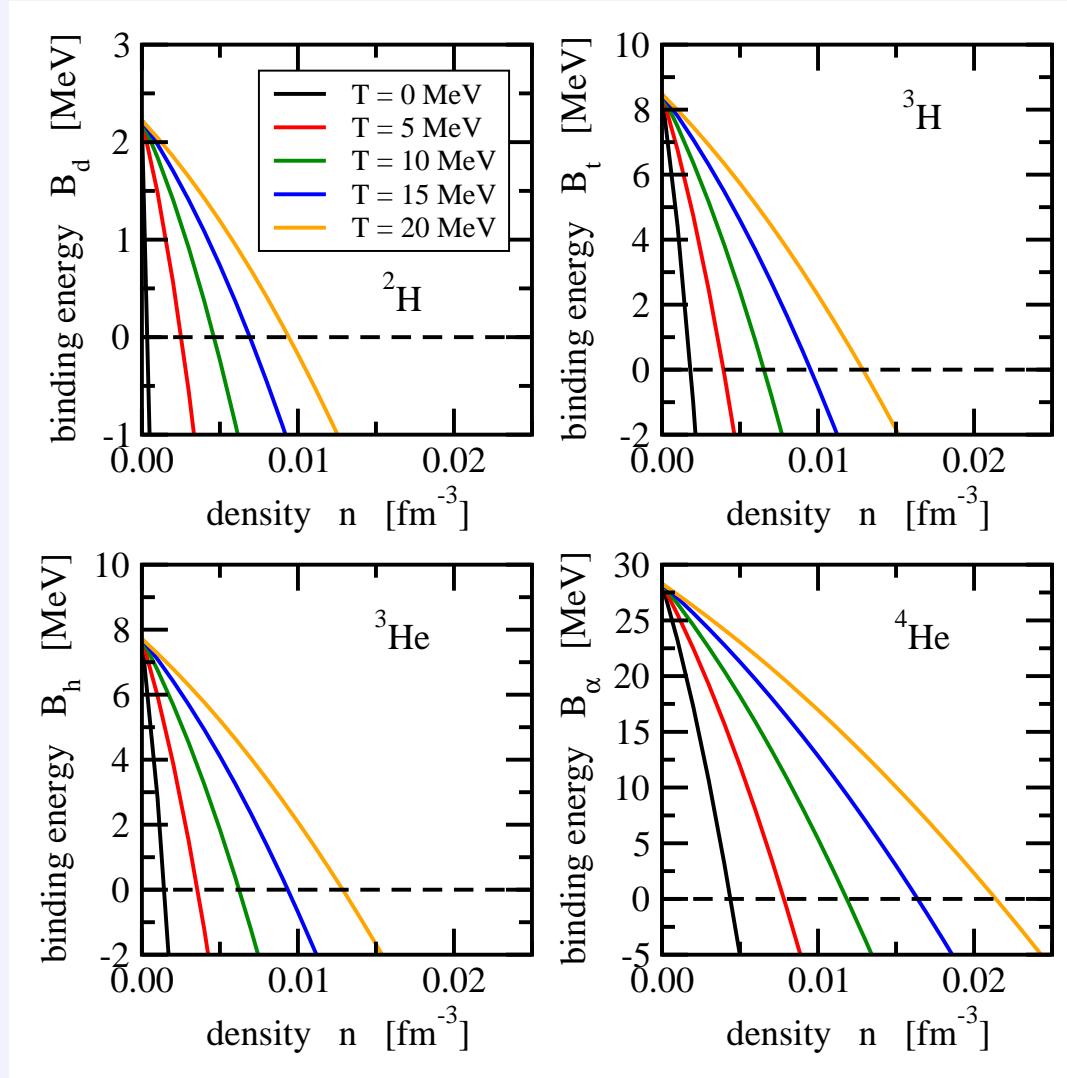
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- example: symmetric nuclear matter, nuclei at rest in medium
- nuclei become unbound ($B_i < 0$) with increasing density of medium
⇒ **dissolution** of nuclei



Mass Shifts III

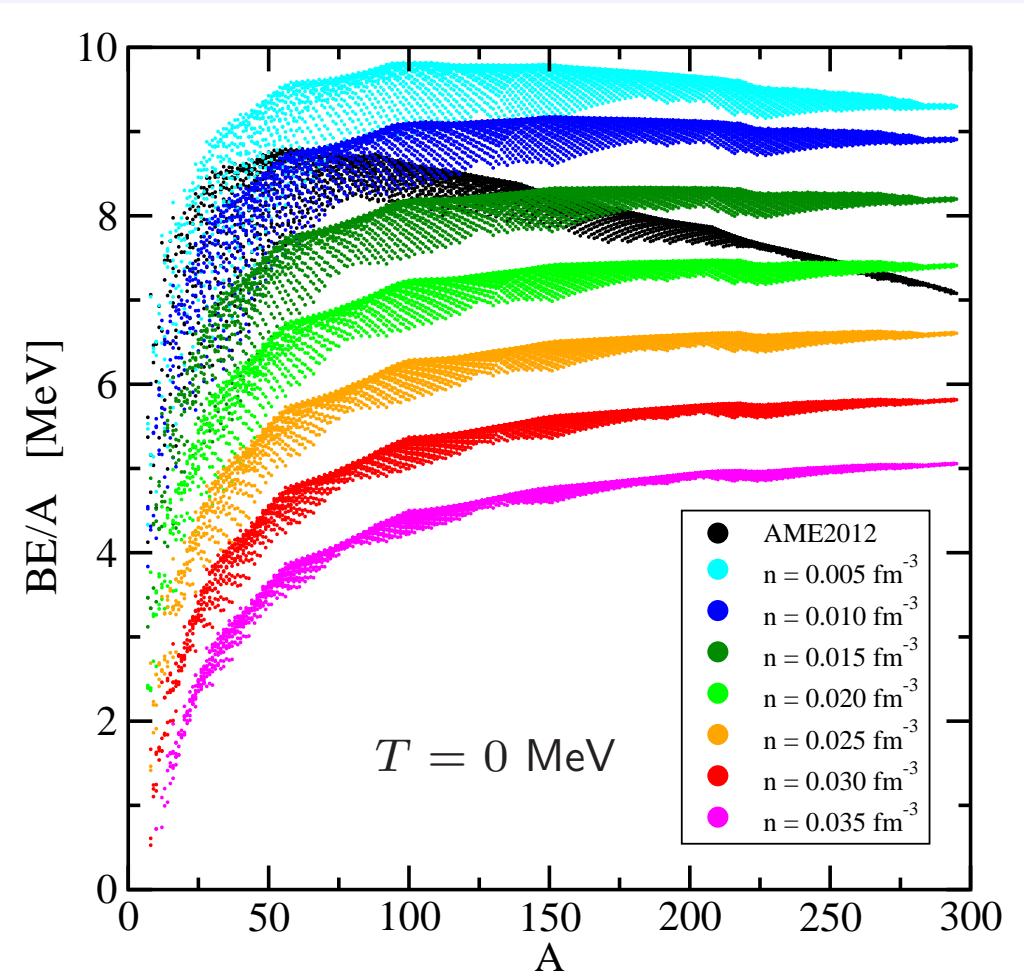
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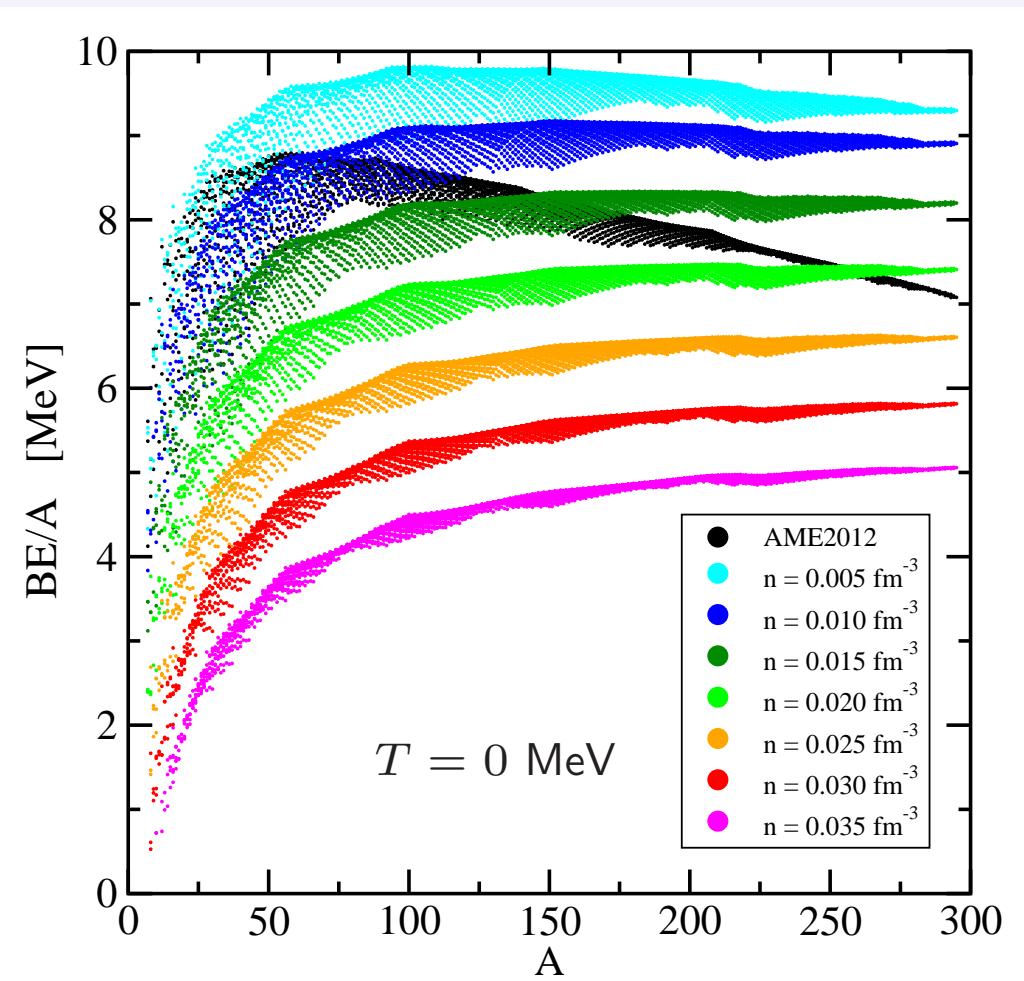
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- preliminary parametrization
⇒ improvement with more systematic
calculations



