

# The determination of incompressibility in nuclei and nuclear matter

1) Incompressibility in nuclei

2) Links with nuclear matter

E. Khan, J. Margueron and I. Vidaña PRL(2012)092501

E. Khan, N. Paar, D. Vretenar, L.-G. Cao, H. Sagawa and G. Colò, PRC87(2013)064311

E. Khan and J. Margueron, PRC88(2013)034319

E. Yüksel, E. Khan and K. Bozkurt, EPJA49(2013)124

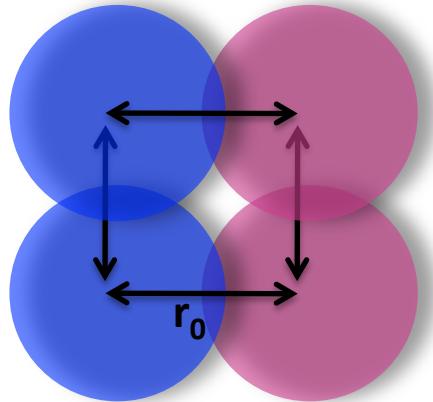
M. Vandebrouck et al., to be published in PRL (2014)

E. Khan



## 1) Incompressibility in nuclei

# Incompressibility of a finite fermionic system (nucleus with $A$ nucleons)



Incompressibility of the system:  $K_A \propto \frac{d^2 E/A}{d\langle r^2 \rangle^2}$

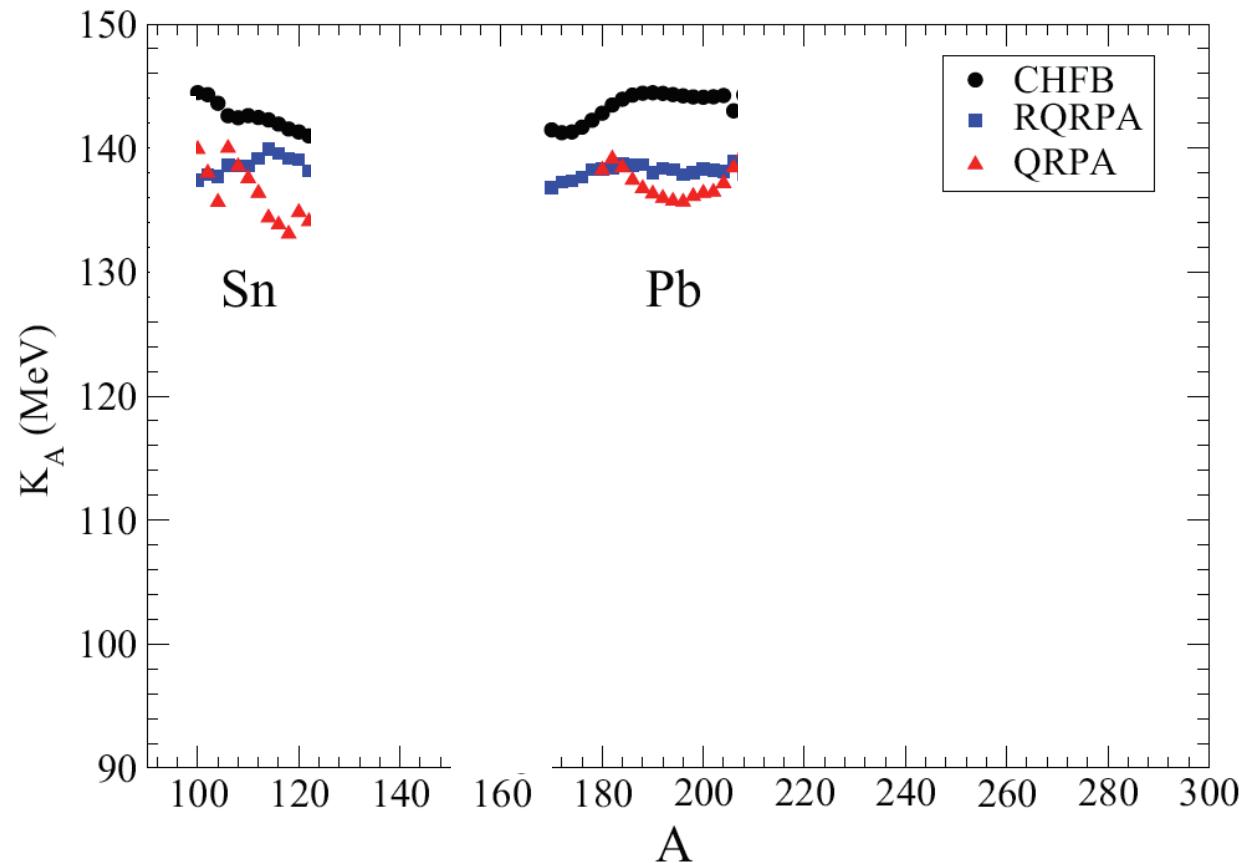
$$E_{GMR} \propto \sqrt{K_A}$$

Zero point kinetic energy:  $T_0 = \frac{\hbar^2}{mr_0^2}$

Incompressibility of the system  $\approx 5 * \text{Zero point kinetic energy}$

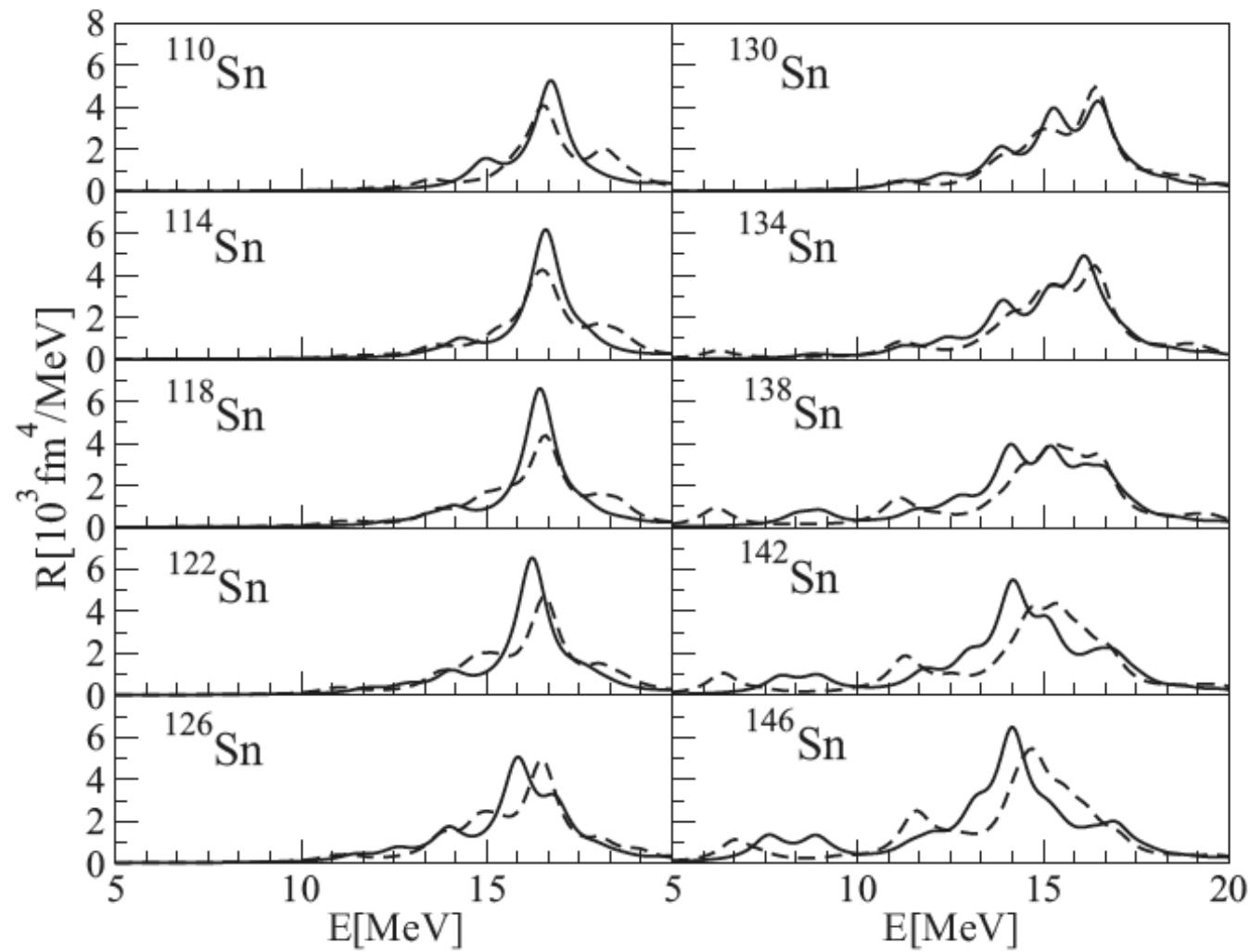
$$150 \text{ MeV} \quad \approx \quad 5 * \quad 30 \text{ MeV}$$

