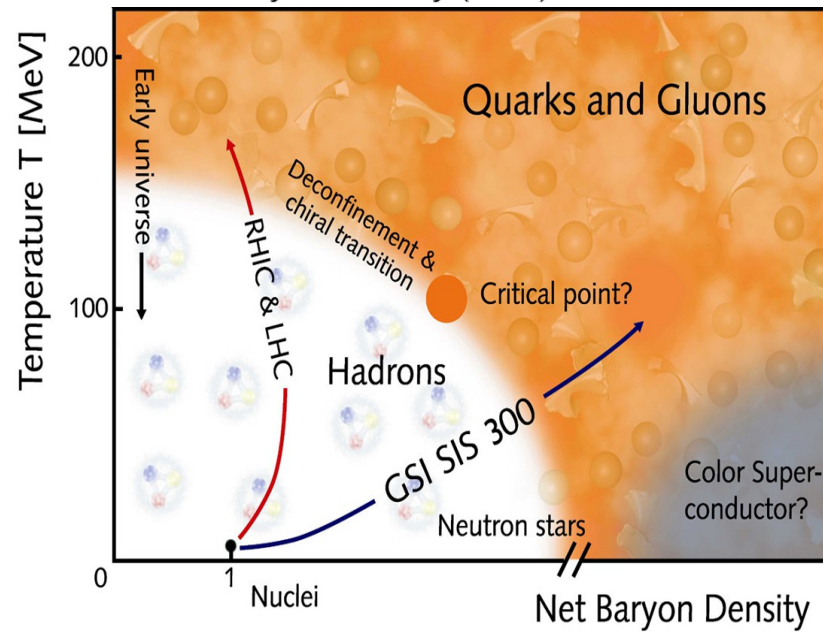
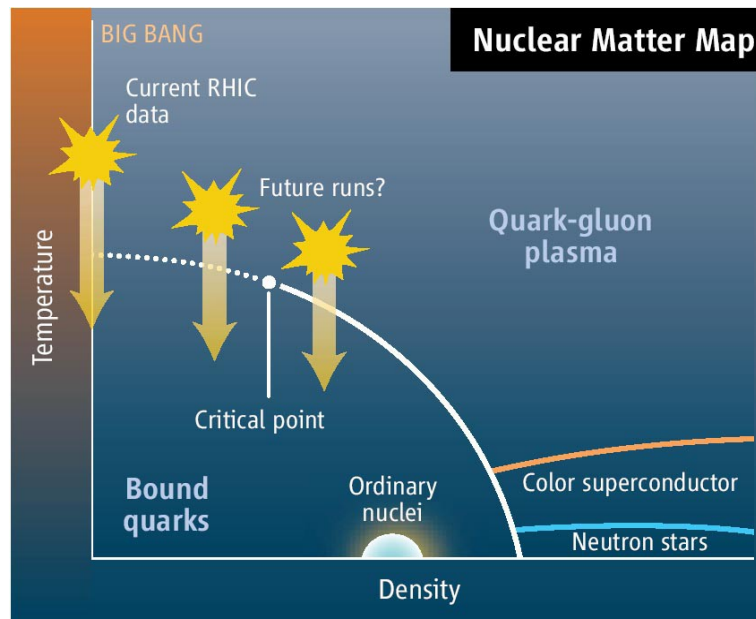
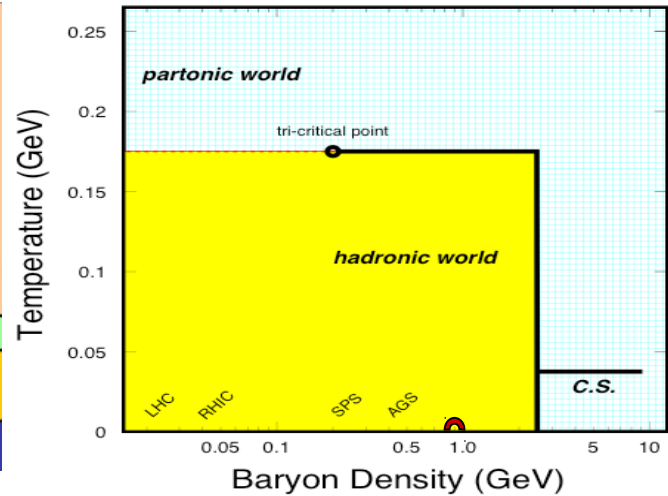
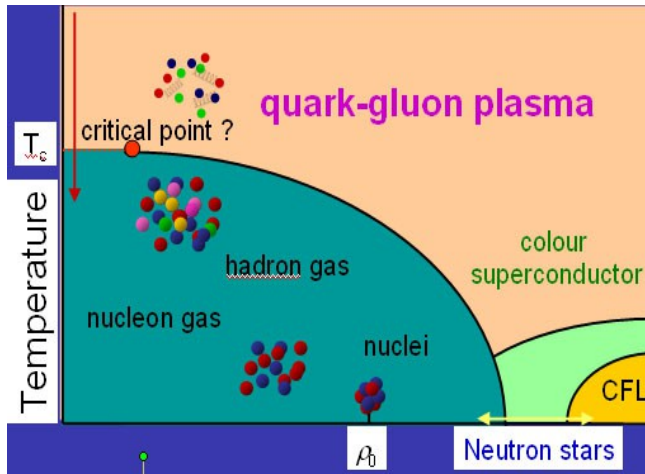


Exploring the QCD Phase diagram



The physics of baryon rich matter



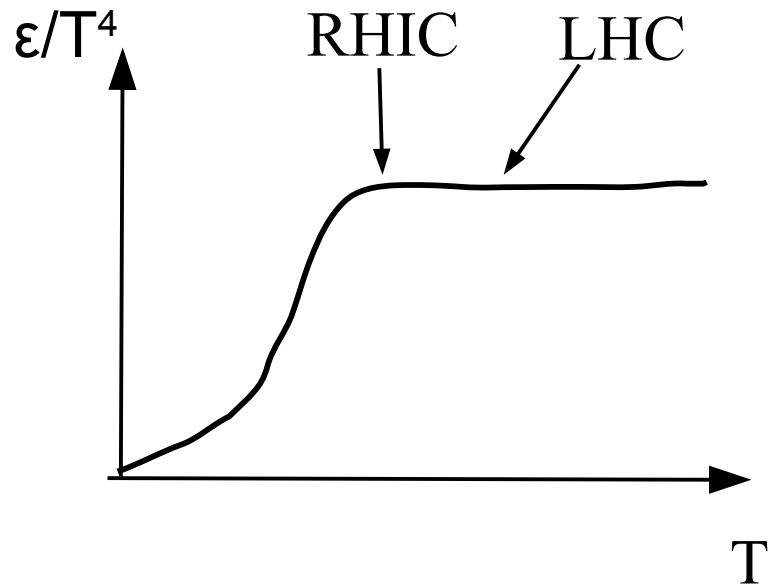
Summary

- Day 1: Flow (barometer)
- Day 2: Cumulants
- Day 3: Dileptons ($M > 1$ GeV, Thermometer)
- Days 4-6: You tell me
- Day 7: Rest and relax

Outline

- Introduction: Why Fluctuations?
- Some remarks about the phase diagram
- First order phase co-existence
 - Dynamical treatment
 - Observables?
- Measuring Fluctuations: Possible pitfalls
- Dilepton
- Charm, exotica: You tell me

The Paradigm

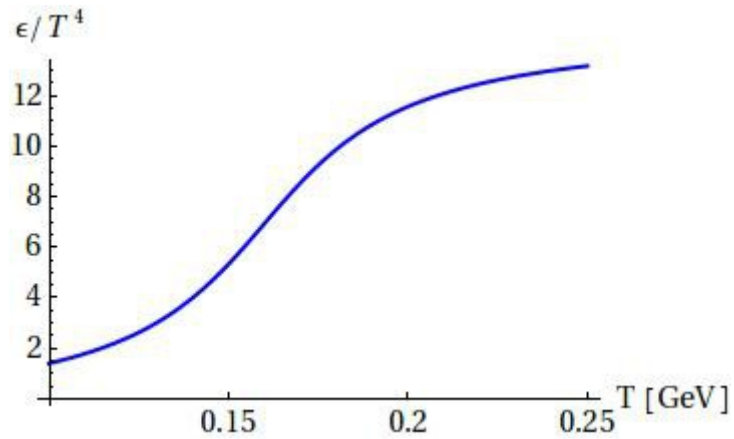


RHIC and LHC look qualitatively similar:

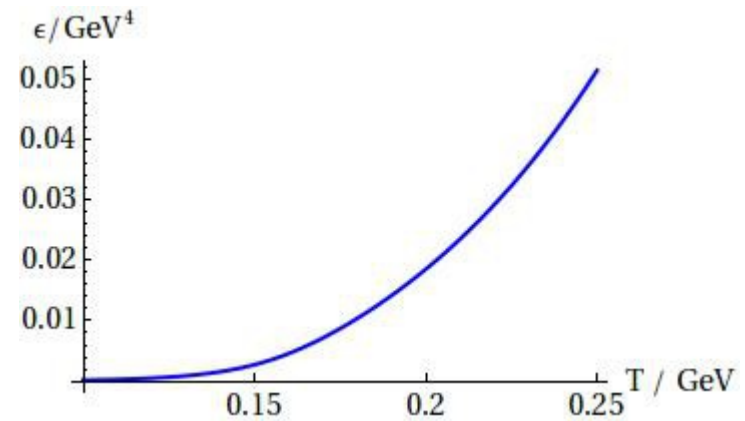
- Flow
- R_{AA}
- Particle production
-

Paradigm seems in good shape
but can we establish that there is indeed a transition

The Lattice EOS



What we always see....

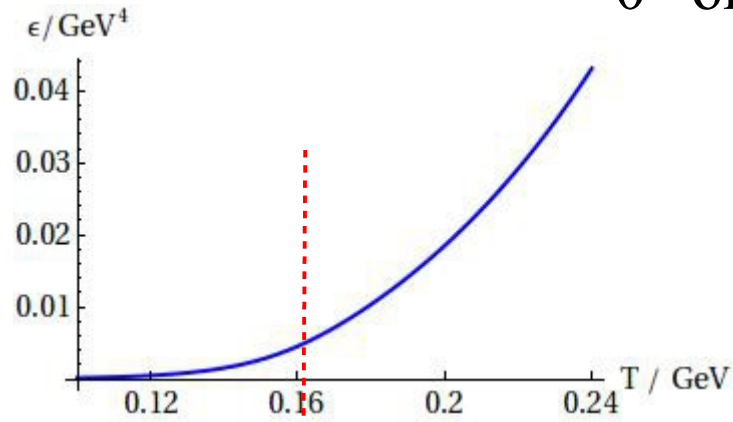


What it really means....

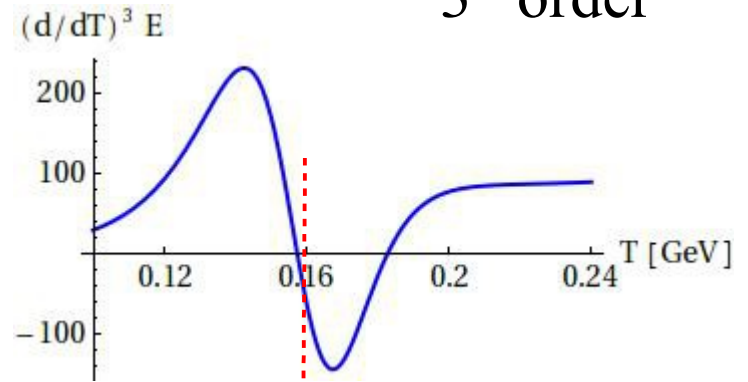
“ T_c ” \sim 160 MeV

Derivatives

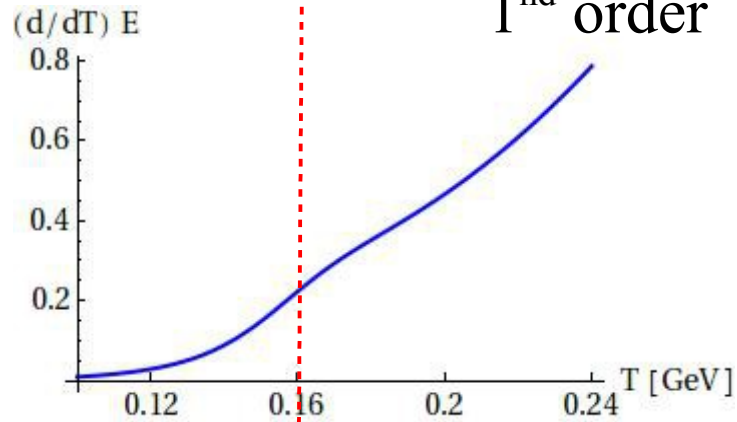
0th order



3th order

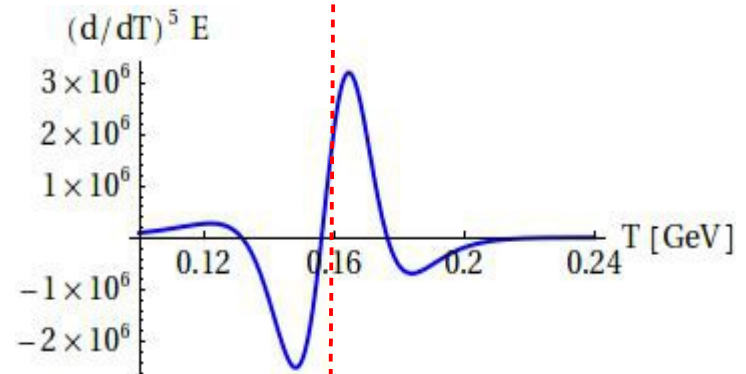


1st order



T_c

5th order



T_c

How to measure derivatives

$$Z = \text{tr} e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

At $\mu = 0$:

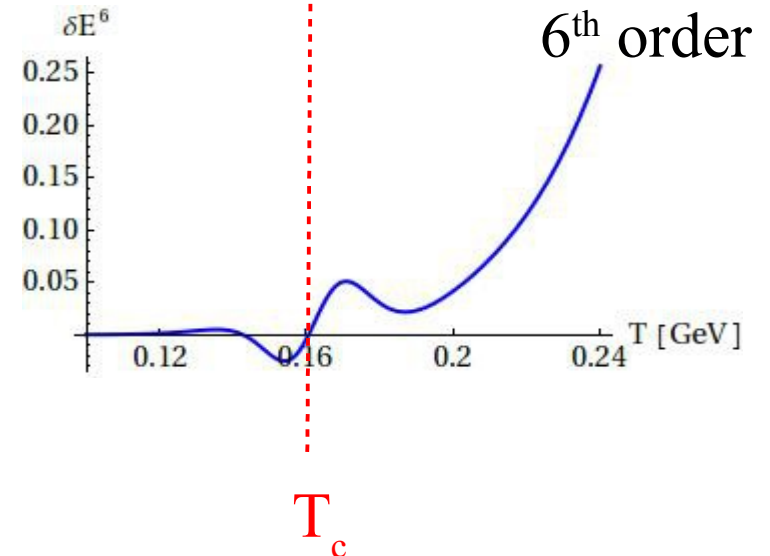
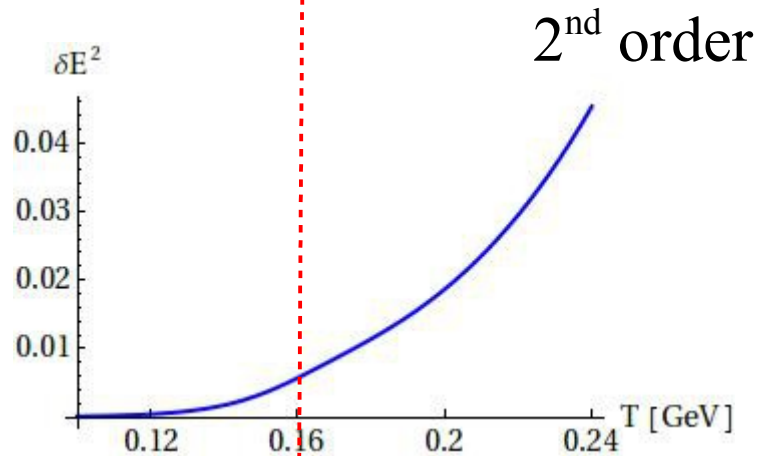
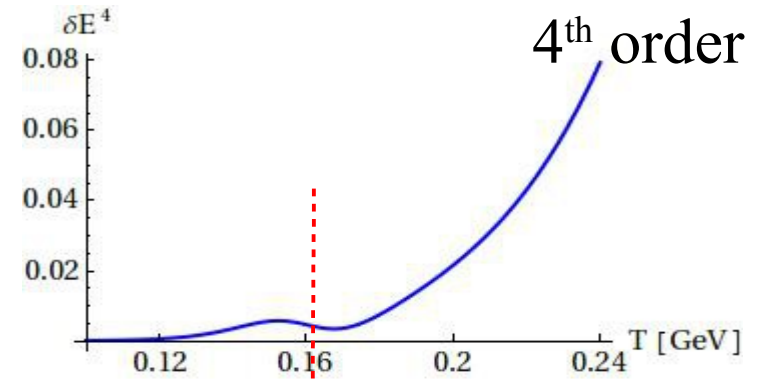
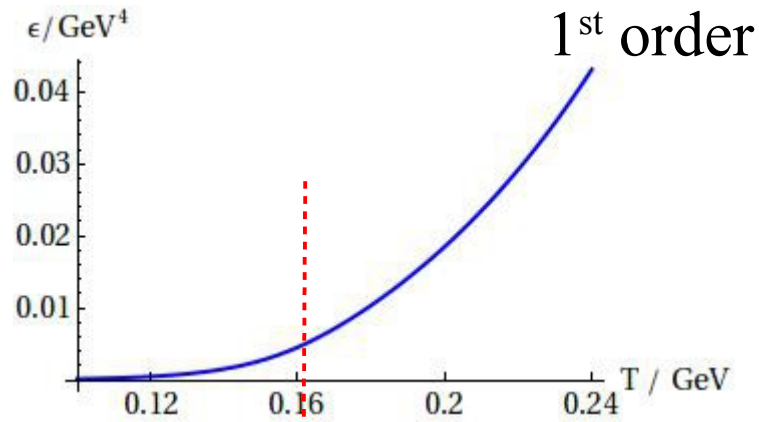
$$\langle E \rangle = \frac{1}{Z} \text{tr} \hat{E} e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the derivatives of the EOS

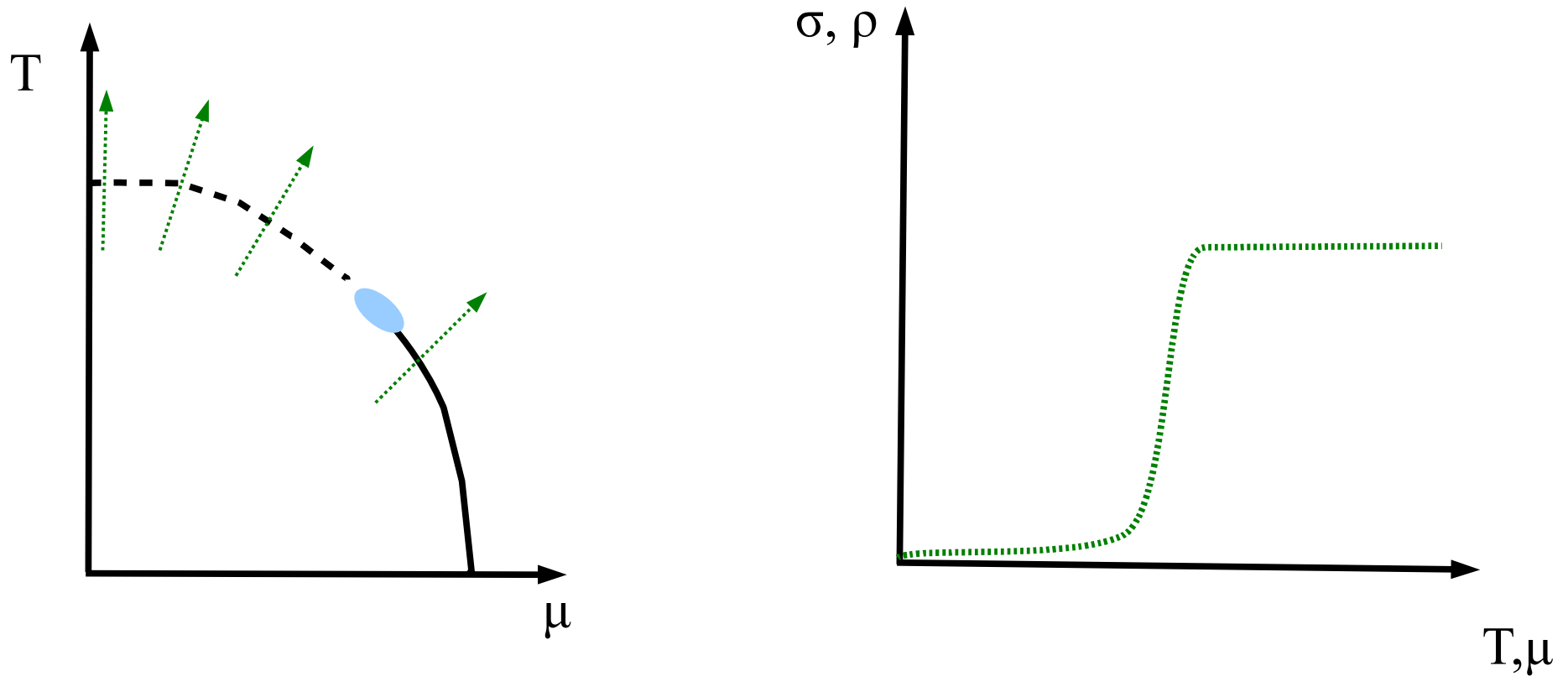
Fluctuations / Cumulants



T_c

T_c

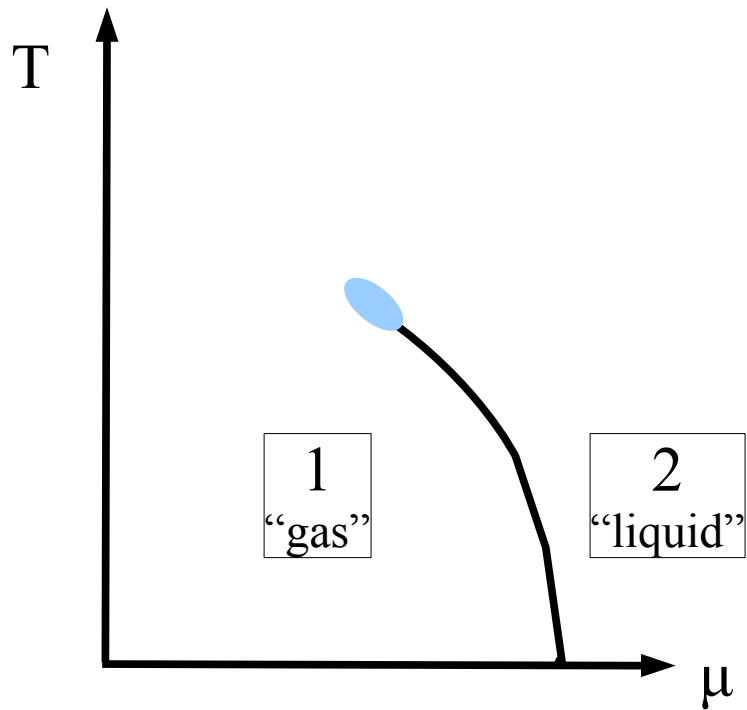
Generic Phase Diagram



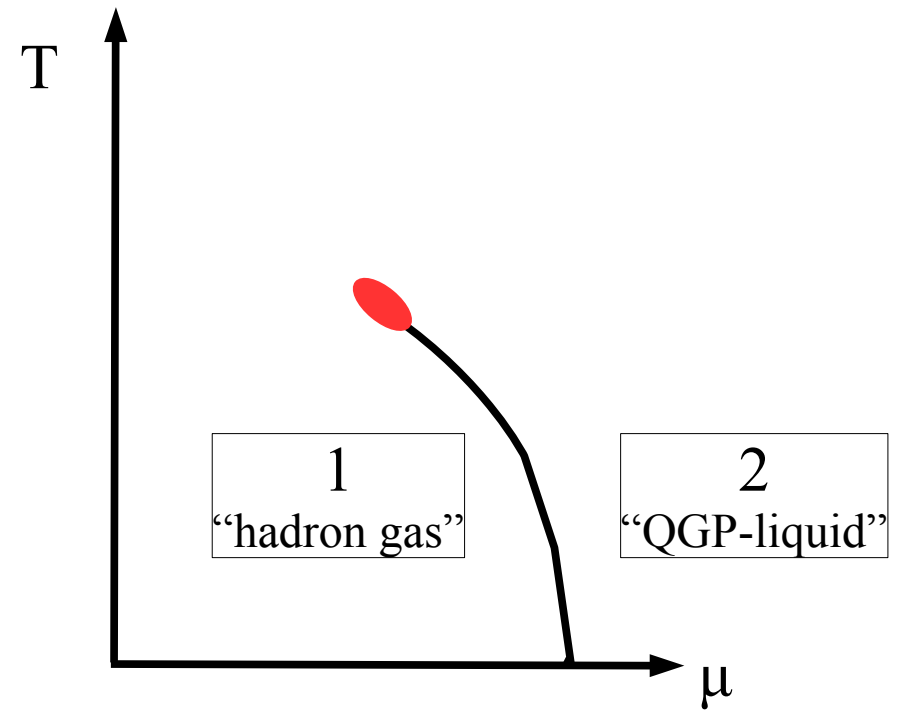
“Simply” use appropriate combination of T and μ

Requires: $\langle (\delta E)^n \rangle$ $\langle (\delta N_B)^n \rangle$ $\langle (\delta E)^m (\delta N_B)^n \rangle$ Mixed cumulants!