Chemical Composition of Nuclear Matter

- mass fractions

$$X_i = A_i \frac{n_i}{n_B} \quad n_B = \sum_i A_i n_i$$

- low densities:

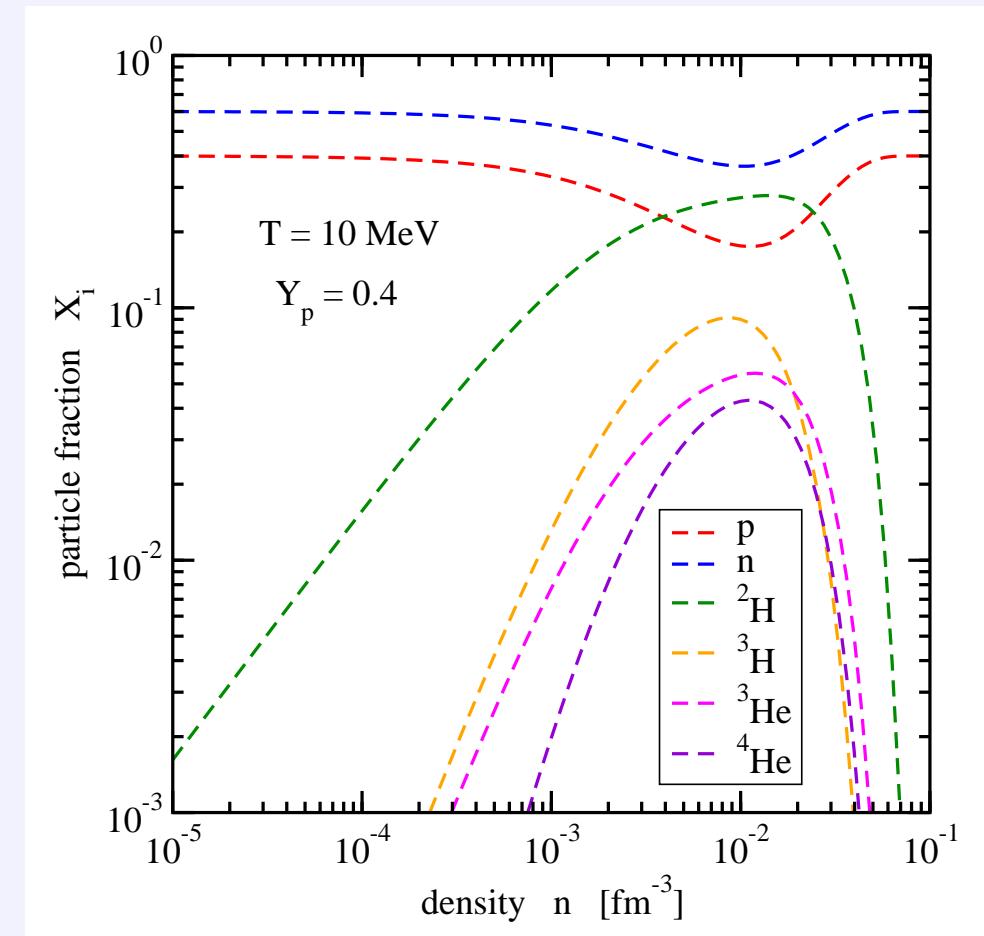
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⇒ Mott effect

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(without heavy clusters)

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- effect of NN continuum correlations

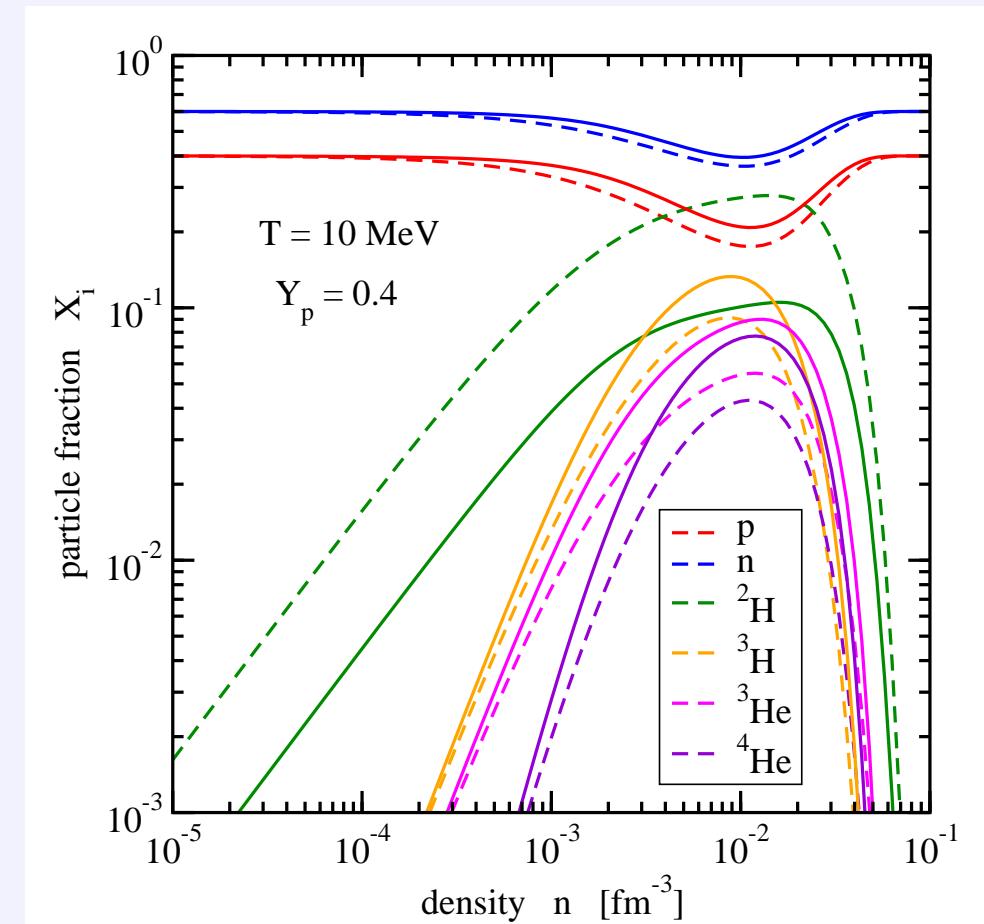
- dashed lines: without continuum

- solid lines: with continuum

⇒ reduction of deuteron fraction,
redistribution of other particles

- essential for correct low-density limit

generalized relativistic density functional

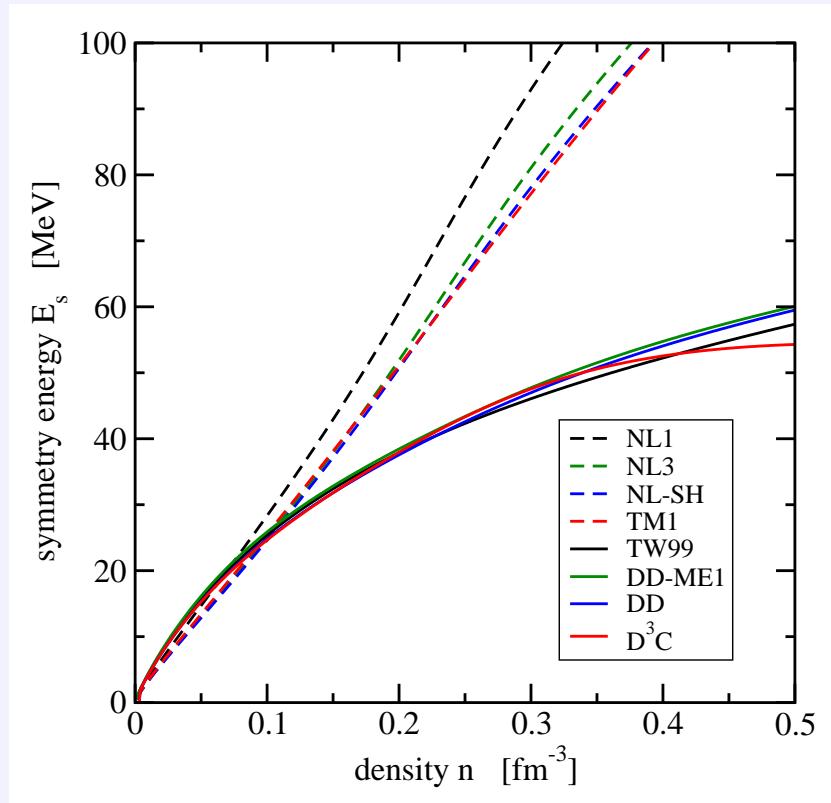


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Symmetry Energy and Neutron Skins

