## Study of the Nuclear Symmetry Energy: Future theoretical directions



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Quest for the density dependence of Nuclear Symmetry Energy (NSE)

- now for more than 20 years of intensive research,
- still not well known, esp. at higher density
- but important for very asymmetric nuclear systems (exotic nuclei), and in astrophysics





#### Why?

Since cannot be reliably calculated, one needs to look for observables in nuclear physics and astrophysics,

strong interdependence of theory and experiment,

- -- experiments and observations: future prospects bright (see talks of E. Brown, G. Verde, Y. Leifels and Bill Lynch).
- -- theory: more expansion desirable, esp. in Europe

challenges in theory:

- microscopic calculation: effective forces and many-body theory
- theoretical interpretation of experiments
  - a) nuclear structure beyond mean field
  - b) HIC: transport approach
  - c) astropysics: structure of NS and dynamics of CCSN



- Note 1: the NSE is a rather simple, stationary concept (a piece of the nuclear EoS) the way to study it involve much more complex systems (finite, dynamical, non-equilibrium)
- Note 2: all the above also true for the EOS in general, i.e. for symmetric nuclear matter. However, the NSE is a subdominant component of the EoS, and thus more difficult to observe and more difficult to calculate.

We are now in the quantative era of the study of the NSE!

This talk: try to identify the challenges in the theoretical treatments of the NSE and the possible future directions: illustrative and incomplete, qualitative, highly personal, but not supposed to be a summary.

**Evolved during workshop but special distracting event** 



#### Symmetry energy reviewed extensively in the past



What do we want to know about the NSE?

$$E(\rho_{B},\delta)/A = E_{nm}(\rho_{B}) + E_{sym}(\rho_{B})\delta^{2} + O(\delta^{4}) + \dots \qquad \delta = \frac{\rho_{n} - \rho_{p}}{\rho_{n} + \rho_{p}}$$
2 ways to define:
$$E_{sym}(\rho) = \frac{1}{2} \frac{\partial^{2}}{\partial \delta^{2}} E(\rho,\delta) \Big|_{\delta=0}$$

$$E_{sym}(\rho) = E(\rho,\delta=1) - E(\rho,\delta=0)$$

not necessarily the same:  $\rightarrow$  higher orders in  $\delta$ 

 $\rightarrow$  change of composition,

e.g. clusterization stronger in SNM

with (solid) and without (dashed) clusters

physics independent of definitions.

but dependent on how this SE is used: Astro (y), HIC (no)

Need further information about the NSE, because of non-static systems

$$U(\rho, k; \delta) = \underbrace{U_{0}(\rho, k) + U_{sym}(\rho, k)(\tau \delta)}_{U_{\tau}(\rho, k)} + \dots$$
$$\frac{m_{\tau}^{*}}{m} = \left(1 + \frac{m}{\hbar^{2} k} \frac{\partial U_{\tau}}{\partial k}\right)^{-1}$$

$$\sigma_{_{NN}}^{(in-med)}(k;
ho)$$

[MeV] 50 2 MeV 4 MeV  $\mathrm{E}_{\mathrm{A}}$ 40 6 MeV 8 MeV 10 MeV 12 MeV 30 14 MeV 16 MeV 20



Connected in a microscopic theory, or in a energy density functional.

Also: Composition of asymmetric matter: important for astrophysical applications

**Special representations of the SE:** 

$$E(\rho_{B},\delta)/A = E_{nm}(\rho_{B}) + E_{sym}(\rho_{B})\delta^{2} + O(\delta^{4}) + \dots$$

$$E_{sym}(\rho) = S + \frac{1}{3} \left(\frac{\rho - \rho_{0}}{\rho_{0}}\right) + \frac{K_{sym}}{18} \left(\frac{\rho - \rho_{0}}{\rho_{0}}\right)^{2}$$

$$\Rightarrow \frac{1}{3} \mathcal{E}_{F}(\rho / \rho_{0})^{2/3} + E_{sym}^{pot}(\rho)$$

$$\boldsymbol{E}_{sym}^{pot} = \boldsymbol{C} \left( \rho / \rho_0 \right)^{\gamma}$$

expansion around  $\rho_0$ 

split into kin. and pot. symm energy

polynomial behavior implies continuity between low and high densities: not necessarily so

kinetic energy in a theoy with correlations not Fermi Gas

a question of mapping microscopic theories to phenomenological approaches

 $\Sigma(\mathbf{k},\rho) \Leftrightarrow \{\mathbf{m}^*, \mathbf{U}(\rho)\}$ 



Carbone, et al., EPJA50;13

## The Nuclear Symmetry Energy in "realistic" models





#### More work has been done since then: Review by Marcello Baldo

Low density symmetry energy behave similarly and are consistent with analyses from nuclear structure and HIC.