# Microscopic predictions

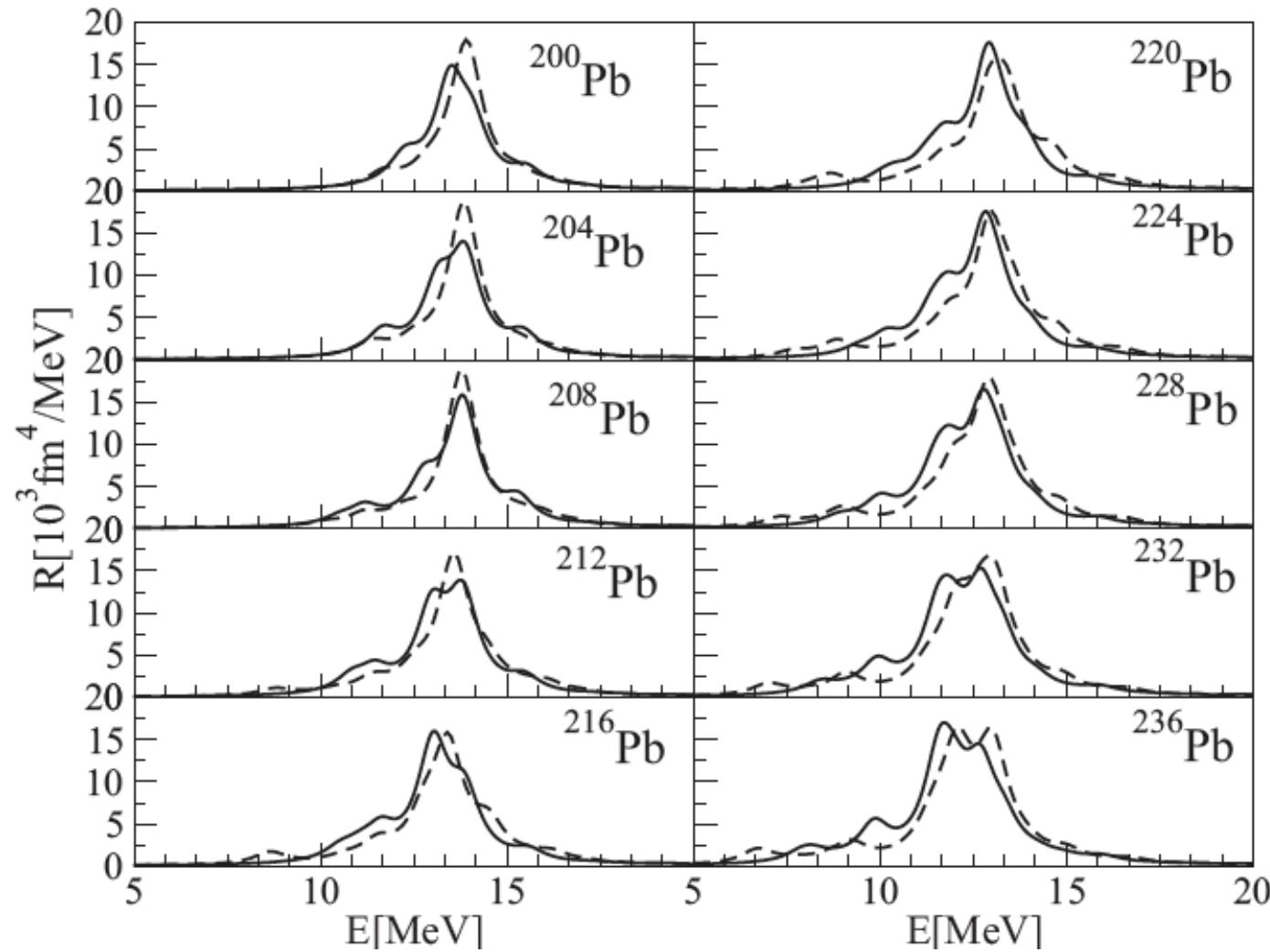


Measurement of the IS monopole strength in unstable Sn and Pb isotopes

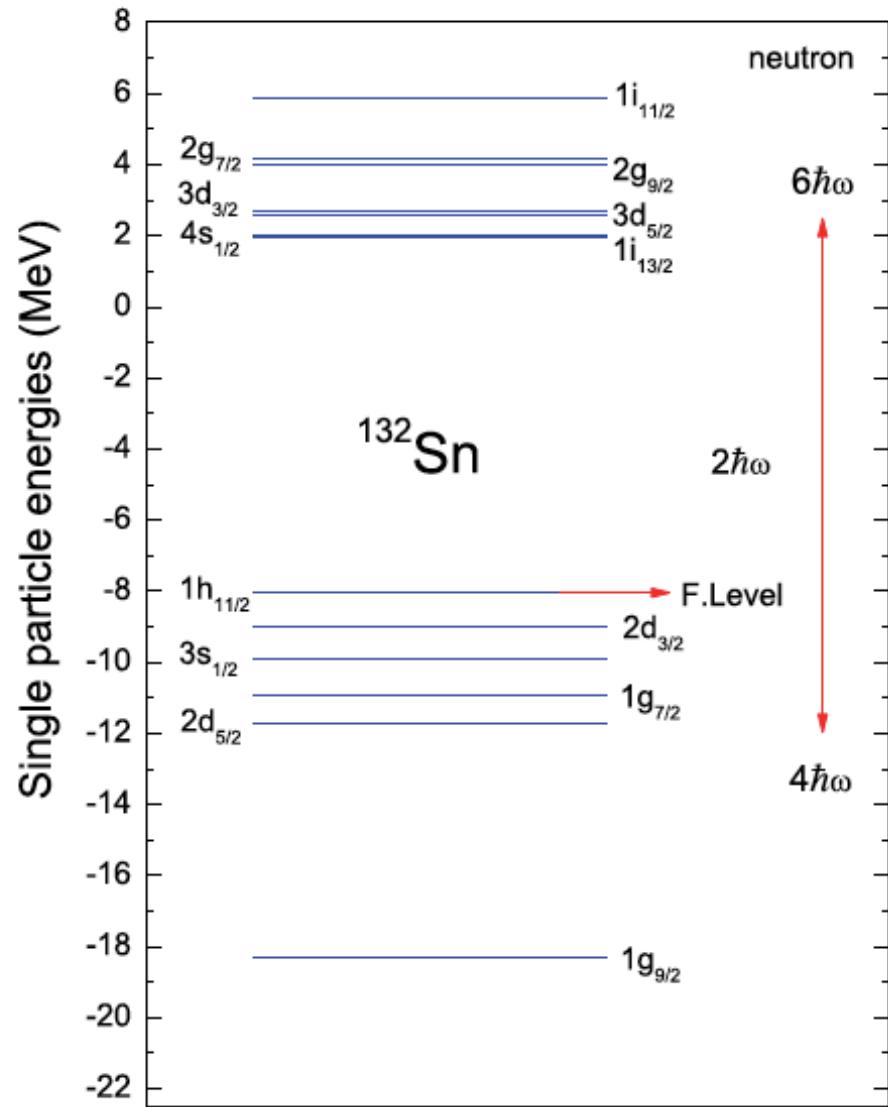
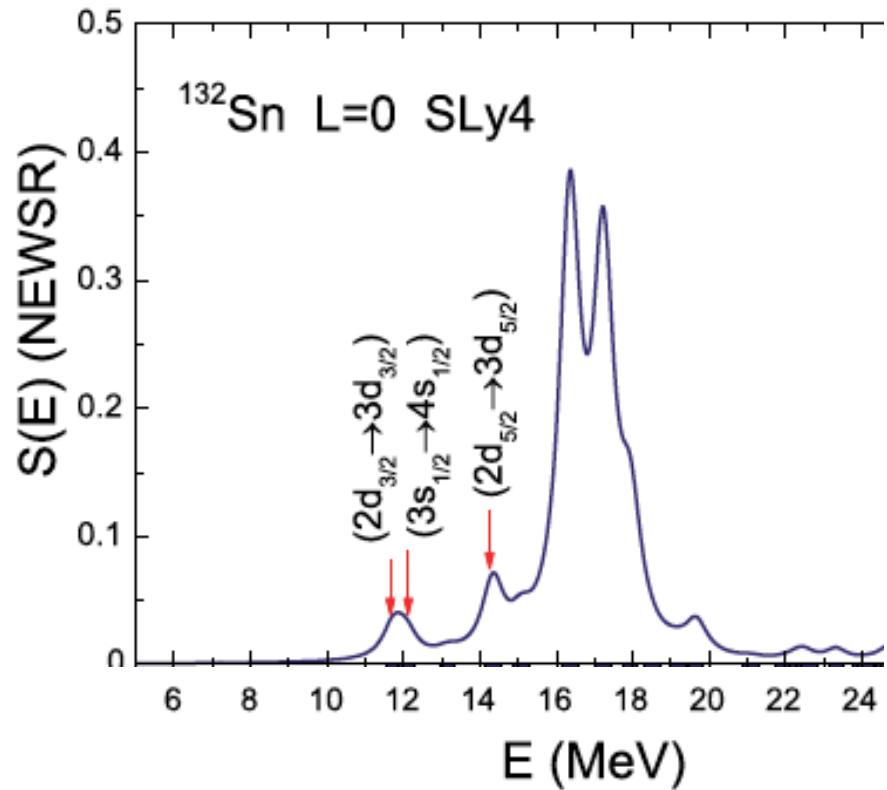
# The soft monopole mode



# The soft monopole mode



# What is the soft monopole mode ?



# Nuclei incompressibility vs. pairing

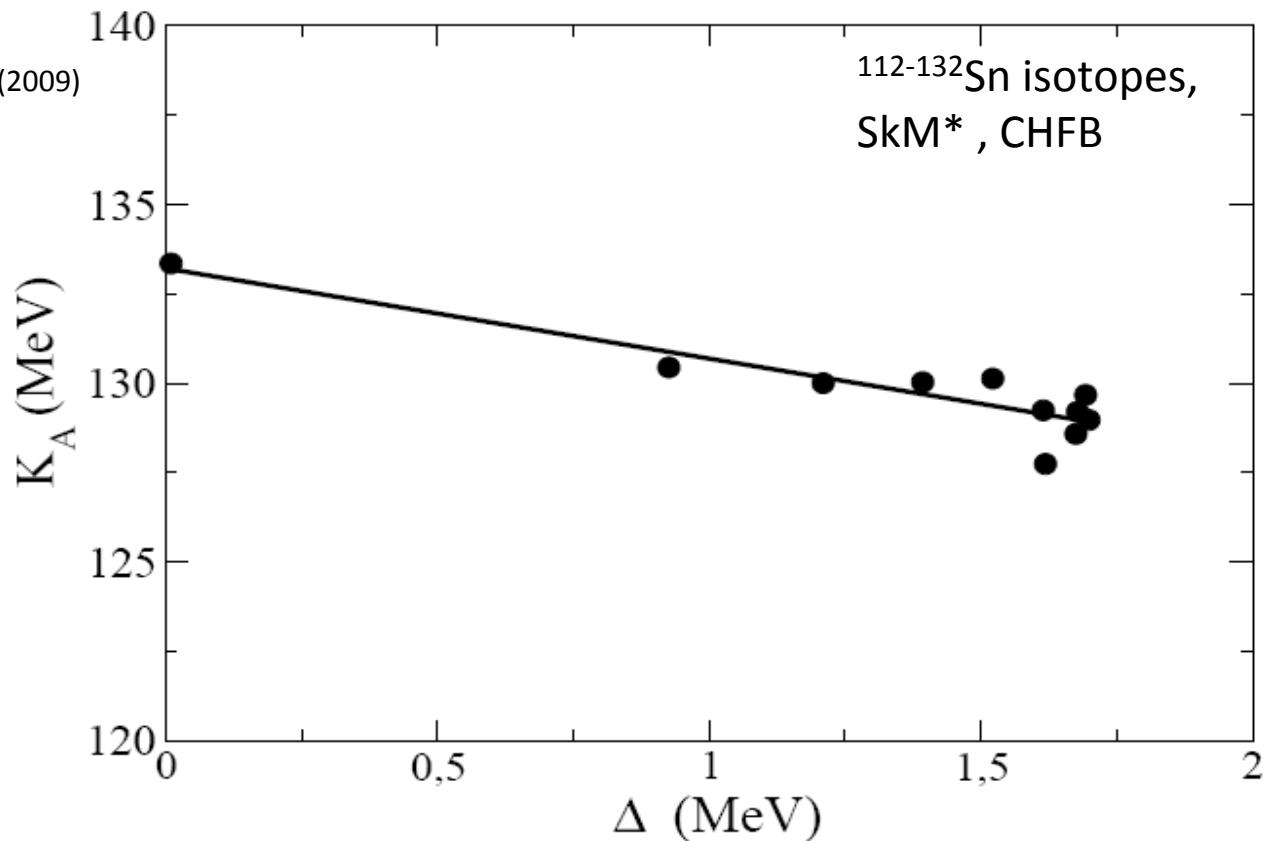
Civitarese et al. (1991)

Li, Colò, Meng (2008)

Khan, Margueron, Colò, Hagino, Sagawa (2009)

Li-Gang Cao, Sagawa, Colò (2012)

$$E_{\text{GMR}} = \sqrt{\frac{\hbar^2 K_A}{m \langle r^2 \rangle}}$$



- Cooper pairs favor compressibility

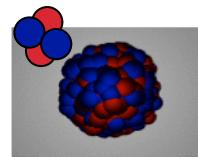
- Incompressibility of **superfluid** nuclear matter:  $K_\infty(\Delta)$

→ Pairing has a small but non negligible effect on the GMR

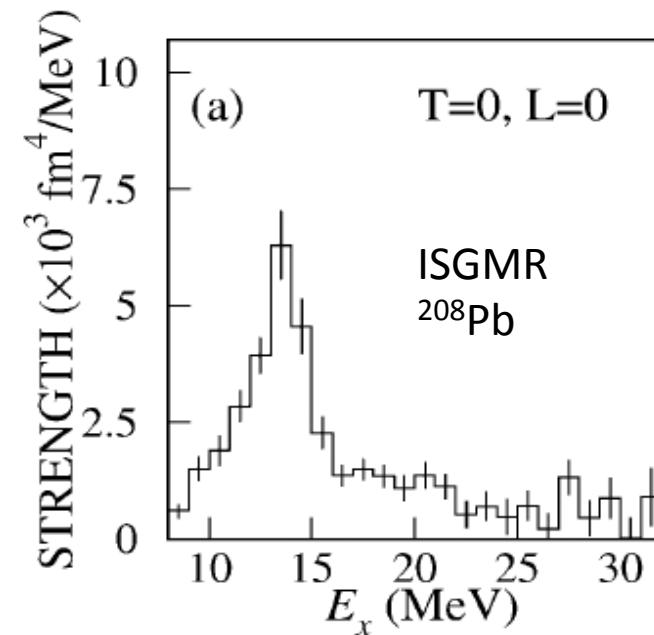
# Method to determine $K_A$

- The nucleus exhibits a collective compression mode (how nice !):  
the **Giant Monopole Resonance**

Inelastic  $\alpha$  scattering



Nucleus (e.g.  $^{208}\text{Pb}$ )