Density Dependence of the Symmetry Energy

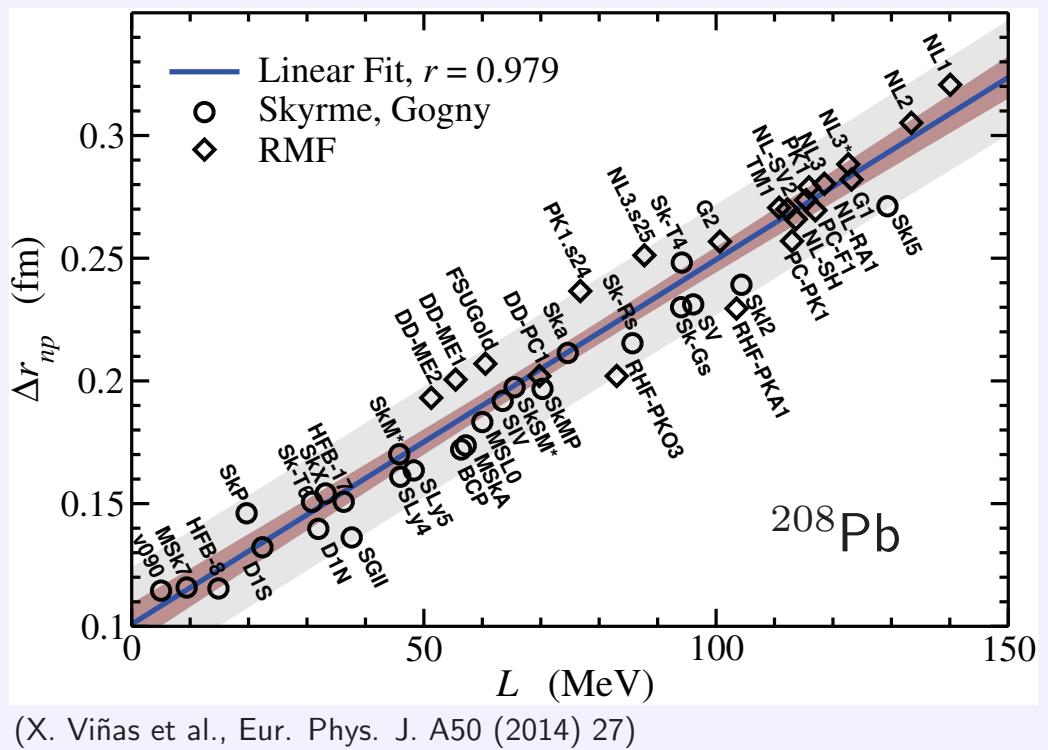
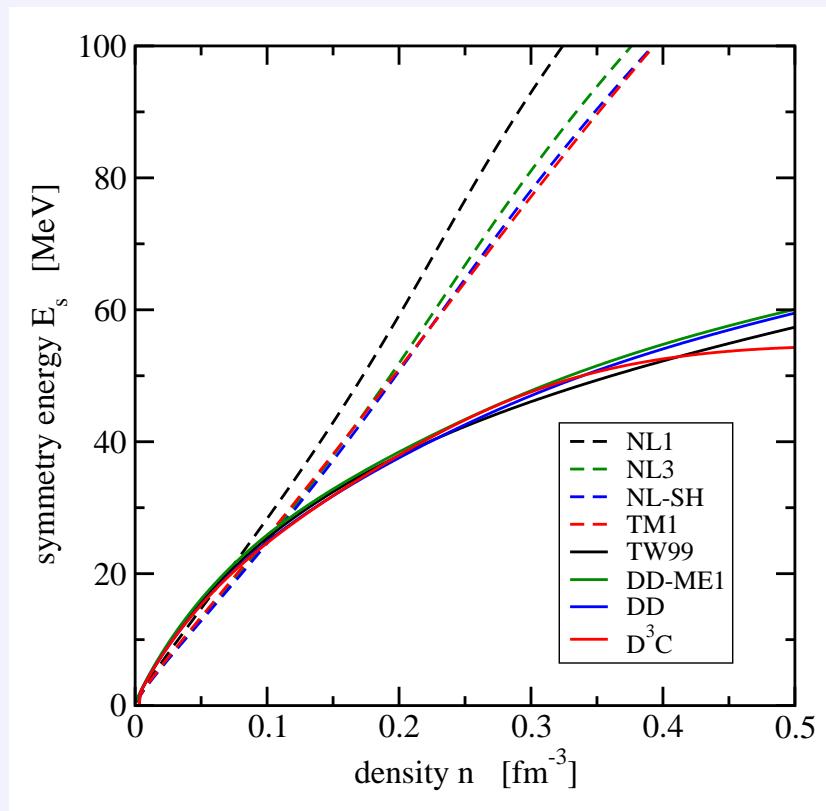
$$E_s(n) = \frac{1}{2} \frac{\partial^2}{\partial \beta^2} \frac{E}{A}(n, \beta) \Big|_{\beta=0} \quad \text{or} \quad \frac{E}{A}(n, 1) - \frac{E}{A}(n, 0) \quad n = n_n + n_p \quad \beta = (n_n - n_p)/n$$



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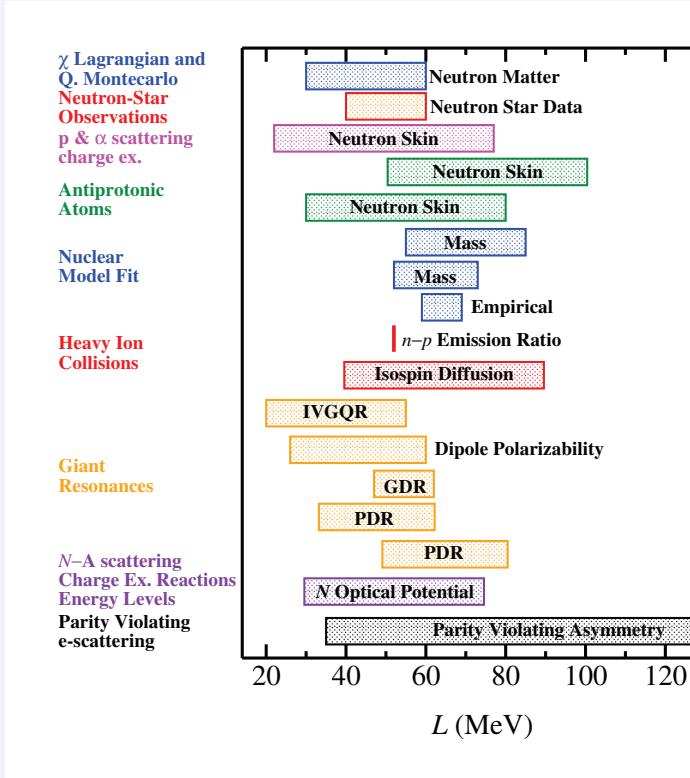
- correlation: neutron skin thickness \Leftrightarrow stiffness of neutron matter EoS \Leftrightarrow slope parameter L of symmetry energy



(X. Viñas et al., Eur. Phys. J. A50 (2014) 27)

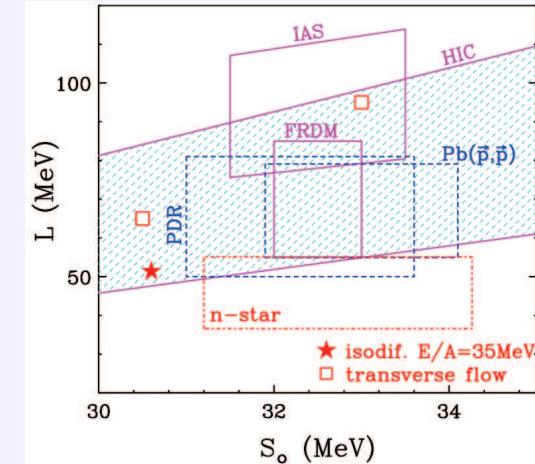
Symmetry Energy Parameters

- many attempts to determine $J = S_0 = S_v$ and L experimentally

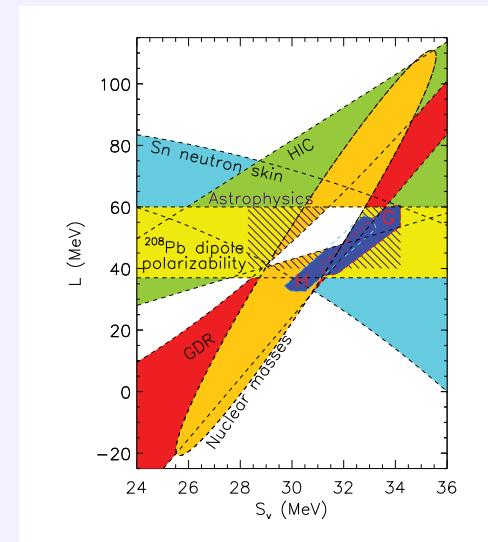


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- measurement of neutron skin thickness (e.g. PREX), correlation with L from mean-field calculations: effects of clustering on surface of nuclei?



