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Constraining the equation of state of neutron stars with gravitational wave observations

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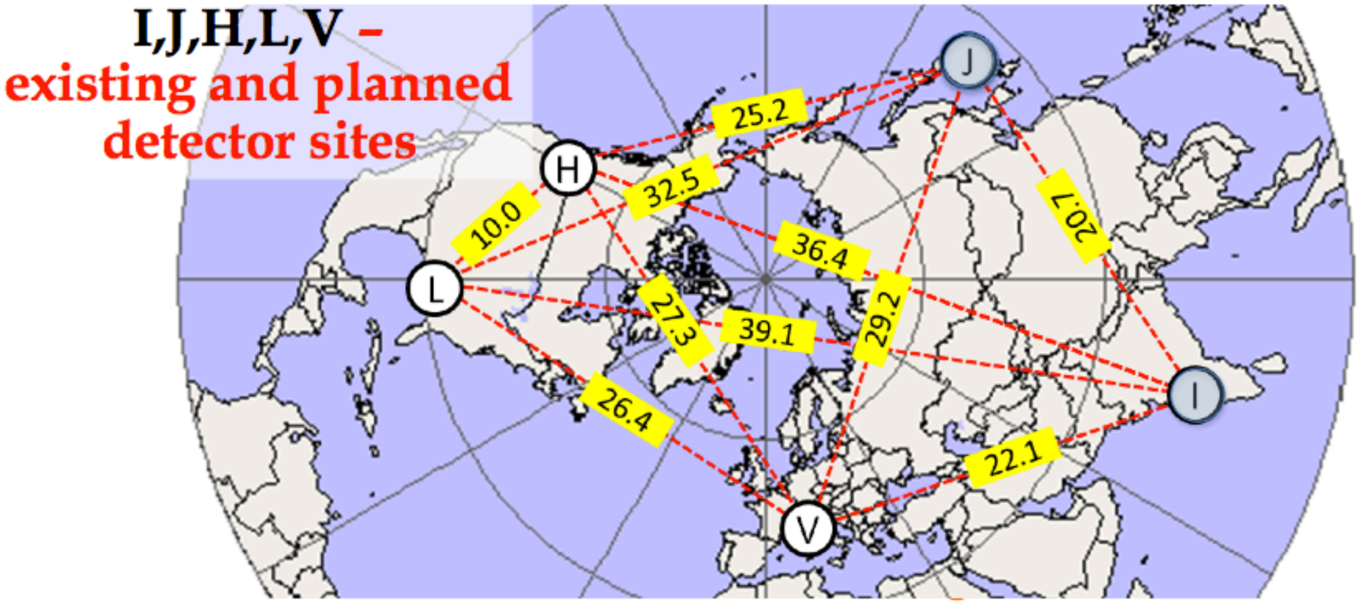
Outline

- The second generation of gravitational waves detectors
- Gravitational wave emission from inspiral of compact binary systems
  - Matter effects
- Inferring the tidal deformability of neutron stars
- Systematics
- Summary

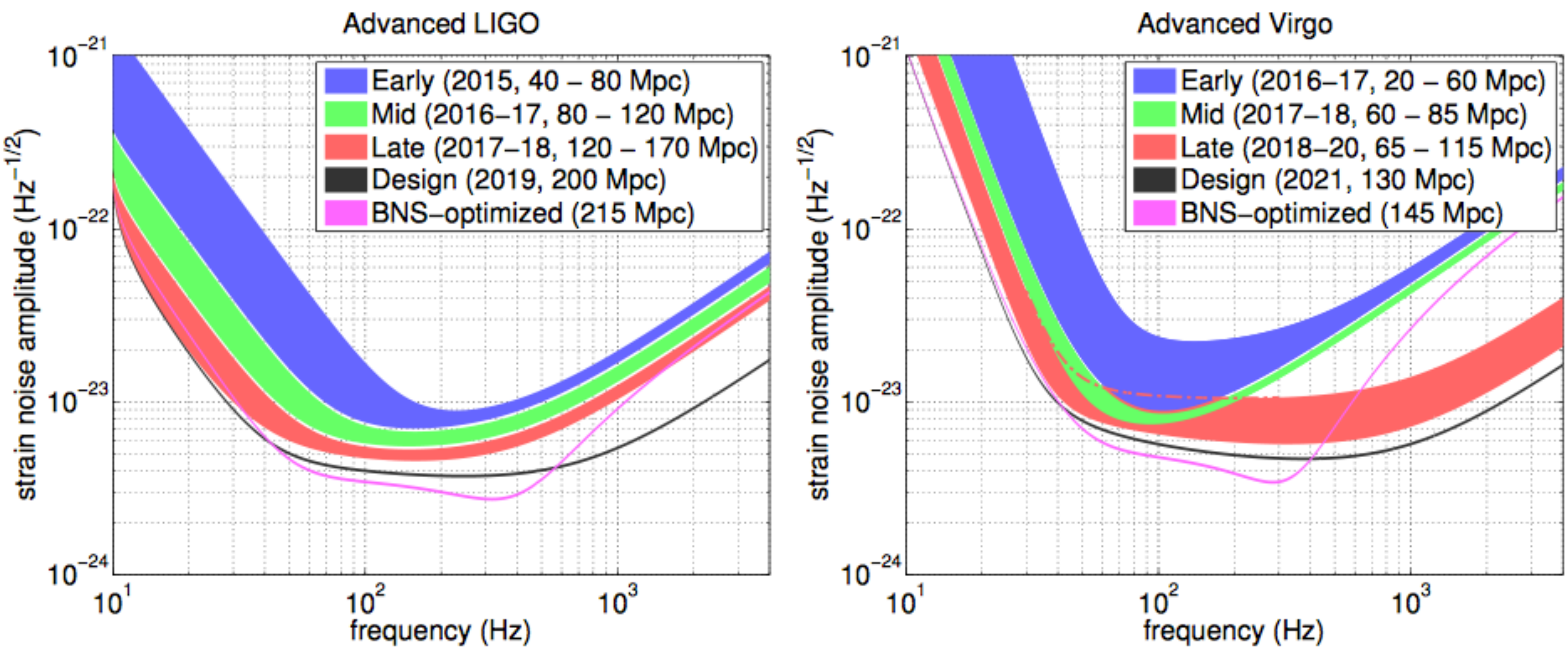
The gravitational wave detectors network



- Two LIGO instruments in Hanford, WA and Livingstone, LO
- Virgo in Cascina, IT
- LIGO India, IN, and KAGRA, JP, will follow

Image Credit: Klimenko, S.