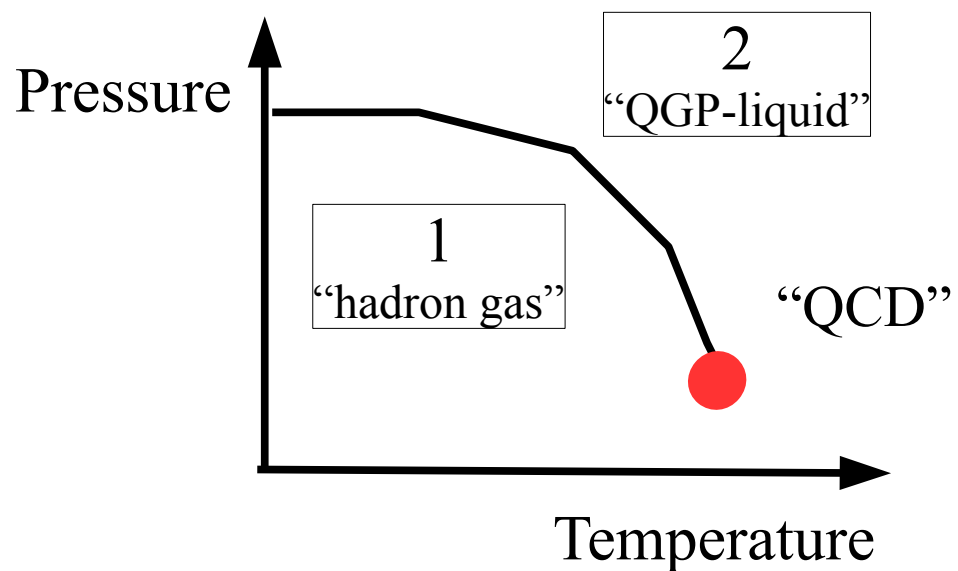
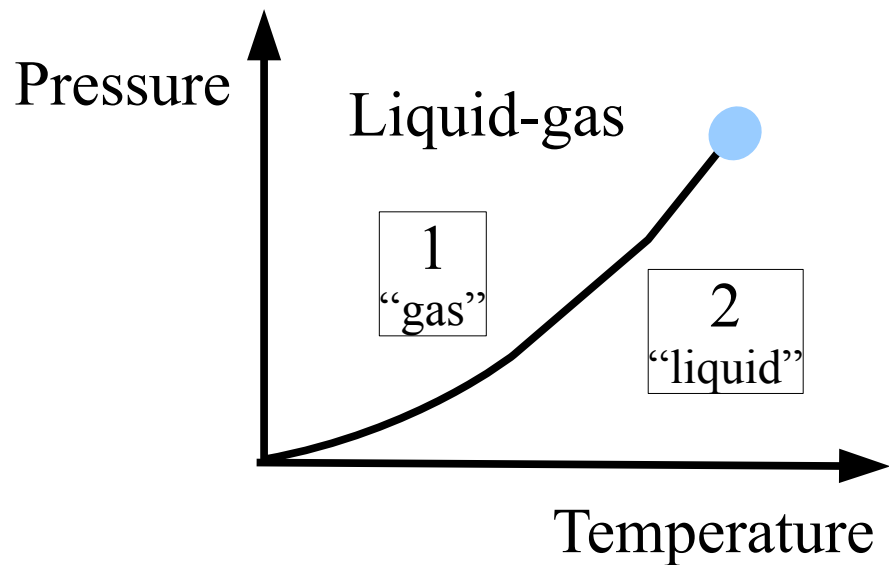
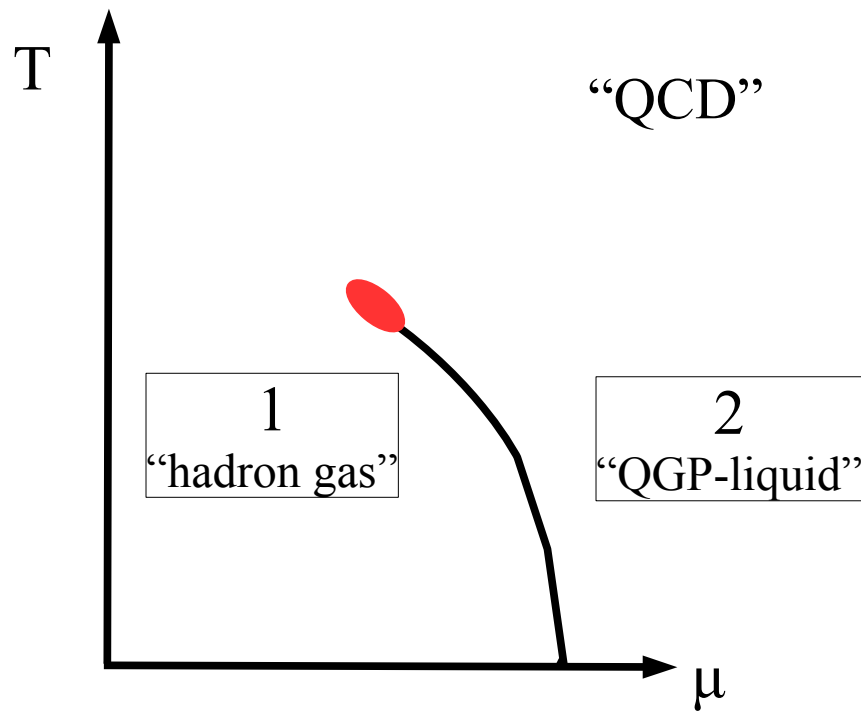
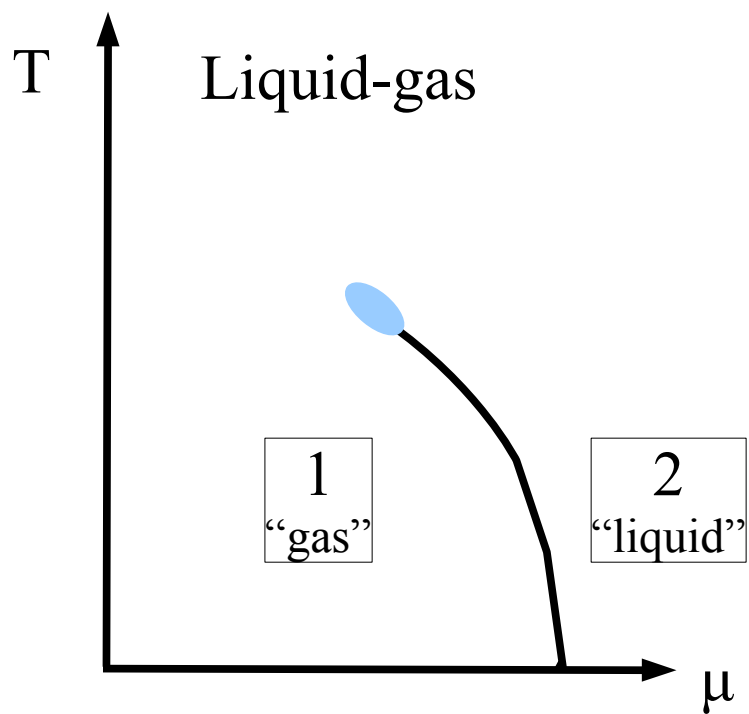
Comments on Phase diagram



Liquid-Gas
Water, nuclear matter, ...

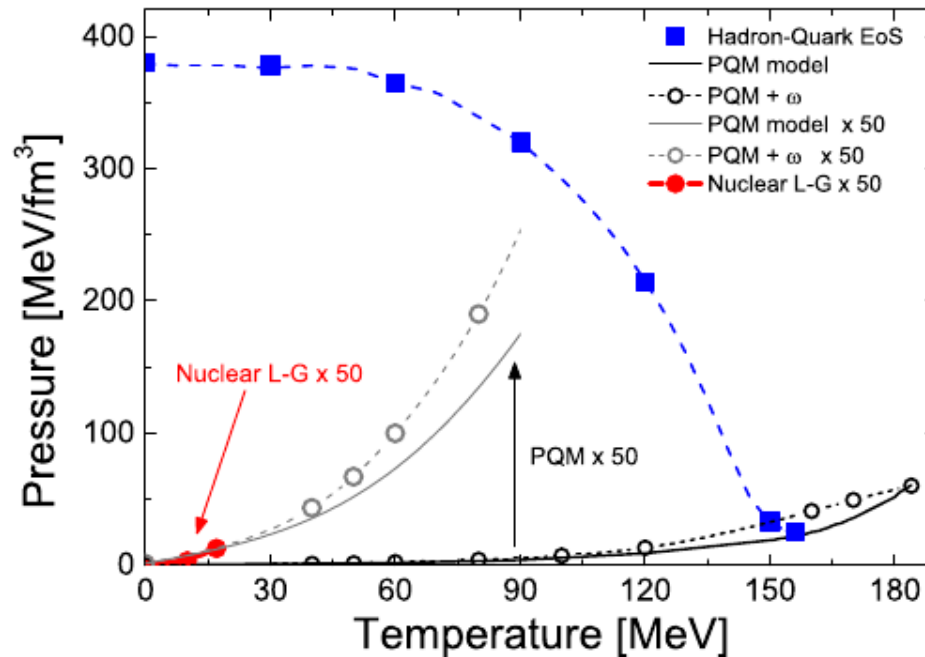


“QCD”



Liquid-gas vs QCD

QCD: pressure at $T=T_c$ and $\mu=0$ same as at $T=0$ and $\rho \sim 2.5 \rho_0$

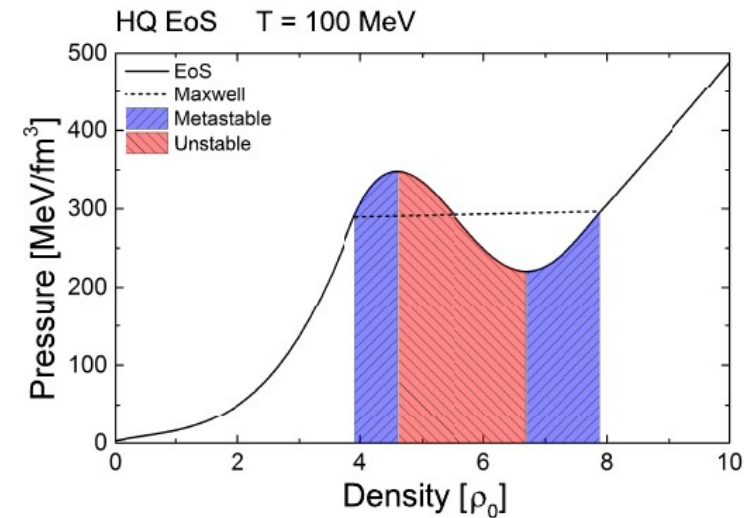
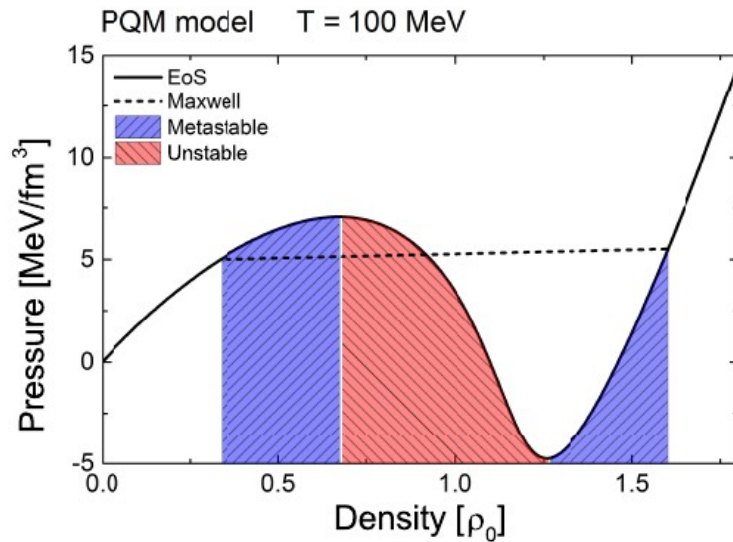


Steinheimer et al,
Phys.Rev. C89 (2014) 034901

If $T=0$ phase transition happens above $2.5 \rho_0 \rightarrow \frac{dP}{dT} < 0$

Note: virtually ALL model predicting a QCD critical point have $\frac{dP}{dT} > 0$

Liquid-gas vs QCD



Liquid Gas:

$T=0$: Liquid co-exists with **vacuum**

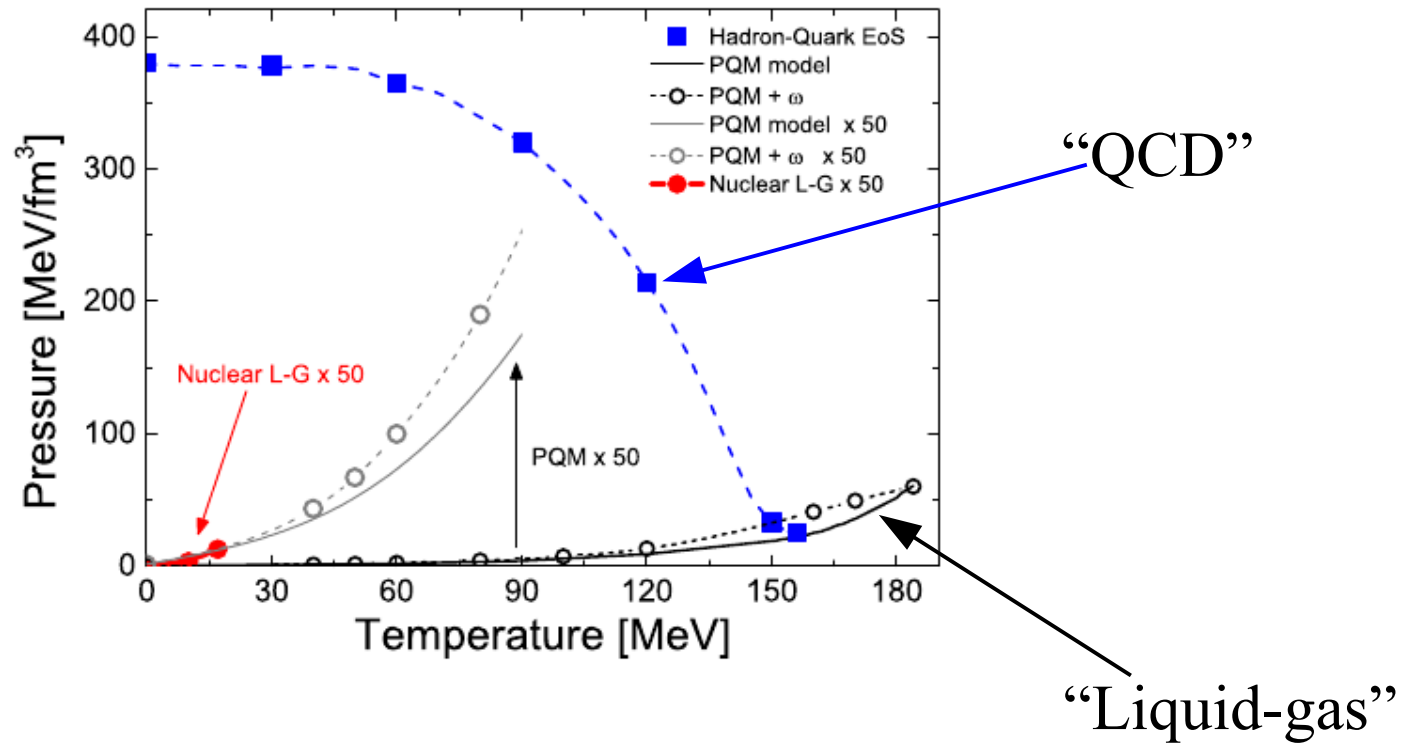
QCD:

$T=0$: Liquid co-exists with high density nuclear matter

Steinheimer et al,
Phys.Rev. C89 (2014) 034901

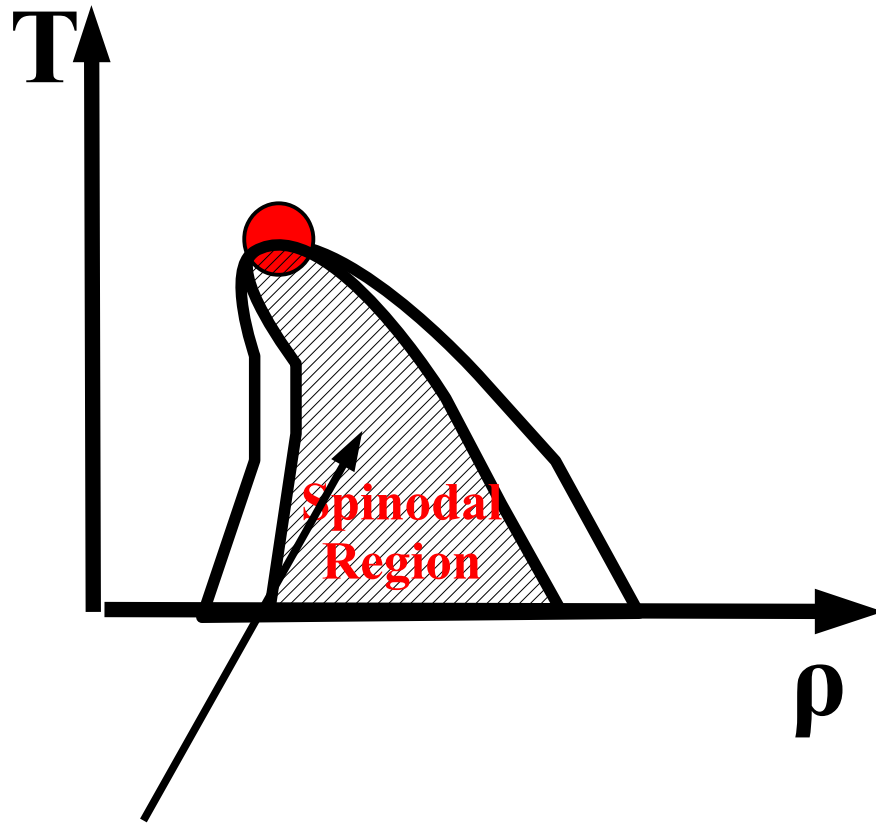
DOES IT MATTER?

Oh, YES!



Measure Pressure (gradients) with flow

Co-existence region



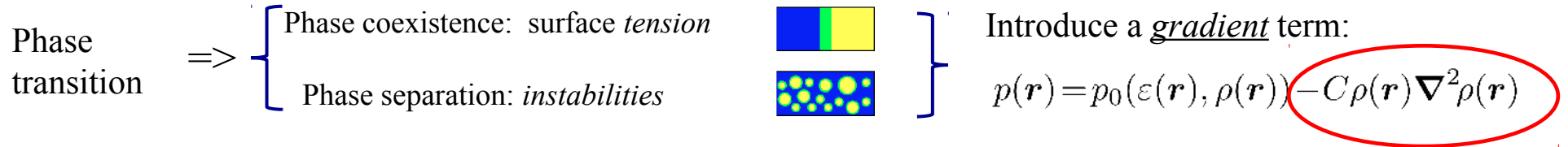
System should spent long time
in spinodal region

Spinodal instability:
Mechanical instability

$$\frac{\partial p}{\partial \epsilon} < 0$$

Exponential growth of clumping
Non-equilibrium phenomenon!

Phase-transition dynamics: Density clumping

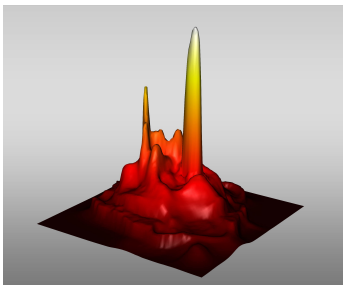


Insert the modified pressure into existing ideal finite-density fluid dynamics code

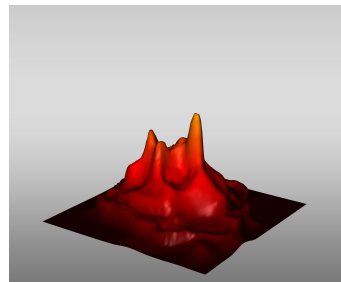
Use UrQMD for pre-equilibrium stage to obtain fluctuating initial conditions

Simulate central Pb+Pb collisions at ≈ 3 GeV/A beam kinetic energy on fixed target, using an Equation of State either *with* a phase transition or *without* (Maxwell partner):

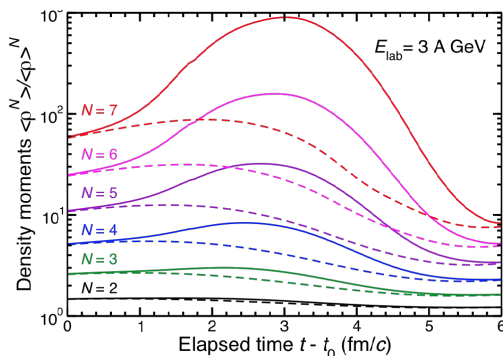
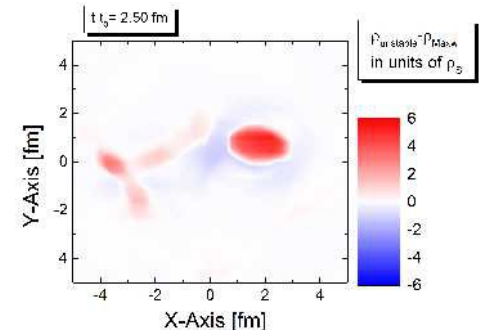
With phase transition:



Without phase transition:



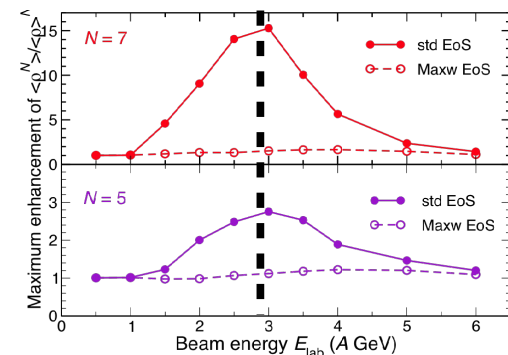
Density enhancement:



Evolution of density moments

$$\langle \rho^N \rangle \equiv \frac{1}{A} \int \rho(\mathbf{r})^N \rho(\mathbf{r}) d^3 r$$

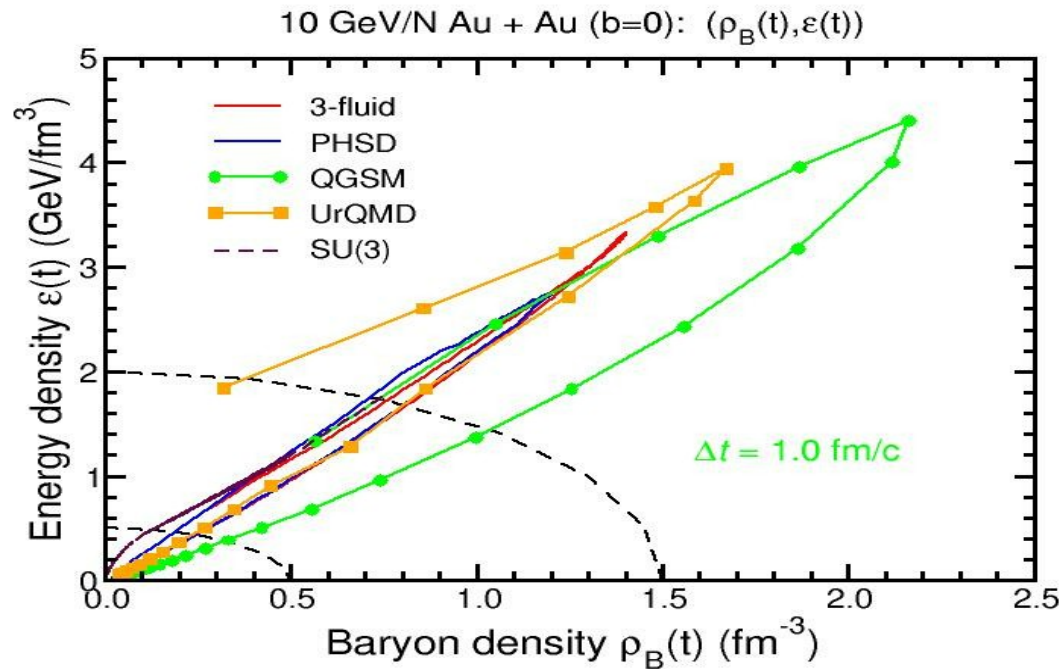
J. Steinheimer & J. Randrup,
PRL 109, 212301(2012)
PRC 87, 054903 (2013)



$E_{Lab} = 3 \text{ GeV}$

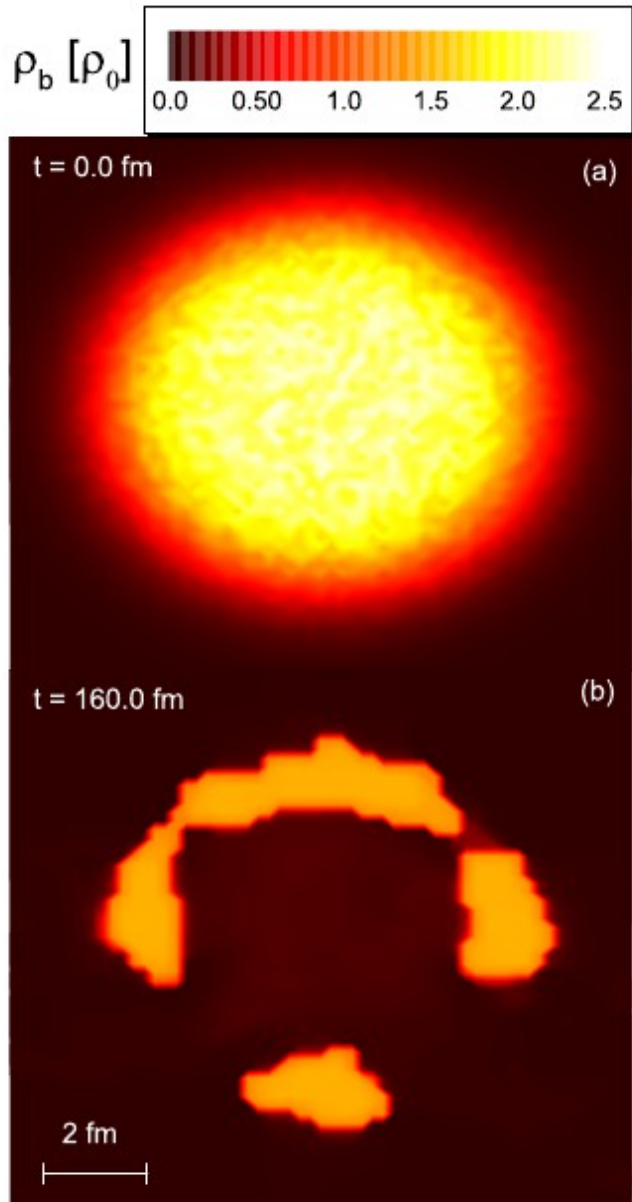
Phase trajectories

(J. Randrup et al)