However, at high densities large differences. -- 3-body forces? (Baldo); scaling with density?



#### further work requred! Note: attempts to derive directly from QCD (QM-BB, QCD sum rules, holographic QCD, Skyrmions)

## Symmetry energy at very low density (< 0.1 ρ<sub>0</sub>) determined by cluster correlations (Typel, et al., PRC81,015803(2010))

RMF model with explicit cluster degrees of freedon with thermal Green function approach to calculate medium modifications of clusters: NSE at  $\rho \rightarrow 0$  finite, because cluster low density symmetric matter gains energy by cluster formation



#### Investigation of very low density NSE in Heavy ion collisions

e.g. experiment 64Zn+(92Mo,197Au) at 35 AMeV S. Kowalski, J. Natowitz, et al.,PRC75 014601 (2007) J. Natowitz, G. Röpke, S. Typel, .. PRL 104, 202501 (2010)



-5<sup>[</sup>0

0.005

ρ, **nuc/fm**³

0.01

0.015

symmetry energy

Assumptions need to be checked in transport calculations.

#### Symmetry energy around saturation density: Determination from nuclear structure and low energy heavy ion collisions

Correlations between characteristic quantities of the SE: e.g. S, L, K<sub>sym</sub> and experimental observables (e.g. neutron skin, polarizability, isospin diffusion,...)



26

28

30

S. (MeV)

32

34

## **Heavy Ion Collisions: Transport Theory**



- → Transport approaches neccessary if system is not always in equilibrium.
- $\rightarrow$  Many observbles are determined during the evolution and not only at the end.
- → Especially interesting questions, like the high density phase, occur when the system is still not equilibrated.
- → Reliable transport approaches crucial to extract physics from heavy ion experiments

## **Transport equations: 2 families**

1. Boltzmann-Uehling-Uhlenbeck (BUU)

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \vec{\nabla}^{(r)} f - \vec{\nabla} U(r) \vec{\nabla}^{(p)} f(\vec{r}, \vec{p}; t) = \int d\vec{v}_2 \, d\vec{v}_1 \, d\vec{v}_2 \, v_{21} \sigma_{12}(\Omega) (2\pi)^3 \, \delta(p_1 + p_2 - p_{1'} - p_{2'}) \\ \left[ f_{1'} \, f_{2'} \, (1 - f_1) (1 - f_2) - f_1 \, f_2 \, (1 - f_{1'}) (1 - f_{2'}) \right]$$

**Derived:** 

→Classically from the Liouville theorem or semiclassically from THDF, collision term added (and fluctuations)

 $\propto \delta(p^{*2}-m^{*2})\Theta(p^{*0})$ 

 $\frac{dr_i}{dt} = \{r_i, \mathcal{H}\}; \quad \frac{dp_i}{dt} = \{p_i, \mathcal{H}\}; \quad H = \sum_i t_i \cdot t_i$ 

 $\rightarrow$  From non-equilibrium theory (Kadanoff-Baym); collision term included mean field and in-medium cross sections consistent, e.g. from BHF

$$\Sigma_{s,\mu}(\mathbf{k};\rho) \approx Tr(Tf); \sigma_{NN}^{(in-med)}(\mathbf{k};\rho) \approx |T^2|$$

Spectral fcts, off-shell transport, quasi-particle approx. (QPA)

$$\begin{array}{c} A(x,p) \propto \frac{2\Gamma(x,p)}{(p^{*2}-m^{*2})+\Gamma^{2}(x,p)} \\ \Gamma(x,p) = m^{*} Im \Sigma_{s}^{+} - p_{\mu}^{*} Im \Sigma^{+\mu} \end{array}$$

2. Molecular Dynamics (QMD)

classical molecular Dynamics with Gaussian particles to reduce fluctuations + collision term

2b) Antisymmetrized MD (AMD, FMD) Gaussians are antisymmetrized wp collision term with stochastic features (wave packet splitting)

Transport theory is on a well defined footing, in principle – but in practice?

