Probing the occurrence of material structures at subnuclear densities with a dynamical self-consistent description

Virginia de la Mota

SUBATECH - Université de Nantes - In2P3 (CNRS) - EMN

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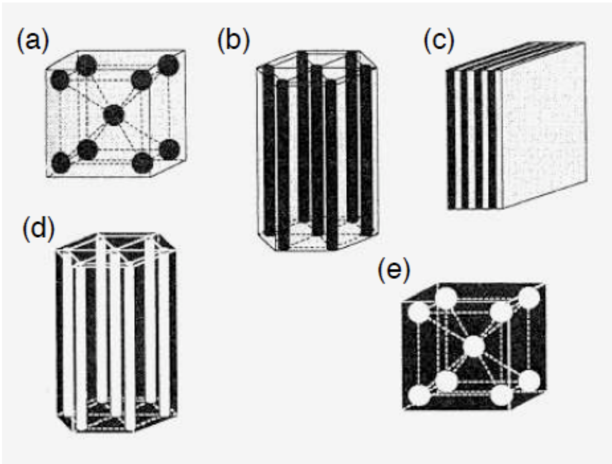
Neutron Star Crusts: Exotic Shapes

$\rho \leq 10^{14}$ g/cm^3

T≈ 0.1MeV

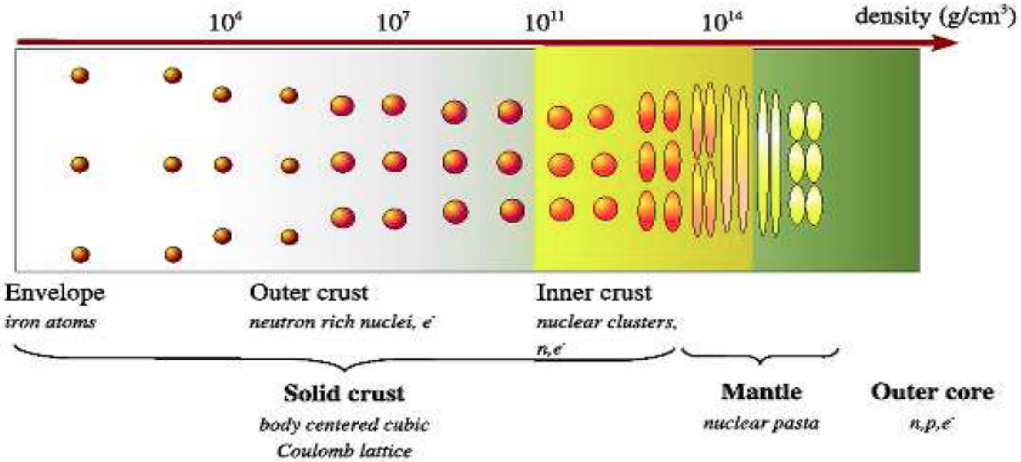
at sub-nuclear densities: frustration



« Nuclear Pasta »

a) spheres
b) rods
c) slabs
d) cylindrical holes
e) spherical holes

K. Oyamatsu, Nucl. Phys. **A561**, 431 (1993)



N. Chamel & P. Hænsel arXiv:0812.3955[astro-ph]

Pasta formation

astrophysical consequences:

* neutrino scattering
* electron scattering
* elastic properties of the crust

connection with nuclear collisions

* fragment formation
* EoS sensitivity, asymmetry energy

The DYnamical WAvelets in Nuclei Model

F. Sébille, V. de la Mota, S. Figerou

Heavy Ion Collisions $\longleftrightarrow$ Neutron Stars

simulation of the dynamical processes in inhomogeneous nuclear matter using a large number of nucleons without any assumptions on the structure of nuclear matter.

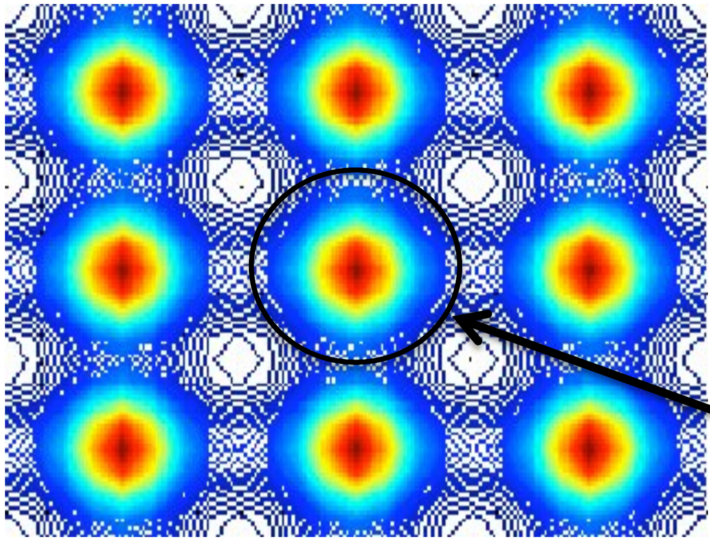
Studying the behavior of matter in the crust:

interacting nucleons in a uniform electron background

initial condition:

periodic boundary conditions

supercell



N lattice sites

$$\langle\rho\rangle = \frac{N\,A}{L^3}$$

nuclear composite

L

Simple Cubic Cell (SCC)

nuclear composites prepared self-consistently

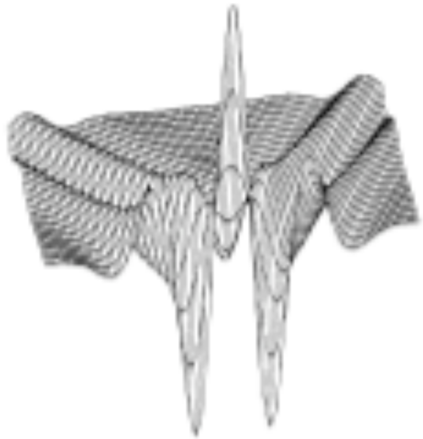
$$[\mathbf{h}(\rho), \rho] = 0$$

$$\mathbf{h} = \frac{\mathbf{p^2}}{2m} + \mathbf{V}^{HF}(\rho)$$

$$\mathbf{V}^{HF} = Tr_2\{\mathbf{V}^A(1,2)\rho(2)\}$$

$$\rho = \sum_{\lambda=0}^{N} \sum_i n_i^\lambda |\alpha_i^\lambda >< \alpha_i^\lambda|$$