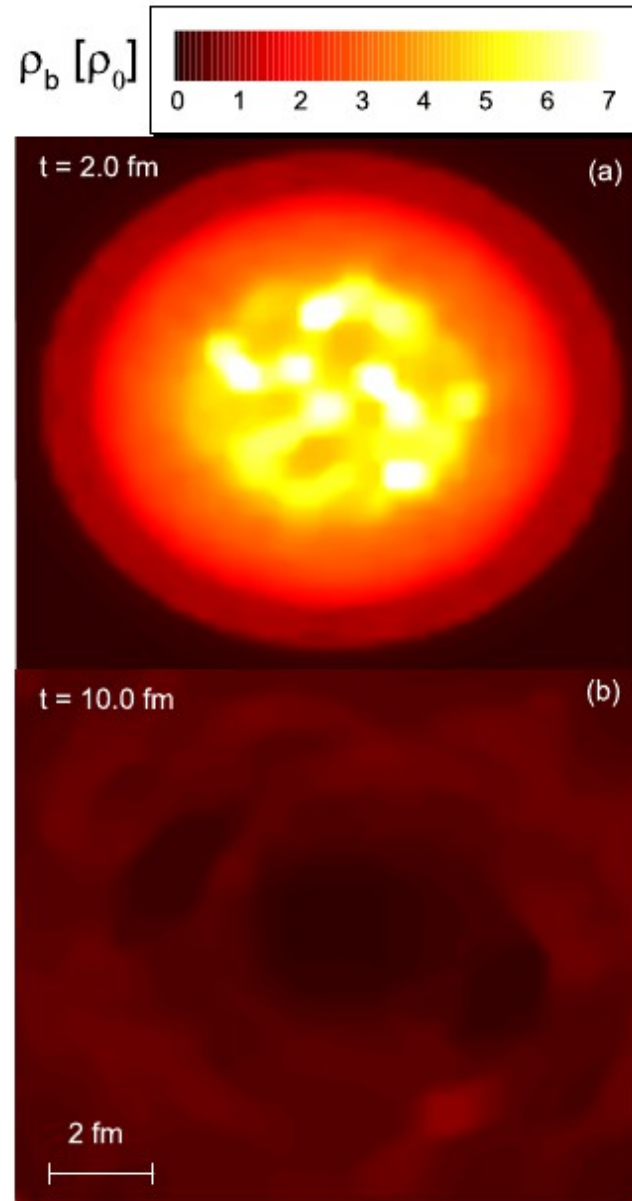


10 AGeV!!!!

SIS 100 territory

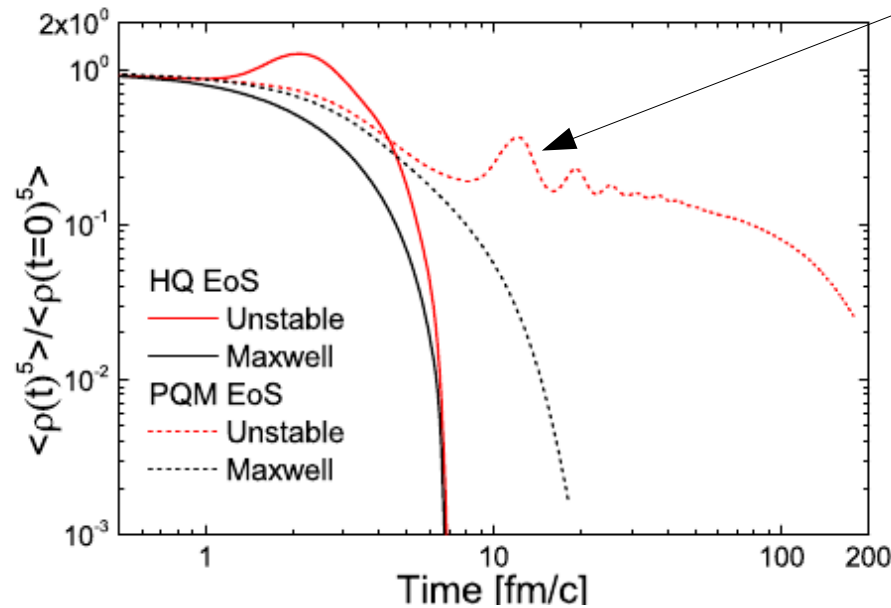


PQM (“liquid-gas”)



“QCD”

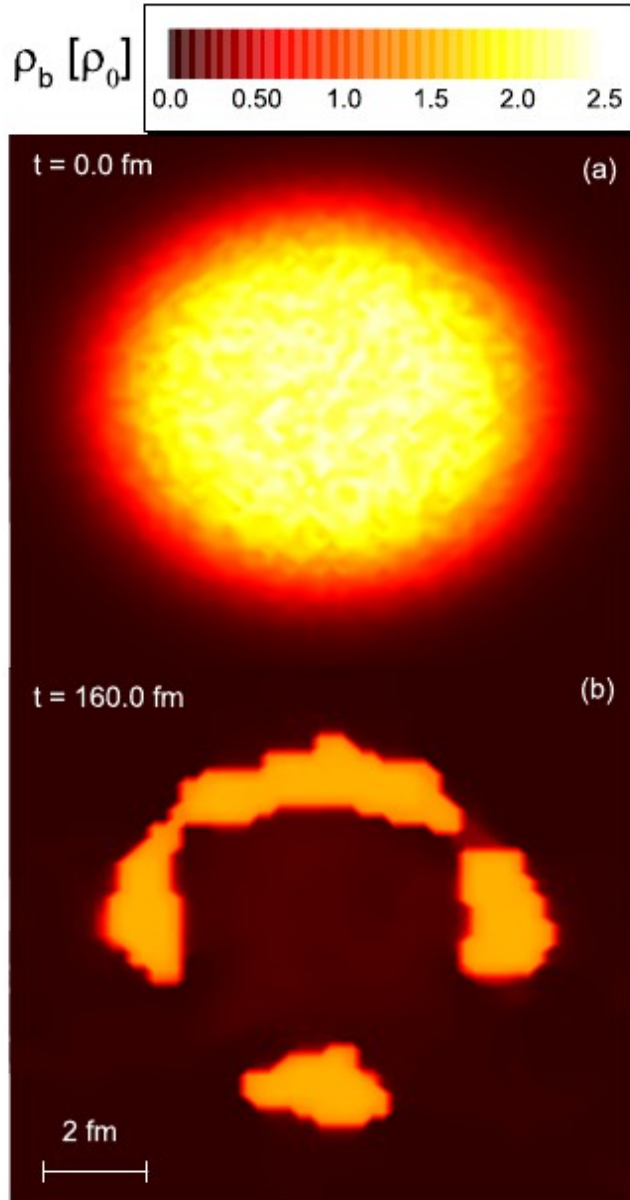
Time evolution



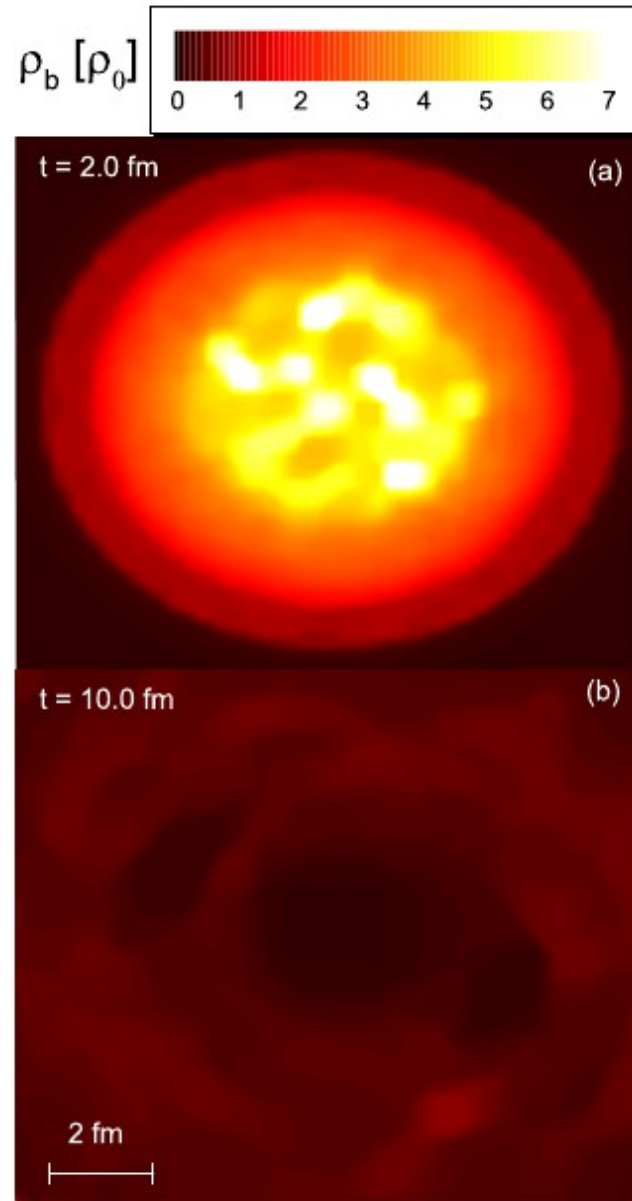
Oscillation of nearly stable droplets for “liquid-gas” EoS

Higher pressure leads to faster evolution of “QCD” EoS.

Flow



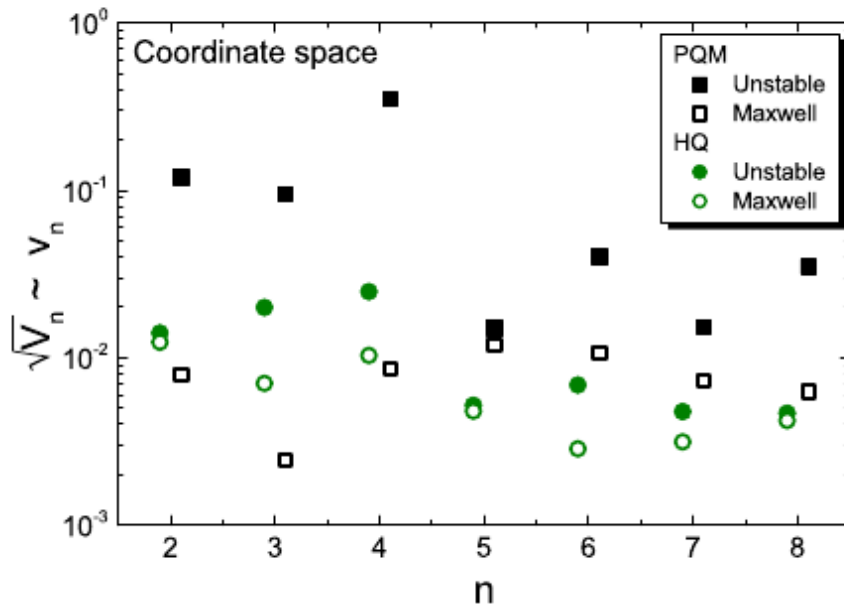
PQM (“liquid-gas”)



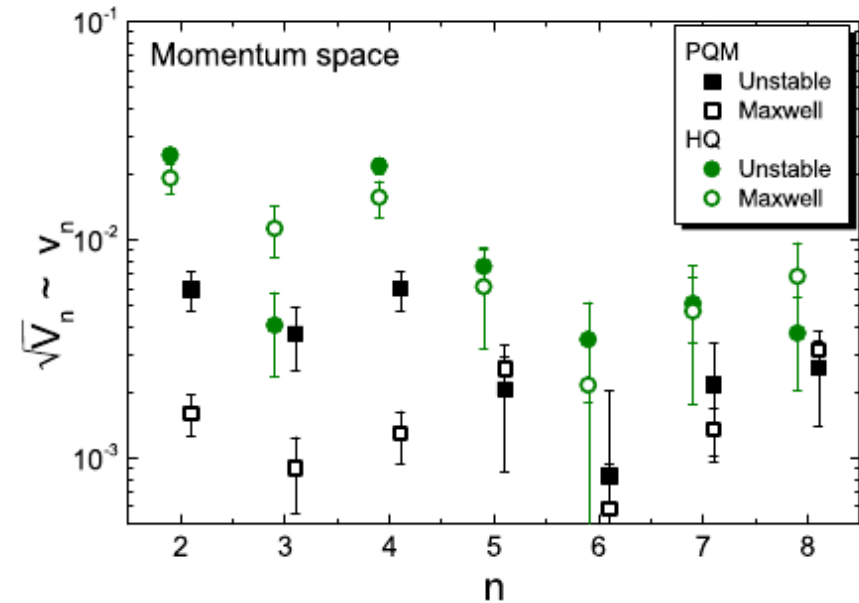
“QCD”

Flow

Coordinate space



Momentum space



Coordinate space asymmetries sensitive to nearly stable droplet formation in “liquid gas” EoS

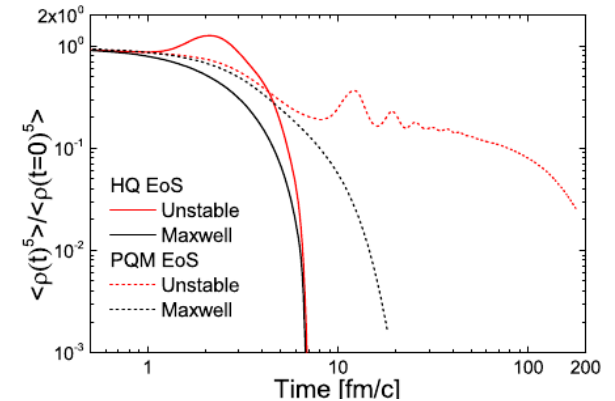
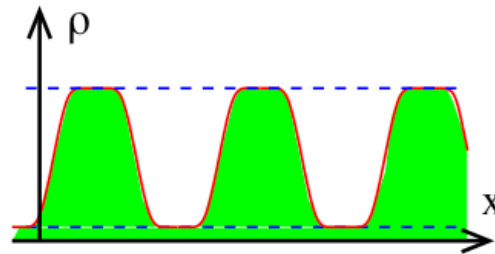
Small pressure of liquid:
Weak mapping into momentum space
hardly any effect of instabilities
In case of “QCD” EoS

V_2 sensitive to pressure!!!!

Steinheimer et al,
Phys.Rev. C89 (2014) 034901

Cluster a.k.a. nuclei

Even if total baryon number does not fluctuate the baryon **density** does



Therefore measure production of NUCLEI: d, ^3He , ^4He , ^7Li

$$\langle d \rangle \sim \langle \rho_B^2 \rangle \quad \langle {}^3\text{He} \rangle \sim \langle \rho_B^3 \rangle \quad \langle {}^7\text{Li} \rangle \sim \langle \rho_B^7 \rangle$$

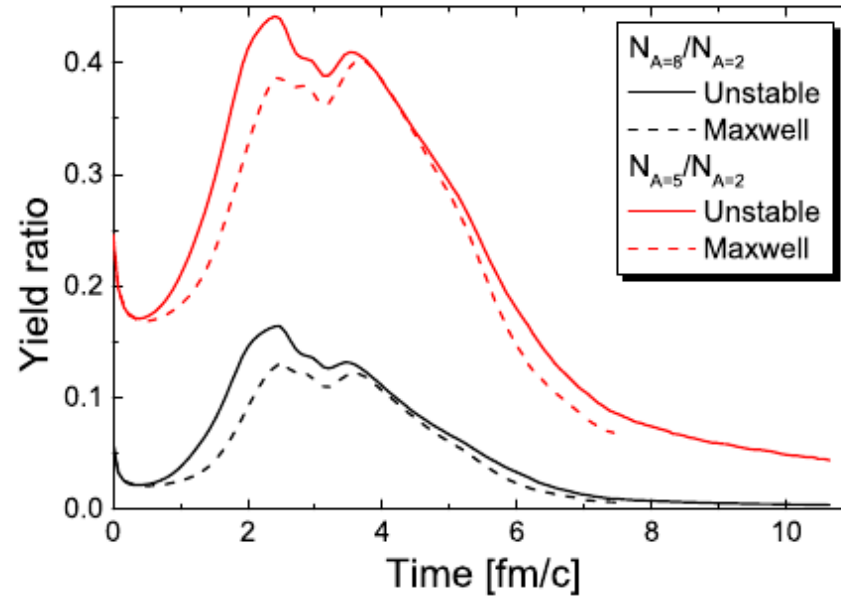
Extracts higher moments of the baryon **density** at freeze out

Nice Idea, but...