QPA

#### **Examples of microscopic input into transport calculations:**



[17] C. Fuchs, et al., Phys. Rev. C 64 (2001) 024003.

#### **Decomposition of DB self energy**

$$\Sigma(p) = \Sigma^{s}(p) - \gamma^{0} \Sigma^{0}(p) + \bar{\gamma} \cdot \bar{p} \Sigma^{v}.$$

Density (and momentum) dependent coupling coeff.

$$\Gamma_i^2(\boldsymbol{k},\rho) = \left(\frac{\boldsymbol{g}_i}{\boldsymbol{m}_i}\right)^2 = \frac{\boldsymbol{\Sigma}_i(\boldsymbol{k},\rho)}{\rho_i}$$
$$\boldsymbol{i} = \boldsymbol{\sigma}, \boldsymbol{\omega}, \boldsymbol{\rho}, \boldsymbol{\delta}$$



Motivation for density dep. RMF model and inclusion of  $\delta$ -meson (J<sup> $\pi$ </sup>=0+,T=1)

#### **Practical transport approaches: somestimes a "fight" between MD and Boltzmann models:**



BUU: Ideal procedure: solve BUU eq. with N<sub>TP</sub>→inf, and add flucuation term into a Boltzmann-Langevin eq. with physically determined fluctation strength. approximations: "gauged" numerical noise, BOB, statistical fluctuations (SMF) Bauer-Bertsch-method (collisions of swarms of TP, BLOB)

$$\frac{df}{dt} = I_{coll} + I_{fluc}$$

MD-models: fluctuation inherent, but determined by a parameter for width of wp

→ Issues now discussed intensively differences mainly in nature and amount of fluctuations

#### Comparison of simulations: SMF-AMD: (Rizzo, Colonna,Ono, PRC76(2007); Colonna et al., PRC82 (2010))



#### Code Comparison Project: Trento, ECT\*, 2006 and 2009 Shanghai, Jan. 2014, (Lanzhou 2014)

check consistency of transport codes in calculations with same system (Au+Au), E=100,400 AMeV, identical (simple) physical input (mean field (EOS) and cross sections)

idea: establish sort of theoretical systematic error or transport calculations (and hopefully to reduceit )



#### 1. step: Initialize colliding nuclei. usually not exact ground states

#### Examples of results: Au+Au, PRELIMINARY

E/A=100 MeV

**BUU models** 

QMD models

E/A=400 MeV BUU and QMD models

Graphs eliminated, since results are preliminary and are still under review of the participating code owners. It is planned to make them available publicly in the near future

- $\rightarrow$  considerable differences
- $\rightarrow$  partly due to initialization, but mainly to collision term
- $\rightarrow$  no essential difference between BUU and QMD models
- → 100 MeV sensitive region for flow because of competition between mf and collisions, better at higher energy

## **Treansport Calculations in a (periodic) Box**

test collision routine and Pauli blocking under controlled conditions; reveal important features of the semiclassical approach. One effect: Initialization, T=0.  $\rightarrow$ Fermi statistics is lost quickly!

> see significant differences between codes!

Graphs eliminated, since results are preliminary and are still under review of the participating code owners. It is planned to make them available publicly in the near future

Broader applications: 1. intialization in spinodal region: fragment formation, check of fluctuations

2. intialize expanded or two interpenetrating Fermi system (two Fermi spheres): check of equilibration 3. others

 $\rightarrow$  interesting for a detailed study



## **Fluctuations in Phase Space**



## Fragment and light clusters in transport calculation

(a)

t=0

fm/c

A fragment represents a many-body correlation, which is not contained in approaches for the one-body density. How to describe anyway? Distinguish Intermediate mass fragments (IMF) and Light Clusters (LC:d,t, ${}^{3}He,\alpha$ )

a) IMF's: formation dominated by mean field,

which favors matter at normal density e.g. BUU calculation in a box (periodic boundary conditions) with initial conditions inside the instability region:  $\rho = \rho_0/3$ , T=5 MeV,  $\beta = 0$ 



