Properties of Infinite Nuclear Matter and Neutron Star Matter with the Gogny Interaction

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Outline

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- The maximum mass and radius of a Neutron Star depends on the Equation of State (EOS) used
- The Gogny force is a finite range force and its ability to predict bulk properties of Infinite Nuclear Matter (INM) is relatively unexplored
- 10 Gogny type forces were examined and their bulk properties were evaluated in a Hartree-Fock framework
- The forces were applied to Neutron Star Matter to produce maximum masses, radii, and composition

We find the total single particle energy within a Hartree-Fock mean field approximation:

$$
\begin{aligned}
\epsilon_{k_1 \sigma_1 \tau_1} &= \langle k_1\ \sigma_1\ \tau_1 | \hat{T} | k_1\ \sigma_1\ \tau_1 \rangle \\
&+ \frac{1}{(2\pi)^3} \sum_{\sigma_2} \sum_{\tau_2} \int_0^{k_F} d^3 k_2\ \frac{1}{2} \langle k_1\ \sigma_1\ \tau_1,\ k_2\ \sigma_2\ \tau_2 | \hat{V} | k_1\ \sigma_1\ \tau_1,\ k_2\ \sigma_2\ \tau_2 \rangle
\end{aligned}
$$

Where k_F is the Fermi momentum

$$k_F = \left(3\pi^2 \rho\right)^{\frac{1}{3}}$$

and ρ is the number density of nucleons in nuclear matter

To find the total energy of the system we now integrate over all k_1:

$$E_{Total} = \frac{1}{(2\pi)^3} \sum_{\sigma_1} \sum_{\tau_1} \int_0^{k_F} d^3k_1 \; \Big[\langle k_1\ \sigma_1\ \tau_1 | \hat{T} | k_1\ \sigma_1\ \tau_1 \rangle$$
$$+ \frac{1}{(2\pi)^3} \sum_{\sigma_2} \sum_{\tau_2} \int_0^{k_F} d^3k_2 \; \frac{1}{2} \langle k_1\ \sigma_1\ \tau_1,\ k_2\ \sigma_2\ \tau_2 | \hat{V} | k_1\ \sigma_1\ \tau_1,\ k_2\ \sigma_2\ \tau_2 \rangle \Big]$$

- Now we are able to examine a two-body interaction, $\hat{V}$
  - Many two body interactions exist, e.g Skyrme
  - This work focusses on the Gogny type interaction

The Gogny Interaction
Beyond Zero Range

The Gogny Force

$$V_{NN}(r) = \sum_{i=1}^{2} \left[W_i + B_i P_\sigma - H_i P_\tau - M_i P_{\sigma\tau}\right] e^{\frac{-\vec{r}^2}{\mu_i^2}}$$
$$+ \sum_{i=1}^{2} t_{0i} \left(1 + x_{0i} P_\sigma\right) \rho^{\alpha_i} \left(\vec{r}\right) \delta \left(\vec{r}\right)$$

- The operator P_σ (P_τ) is the spin (isospin) exchange operator

$$\langle \sigma_1,\ \sigma_2 | P_\sigma | \sigma_1,\ \sigma_2 \rangle = \langle \sigma_1,\ \sigma_2 | \sigma_2,\ \sigma_1 \rangle = \delta_{\sigma_1 \sigma_2}$$

- Remaining 14 variables are free parameters used to define the force
  - Fit to properties of finite nuclei
  - Only 10 appear so far in literature compared to >100 for Skyrme
  - Application to Neutron Star Matter not well explored

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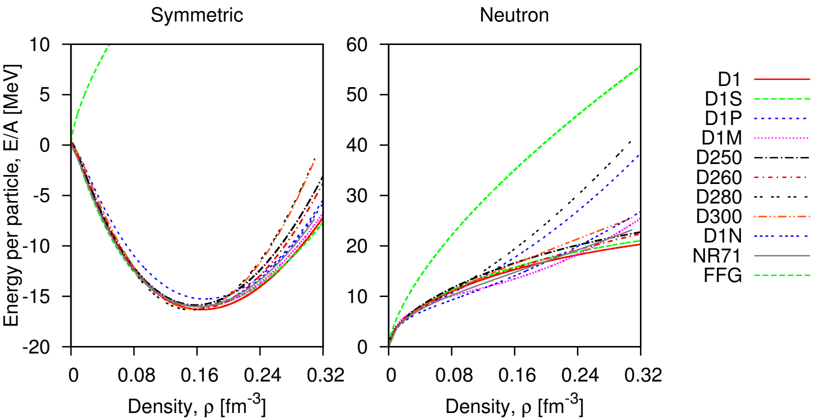
The Gogny Interaction
Energy Per Particle



Normal nuclear matter has a saturation density at around 0.16 nucleons per fm^3 and has an energy of approximately -16 MeV

The Gogny Interaction
Symmetry Energy and Slope

Symmetry Energy:

$$E_{Sym}\left(\rho\right) = \frac{1}{2!}\left.\frac{\delta^2 E\left(\rho,\beta\right)}{\delta\beta^2}\right|_{\beta=0}$$

- Influences ratio of protons to neutrons in a Neutron Star

Slope Parameter, L:

$$L = 3\rho_0 \left.\frac{\delta E_{Sym}\left(\rho\right)}{\delta\rho}\right|_{\rho_0}$$

- Generally accepted to lay between 30 MeV and 100 MeV
- Influences how pressure changes with density (stiffness)