$\{\alpha(\vec{r}, t)\}$: moving basis



$$\alpha(\vec{r}) = \alpha_x(x)\alpha_y(y)\alpha_z(z)$$

$$\alpha_x(x) = \mathcal{N}\exp\{-a(\chi,\phi)(x-\langle x\rangle)^2 + i\frac{\langle p_x\rangle}{\hbar}(x-\langle x\rangle)\}$$

$$\{\langle x\rangle, \langle p_x\rangle, \chi, \gamma\}$$

$$\chi = \langle (x-\langle x\rangle)^2\rangle$$

$$\phi = \langle (p_x-\langle p_x\rangle)^2\rangle$$

$$\sigma = \langle [(x-\langle x\rangle),(p_x-\langle p_x\rangle)]_+\rangle$$

$$\gamma = \frac{\sigma^2}{2\chi}$$

$$\Delta = \chi\phi - \sigma^2 = \frac{\hbar}{4}$$

dynamical evolution:

$$i\hbar\dot{\rho} = [\mathbf{h}, \rho]$$

TDHF equation

$$\mathbf{h} \ = \frac{\mathbf{p^2}}{2m} + V^{HF}(\rho(t))$$

time dependent one-body Hamiltonian

$$\rho = \sum_{\lambda=0}^{N} \sum_{i} n_i^\lambda |\alpha_i^\lambda >< \alpha_i^\lambda|$$

c.s. expansion of the one-body density matrix

$$i\hbar\frac{\partial|\alpha_k^\lambda(t)\rangle}{\partial t} = \mathbf{h}\ |\alpha_k^\lambda(t)\rangle$$

equation of motion for the basis elements

$$\mathcal{A} = \int_{t_1}^{t_2} \langle\alpha|i\hbar\frac{\partial}{\partial t} - \mathbf{h}|\alpha\rangle$$

variational principle

evolution of wavelet centroids and widths:

$$\frac{\mathrm{d}\langle x\rangle}{\mathrm{d}t} = \frac{\langle p_x\rangle}{m} \qquad \frac{\mathrm{d}\langle p_x\rangle}{\mathrm{d}t} = -\frac{\partial\langle V^{HF}\rangle}{\partial\langle x\rangle}$$

$$\frac{\mathrm{d}\chi}{\mathrm{d}t} = \frac{4\gamma\chi}{m}$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{\hbar^2}{8m\chi^2} - \frac{2\gamma^2}{m} - \frac{\partial\langle V^{HF}\rangle}{\partial\chi}$$

$$\langle \quad \rangle \equiv \langle\alpha| \quad |\alpha\rangle$$

effective force:

$$V_q^{HF}(\rho,\xi) = \frac{t_0}{\rho_\infty}\rho + \frac{t_3}{\rho_\infty^{\nu+1}}\rho^{\nu+1} + \frac{c}{{\rho_\infty}^2}\xi^2 + \frac{4qc}{{\rho_\infty}^2}\rho\xi + \frac{\Omega}{3{\rho_\infty}^2}\xi^2 + \frac{4q\Omega}{3{\rho_\infty}^2}(\rho-\rho_\infty)\xi + V_q^C$$

$$\rho = \rho_n + \rho_p$$

$$\xi = \rho_n - \rho_p$$

$q=$ ½ neutrons
- ½ protons

$$t_0 = -356/\rho_\infty(MeV/fm^{-3})$$

$$t_3 = 303/\rho_\infty^{\nu+1}(MeV/fm^{-3})$$

$c = 20\ MeV$ ($J=31.5\ MeV$)

ρ_∞ = 0.145 fm^{-3}

$\nu = 1/6$

V_q^C lattice calculation with Ewald summation technique macroscopic properties of nuclear matter

$$\omega = \frac{\int V_q^{HF} \ \mathrm{d}\rho}{\rho} + \omega_{kin} \qquad \delta = \frac{\xi}{\rho} \qquad \omega_\delta = \frac{1}{2}\lim_{\delta\to 0}\frac{\partial^2\omega}{\partial\delta^2}$$

$$\omega(\rho,\delta) = \frac{t_0}{2\rho_\infty}\rho + \frac{t_3}{(\nu+2)\rho_\infty^{\nu+1}}\ \rho^{\nu+1} + \frac{c}{\rho_\infty^2}\delta^2\rho^2 + \frac{\Omega}{3\rho_\infty^2}(\rho-\rho_\infty)\rho\delta^2 + \omega_{kin}$$

$$\boxed{\omega = \omega_0 + \frac{K_0}{18\rho_\infty^2}(\rho-\rho_\infty)^2 + \left[J + \frac{L}{3\rho_\infty}(\rho-\rho_\infty)\right]\delta^2}$$

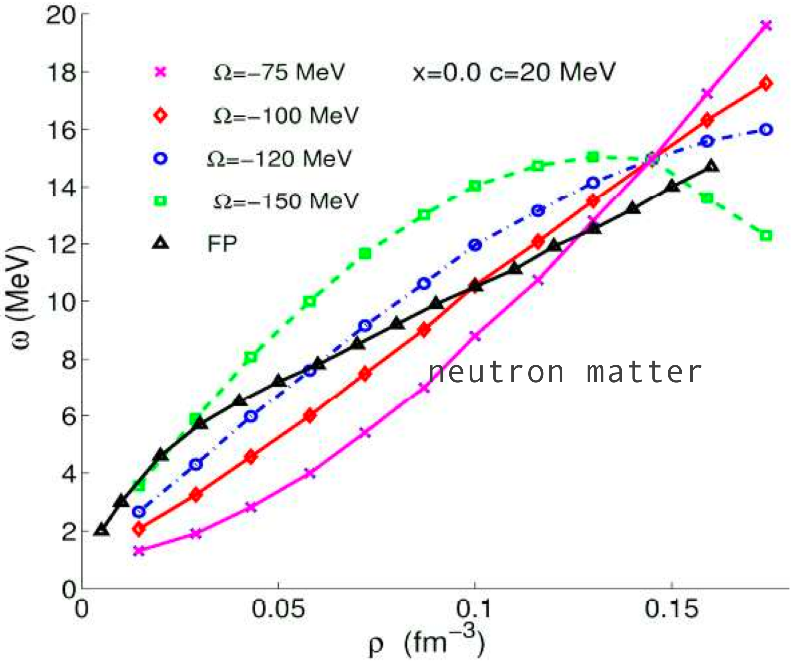
J.M. Lattimer, Ann. Rev. Nucl. Part. Sci. 31 (1981) 337
K. Oyamatsu, Pr. Th. Phys. 109 (2003) 631

$$J = \omega_\delta(\rho_\infty) \qquad L = 3\rho\frac{\partial\omega_\delta}{\partial\rho}\Big|_{\rho=\rho_\infty} \qquad L_{kin} = 3\rho_\infty\left[\frac{\mathrm{d}\omega_{kin}(\rho)}{\mathrm{d}\rho}\right]_{\rho_\infty}$$