“Cluster” formation

“QCD” EoS

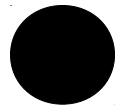
$$\left(\frac{S}{B}\right)_{\text{hadron-gas}} < \left(\frac{S}{B}\right)_{\text{QGP-liquid}}$$



Clumping in coordinate space is compensated by dilution in momentum space → tiny effect

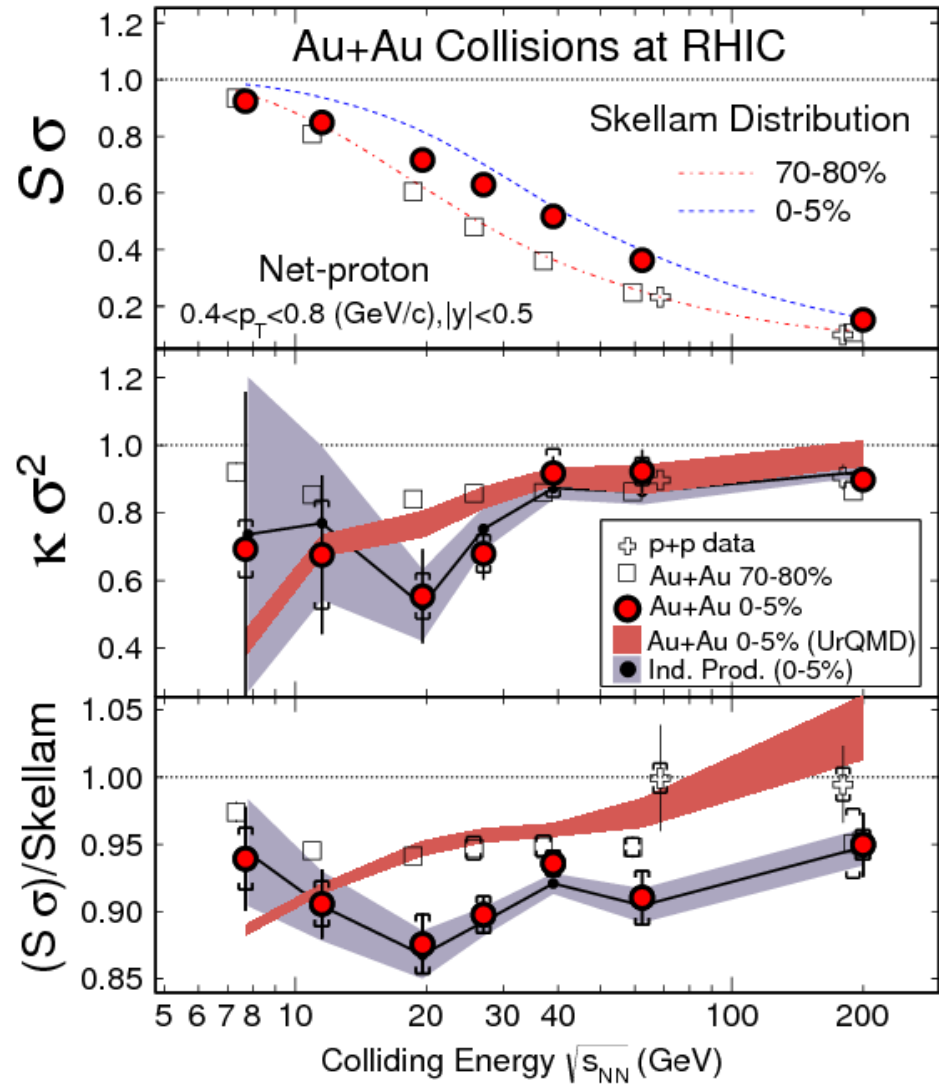
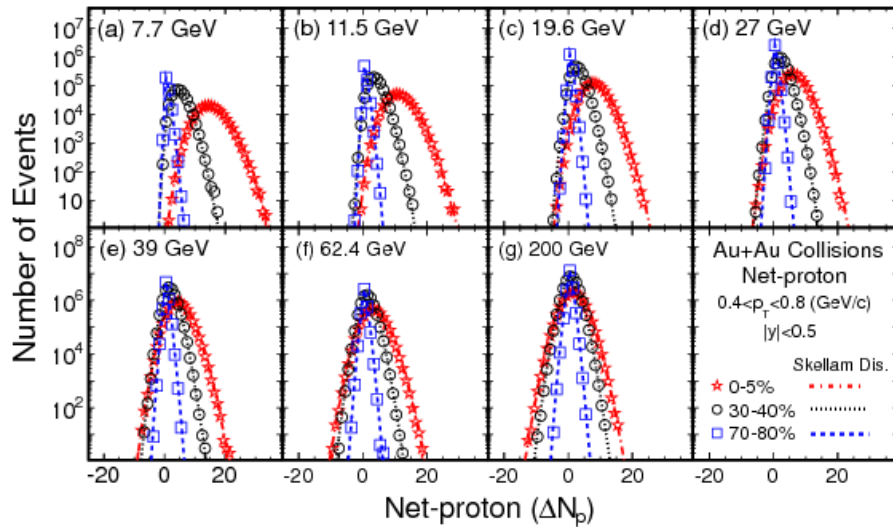
Steinheimer et al,
Phys.Rev. C89 (2014) 034901

Back to



STAR net-proton cumulants

(Phys.Rev.Lett. 112 (2014) 032302)

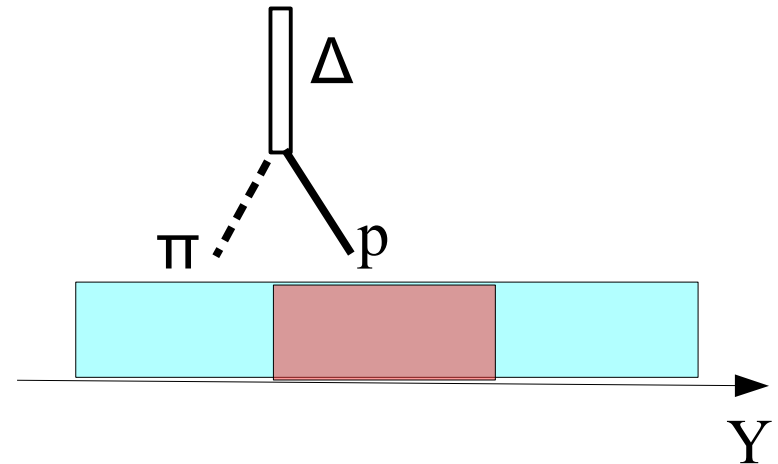
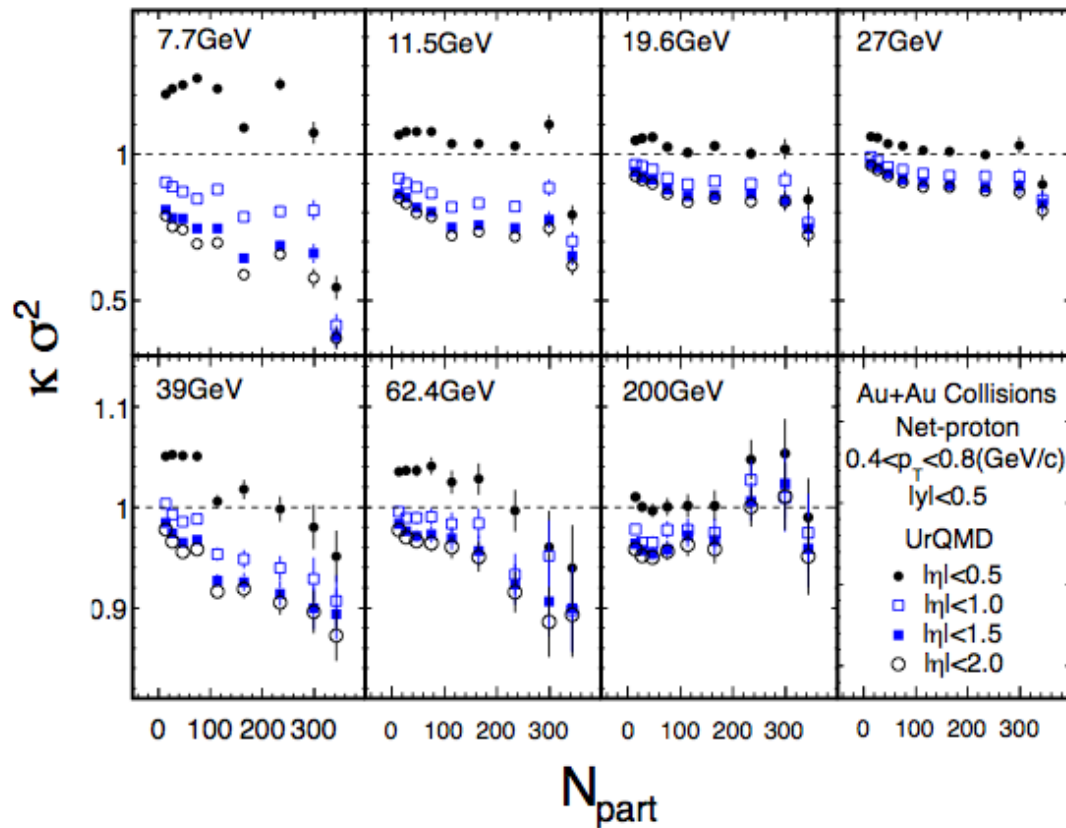


Things to consider

- Fluctuations of conserved charges ?!
- Higher cumulants probe the tails. Statistics!
- The detector “fluctuates” !
- Net-protons different from net-baryons
 - Isospin fluctuations
- Auto-correlations
- Beware of the “Poissonizer”
- “Stopping” Fluctuations

Auto Correlations

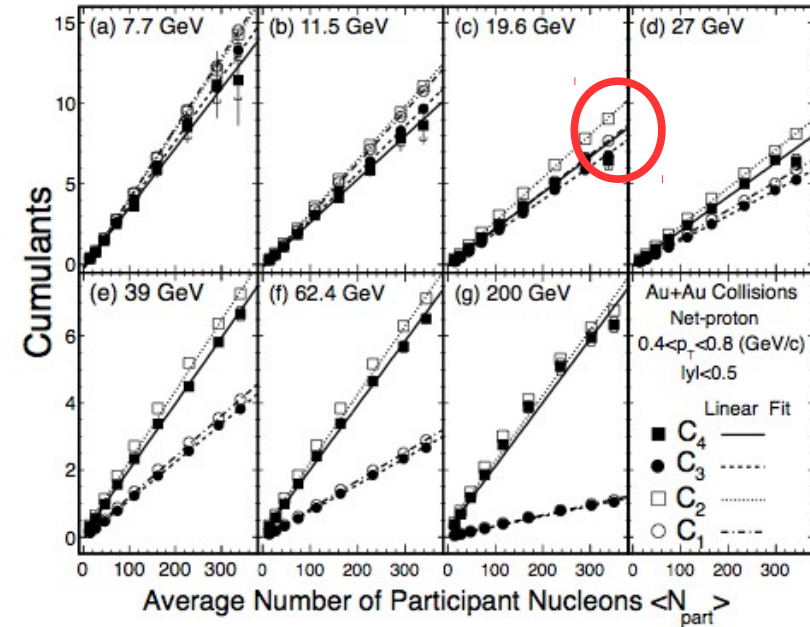
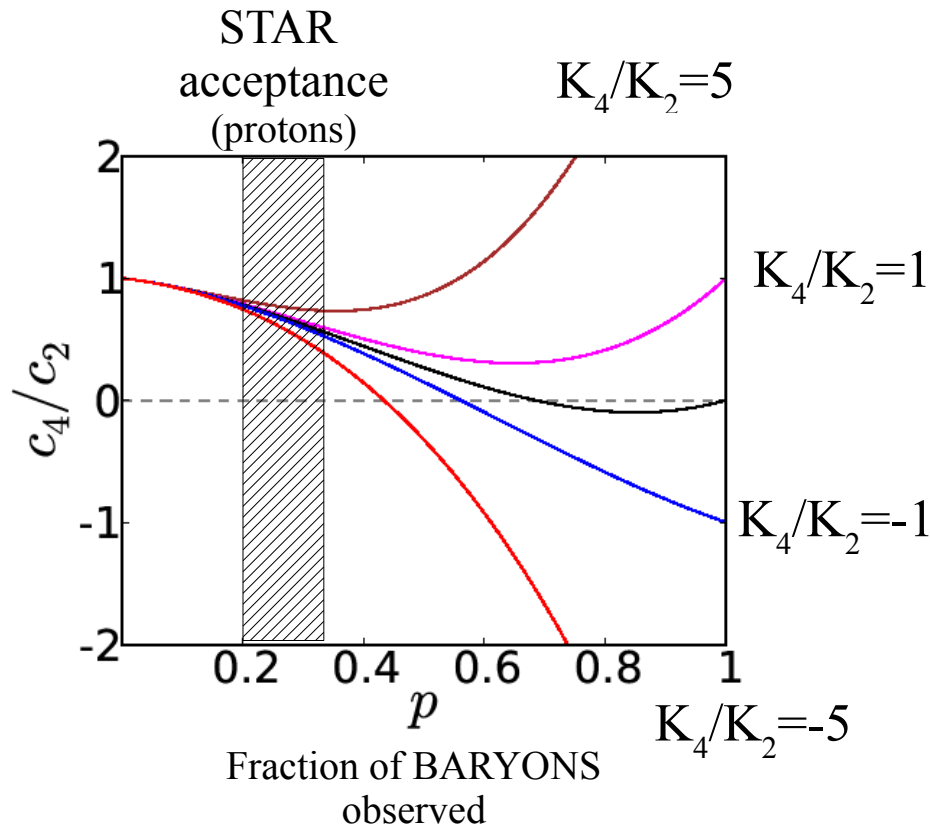
Luo et al, arXiv:1303.2332