M. Uchida, H. Sakaguchi et al., PLB557(2003)12

# Experimental cornerstones

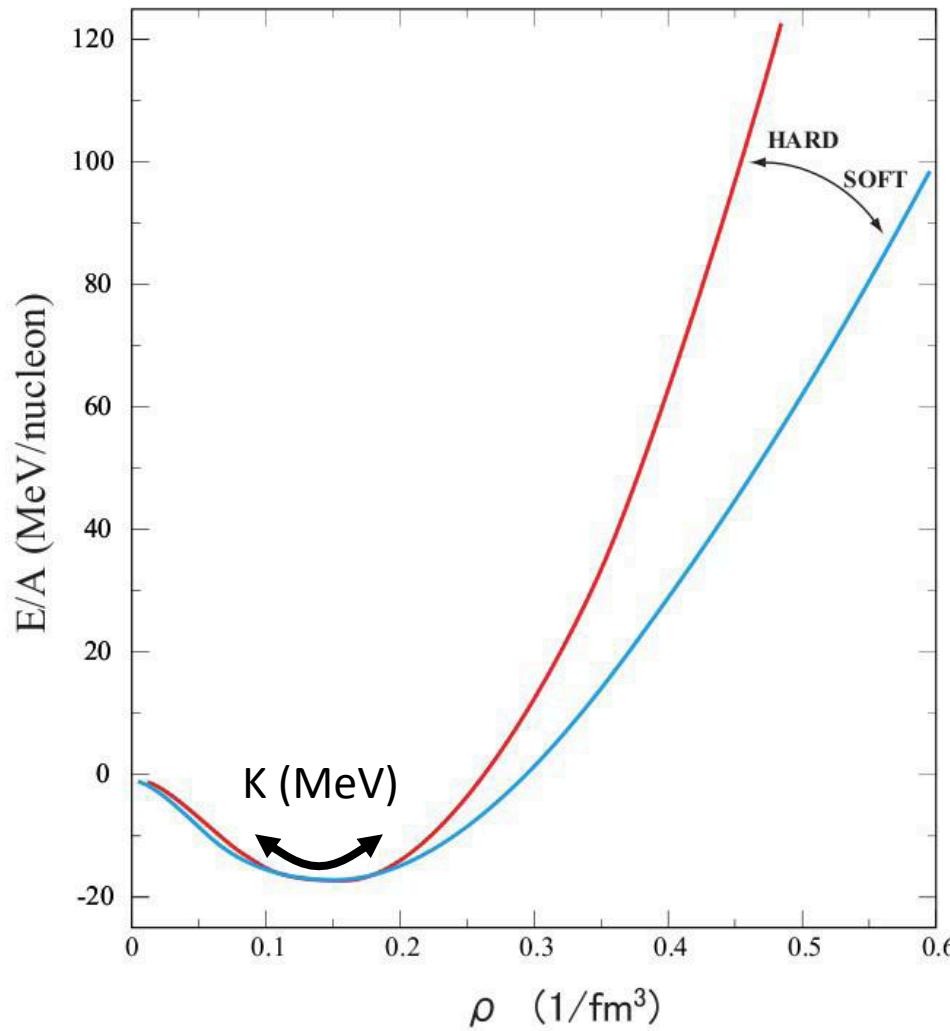
- Is it possible to measure the IS monopole strength neutron-rich exotic nuclei ?
- Is it possible to get experimental clues on a soft monopole mode ?



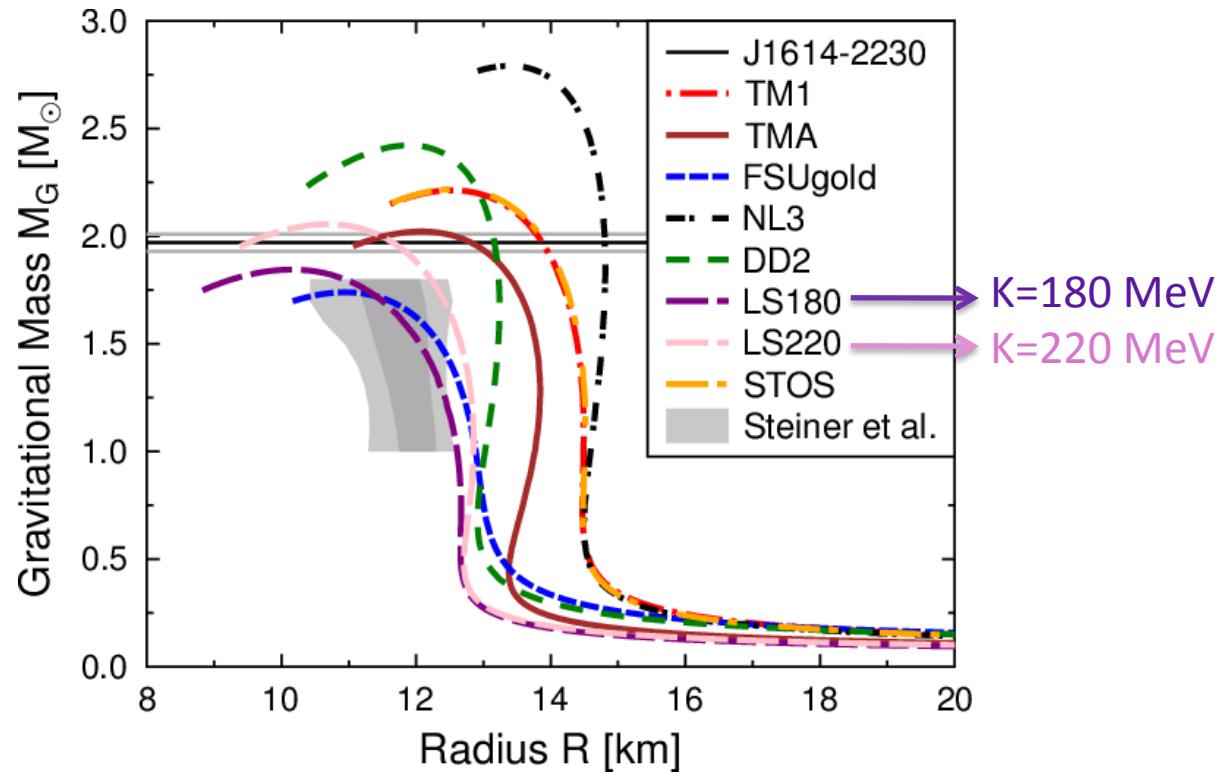
See Marine Vandebrouck's talk

## 2) Links with nuclear matter

# Nuclear incompressibility

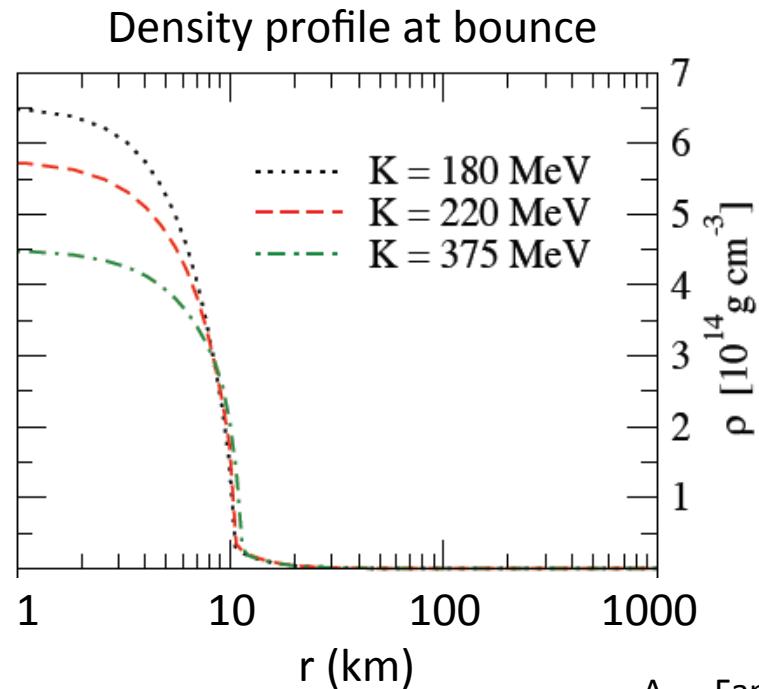
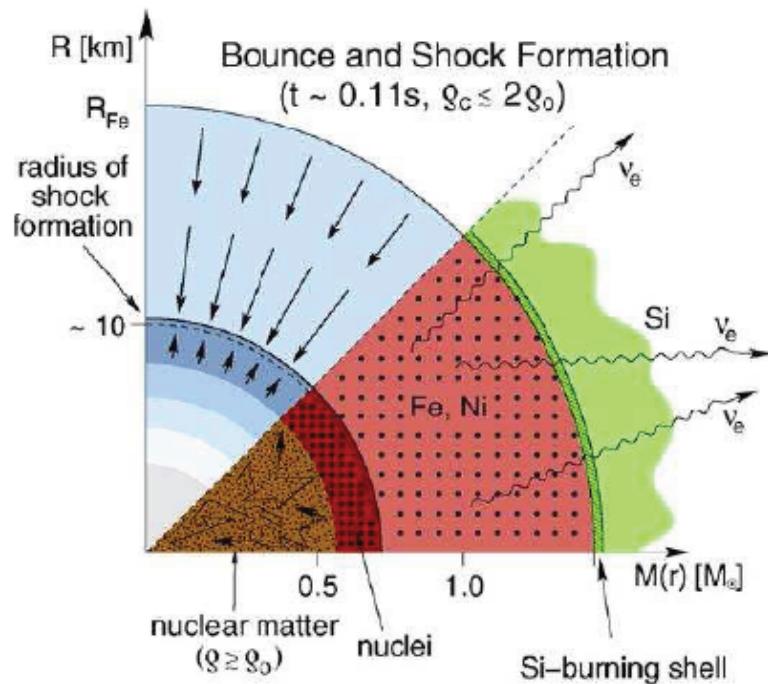