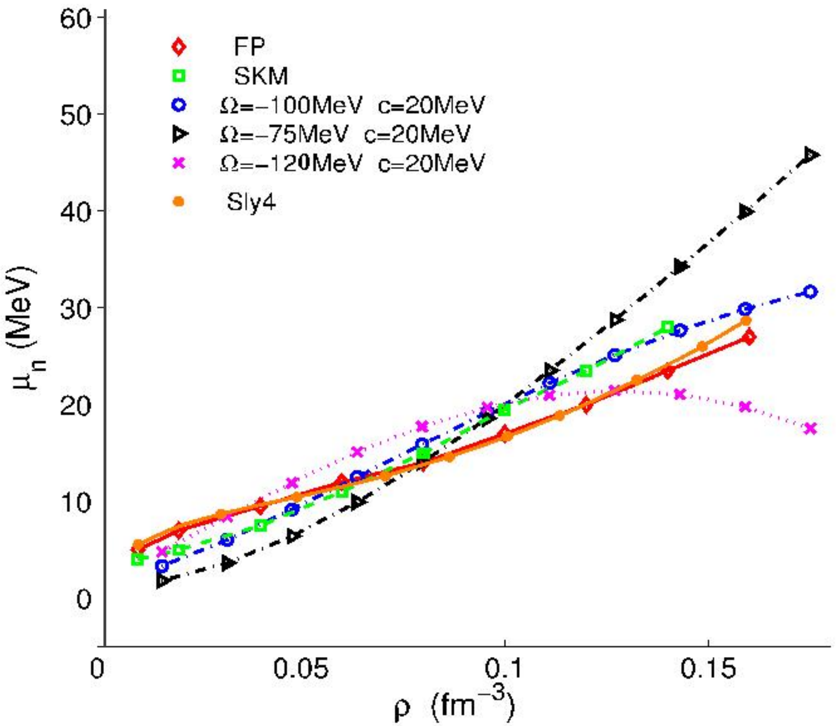
$$K_{sym} = 9\rho^2\frac{\partial^2\omega_\delta}{\partial\rho^2}\Big|_{\rho=\rho_\infty} \qquad K_0 = K_{sym}(\delta=0)$$

$$L = 6c + \Omega + L_{kin}$$

neutron matter properties

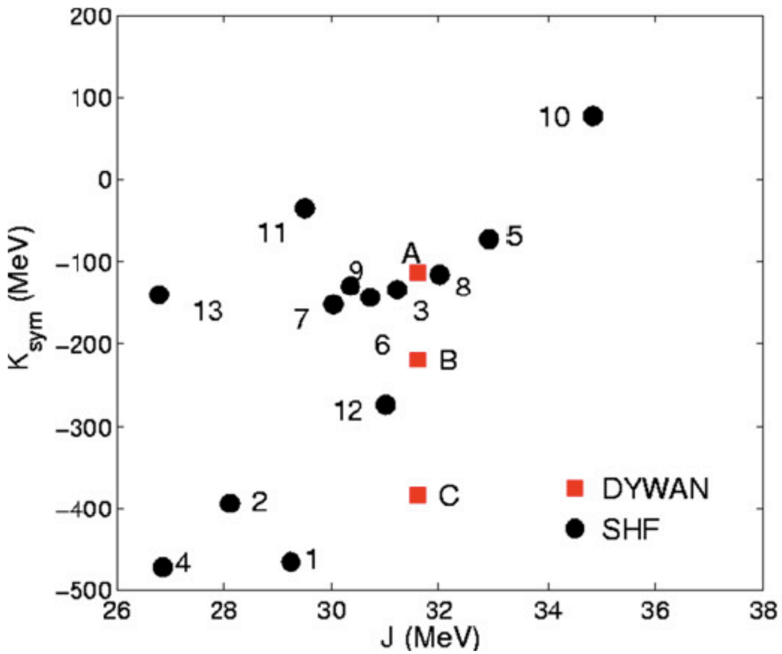


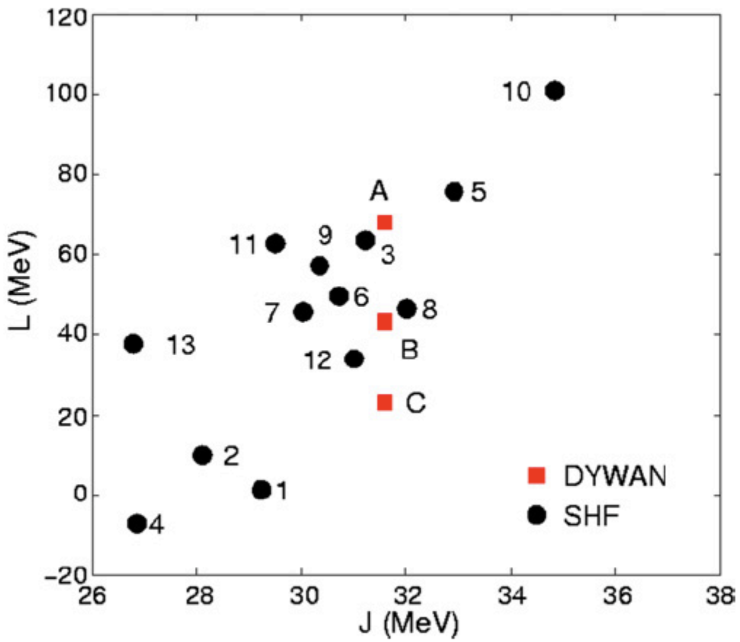
$$\mu_i = \frac{\partial \varepsilon}{\partial \rho_i} \qquad i = n, p.$$



FP B. Friedman et B.R. Pandharipande, Nucl. Phys. 361(1981) 501
SKM H. Krivine et al, Nucl. Phys. A 336 (1980) 155
Sly4 F. Douchin et al, Phys. Lett. B 435 (2000) 107