Expected sensitivity



Abadie et al, arxiv:1304.0670

Detection rates

- Horizon distance ~ 200 Mpc for binary neutron star systems (BNS)
- BNS coalescences are the most likely sources for first detection

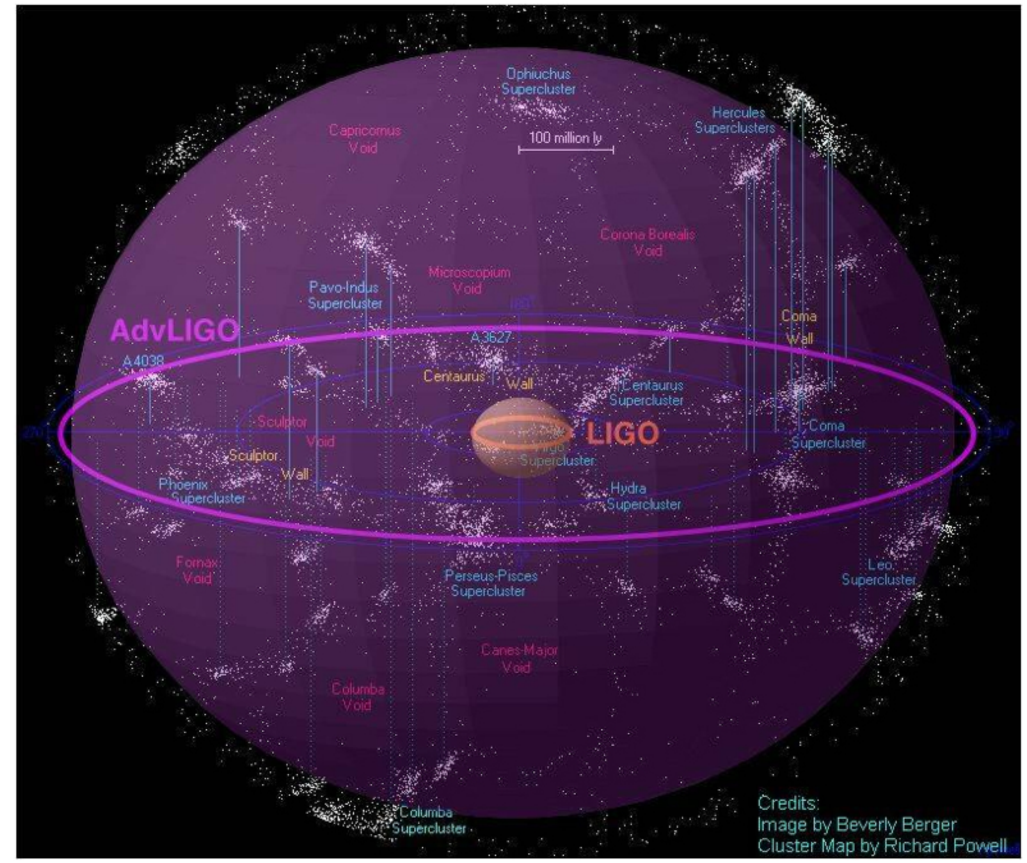
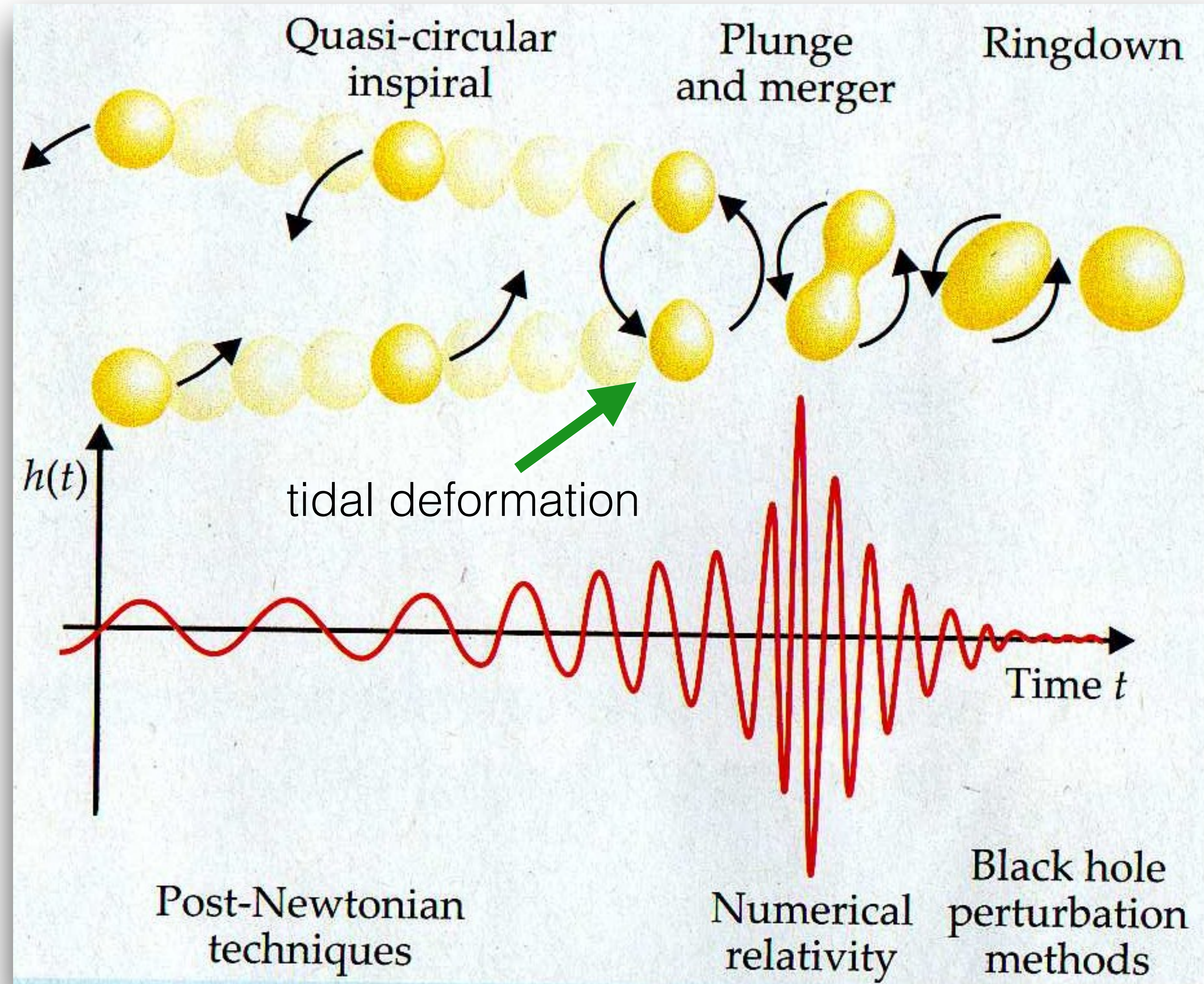


TABLE V: Detection rates for compact binary coalescence sources.