Strong correlation between multiplicity determination and proton cumulants
Due to baryon resonances

Need to determine multiplicity far away in rapidity from cumulants

The “Poissonizer”

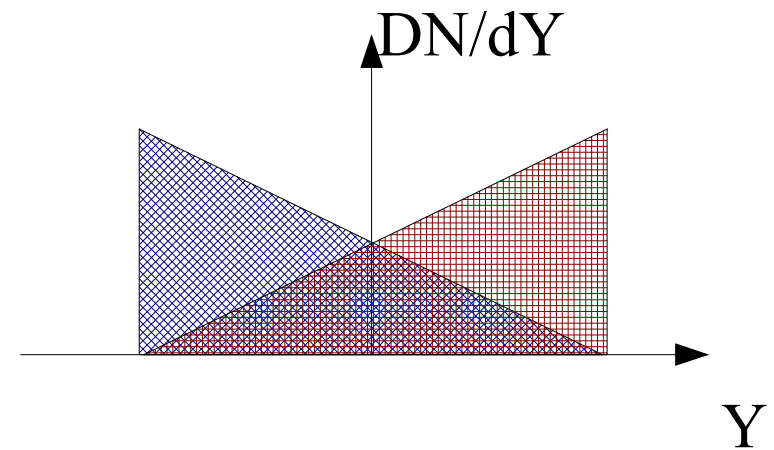
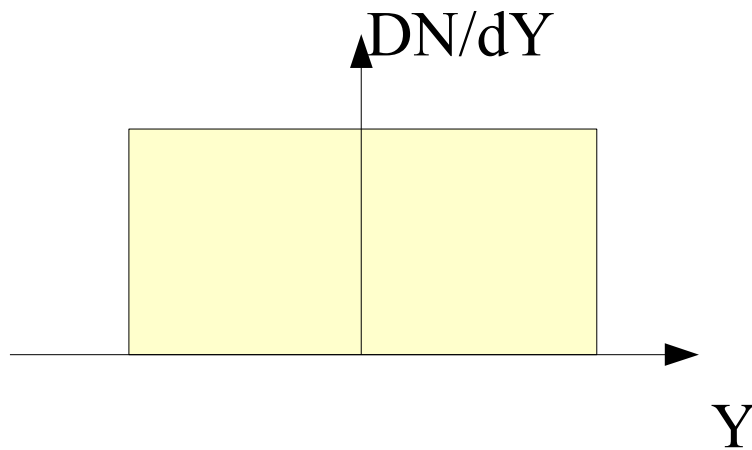


NA49: 32 protons per unit rapidity at top SPS energies!!!
STAR “sees” 8

“Stopping” Fluctuations

At low energy most of the baryon number (isospin) is brought in from the colliding nuclei.

Need to control the fluctuations to due baryon stopping

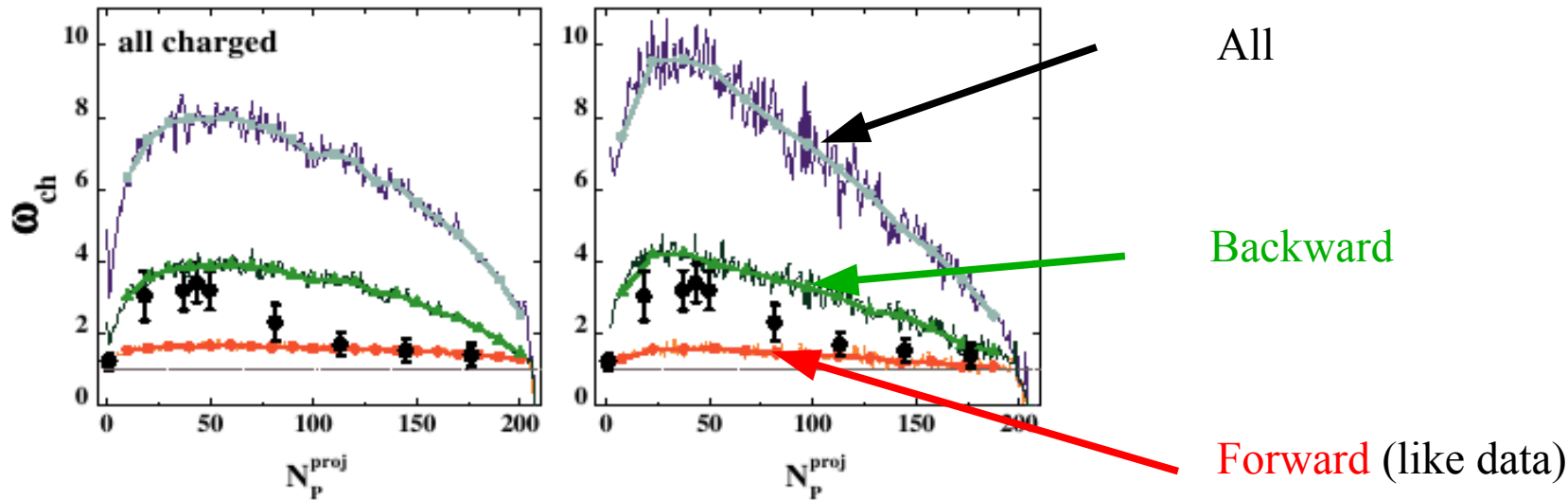


These fluctuations may also be biased by multiplicity selection.

Dynamics, event selection ...

(or why a symmetric detectors are good)

Konchakovski et al, nucl-th/0511083



Fluctuations are sensitive to dynamics (mixing of projectile and target material?)

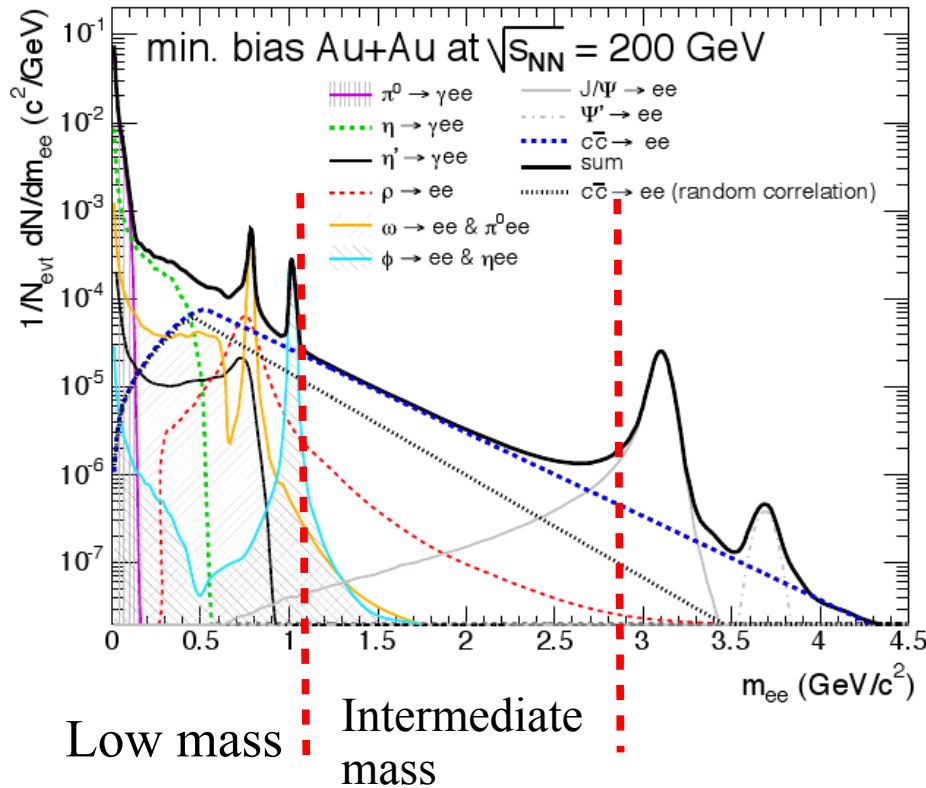
Event selection/trigger affects fluctuations → **large Acceptance!**

Need backward and forward multiplicity detectors!

Need Backward and forward particle ID (protons) !

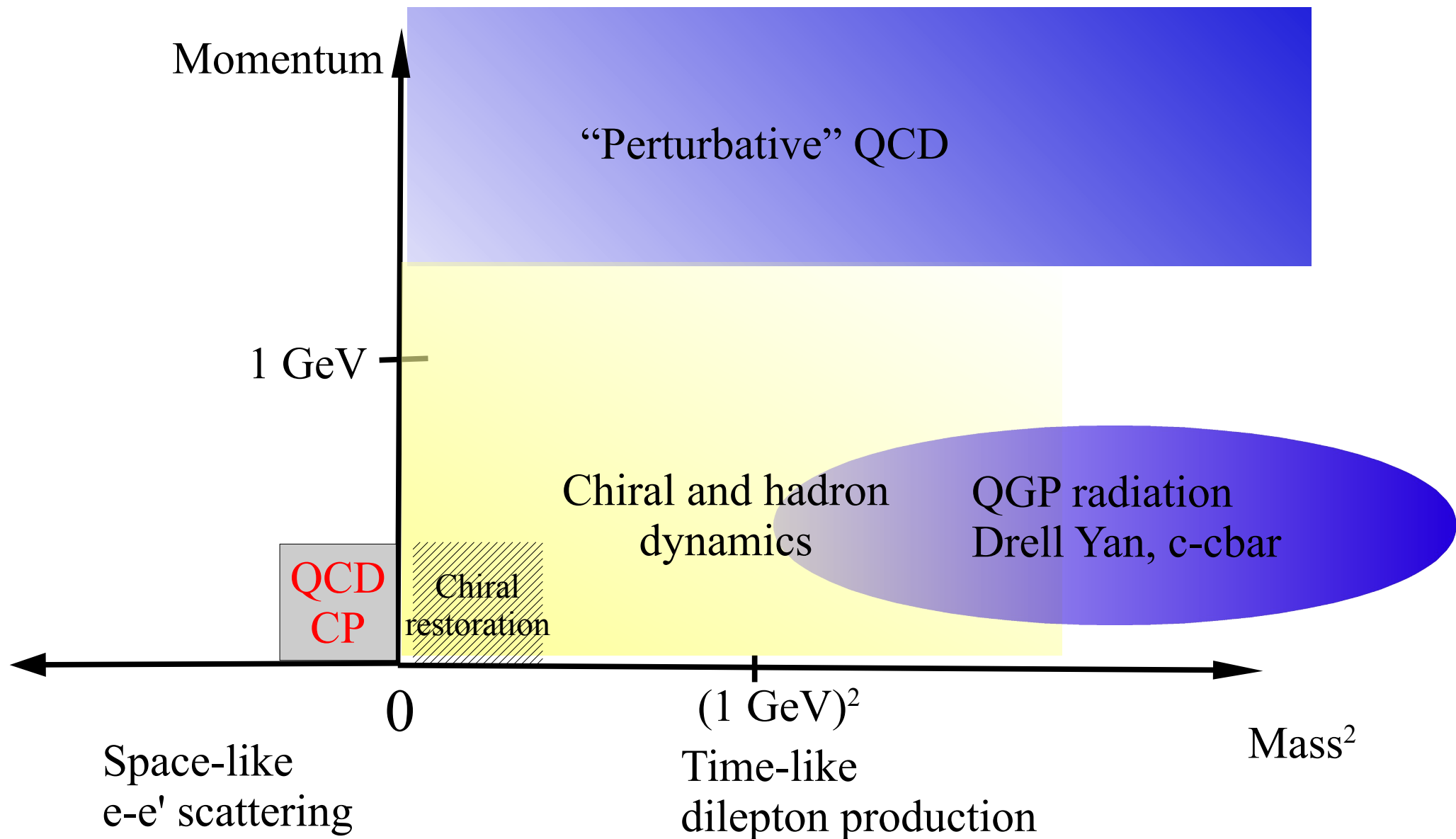
Dileptons

- Low mass (done deal... → Chronometer)
- Intermediate mass: Interesting (Thermometer)