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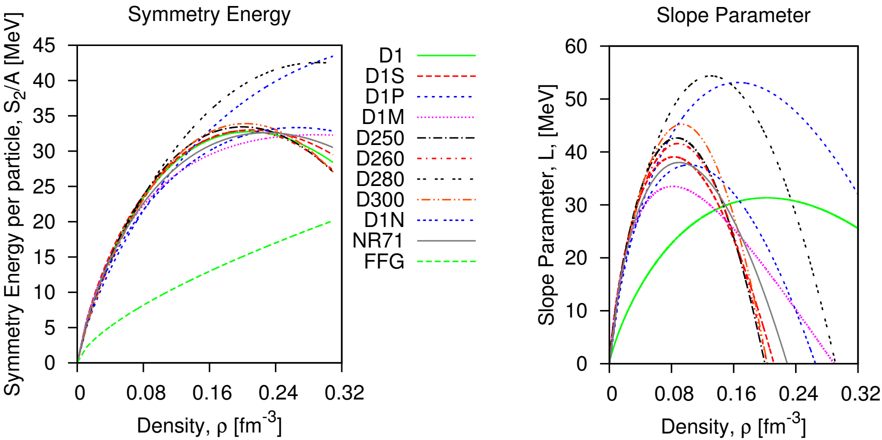
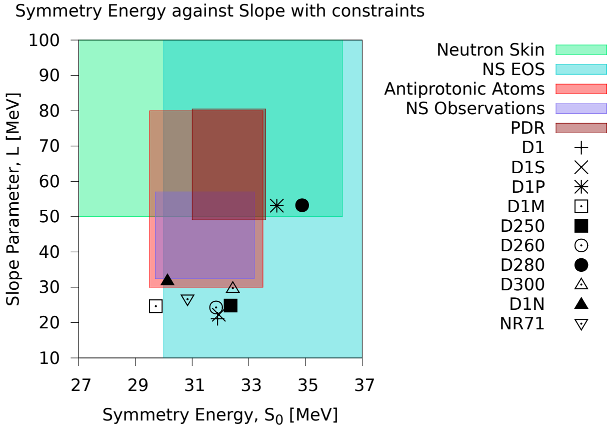
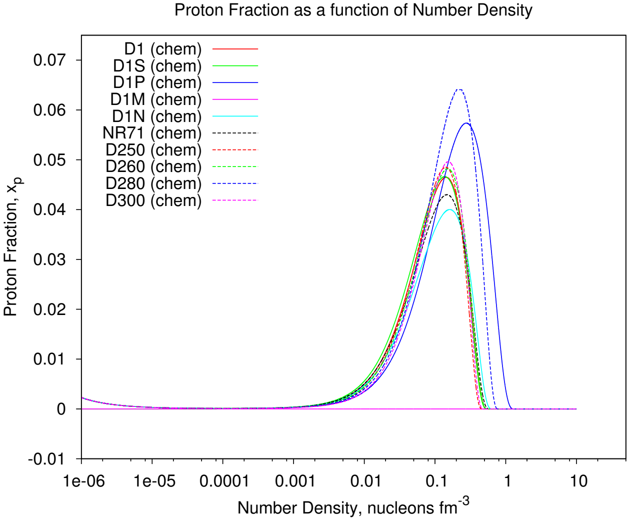


- Stiffer EOS is predicted to provide higher mass neutron stars
- Larger Symmetry Energy is predicted to yield higher proton fraction

The Gogny Interaction
Symmetry Energy and Slope



- We predicted D1P and D280 would give the highest mass Neutron Stars and highest proton fractions



- Proton fraction was calculated by equalising chemical potentials:

$$\mu_N - \mu_P - \mu_e = 0$$

Building a Neutron Star

The TOV Equations

To construct a Neutron Star from an EOS we use the Tolman Oppenheimer Volkoff equations:

$$\frac{dm(r)}{dr} = 4\pi r^2 \frac{\epsilon}{c^2}$$

$$\frac{dp(r)}{dr} = \frac{G\epsilon(r)m(r)}{c^2 r^2}\left[1 + \frac{p(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 p(r)}{m(r)c^2}\right]\left[1 - \frac{2Gm(r)}{c^2 r}\right]^{-1}$$

$$\rho(r) = \frac{\epsilon(r)}{c^2}$$

Pressure as a function of density was calculated from the Gogny EOS as

$$p(\rho) = \rho^2(r)\frac{\delta E\,(\rho)}{\delta\rho}$$

- We compiled a lookup table of pressures
- The table was then used to interpolate density as a function of pressure

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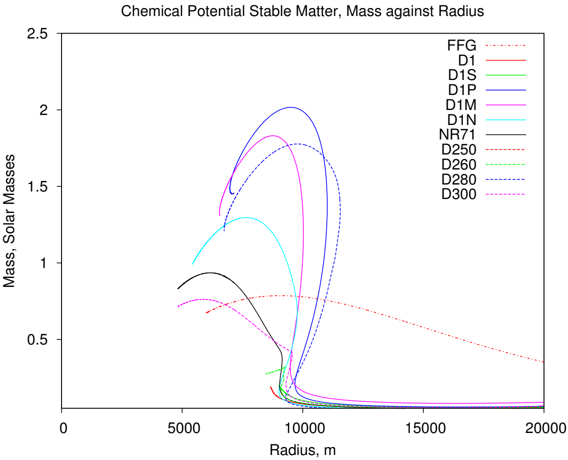
- D1P gives largest maximum radius at 2.01 Solar Masses



Demorest et al. (2010) measured a 1.97(4) mass neutron star!

- Most, if not all, Gogny EOS are too soft to produce a Neutron Star in keeping with current observations
- Isovector properties of Gogny forces should be improved if they are to be applied to Neutron Star matter

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- Partial wave analysis of the Gogy force is underway
- Pairing with the Gogny force will be explored

Questions

Any Questions?