# Neutron stars



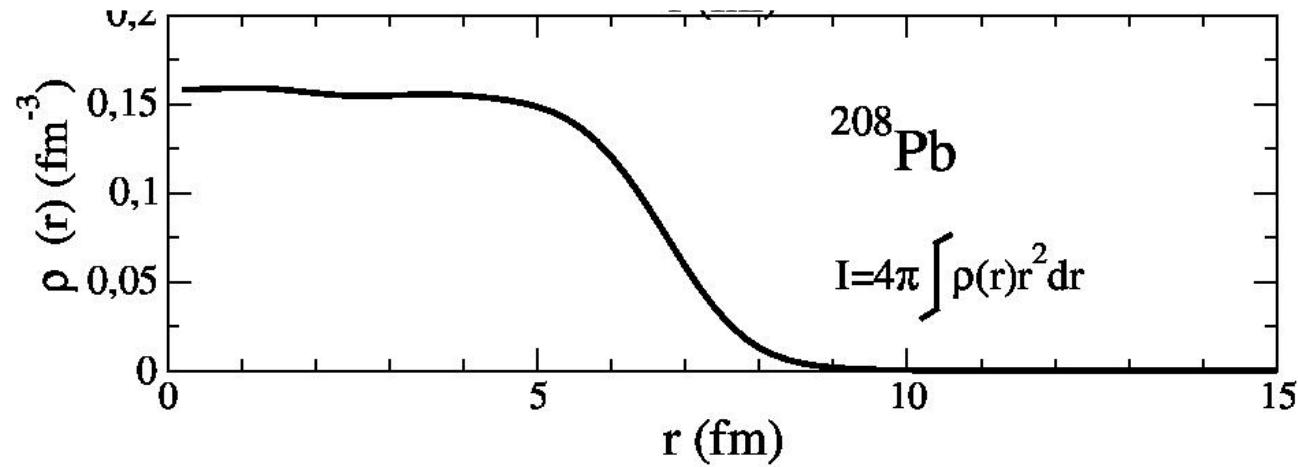
Matthias Hempel  
ITP Frankfurt

# SuperNovaе bounce

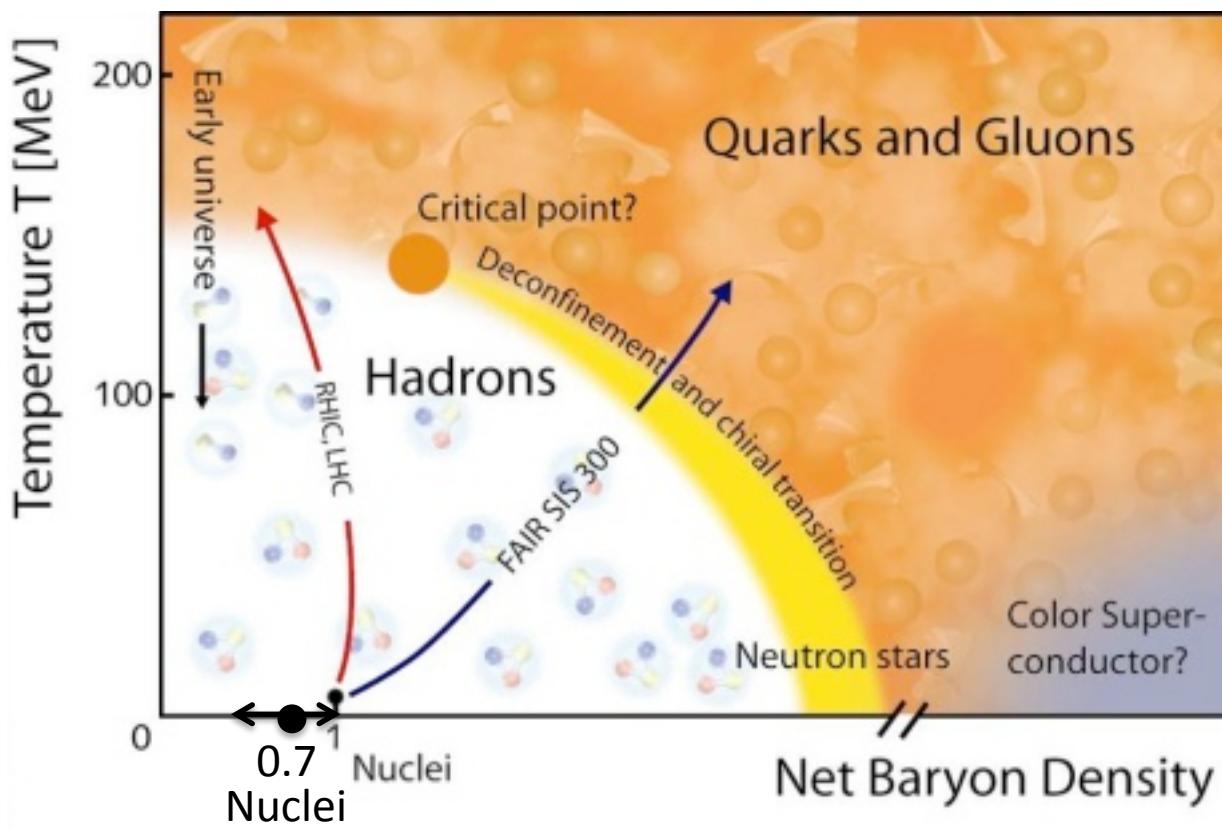


A. Fantina  
PhD (2010)  
IPNO-IAA

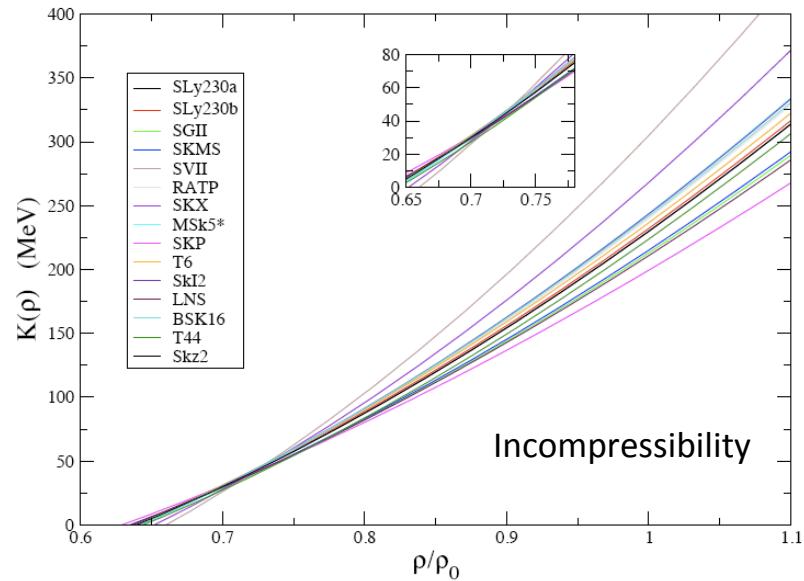
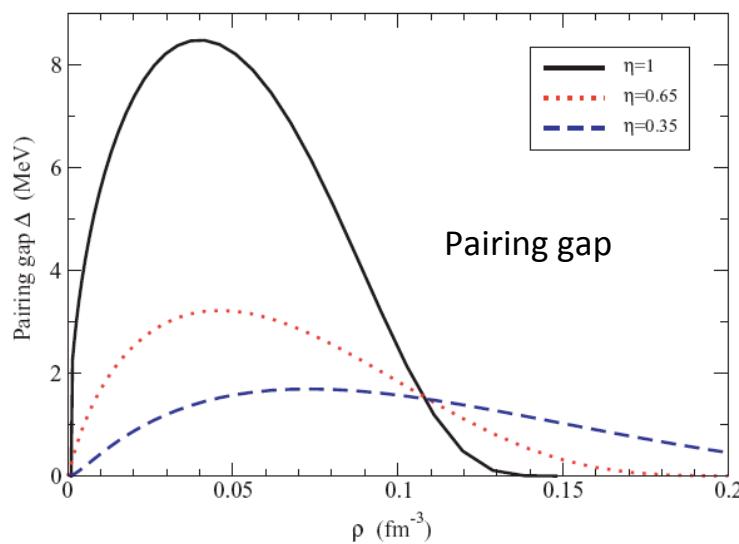
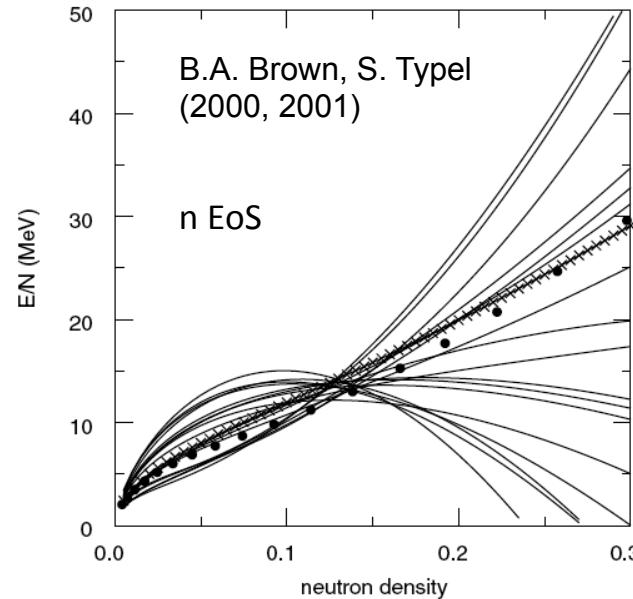
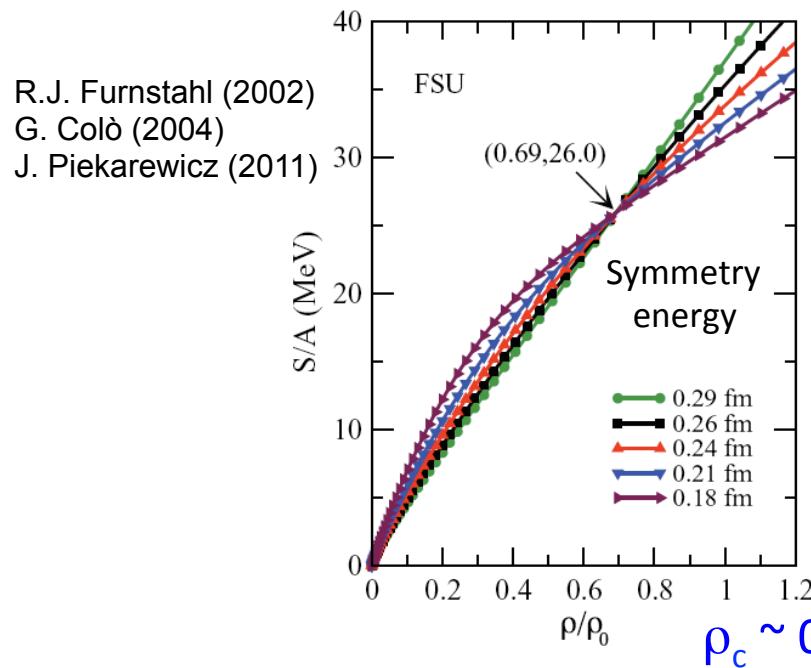
# Saturation ?



- Surface: 2/3 of nucleons in  $^{208}\text{Pb}$
- Saturation density area may not be the most probed



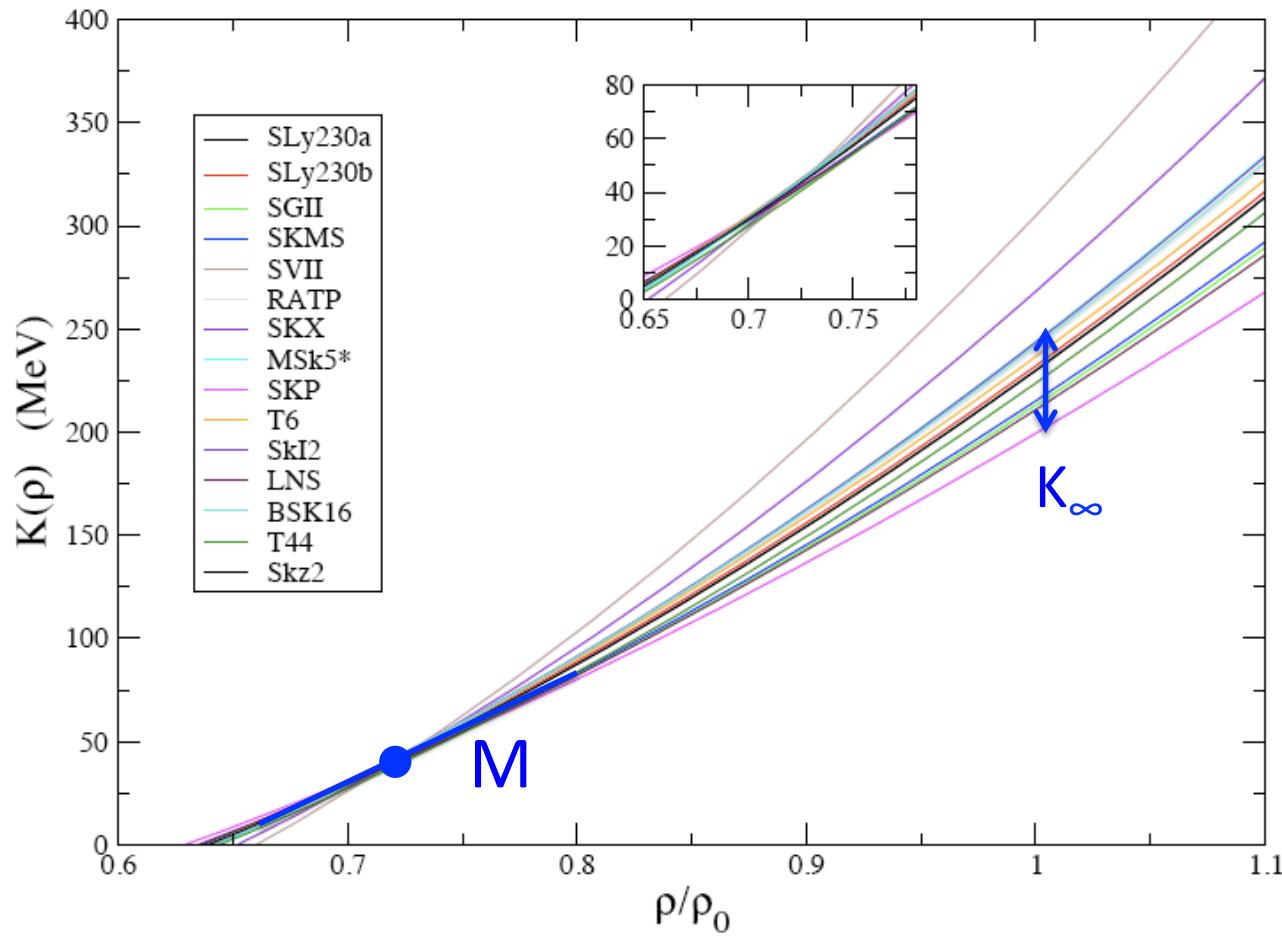
# The crossing density



E. Khan, M. Grasso, J. Margueron (2010)

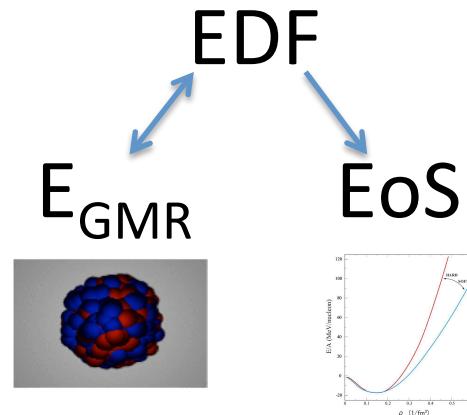
**M**

$M = 3\rho K'(\rho)|_{\rho=\rho_c}$  : third derivative of EoS at the crossing density