correlations between macroscopic properties





A Ω=-75 MeV

B Ω=-100 MeV

C Ω=-120 MeV

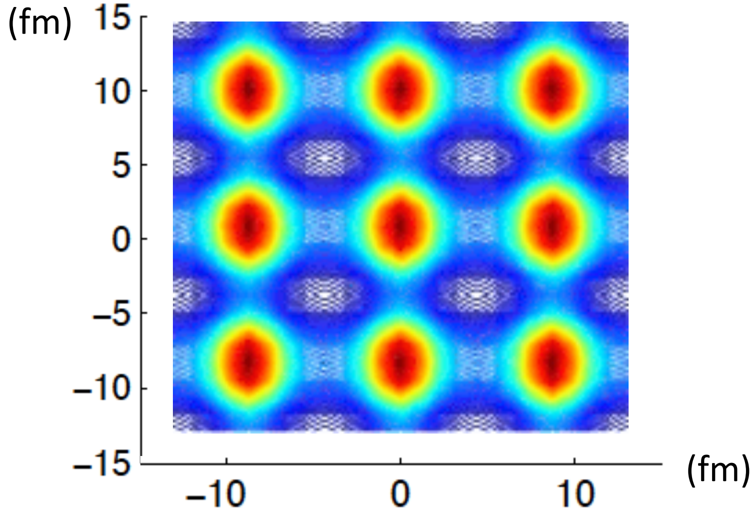
SHF calculations

1 SI	6 SkM
2 SIII	7 SkM*
3 SIV	8 SLy4
4 SVI	9 MSkA
5 Skya	10 SkI3
	11 SkI4

S. Yoshida, H. Sagawa, Phys. Rev. C 73 (2006) 044320.

dynamical evolution

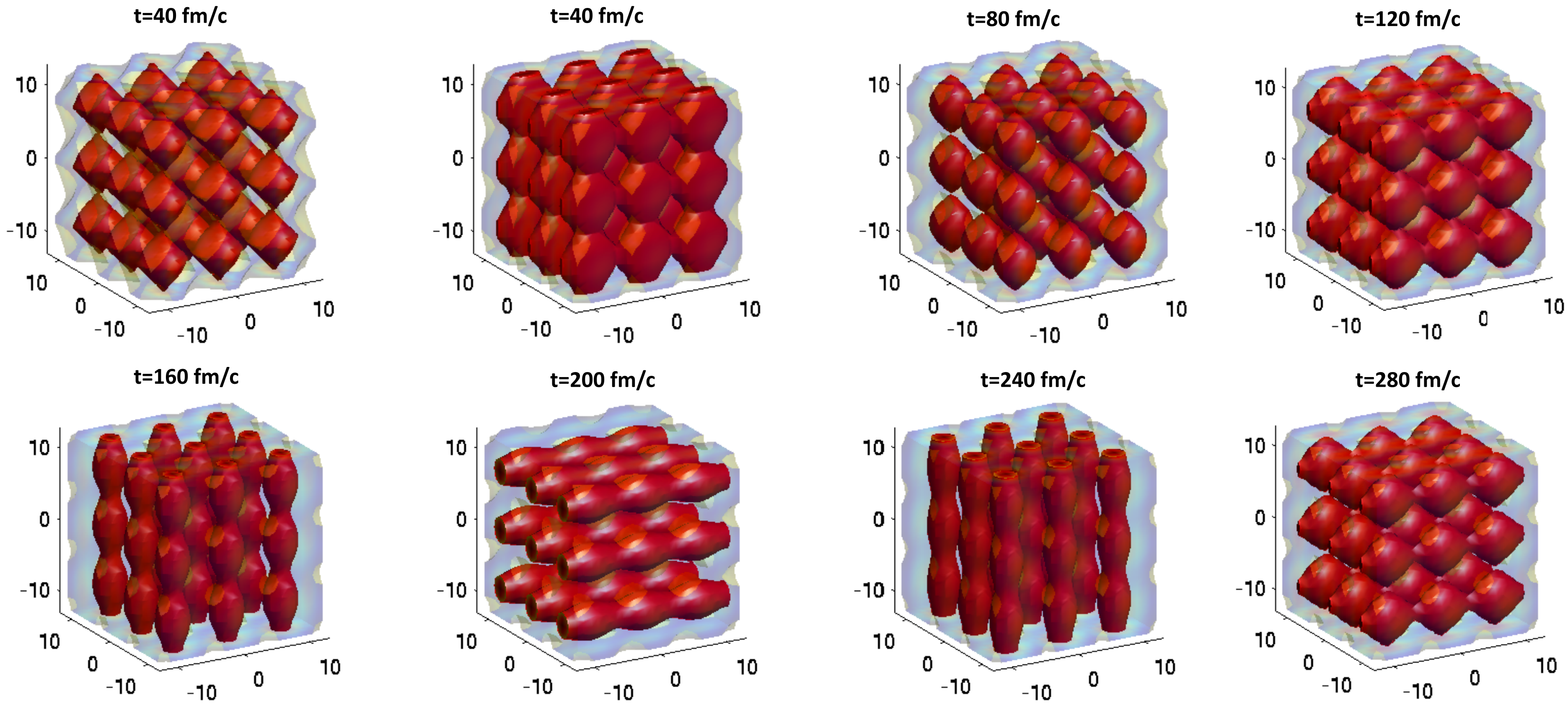
Ω= -100 MeV



Oxygen x=0.2

$\langle\rho\rangle$=0.072 fm^{-3}

ρ_t =0.08 fm^{-3}



neutron phase diagrams

Structure patterns in the ρ_t $\langle\rho\rangle/\rho_\infty$ plane
for different values of Ω