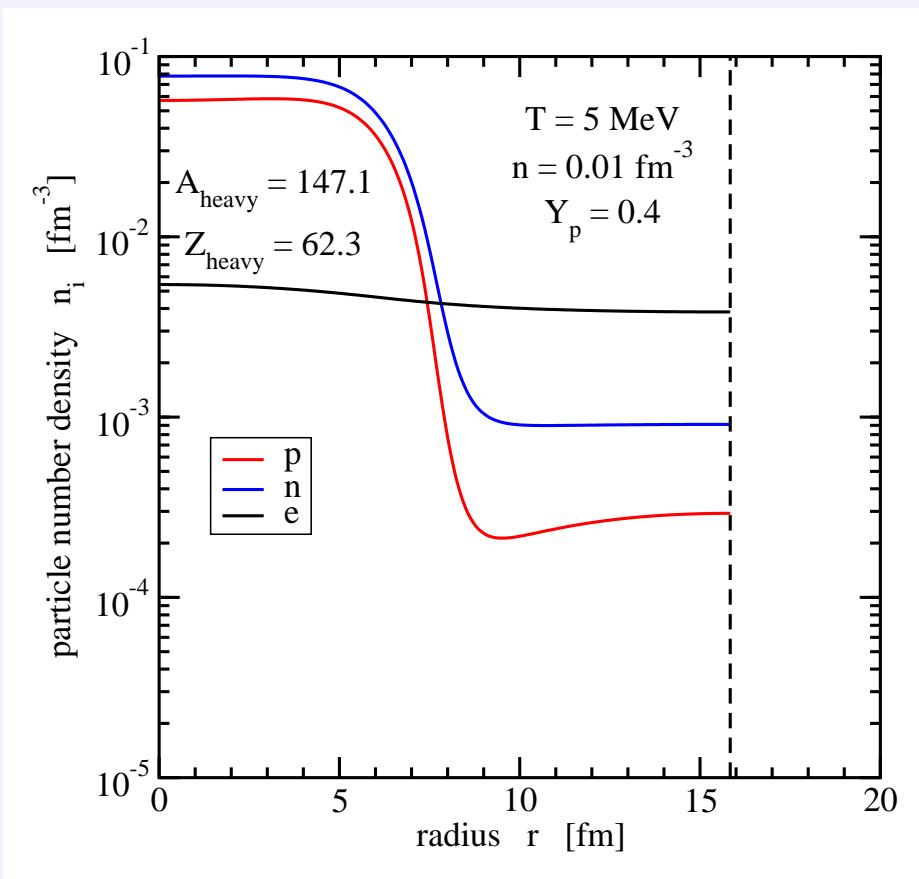
(M.B. Tsang *et al.*, arXiv:1204.0466 [nucl-ex])



(J.M. Lattimer, Y. Lim, ApJ. 771 (2013) 51)

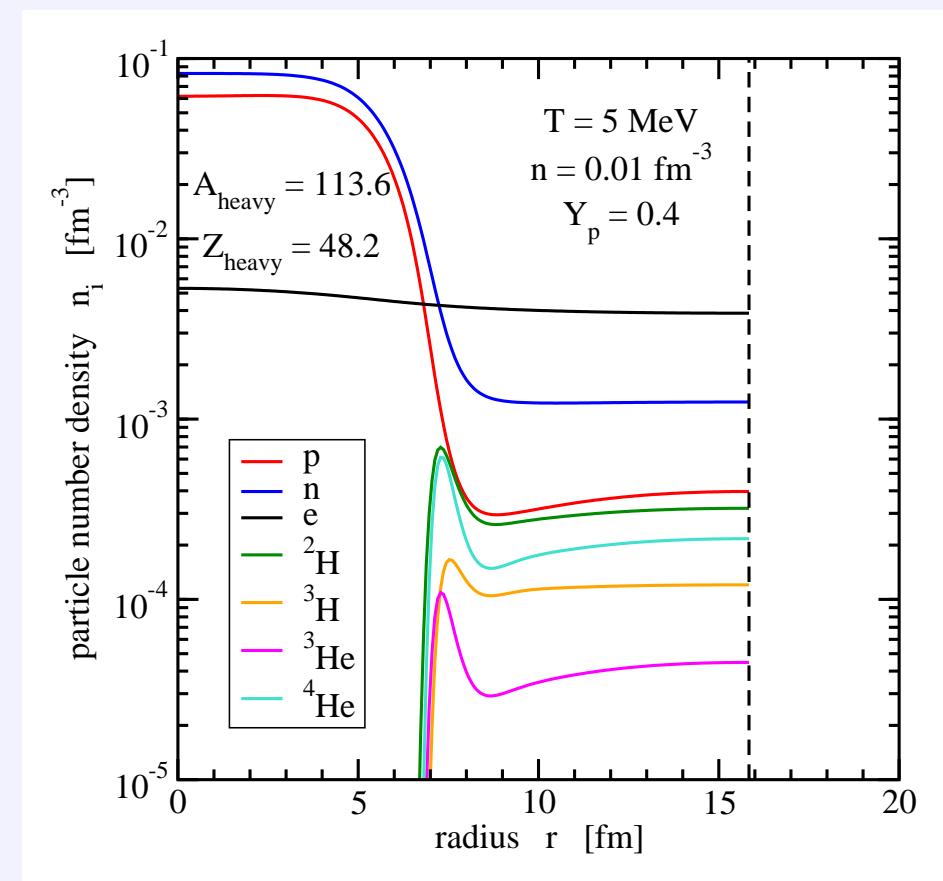
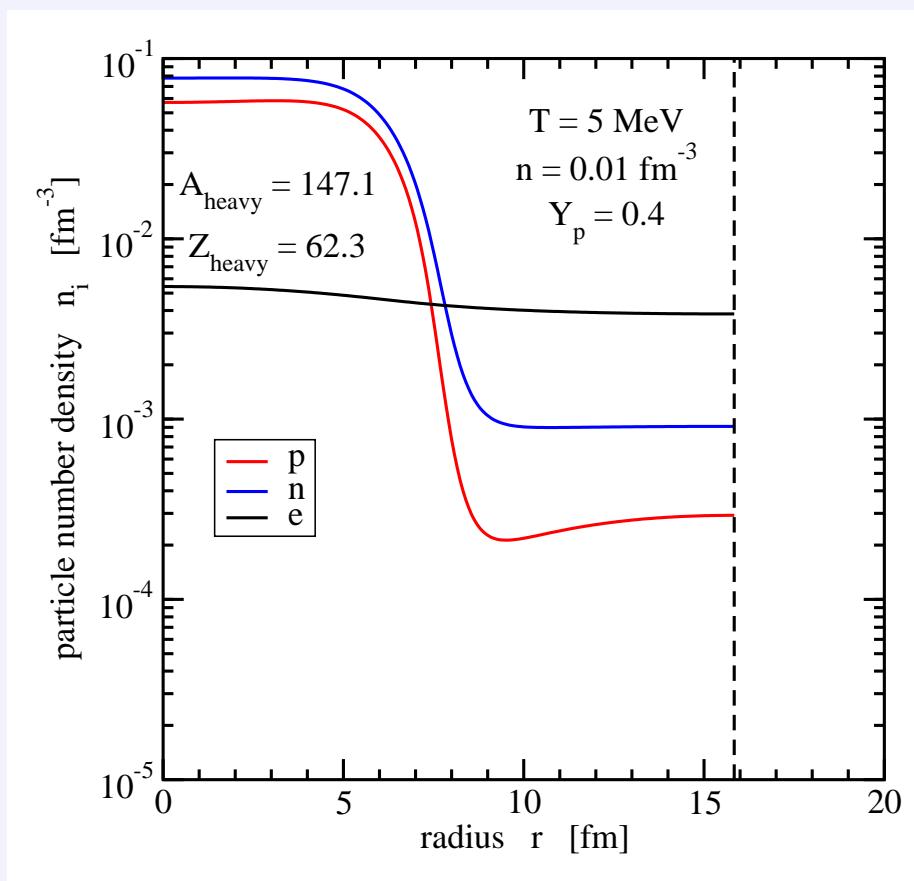
Neutron Skins with Cluster Correlations

- finite temperature gRDF calculations in spherical Wigner-Seitz cell, extended Thomas-Fermi approximation



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effects for **heavy nuclei in vacuum at zero temperature?**



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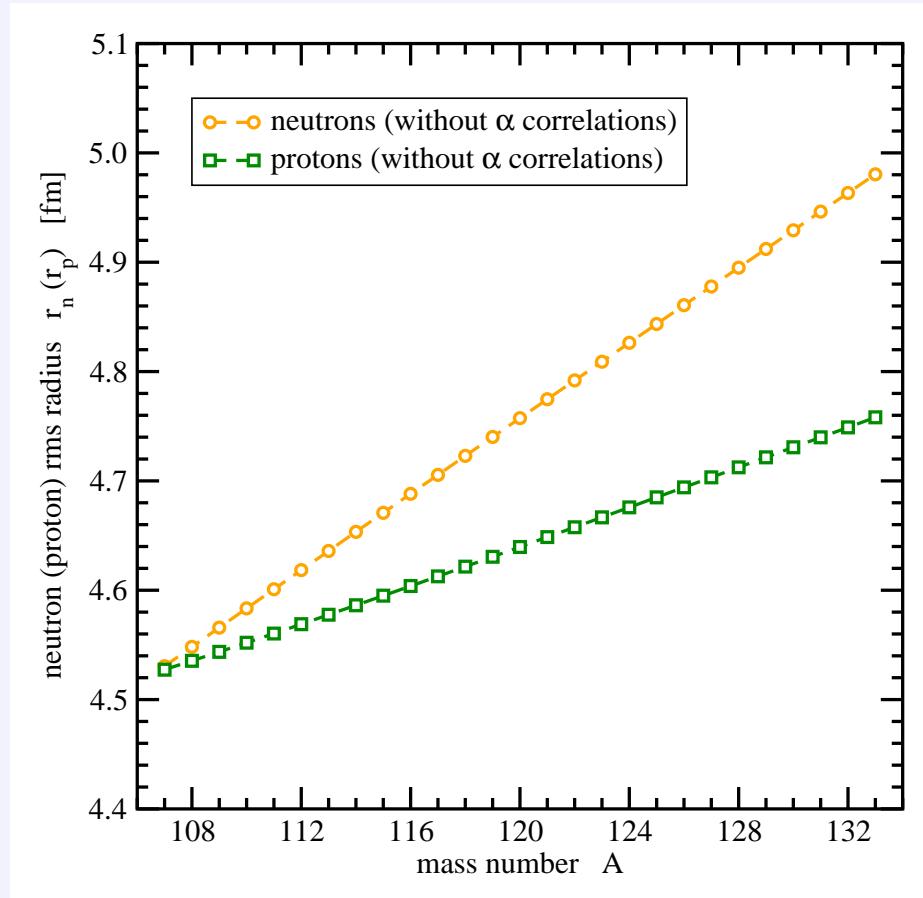
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- variation of isovector interaction ⇒ modified parametrizations