IFO	Source[a]	$N_{\rm low}$ yr^{-1}	$N_{\rm re}$ yr^{-1}	$N_{\rm high}$ yr^{-1}	$N_{\rm max}$ yr^{-1}
Initial	NS-NS	2×10^{-4}	0.02	0.2	0.6
	NS-BH	7×10^{-5}	0.004	0.1	
	BH-BH	2×10^{-4}	0.007	0.5	
	IMRI into IMBH			$< 0.001^{b}$	0.01^{c}
	IMBH-IMBH			$10^{-4\,d}$	$10^{-3\,e}$
Advanced	NS-NS	0.4	40	400	1000
	NS-BH	0.2	10	300	
	BH-BH	0.4	20	1000	
	IMRI into IMBH			10^{b}	300^{c}
	IMBH-IMBH			0.1^{d}	1^{e}

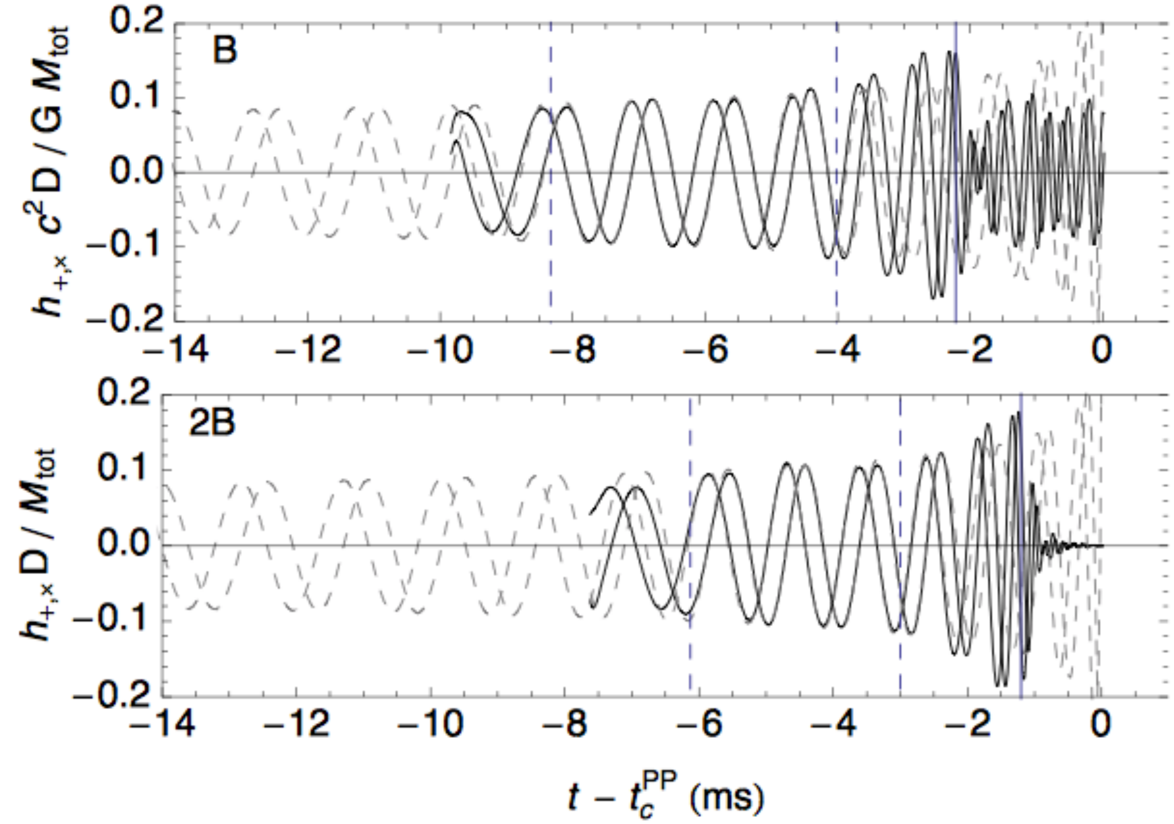
Abadie et al, arxiv:1003.2480

BNS coalescence



Merger

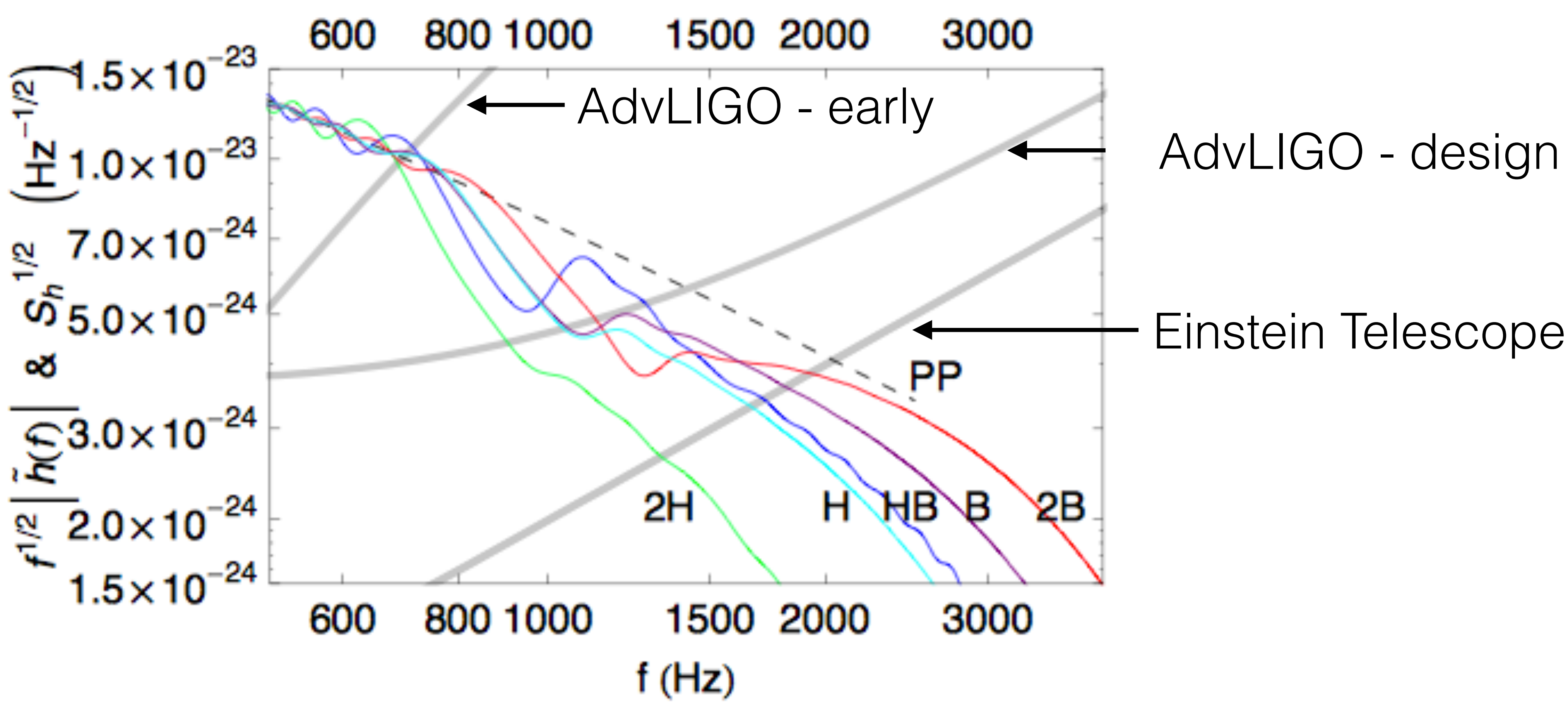
- The merger of a BNS systems yields the most information about the equation of state (EOS)
    - complicated physics
    - little analytical insight
- From numerical simulations:
    - Hard EOS (top): milliseconds lived hypermassive NS then collapse to a BH
    - Soft EOS (bottom): prompt collapse to a black hole (BH)



Read et al, arxiv:0901.3258

Merger

- The merger happens at high frequency (~2000 Hz)
- Poor sensitivity of the detectors