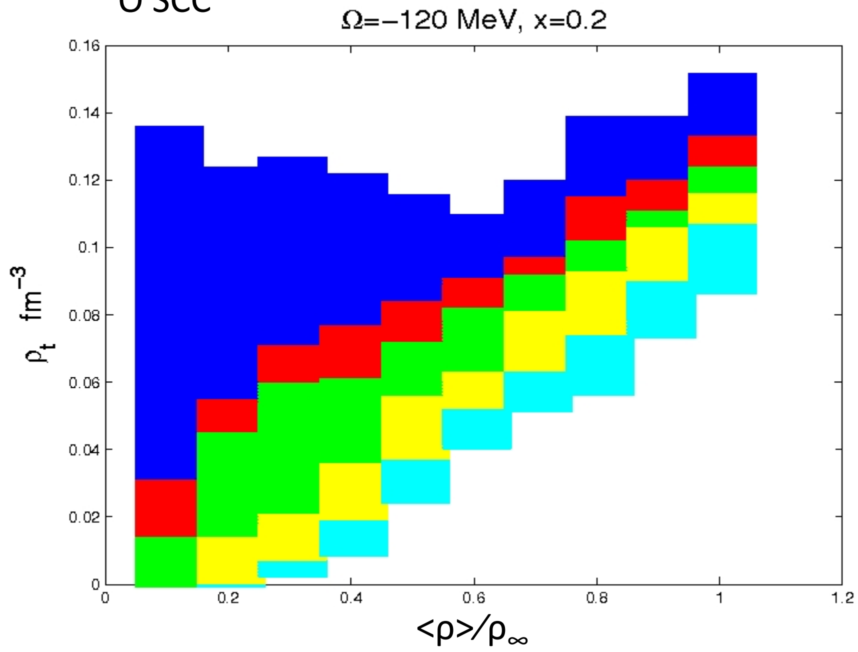
structures characterization

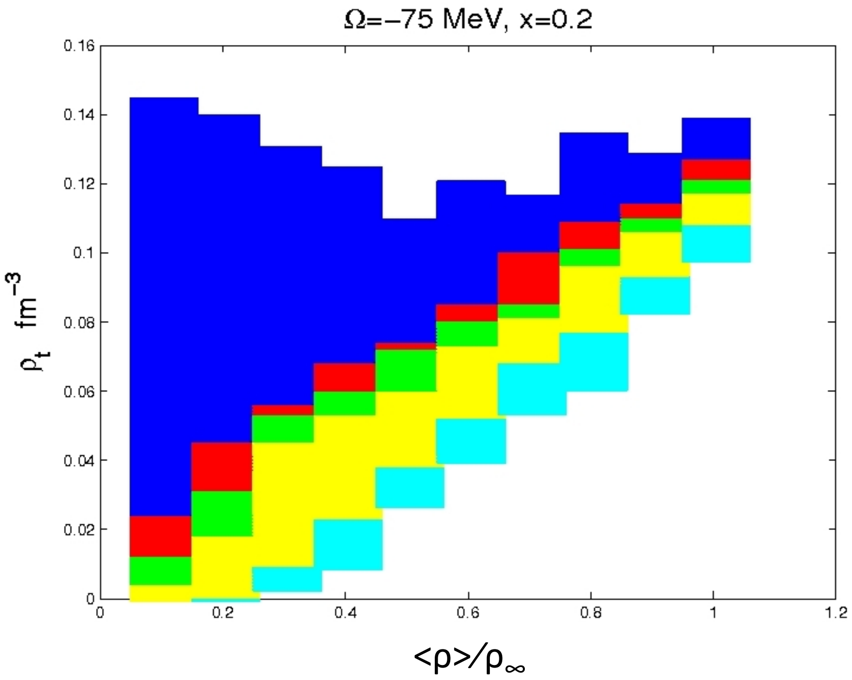
Morphological
Image Analysis

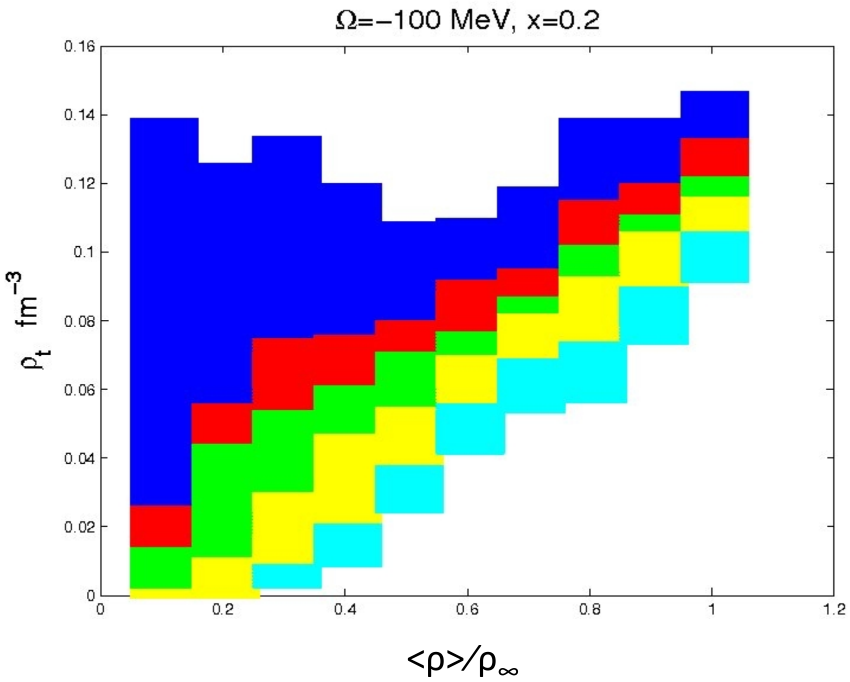
ρ_{th}

- spheres
- rods
- slabs
- sponges
- bubbles

O SCC

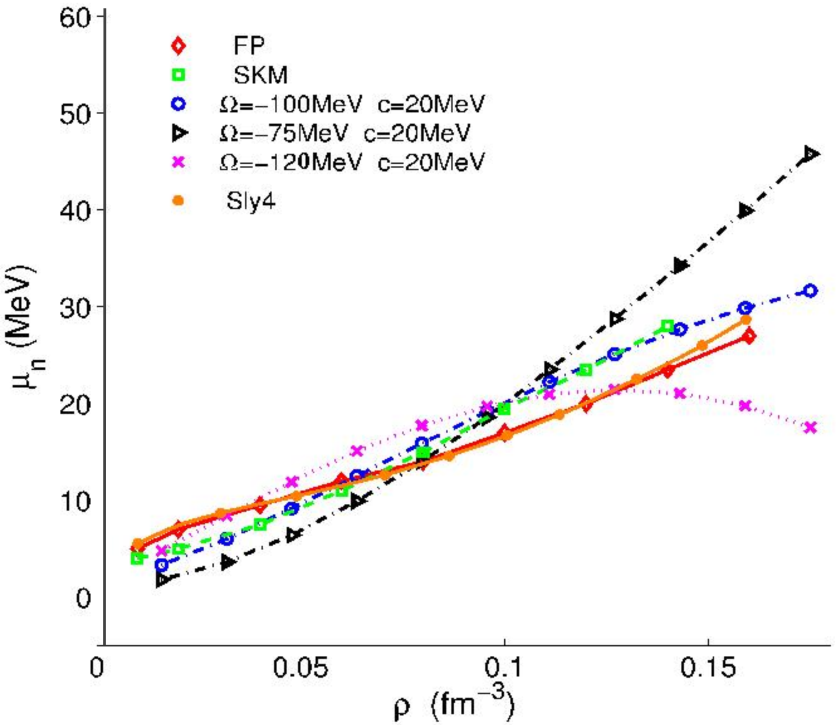






influence of Ω

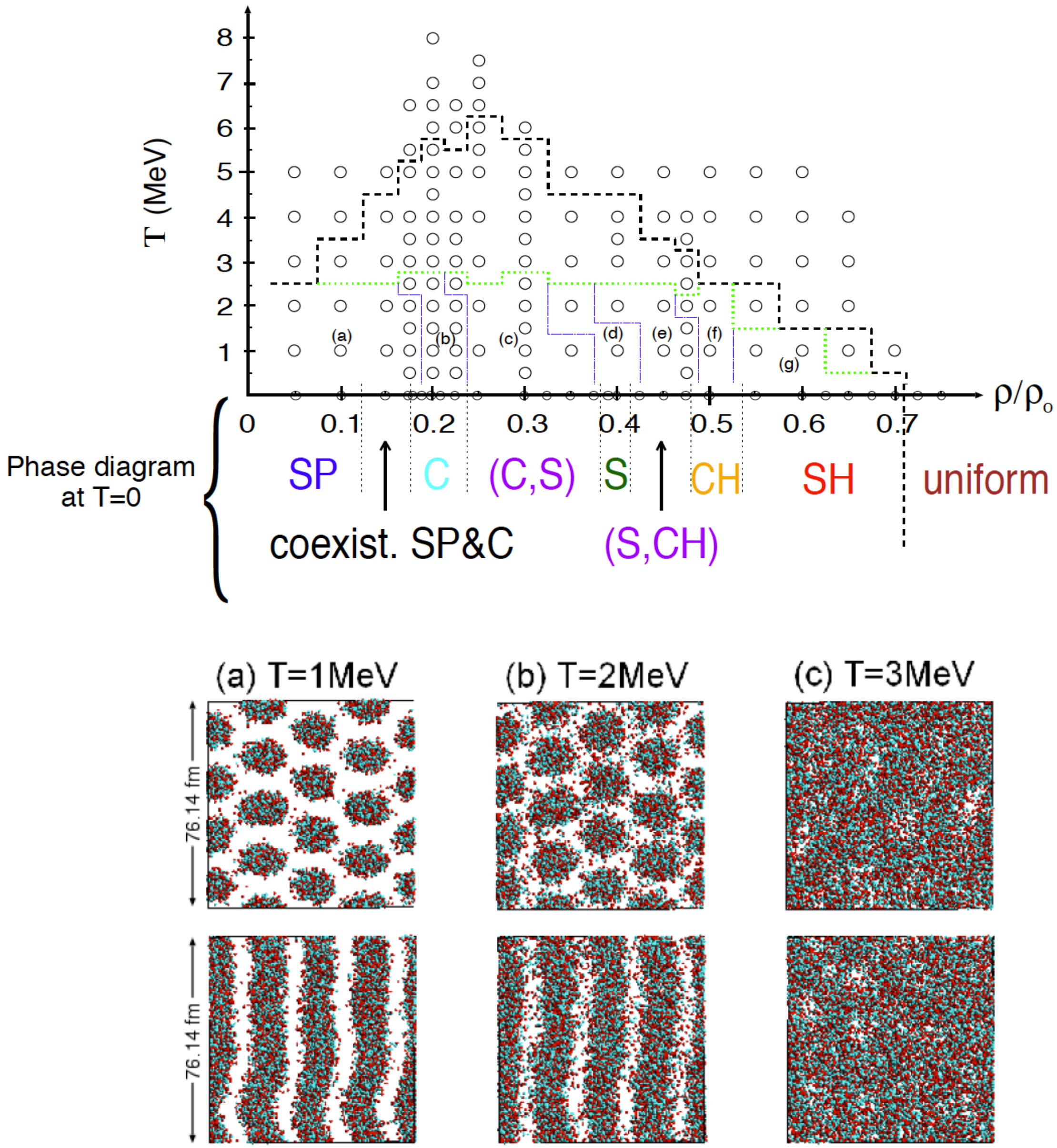
stiffer force
favours spherical & sponge-like structures
restrains cylinder & slabs.