Correlation dominated (esp.Pauli-Correlation). not good in BUU and MD models, except(!) for AMD: can define realistic wave functions for LC with reasonable BE;

#### Solution for BUU models:

LC distribution functions as explicit degrees of freedom coupled to nucleon distribution functions by 3-body collisions of type NNN $\rightarrow$ ND  $\rightarrow$  P. Danielewicz and Q. Pan, PRC 46 (1992) (d,t,<sup>3</sup>He, but no  $\alpha$ !) coupled transport equations



t=100

fm/c

t=200

fm/

#### Caveat: Medium properties of LC: see discussion of low density matter,

refs.: C. Kuhrts, Beyer, Danielewicz,..PRC63 (2001) 034605, Typel, Röpke, et al., PRC81 (2010)

## Particle production: pions, Kaons, ....

#### Particularly interesting in view of the search for the high density symmetry energy

D\*2 +2

- 1. "direct effects": difference in proton and neutron (or light cluster) emission and momentum distribution
- 2. "secondary effects": production of particles, isospin partners  $\pi^{,+}$ , K<sup>0,+</sup>



usual treatment: sample spectral fct



#### Mass-radius relation of neutron stars and NSE



Small radii together with 2 solar mass NS seemed to imply a special behavior of the NSE (soft at ~  $2\rho_0$  and stiffer afterwards, like WFF1.

Diskussion by A. Steiner seems to make this conclusion less stringent. However, a non-polynomial behavior of  $E_{sym}(\rho)$  is seen also in other cases (e.g.DB)

#### The NSE energy in Core-Collapse Supernovae (CCSN)

Workshop: Simulating the Supernova neutrinosphere with heavy ion collision, Trento, ECT\*; April 2014



### Correlations & Neutrino Scattering

- Neutrinos "see" more than one particle in the medium.
- Nature of spatial and temporal correlations between nuclei, nucleons and electrons affect the scattering rate.



At small  $q_0$  and  $\mathbf{q}$  the neutrino cannot resolve single particles.

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Sawyer (1975, 1989)
Iwamoto & Pethick (1982)
Horowitz & Wherberger (1991)
Raffelt & Seckel (1995)
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(S. Reddy)

#### Neutrino opacities and neutrino spectra dependent on low density symmety energy

Most of Equations of State treat neutrons and protons as "non-interacting" (quasi)particles that move in a mean-field potential  $U_{n,p}(\rho, T, Y_e)$ .



- Energy shift helps overcome electron final state blocking.
- Enhances ν<sub>e</sub> absorption
- Larger energy needed to produce neutrons suppresses anti-Ve absorption.



solid=with symm energy shift, dashed without

#### Nucleosynthesis



- Elements between Zn and Mo, including <sup>92</sup>Mo, are produced
- Mainly neutron-deficient isotopes are produced
- No elements heavier than Mo (Z = 42) are produced.



Interesting to consider Homework from ECT\* workshop:

1. Use transport model (calibrated on HIC data) to simulate warm matter at low density in box calculation and extract dynamical neutrino response functions

2. Check freeze-out densities from coalescence and particle correlation methods. (Re-)analyze more data sets of HIC.

- 3. Explore themodynamical conditions of freeze-out configuration for larger N/Z to come closer to neutrinosphere conditions
- 4. Improve light cluster description in transport codes
- 5. Establish a kind of collaboration:
  - website,
  - white paper for the Texas low energy community meeting,
  - another workshop in about 2 years

Important point:

Opportunity, where nuclear physics can make a concrete contribution to an important astrophysical problem!

#### **Final Remarks:**

NSE: a field of strong exchange between theory and experiment

largest uncertainties at very low ( $\rho < 0.1 \rho_0$ ) and at high density ( $\rho > 2 \rho_0$ )(clusters)(strongly correlated)

mapping of microscopic models to phenomenological approaches (both in nuclear structure and in transport calculation): (e.g. effective masses, mean field potentials, kinetic energies, medium cross sections, medium modification of clusters)

development of transport approaches: fluctuations and fragmentation dynamical role of light clusters

Direct confrontation with astrophysical questions: NS, e.g. mass radius relation and other observables CCSN: neutrino opacities in the v-sphere

many things to do in the future