Read et al, arxiv:0901.3258

EOS signature during the inspiral

- The detectors will be sensitive to the inspiral part of the signal

$$h(t) = A(t)\cos(\Phi(t))$$

- Tidal deformations change the phase of the gravitational wave

$$\Phi(t) = \Phi_{\mathrm{PP}}(t) + \Phi_{\mathrm{tidal}}(t)$$

EOS signature during the inspiral

- Tidal effects enter through the tidal deformability

$$Q_{ij} = -\lambda(\mathrm{EOS}; m)\tau_{ij}$$

quadrupole moment

tidal field of companion star

- Tidal deformability function

$$\lambda(m) = \frac{2}{3}k_2 R^5(m)$$

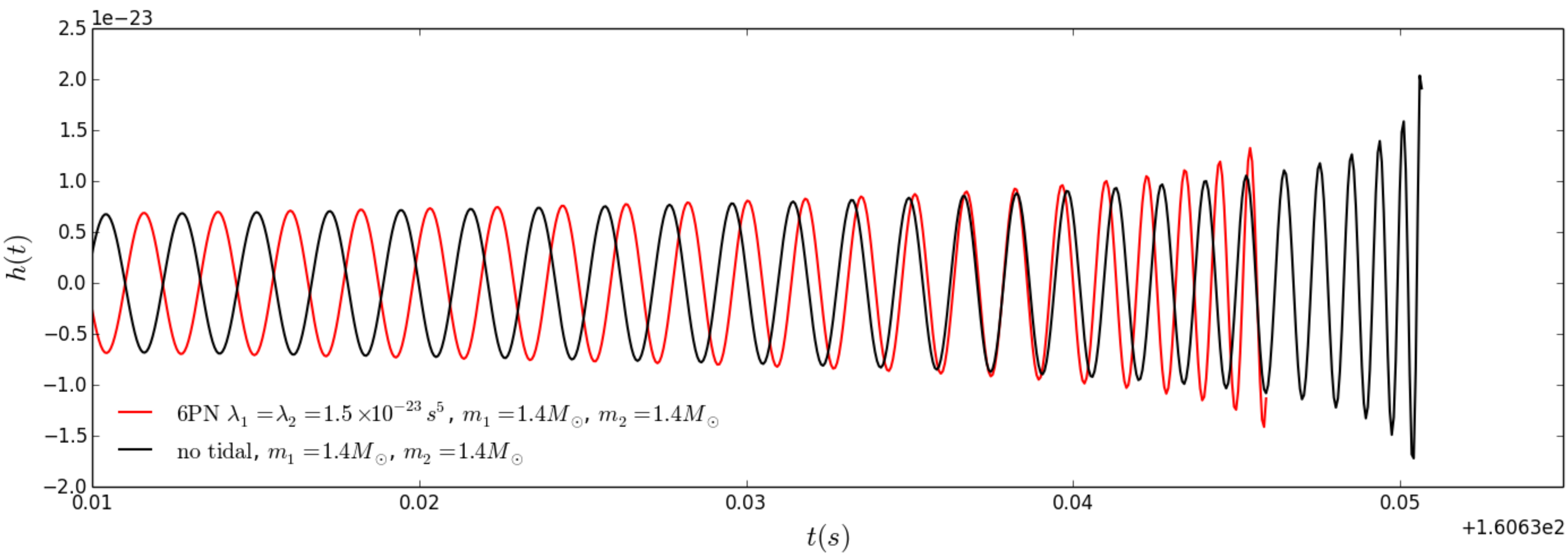
second Love number

neutron star radius

Hinderer et al, arxiv:0711.2420

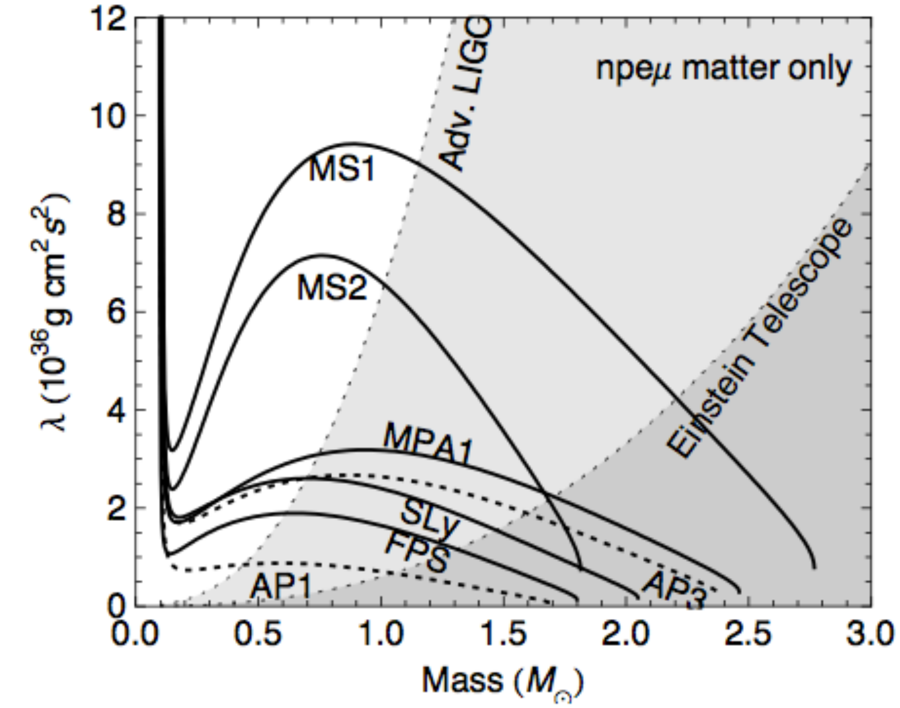
Tidal deformability

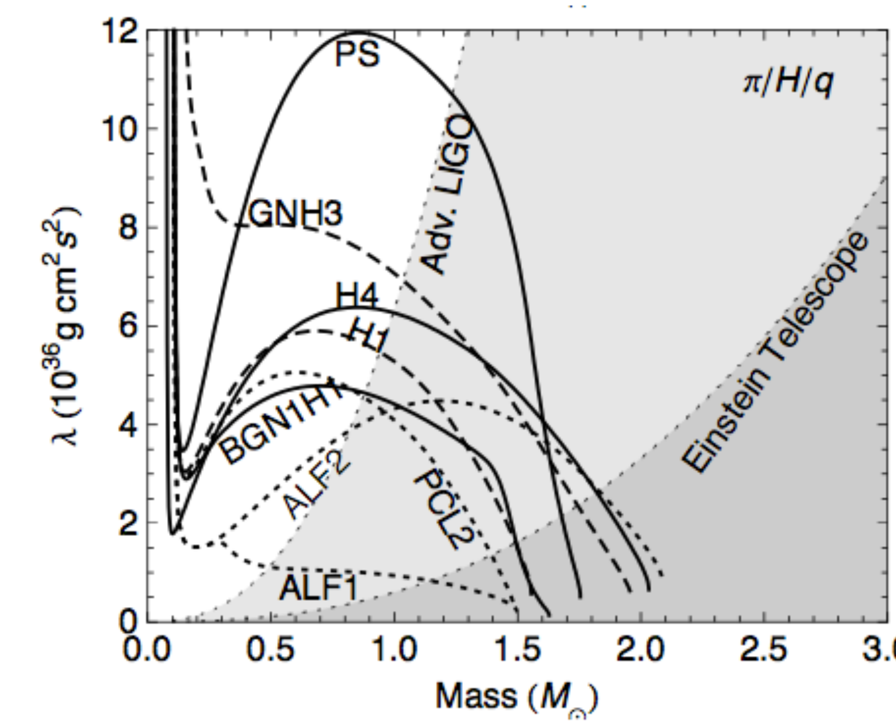
- Tidal effects show at O((v/c)^{10}) (5th post-Newtonian order)
- Known up to O((v/c)^{12}) (6th post-Newtonian order)



Measurability of tidal deformability

- Preliminary studies (based on Fisher information matrix) indicate that adLIGO/Virgo might be able to measure the softest EOS (e.g. Hinderer et al, arxiv:0911.3535)
- Fisher matrix studies find the minimum uncertainty attainable in measuring a parameter
  - Cramer-Rao lower bound





Hinderer et al, arxiv:0911.3535

Measurability of tidal deformability

- Bayesian formalism allows straight-forward computation of constraints from multiple observations:

$$p(EOS|e_1,\ldots,e_n) = p(EOS)\prod_{i=1}^{n}\frac{p(e_i|EOS)}{p(e_i)}$$

- Bayesian analysis of astrophysical population of BNS
  - non-spinning signals analysed in synthetic advanced LIGO/Virgo noise

Del Pozzo et al, arxiv:1307.8338

Series expansion of λ(m)