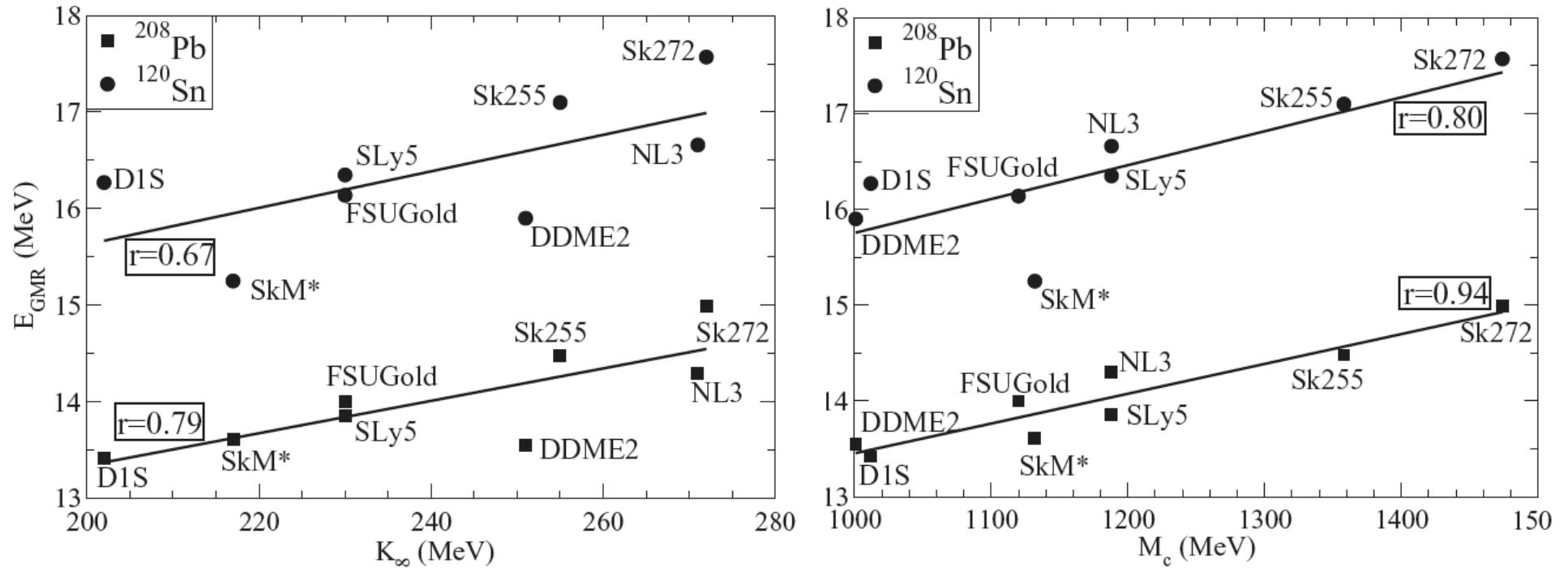
# Microscopic method

$$V = \int d\mathbf{r} \sum_{T=0,1} \left[ C_T^\rho \rho_T^2 + C_T^{\Delta\rho} \rho_T \Delta\rho_T + C_T^\tau \rho_T \tau_T + C_T^J J_T^2 + C_T^{\nabla J} \rho_T \nabla \cdot \mathbf{J}_T + C_T^s s_T^2 + C_T^{\Delta s} s_T \cdot \Delta s_T - C_T^J s_T \cdot \mathbf{T}_T + C_T^{\nabla s} (\nabla \cdot s_T)^2 - C_T^\tau J_T^2 + C_T^{\nabla J} s_T \cdot \nabla \times \mathbf{j}_T \right]$$



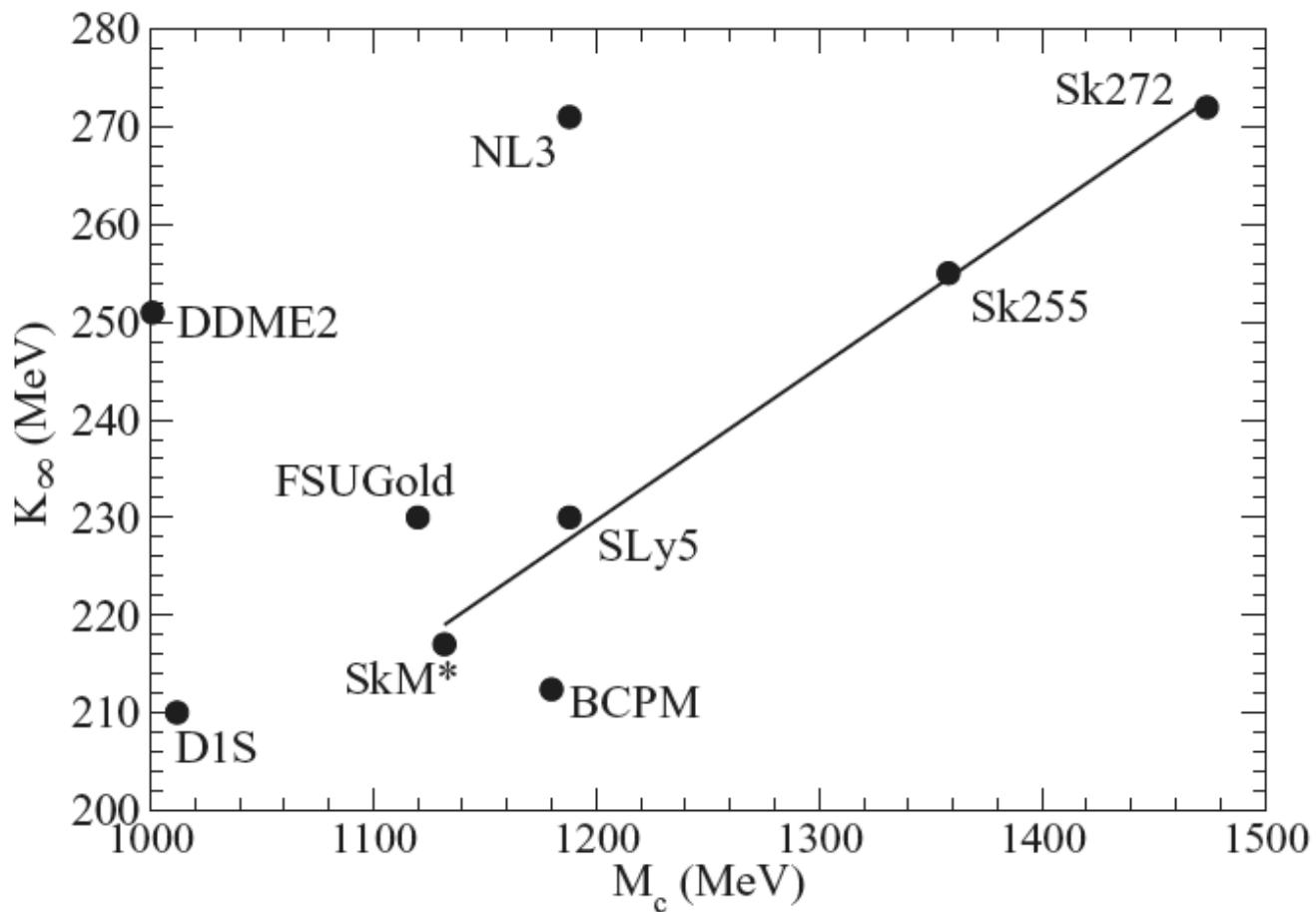
- Nuclear structure models: from EDF to E<sub>GMR</sub>
- Limitations : all the terms (EoS) have to be correctly predicted at once

# From the GMR to nuclear matter



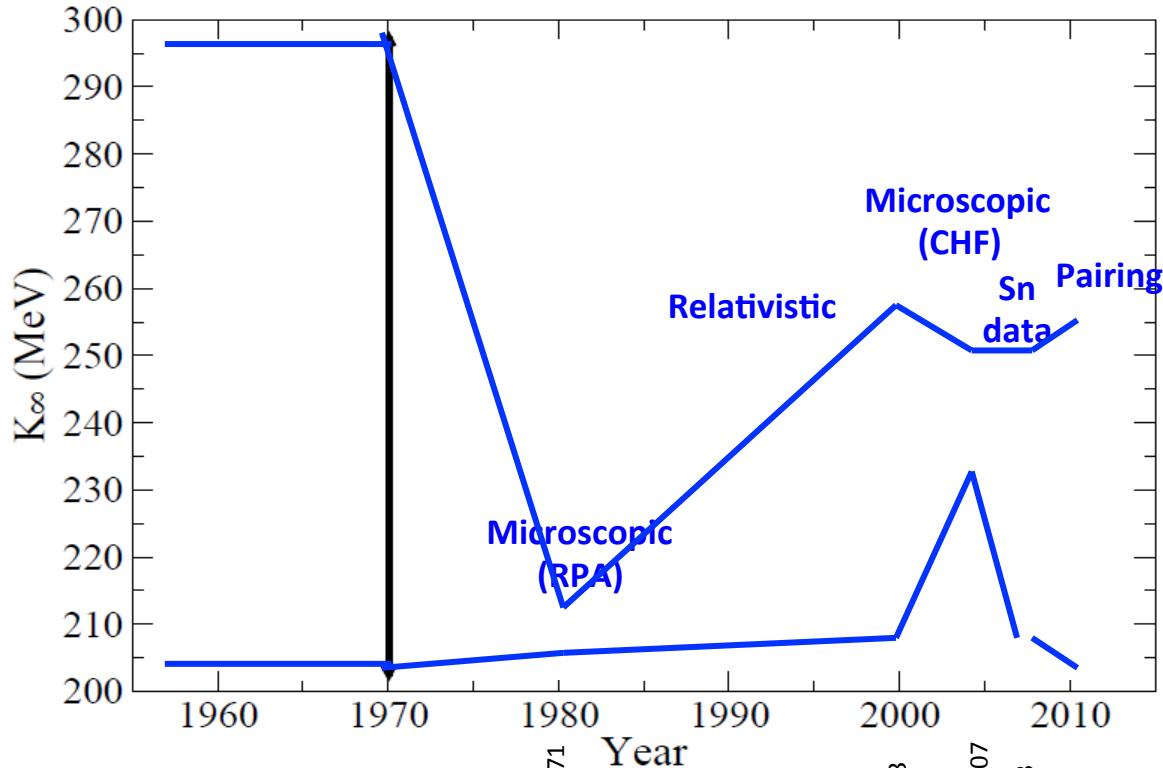
$M_c$  better constrained by the data than  $K_\infty$

# The reason for dispersed $K_\infty$ values



Lack of constraints on the density dependence of the EDF generates dispersion of  $K_\infty$