parametrization	symmetry energy J [MeV]	slope coefficient L [MeV]	ρ -meson coupling $\Gamma_\rho(n_{\text{ref}})$	ρ -meson parameter a_ρ
DD2 ⁺⁺⁺	35.34	100.00	4.109251	0.063577
DD2 ⁺⁺	34.12	85.00	3.966652	0.193151
DD2 ⁺	32.98	70.00	3.806504	0.342181
DD2	31.67	55.04	3.626940	0.518903
DD2 ⁻	30.09	40.00	3.398486	0.742082
DD2 ⁻⁻	28.22	25.00	3.105994	1.053251

$$\Gamma_\rho(n) = \Gamma_\rho(n_{\text{ref}}) \exp \left[-a_\rho \left(\frac{n}{n_{\text{ref}}} - 1 \right) \right]$$

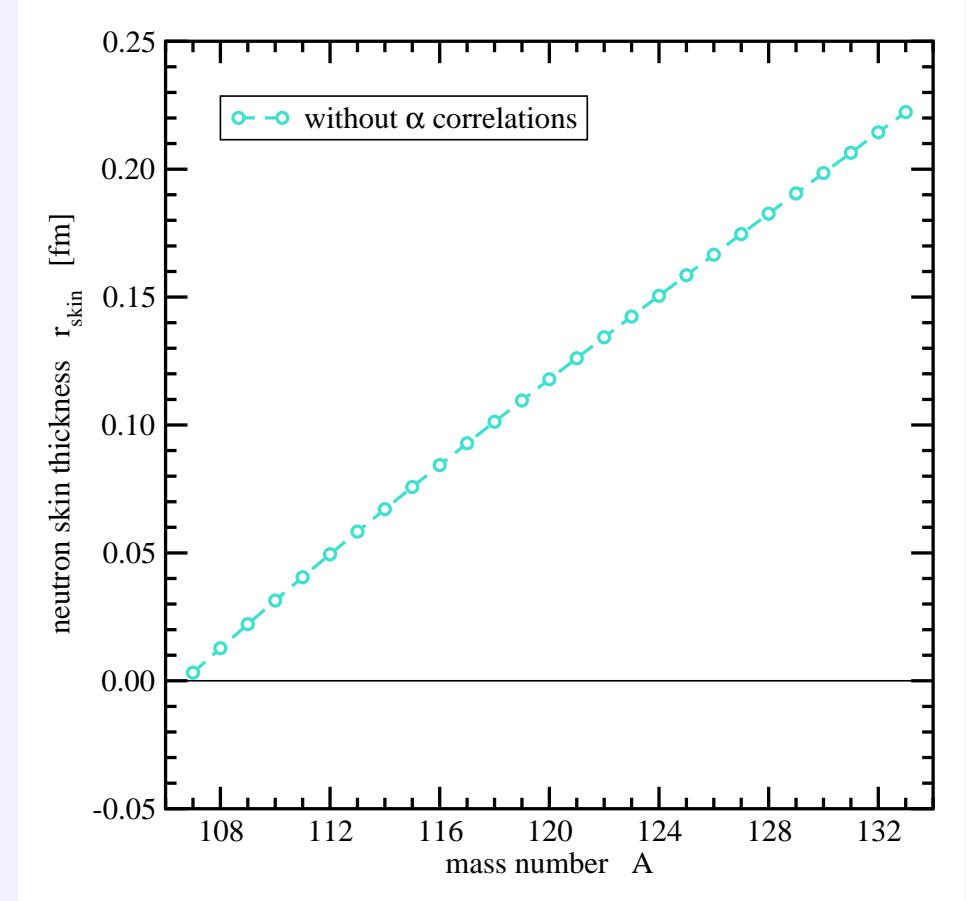
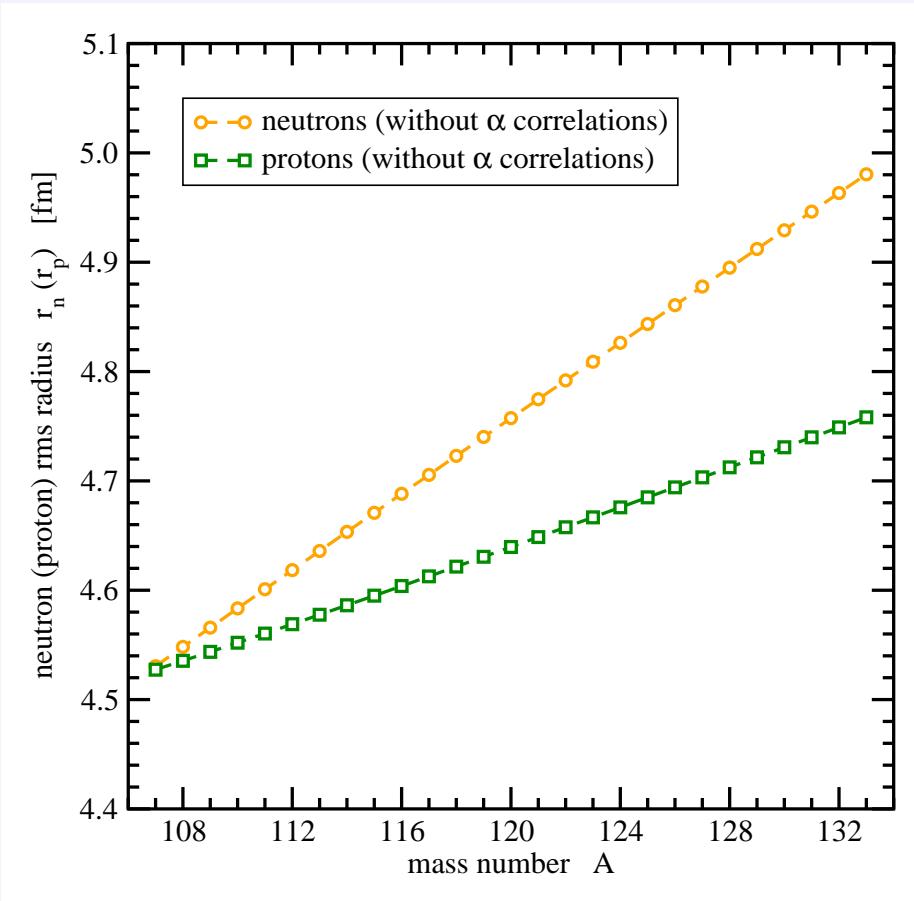
Neutron Skin of Sn Nuclei