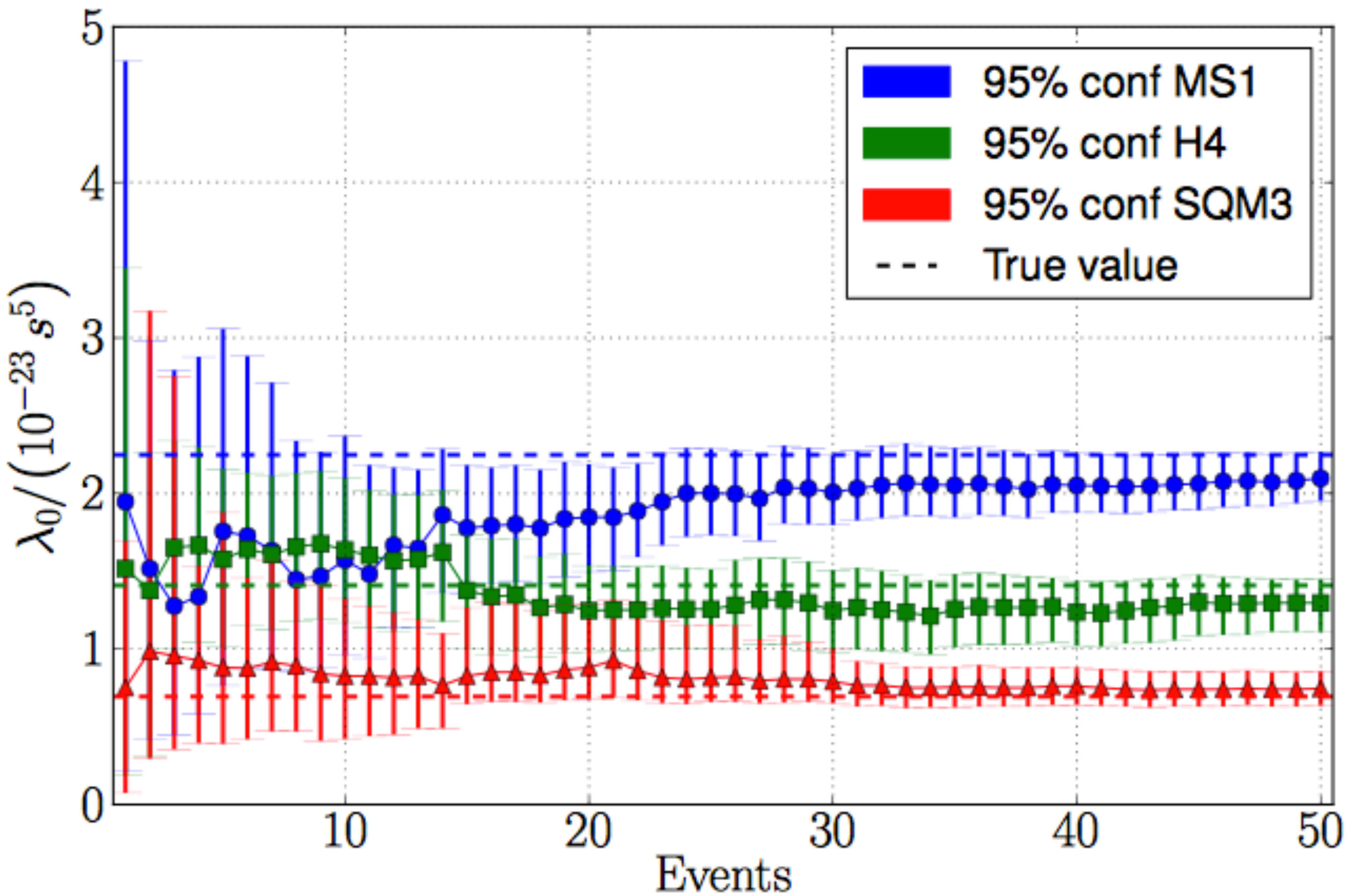
- Inferring directly the function λ(m) is difficult
- Expand it in Taylor series and measure its coefficients:

$$\lambda(m) = \sum_{k=0}^{\infty} \frac{\lambda_k}{k!} \left(\frac{m - m_0}{M_\odot} \right)^k$$

- For a given equation of state, the λ_k are unique, hence the same for all sources

Series expansion of λ(m)

- The zeroth order coefficient at 1.4M$_\odot$ is measurable
- Discriminate among soft, hard and intermediate EOS with O(20) events



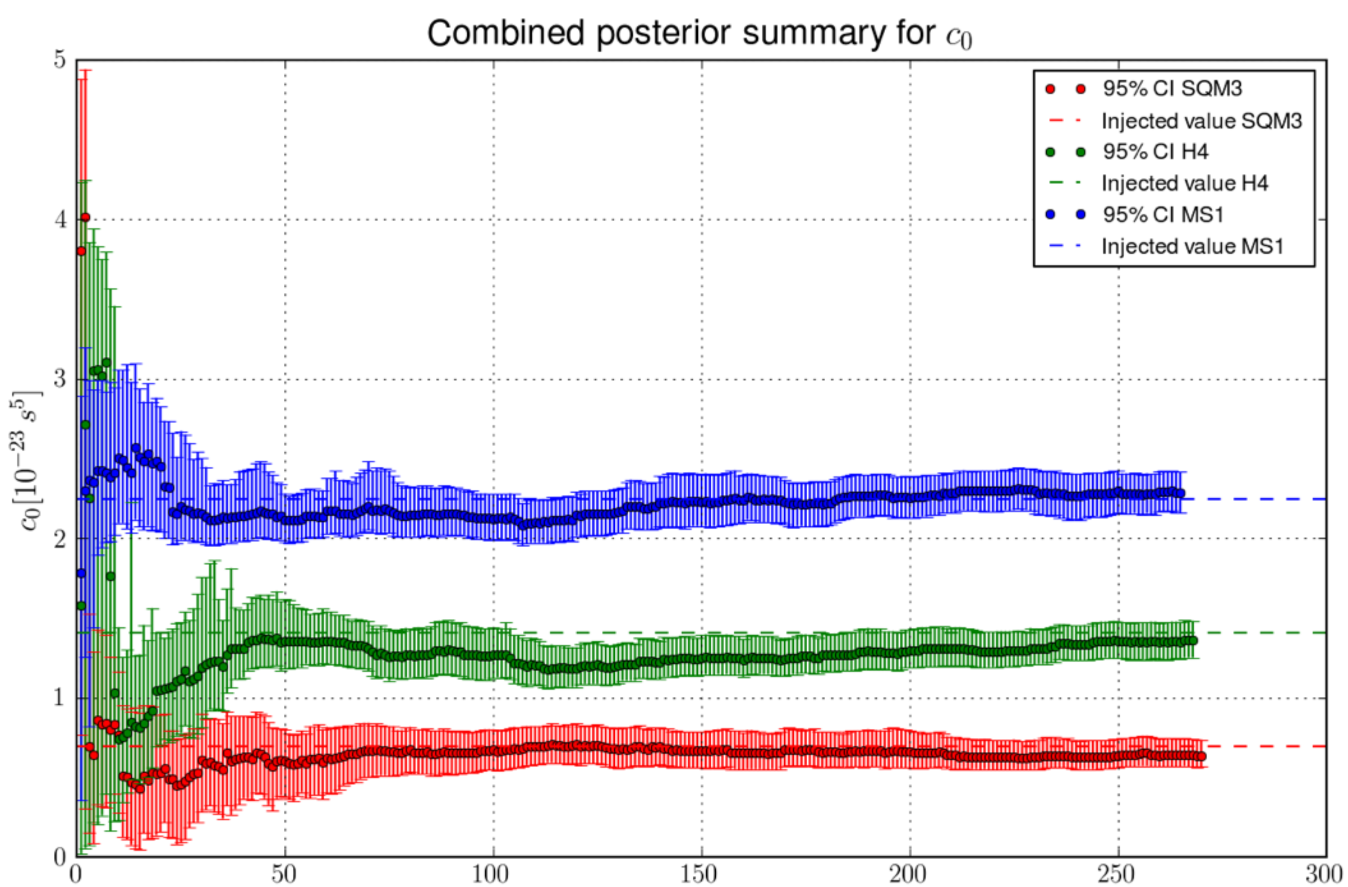
Del Pozzo et al, arxiv:1307.8338

Extensions

- Neutron stars have spins
  - oblate due to their rotation, determined by their EOS
  - it affects their gravitational wave emission
- Depending on the EOS, the inspiral terminates earlier than the last stable orbit
  - contact between the two stars