# Uncertainties on $K_\infty$



$$K_\infty = 240 \pm 30 \text{ MeV}$$

J.P. Blaizot, Phys. Rep. **64**(1980)171

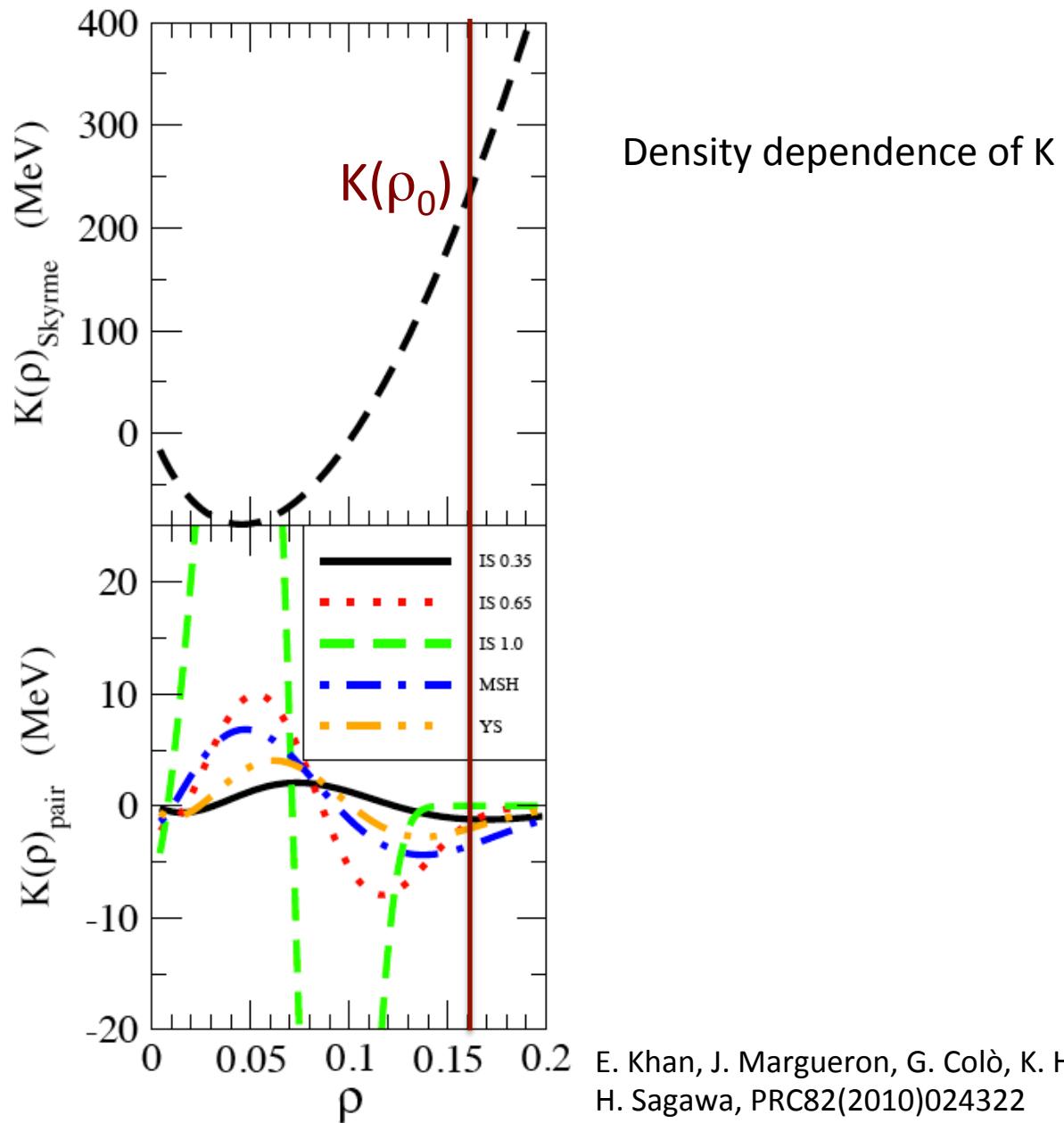
Z.Y. Ma et al, NPA**86**(2001)173

G. Colo et al, PRC**70**(2004)024307

J.Li et al, PRC**78**(2008)064303

E. Khan PRC**80**(2009)011307(R)

# Origin of the pairing effect



# Some clues on N vs. Z asymmetry

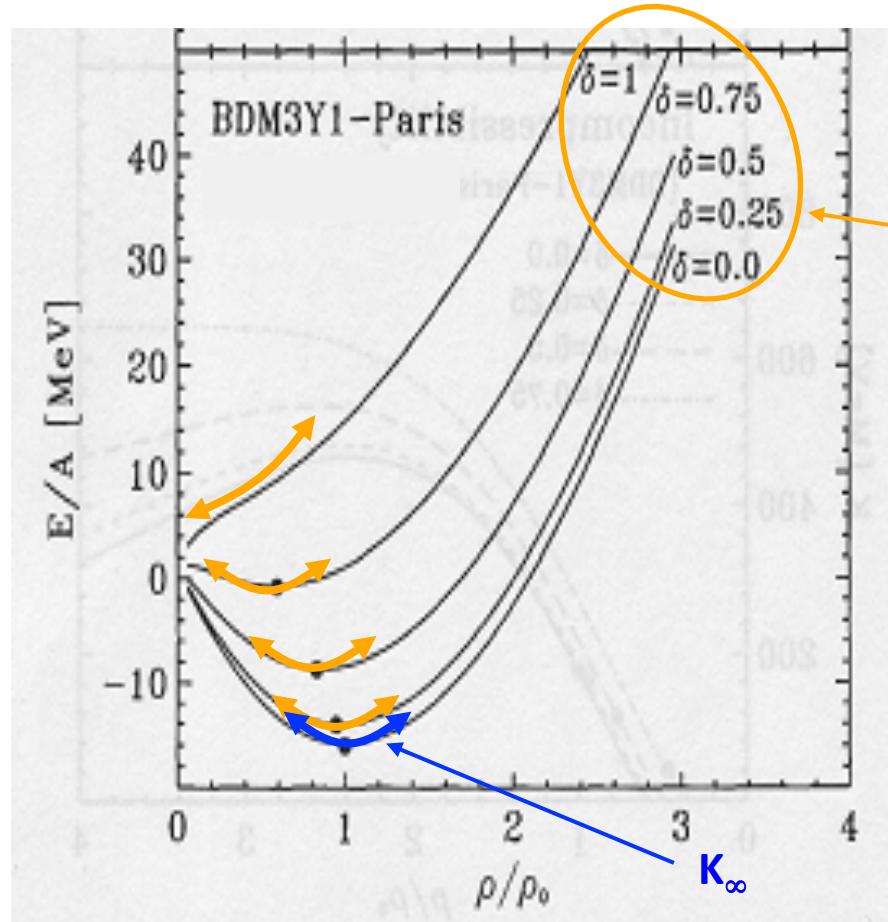
$$E(\rho, \delta) = E(\rho, 0) + a_{sym}(\rho)\delta^2$$

: density and neutron excess

$$\delta = (N-Z)/A$$

$$\begin{aligned}\delta(^{100}\text{Sn}) &= 0 \\ \delta(^{132}\text{Sn}) &= 0.24 \\ \delta(^{208}\text{Pb}) &= 0.21\end{aligned}$$

$$\delta(^{180}\text{Sn}) = 0.44$$



evolution of  
incompressibility  
with asymmetry:

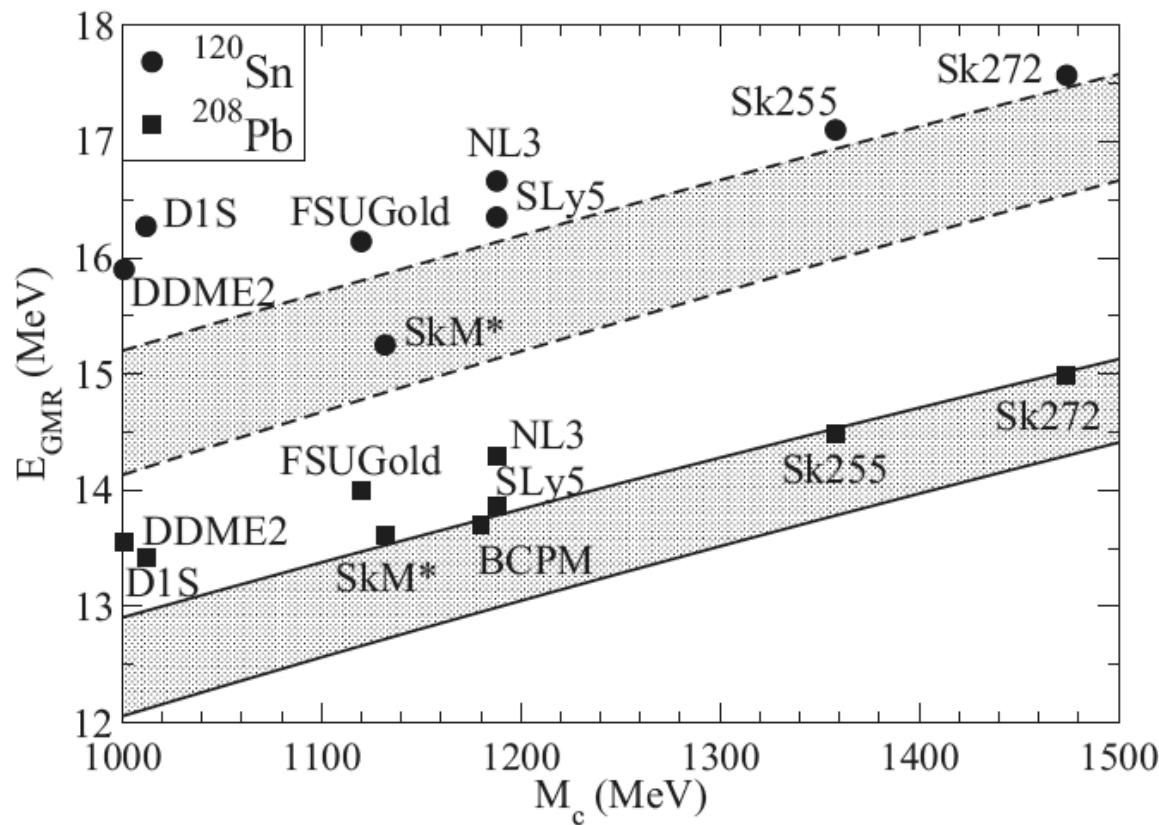
$$\begin{aligned}K(\rho, \delta) &\sim \frac{1}{2} \frac{\partial^2 E(\rho, \delta)}{\partial \rho^2} \\ &= K(\rho) + K_{sym}(\rho)\delta^2\end{aligned}$$

$$K_{sym} = \left. \frac{1}{4} \frac{\partial^4 E(\rho, \delta)}{\partial \rho^2 \partial \delta^2} \right|_{\rho=\rho_0, \delta=0}$$

# Conclusions

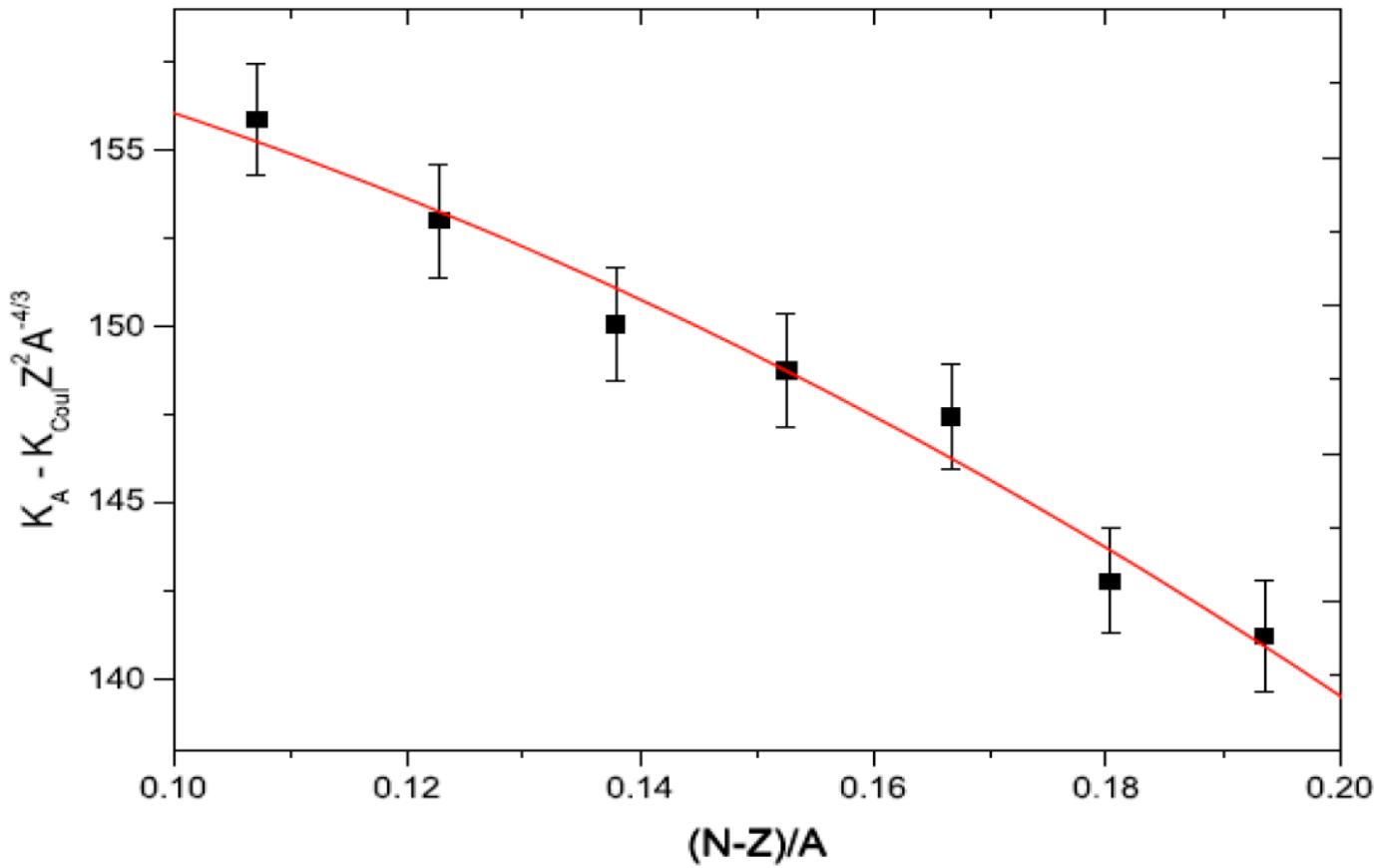
- Incompressibility in nuclei originates from the ZPE
- Concomitant exp. and theo. Indications for a soft monopole mode
- Measurement of the IS L=0 in  $^{132}\text{Sn}$ ,  $^{210,212}\text{Pb}$
- The GMR measurement constrains Mc before Kinf
- The density dep. of the EDF is critical to better constrain Kinf.
- Far beyond  $^{132}\text{Sn}$  to grasp the isospin dep. of nuclear incompressibility