- neutron and protons rms radii r_n and r_p



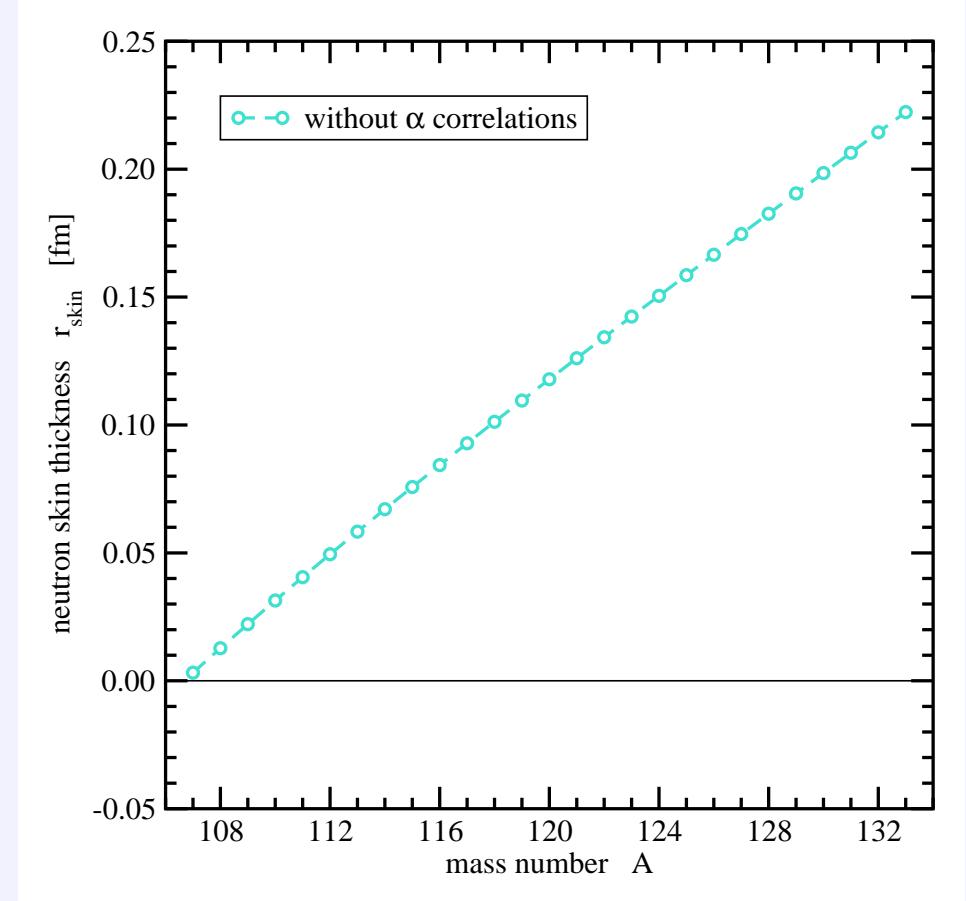
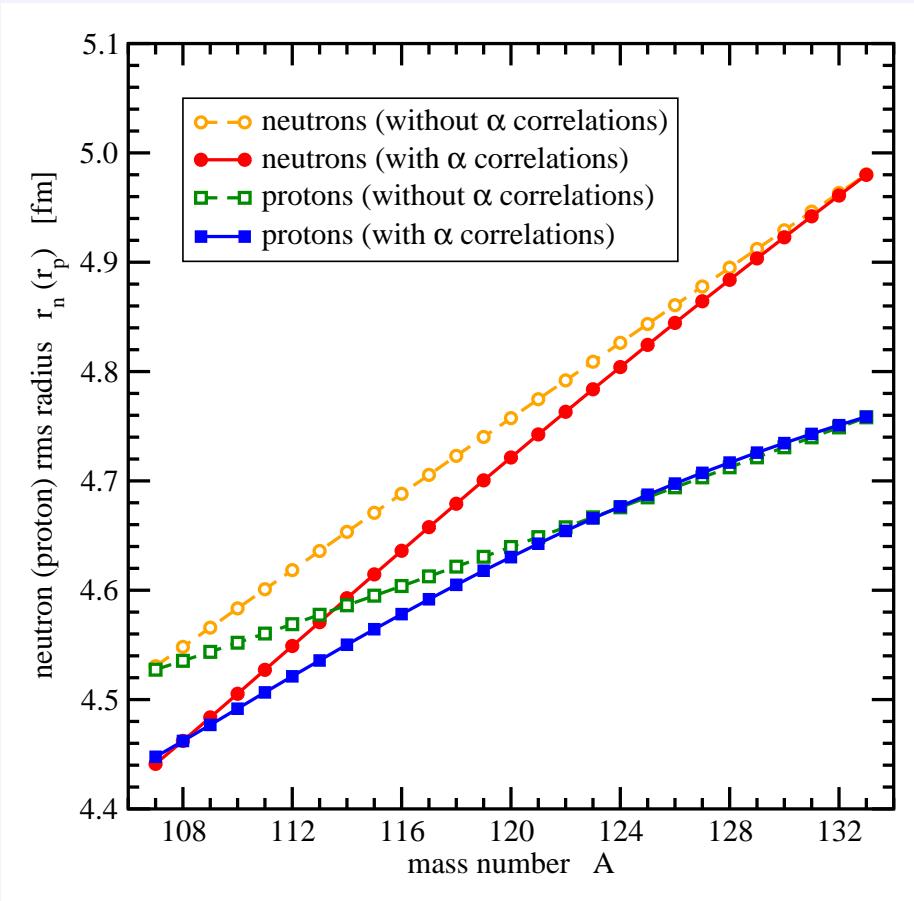
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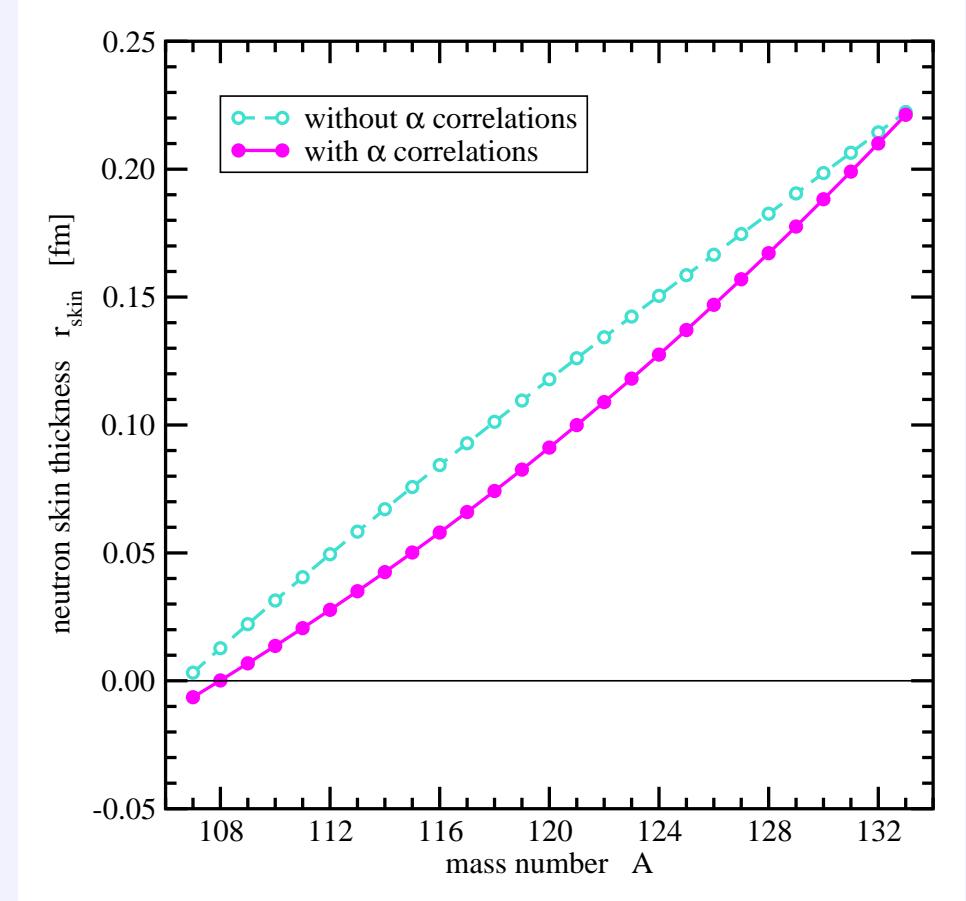
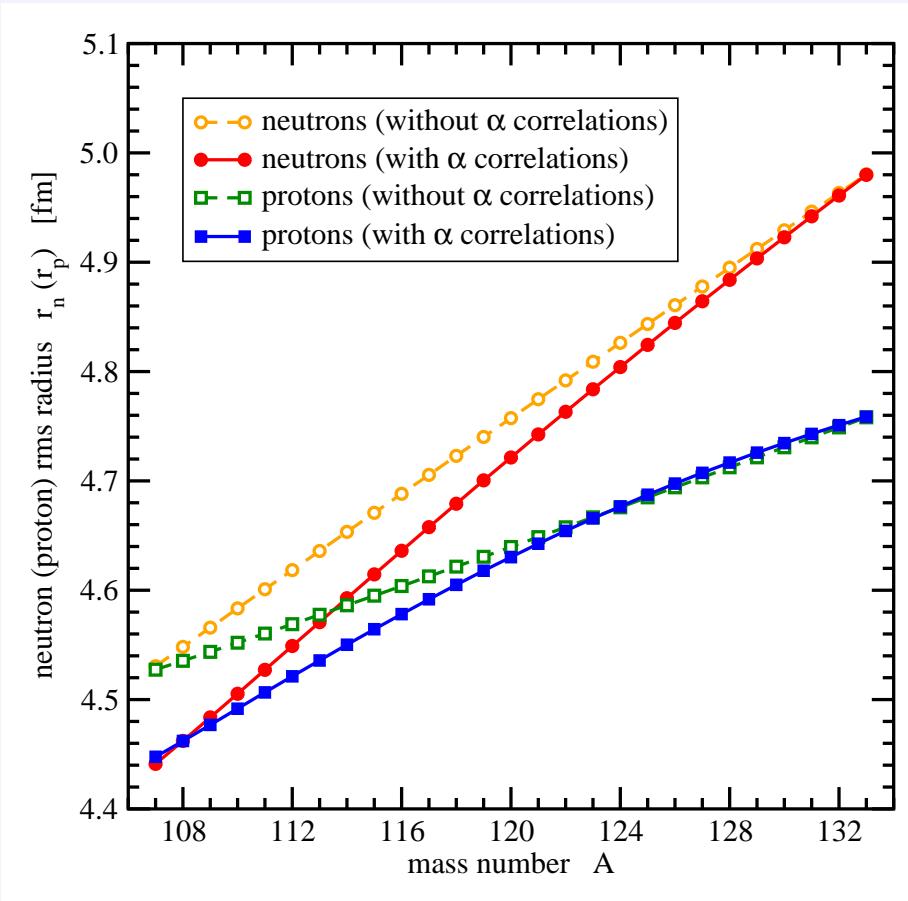
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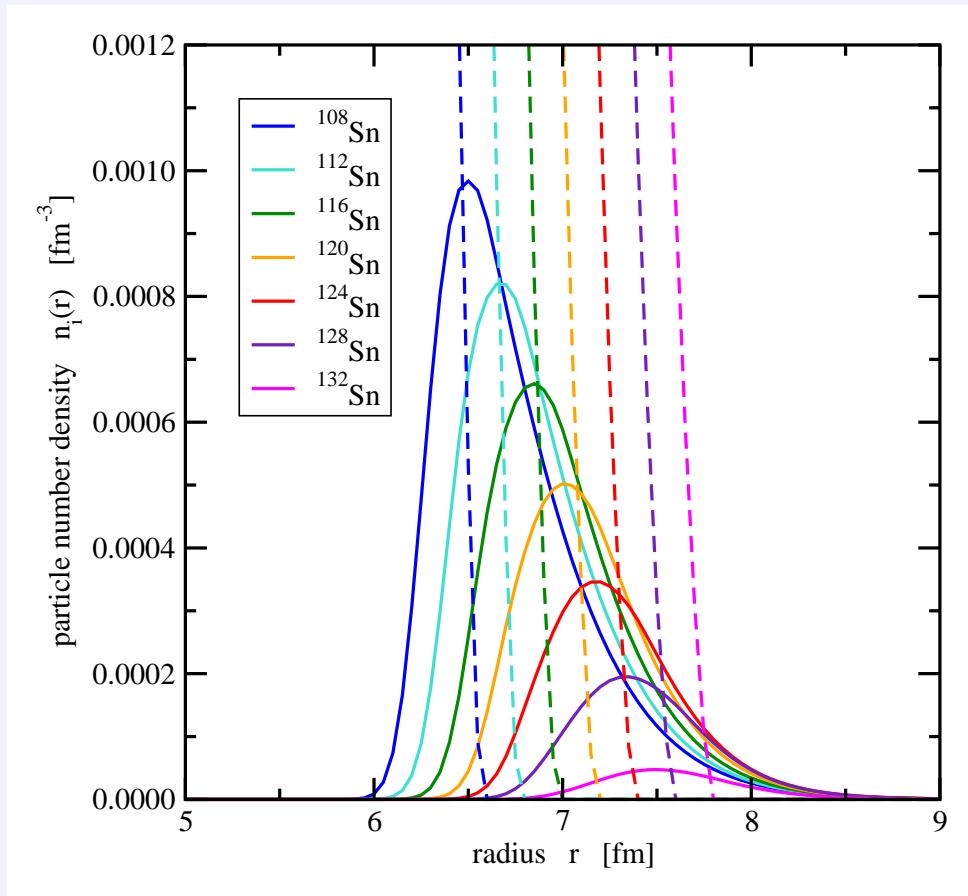
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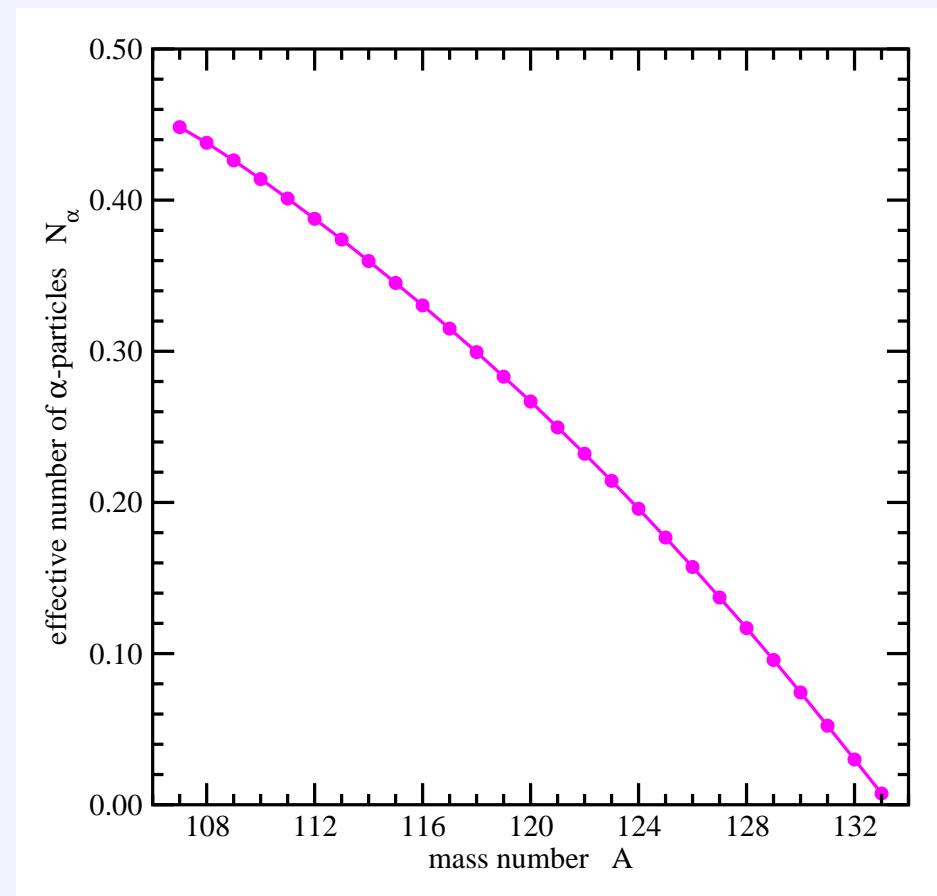
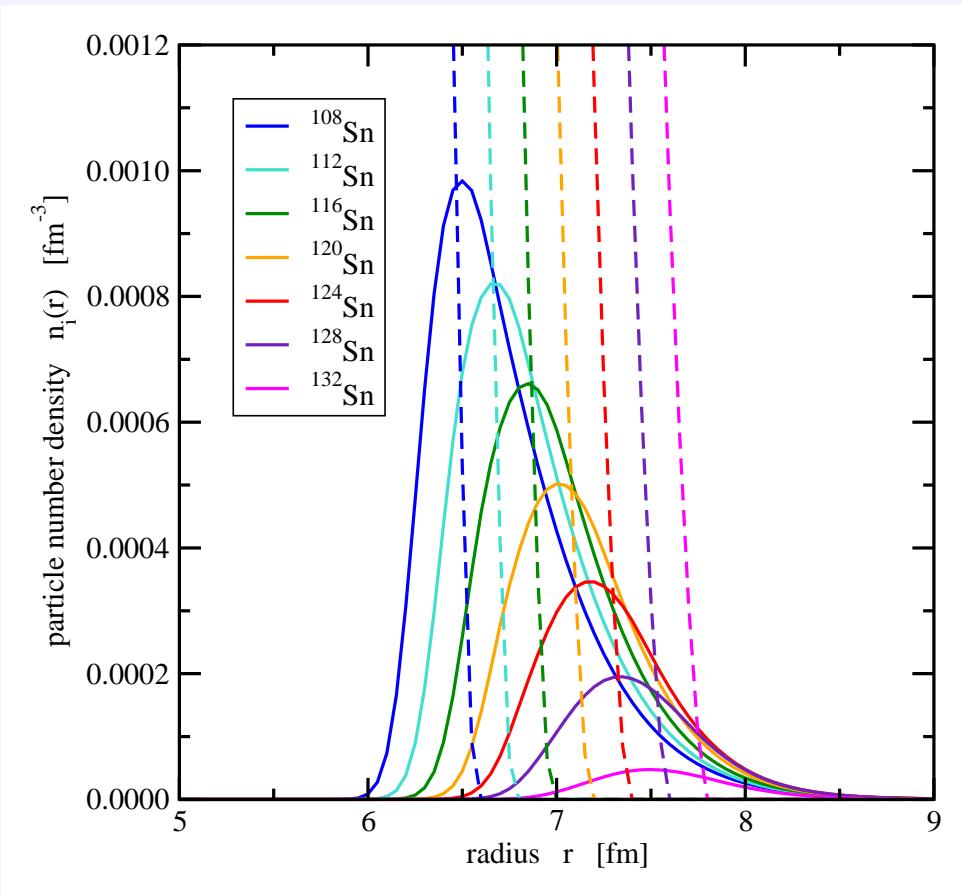
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- density distribution of α -particles (full lines)
- density distribution of neutrons (dashed lines)