Extensions

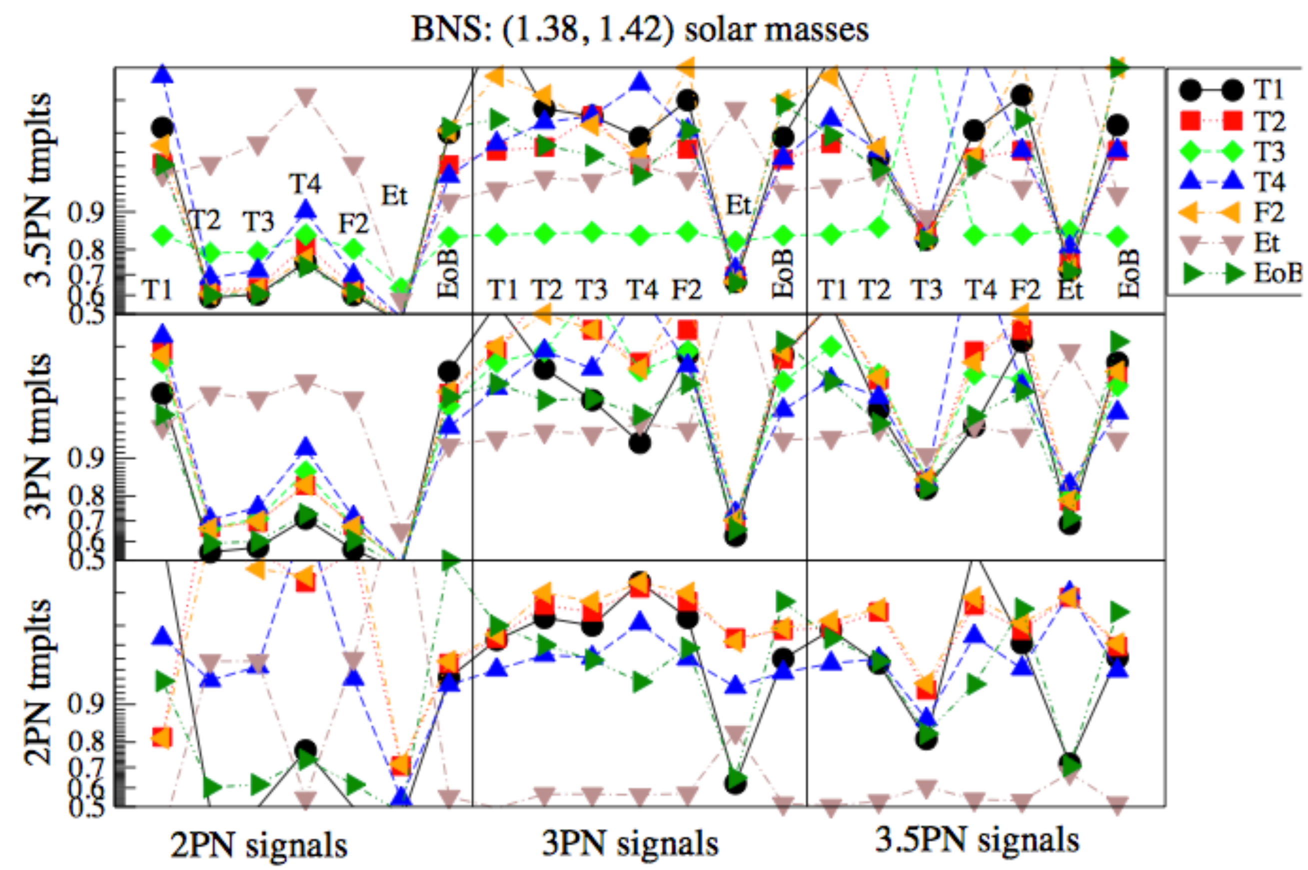
- Depending on the typical spin magnitude, the measurability of λ(m) is degraded
- However < O(50) events seem sufficient for a 95% CL statement



Agathos et al, in preparation

Caveats

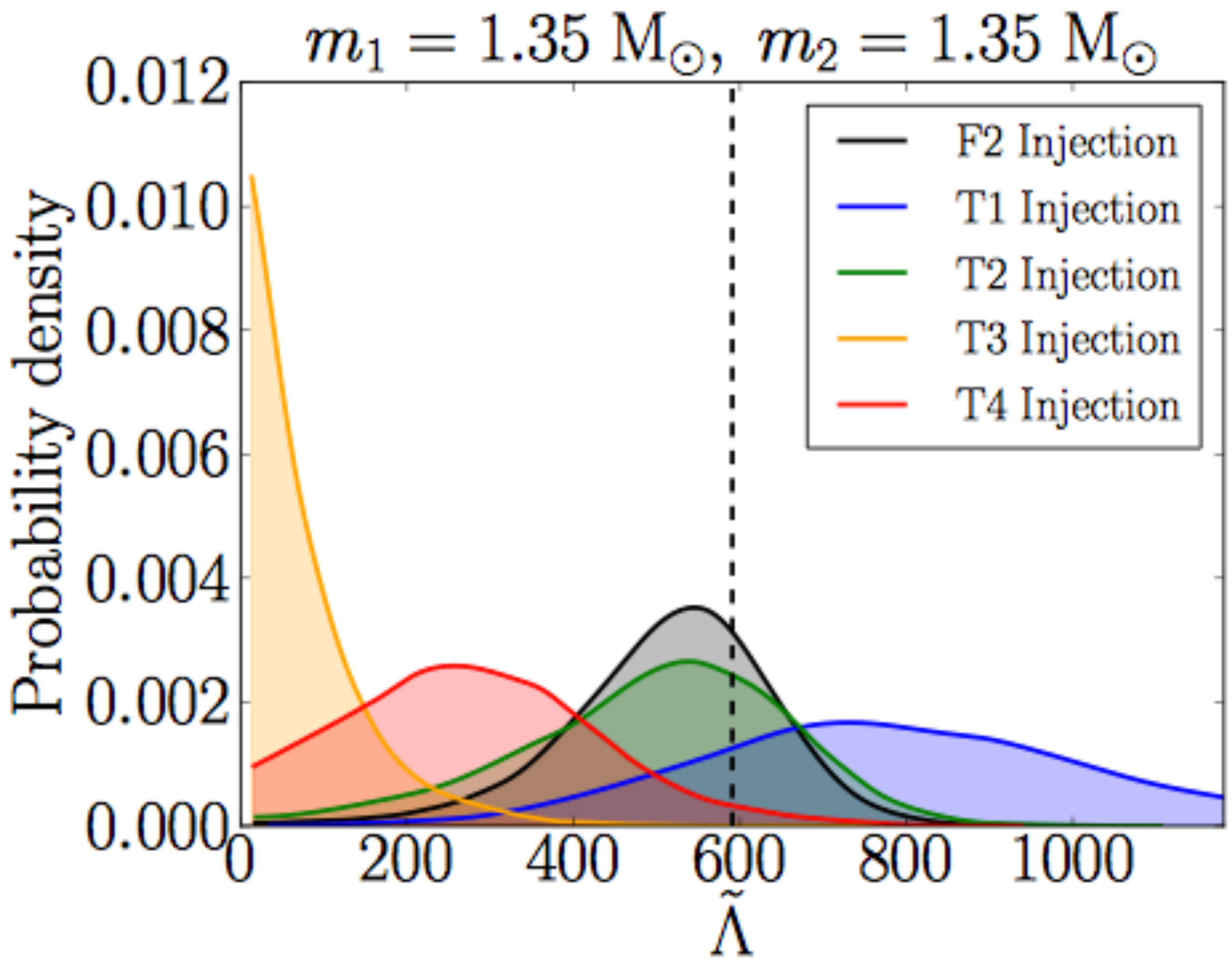
- All studies assume assume that the waveform is known *perfectly*
- This is not the case, already serious uncertainties in the post-Newtonian expansion
- Mismatches in waveform models and unknown high post-Newtonian orders induce systematic effects which hinder our capacity to measure tidal effects (Favata, arxiv:1310.8288)