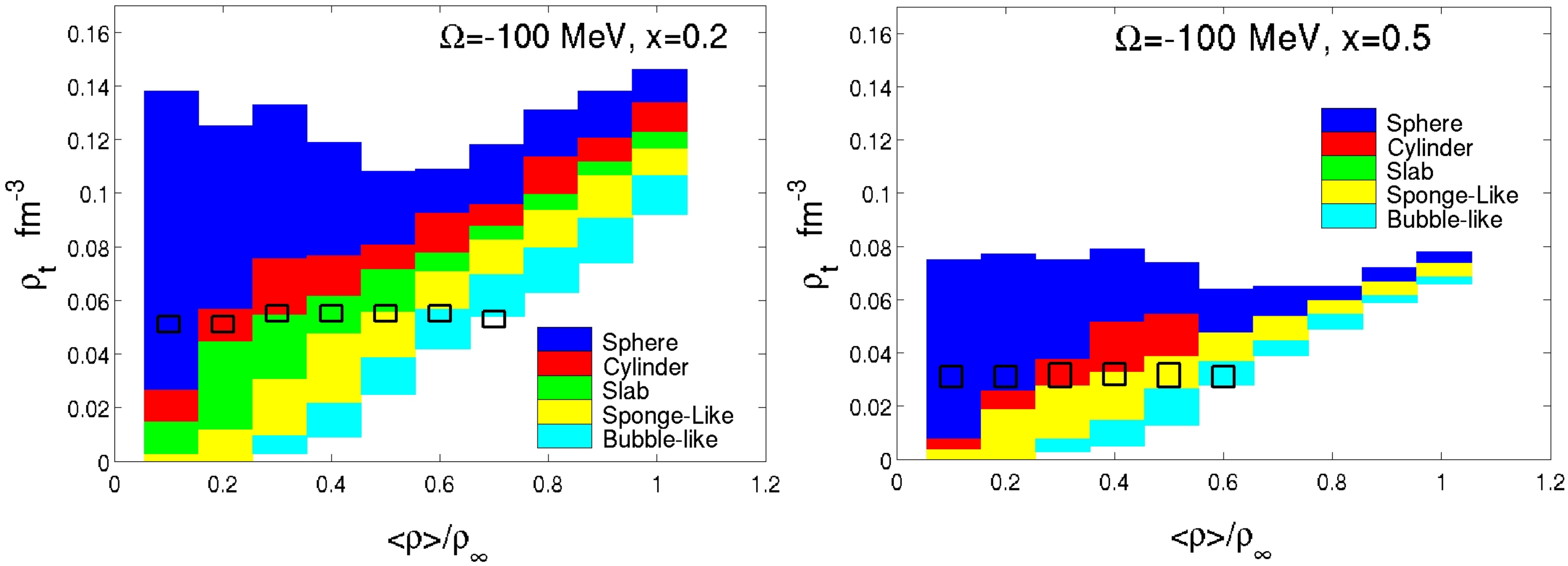
neutron chemical potential

Quantum Molecular Dynamics

SP sphere
C cylinder
S slab
CH cylindrical holes
SH spherical holes



G. Watanabe et al. Phys. Rev. C**69** 055805 (2004)



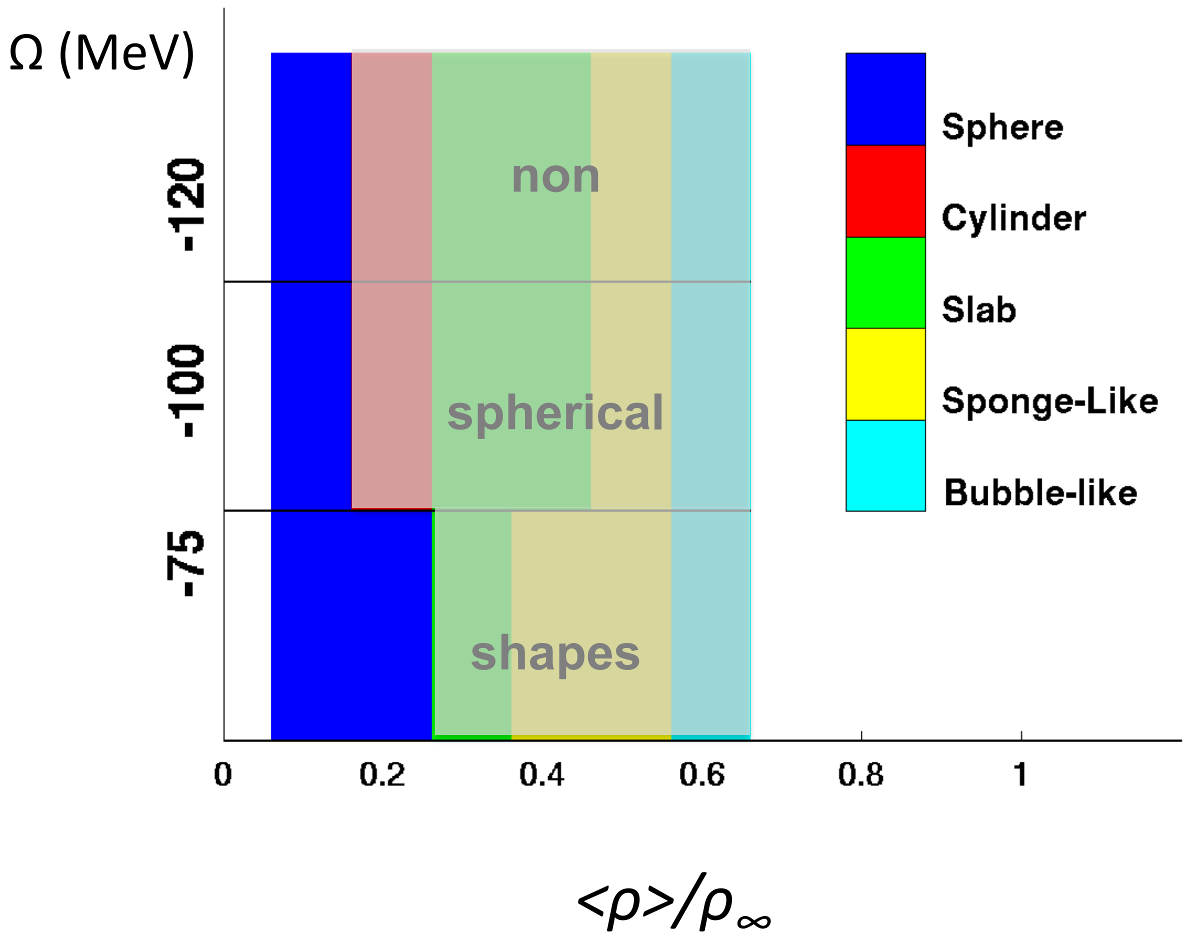
0.03 fm^{-3}<ρ_t^{ref}<0.08 fm^{-3} B. Schuetrumpf et al. PRC 87 055805 (2013)

□ ρ_t^{ref} satisfies the condition: $$\left(\frac{d^2(\mathcal{V}/\mathcal{A})}{d\rho_t^2}\right)_{\rho_t^{ref}} = 0$$

G. Watanabe et al.
PRC **66** 012801 (R) (2002)

- ρ_t^{ref} ~ independent of <ρ>
- classical sorting of structures
- different structures exist for the same <ρ> value: embedded
- sensitivity to isotope composition

synopsis: shapes vs Ω at ρ_t^{ref}



the density region
with non spherical
shapes
is narrow for
the stiffer force