# Analytical $E_{GMR}(M_c)$



$$E_{GMR} = \frac{\hbar}{R} \left\{ \frac{20\pi}{3mA} \int_{\rho_0/2}^{\rho_0} \left[ a \ln \left( \frac{\rho_0}{\rho} - 1 \right) + R \right]^2 \left( \frac{M_c \rho}{3\rho_c} - \frac{M_c}{3} + K_c \right) \frac{a}{1 - \rho/\rho_0} \frac{\rho_0^2}{\rho^2} d\rho + \frac{5K_{\text{Coul}}}{3m} Z^2 A^{-4/3} \right\}^{1/2}$$

# Determining the isospin dependence of the incompressibility



T. Li *et al.*, PRL99(2007)162503

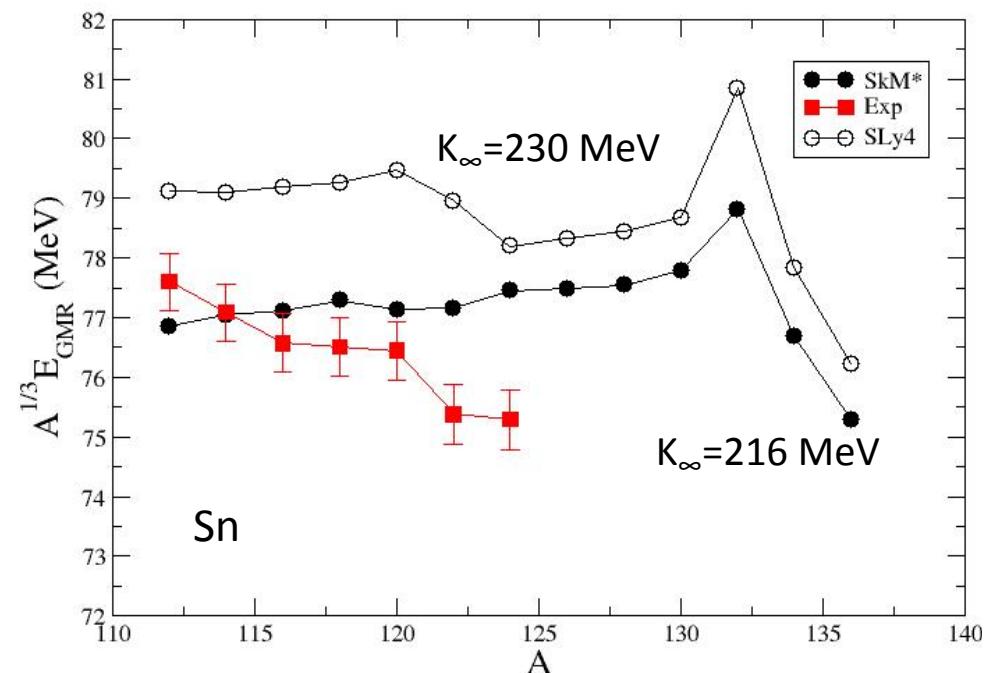
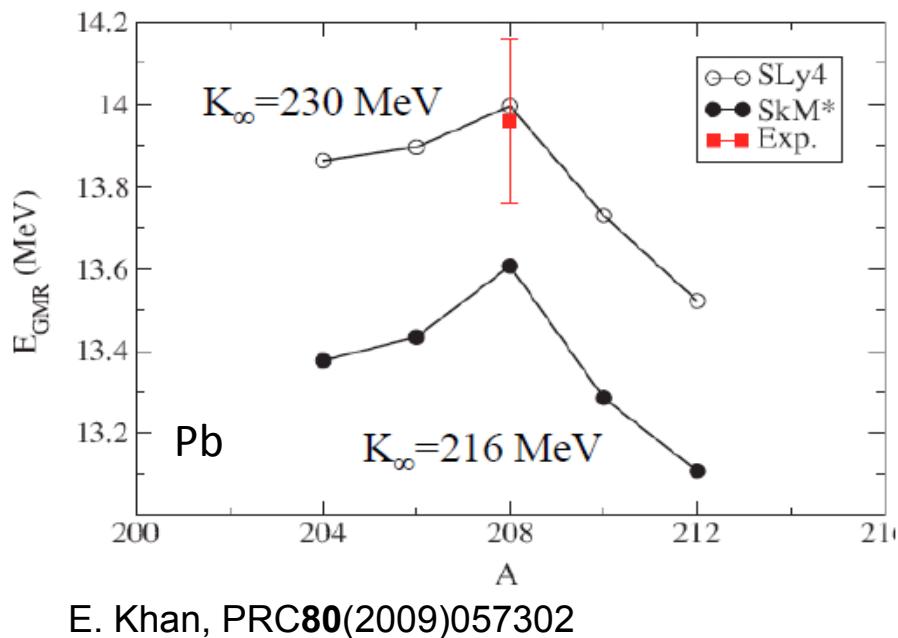
- $K_\tau = -550 \pm 100$  MeV

- However the method is macroscopic

$$K_A = K_\infty + K_{surf} A^{-1/3} + K_\tau \delta^2 + K_{Coul} \frac{Z^2}{A^{4/3}},$$

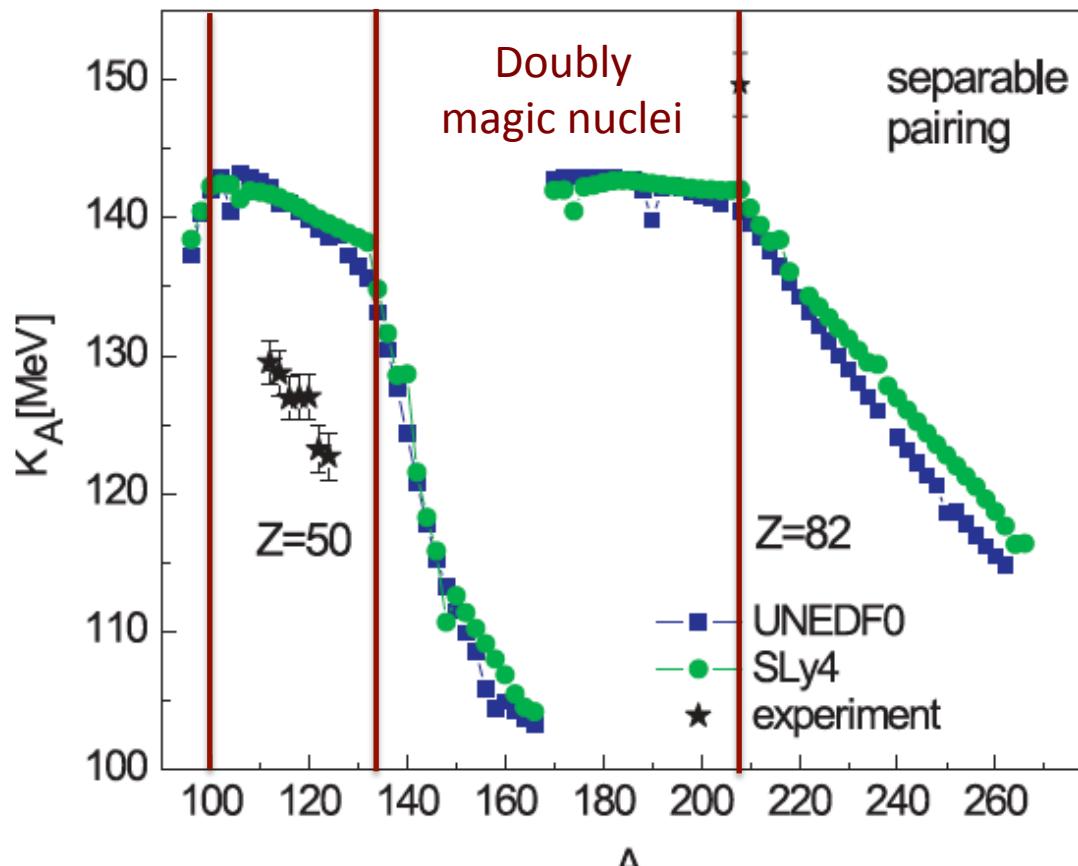
# Pairing and shell effects on the GMR

- Pairing  $\Rightarrow$  shell effects on the GMR value



- Softness of Sn
- Necessity to measure isotopic chains, including unstable neutron-rich nuclei

# The shell effect on the GMR

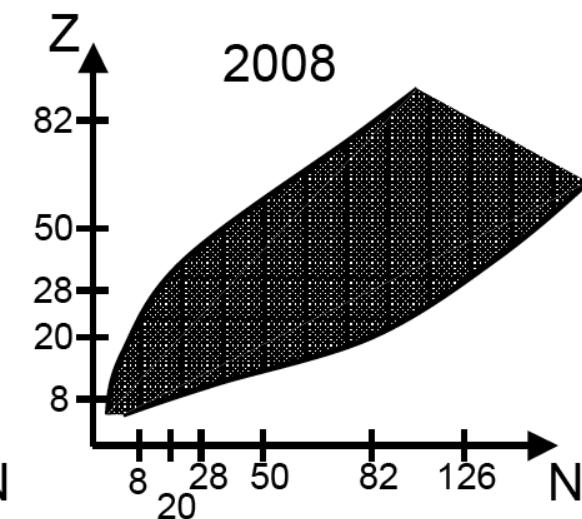
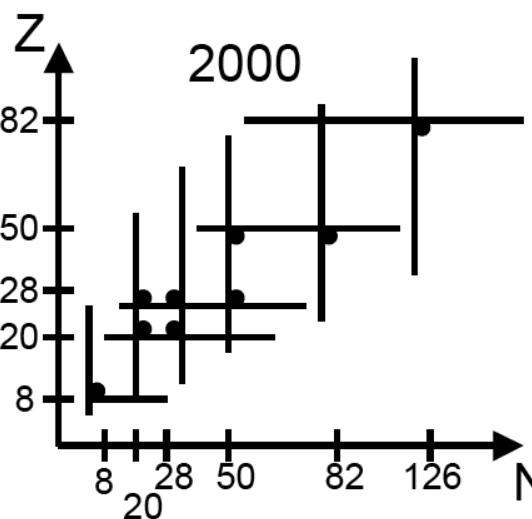
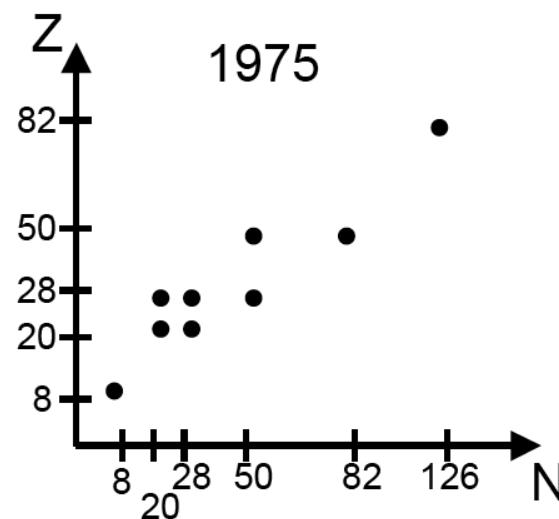