Buonanno et al,arxiv:0907.0700

Systematic effects

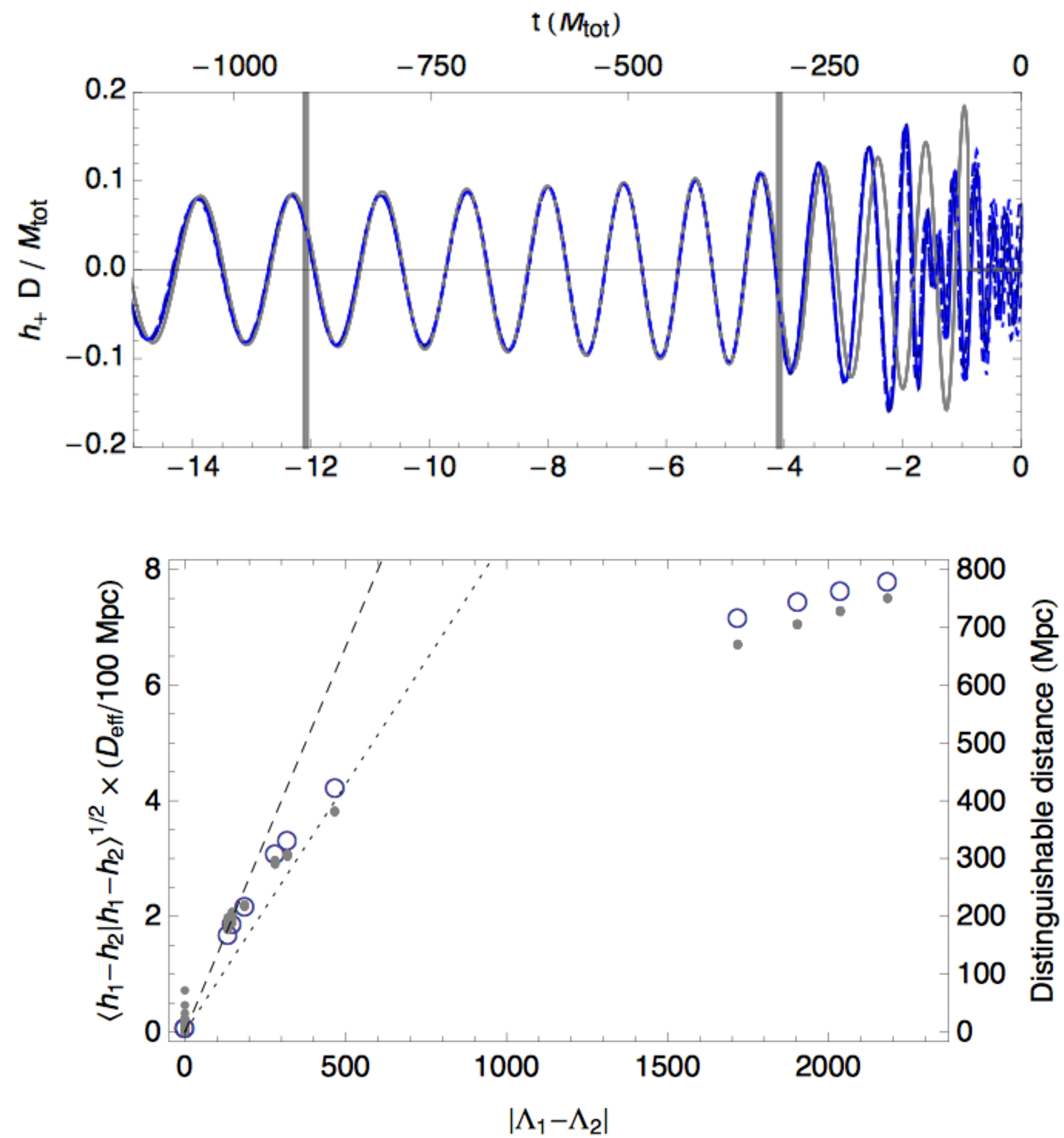
- Bayesian study from Wade et al confirms this picture



Wade et al, arxiv:1402.5156

Hybrid waveforms

- Promising route is the hybridisation of numerical waveforms
  - procedure consists in "stitching" together a low frequency analytical model and a high frequency numerical waveform
- Analysis of the systematics shows that analytical waveforms give the right idea for the accuracy of λ(m) measurement



Read et al, arxiv:1306.4065

Summary

- Gravitational waves can probe the nuclear physics of neutron stars
- Merger and post-merger dynamics potentially offers deepest insights
    - no parametrised model, only numerical
    - hybrid waveforms?
- Inspiral phase yields weaker measurements
    - few tens sources should constrain the tidal deformability function
- Uncertainty in the waveform models
    - Systematics