agreement with Sonoda et al.
PRC 77 035806 (2008)

Summary and Perspectives

self-consistent dynamical description of matter has been performed

a small initial deposited energy permits to explore a landscape of meta-stable structures with constant energy preserving the initial symmetries

the ordered standard types of pasta emerge naturally whatever the threshold density is

sensitivity to the asymmetry dependent term of the potential: the stiffer force favours the occurrence spherical and sponge-like structures, restraining rods and slabs

effective force drawbacks: non-locality effects, spin-orbit, pairing

temperature: effects of varying T

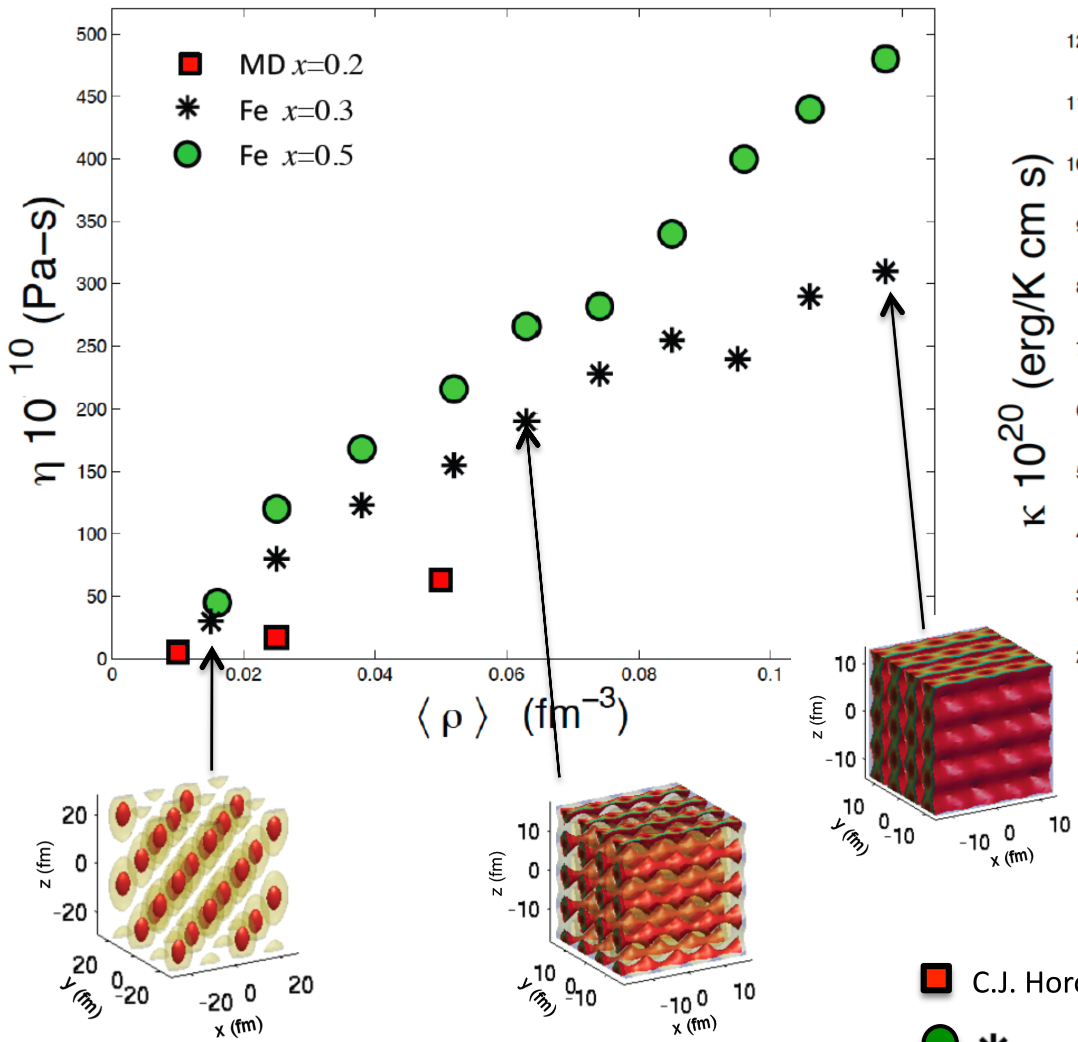
inclusion of residual interactions in the dynamical description
dissipation: influence on pasta formation