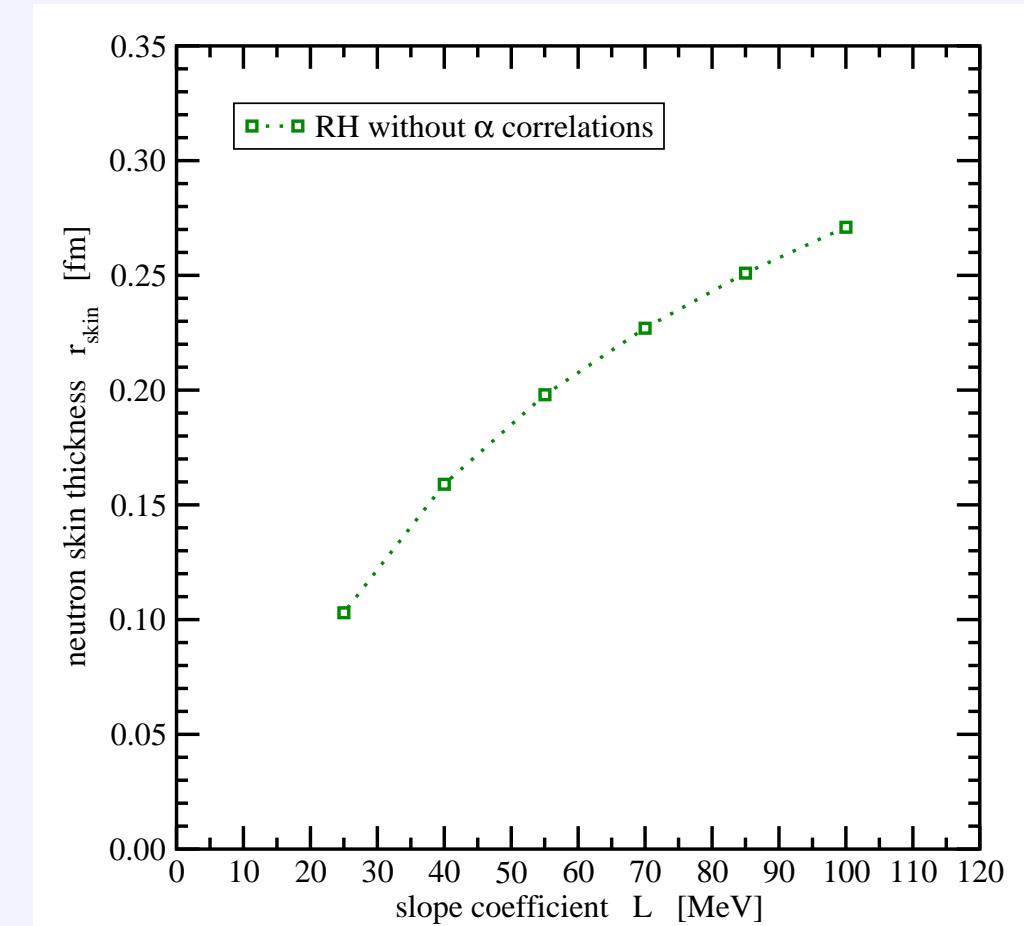
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- effective number of α -particles N_α