P. Vesely, J. Toivanen, B.G. Carlsson, J. Dobaczewski,  
N. Michel, A. Pastore, PRC86(2012)024303

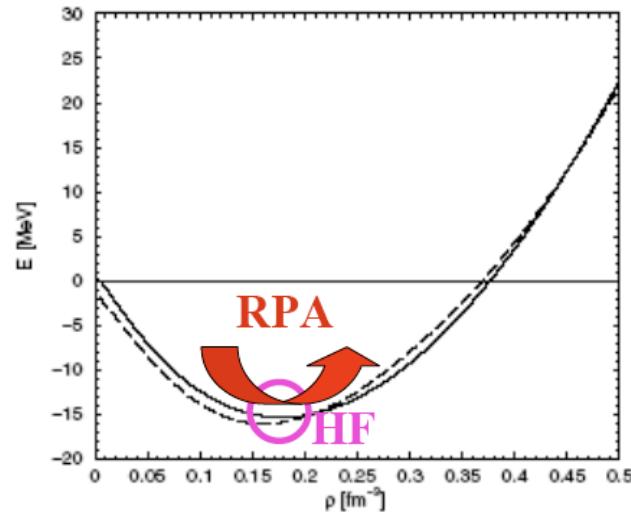
→ shell effects (MEM ?)

Pairing and shell effects not enough to explain Pb/Sn difference on  $K_\infty$

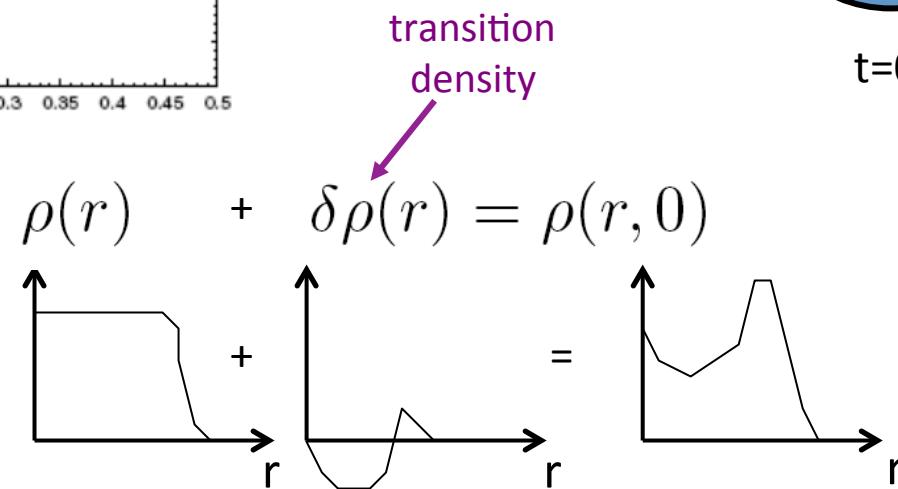
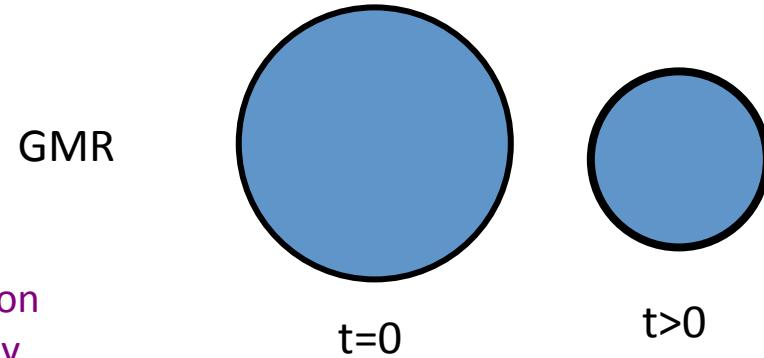
# Nuclear excitations



# Picture of a GMR



$$\rho(r, t) = \rho(r) + \delta\rho(r)\cos(\omega t)$$

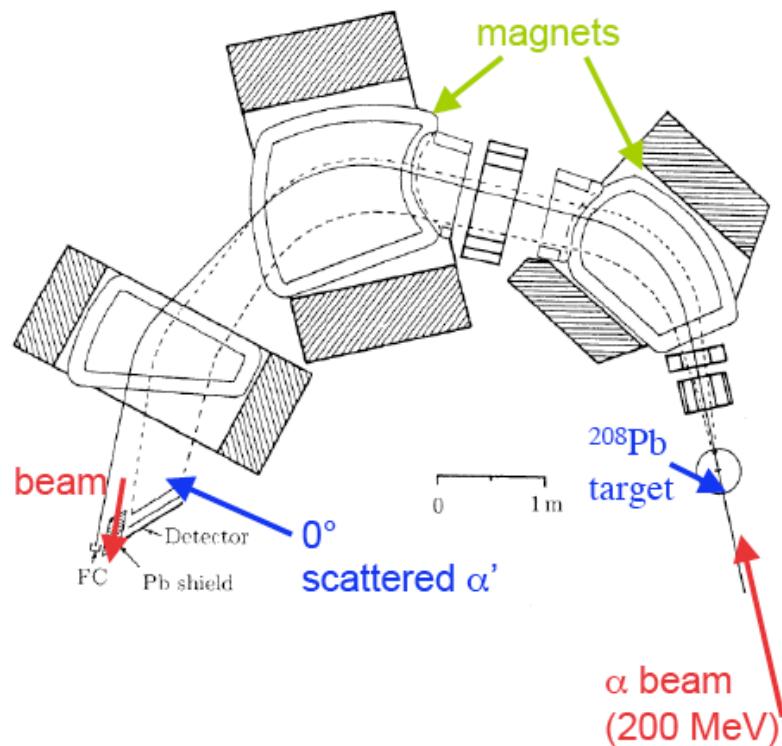


- GR are collective (many ph pairs involved)
- Small amplitude vibration:  $\delta\rho \ll \rho$

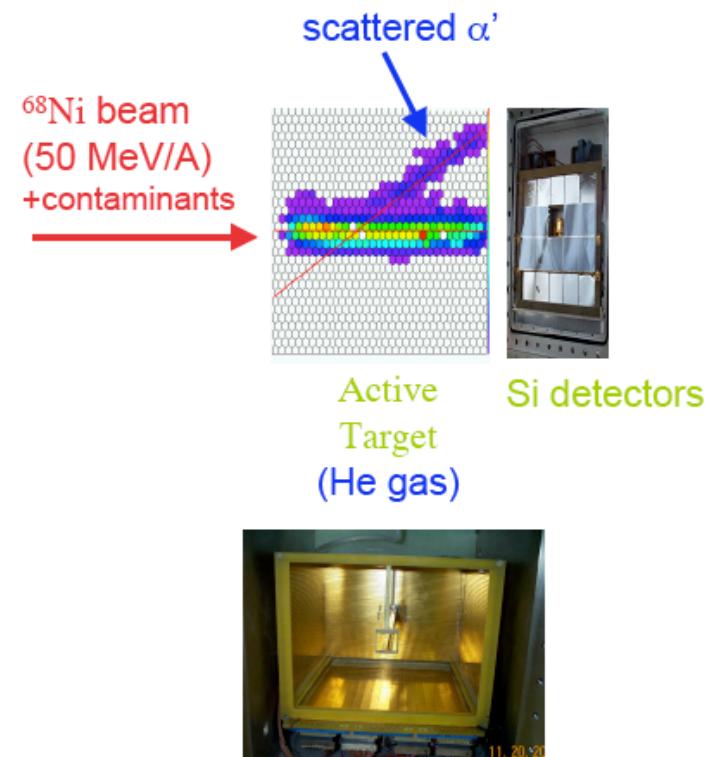
$$\delta\rho(r) = \sum_{mi} (X_{mi} - Y_{mi}) \phi_i^*(r) \phi_m(r)$$

# Measurement of IS monopole modes

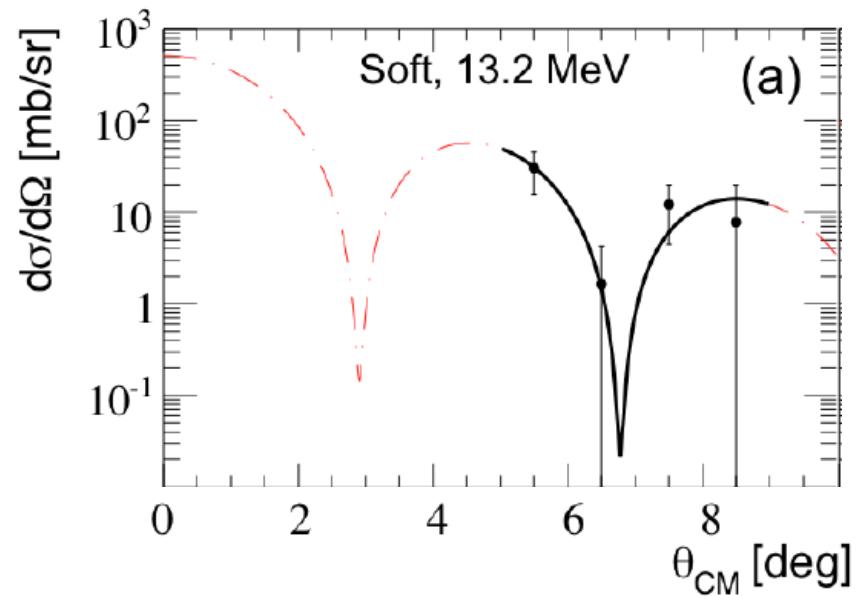
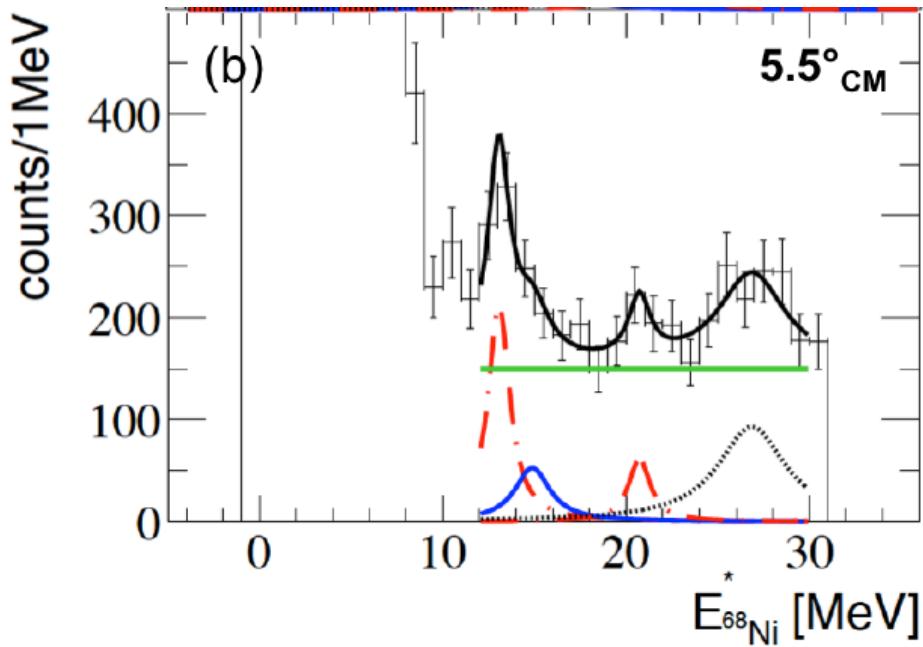
With stable nuclei



With exotic nuclei



# Is the soft monopole mode detected?



# Exp. collaboration

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+ G. Colò (Milano) and N. Keeley (Warsaw) for theo. predictions of the angular distributions