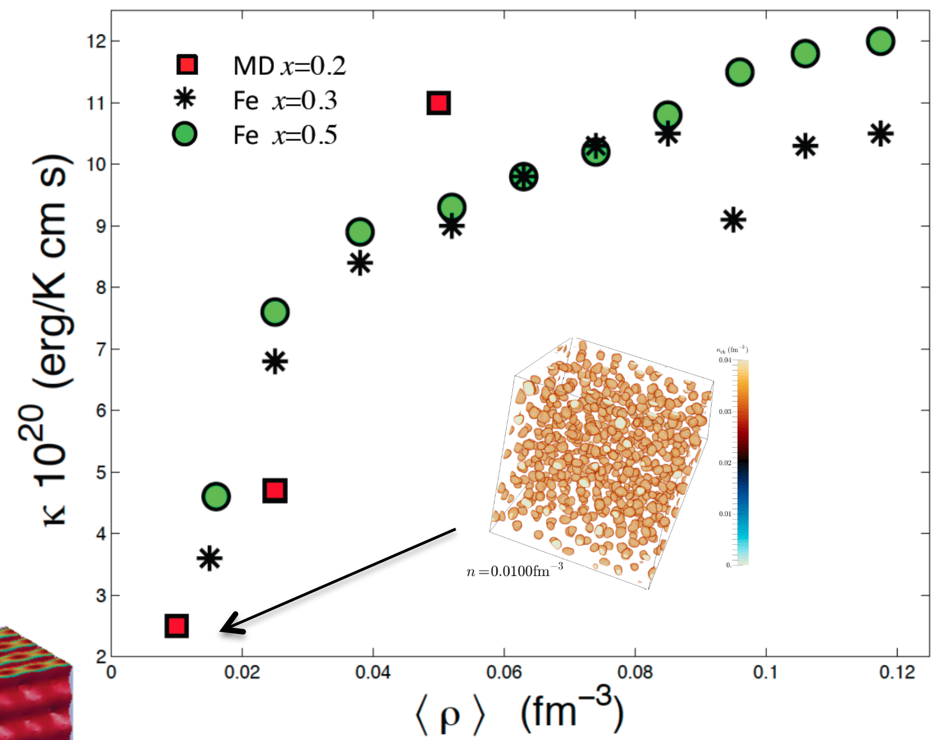
work in progress

bonus ...

shear viscosity

thermal conductivity





C.J. Horowitz and D.K. Berry, Phys. Revv. C 78 (2008) 035806

DYWAN calculations Fe FCC

The Coulomb potential

charged particles $\left\{ \begin{array}{l} \text{protons} \\ \text{electrons} \end{array} \right.$

protons

wavelet expansion of the density

$$\rho(r) \sim \sum_i \beta_i \; g_i(r)$$

$$V^C(r) = \sum_i V_i(r)$$

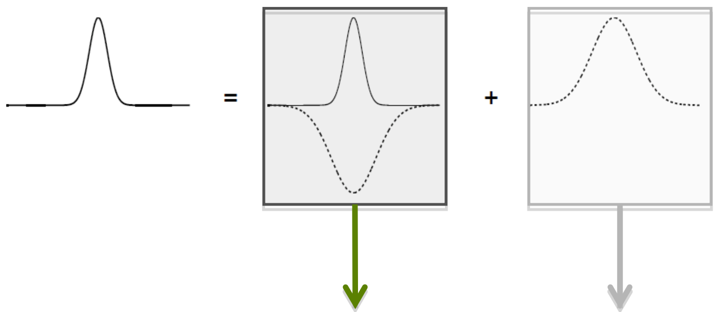
electrons

$$\rho_{\text{electron}} = Q/\mathcal{V}$$

$$\rho_T = \rho_{\text{proton}} + \rho_{\text{electron}}$$

total density decomposition

potential decomposition



$$V^C = V_d + V_r$$

V_d : direct term → « cut-off » analytic

V_r : reciprocal term → solution of the Poisson equation with a FFT

[1]

MORPHOLOGICAL IMAGE ANALYSIS (MIA)

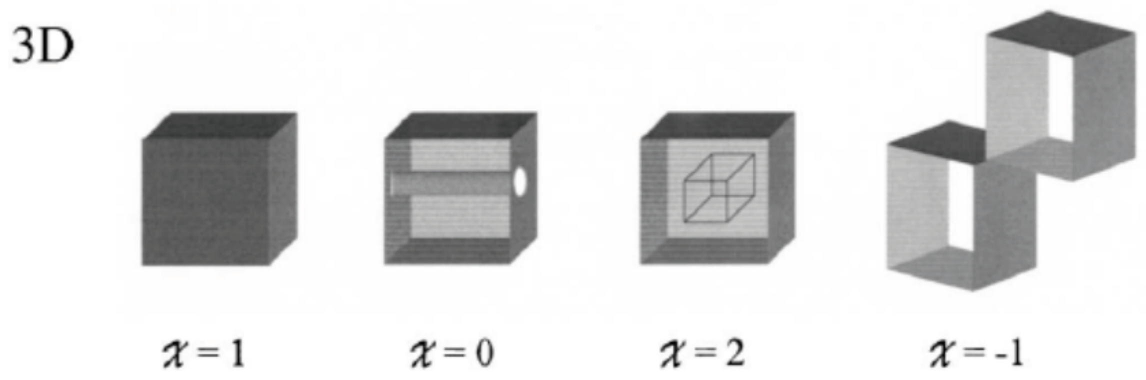
convex ensemble K → Minkowski Functionals:

$\mathcal{V}$ volume
$\mathcal{S}$ surface
$\mathcal{B}$ curvature
χ Euler char.

additive: $K = \mathbf{U}\, K_i$, K_i=unitary

they provide a complete mesure of K mophology

e
x
a
m
p
l
e



$$\chi = N_{elements} - N_{tunnels} + N_{cavities}$$

[1] INTEGRAL-GEOMETRY MORPHOLOGICAL IMAGE ANALYSIS
K. Michielsen, H. De Raedt / Physics Reports 347 (2001) 461}538 461