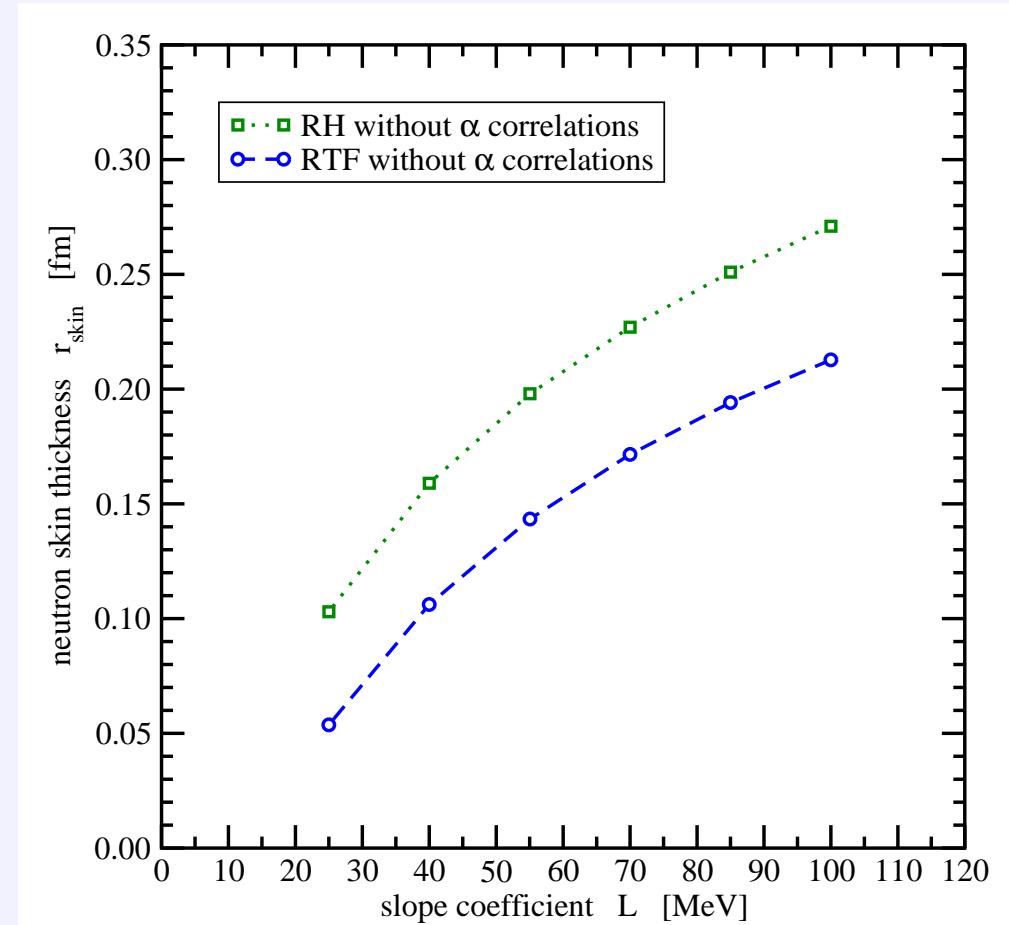
Neutron Skin of ^{208}Pb

- dependence on symmetry energy slope coefficient L
⇒ use parametrizations DD2⁺⁺⁺, ..., DD2⁻⁻
- relativistic Hartree (RH) calculation used in original fit of model parameters
(correlation $r_{\text{skin}} \leftrightarrow L$ not linear because no complete refit of model parameters, only of effective isovector interaction)



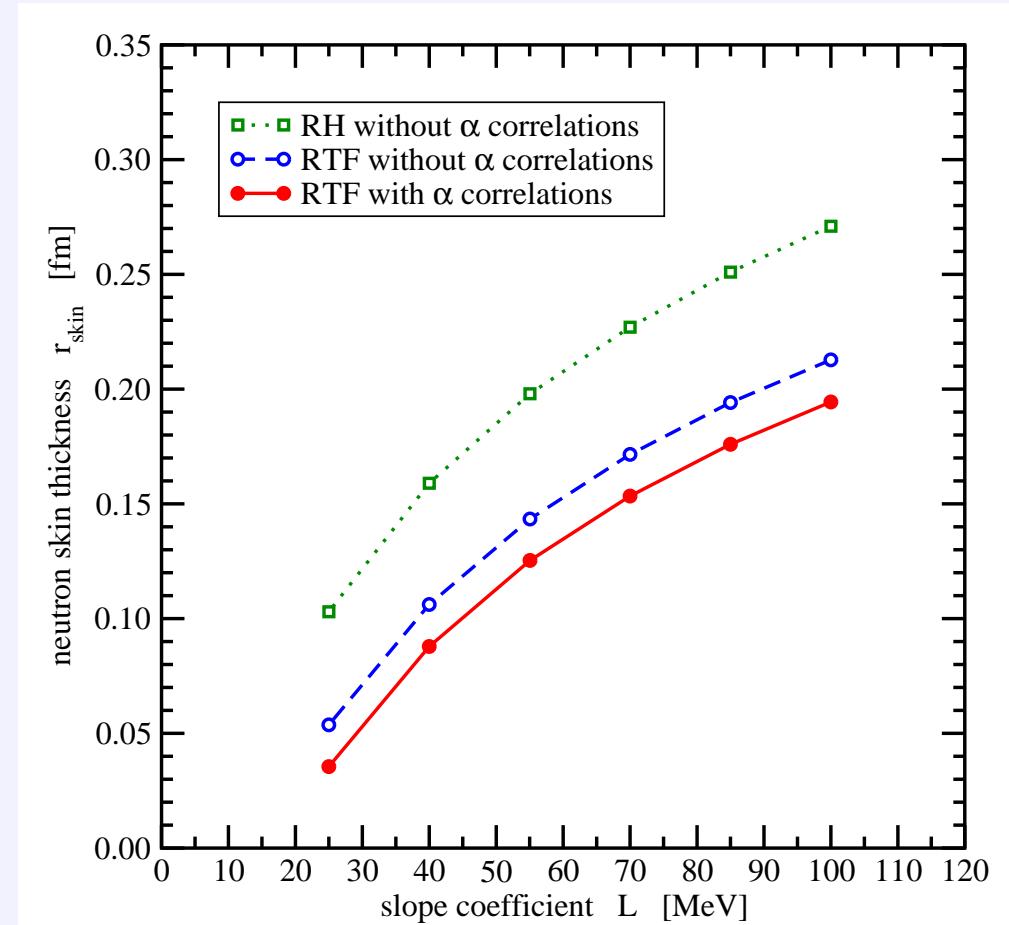
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⇒ underestimate of neutron skin thickness but similar correlation as in RH calculation
- with α -particles at surface
⇒ reduction of neutron skin
⇒ consequences for determination of L from r_{skin} measurements?



Conclusions

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- nuclear/stellar matter: correlations in many-body system essential
⇒ clustering, phase transitions
 - modification of chemical composition and thermodynamic properties
- generalized relativistic density functional for dense matter
 - density-dependent couplings, well-constrained parameters
 - extended set of constituents: explicit cluster degrees of freedom, quasiparticle description
 - medium-dependent properties (mass shifts!) of composite particles
⇒ formation and dissolution of clusters, correct limits
 - thermodynamic consistency ⇒ rearrangement contributions
 - Coulomb correlations ⇒ phase transition to crystal
- applications:
 - equation of state of stellar matter ⇒ astrophysical simulations
 - nuclear structure ⇒ reduction of neutron skin
- future:
 - minor improvements of model ⇒ preparation of global EoS table
 - experimental study of α -particle correlations on surface of Sn nuclei
⇒ experiment with quasifree ($p,p\alpha$) reactions at RCNP Osaka

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- **to you, the audience**

for your attention and patience



Exzellente Forschung für
Hessens Zukunft

Excellence Cluster Universe