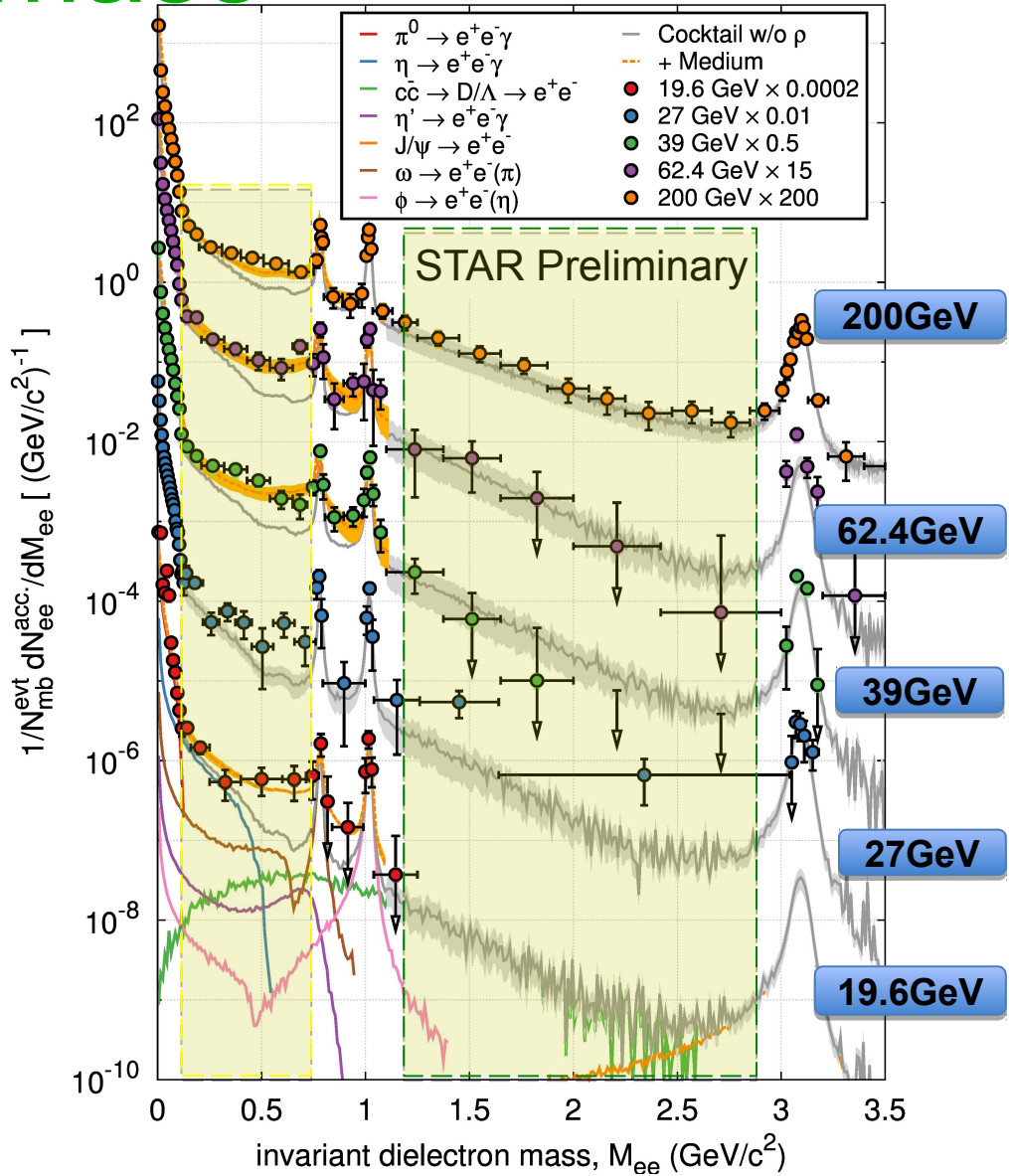
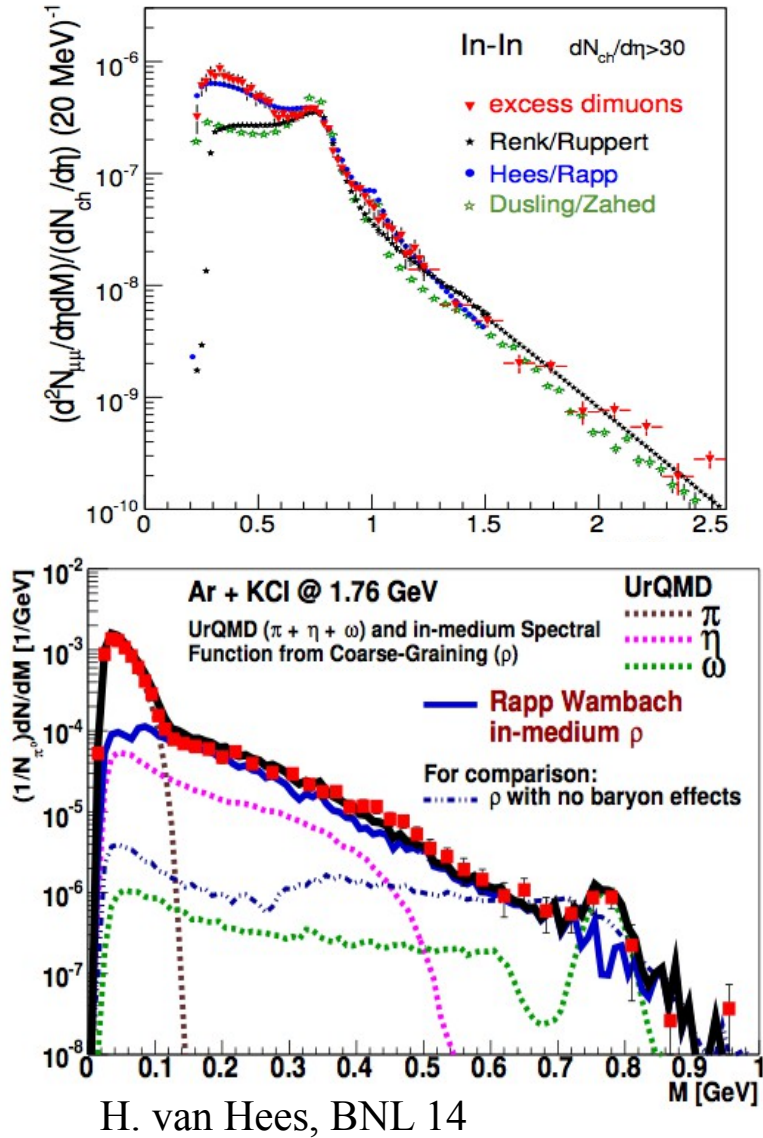


Thanks to
T. Ullrich

The Dilepton production landscape



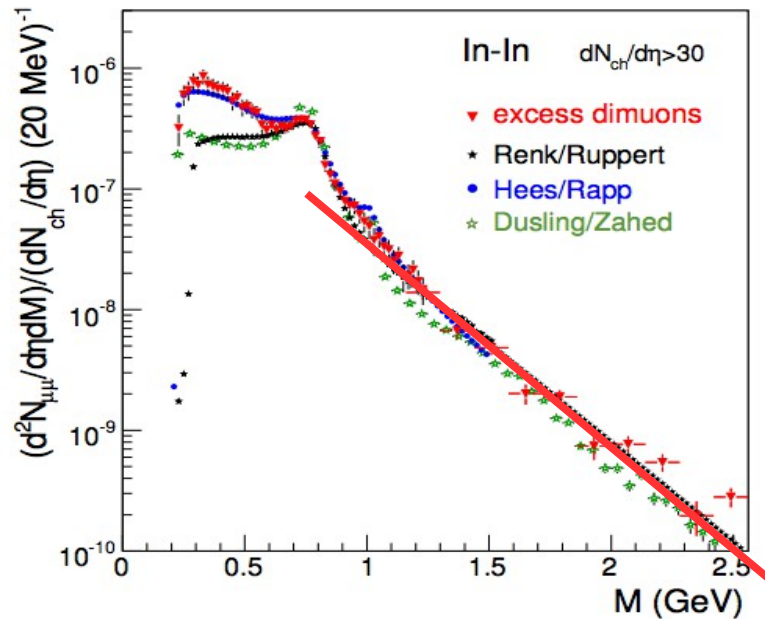
Low mass



Low Mass region is understood:
broadening through mixing

Baryon resonances plus

Intermediate Mass



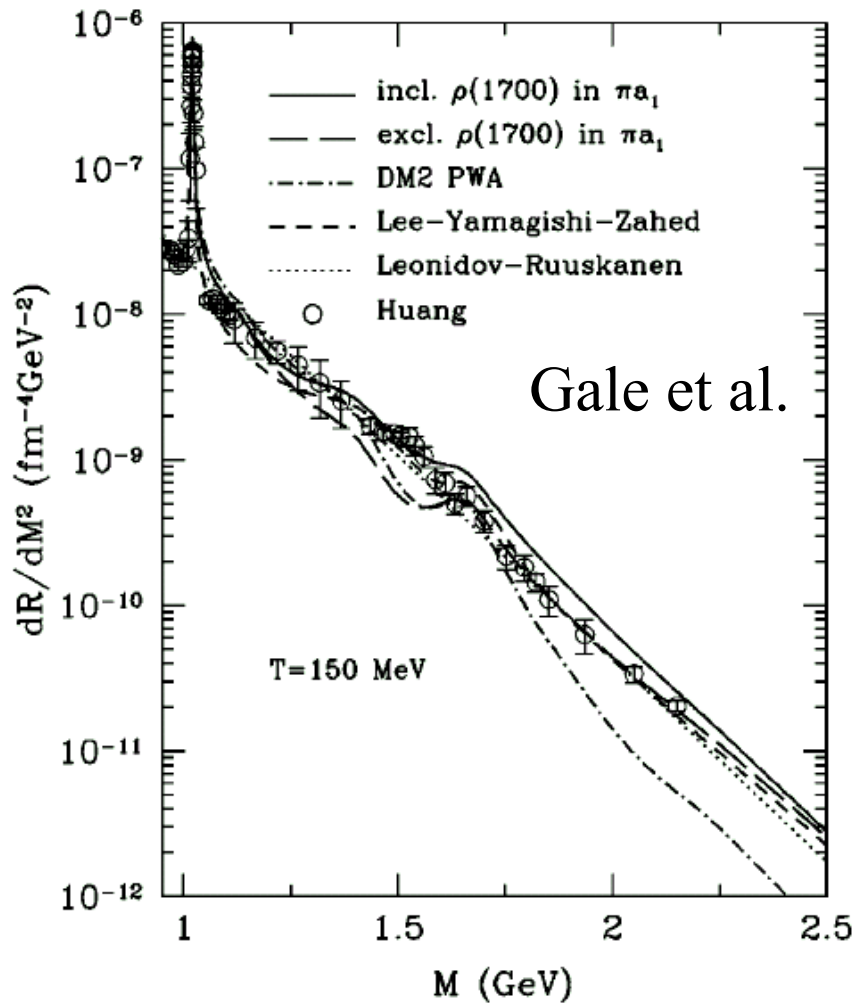
Intermediate mass sensitive to “QGP” radiation. NA60: $T_{\text{eff}} \sim 200 \text{ MeV}$

What to expect at lower energies?

Should we see any radiation if no QGP? YES!

Will we see simply a lower temperature? (Hopefully !)

Duality



$$\text{Rate(QGP)} = \text{Rate(Hadrons)}$$

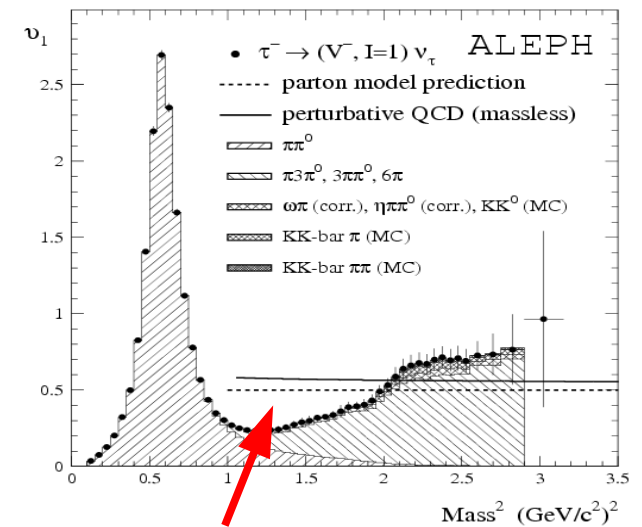
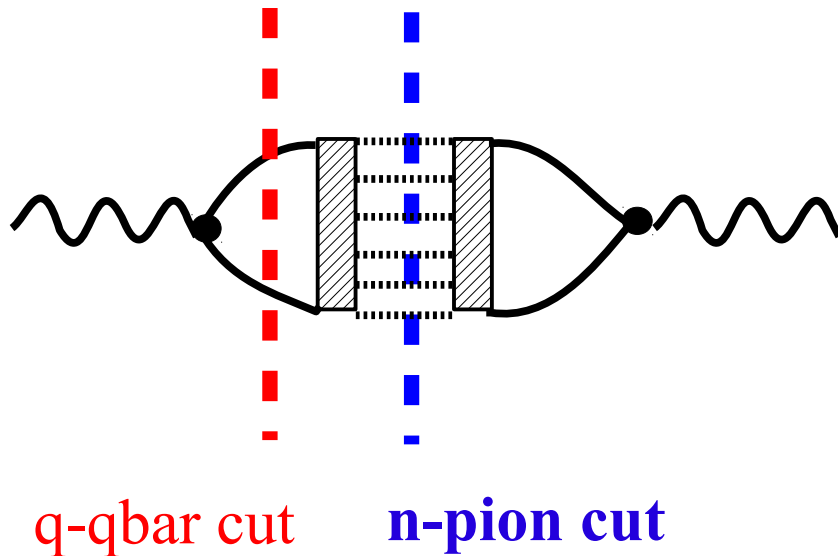
If quarks radiate so do hadrons!

→ We will see dileptons above
 $M > 1$ GeV

Duality

$$E_+ E_- \frac{d^6 R}{d^3 p_+ d^3 p_-} = \frac{2}{(2\pi)^6} \frac{e^2}{k^4} \left[p_+^\mu p_-^\nu + p_+^\nu p_-^\mu - g^{\mu\nu} (p_+ \cdot p_- + m_1^2) \right] \text{Im} \Pi_{\mu\nu}^R(k) \frac{1}{e^{\beta\omega} - 1}$$

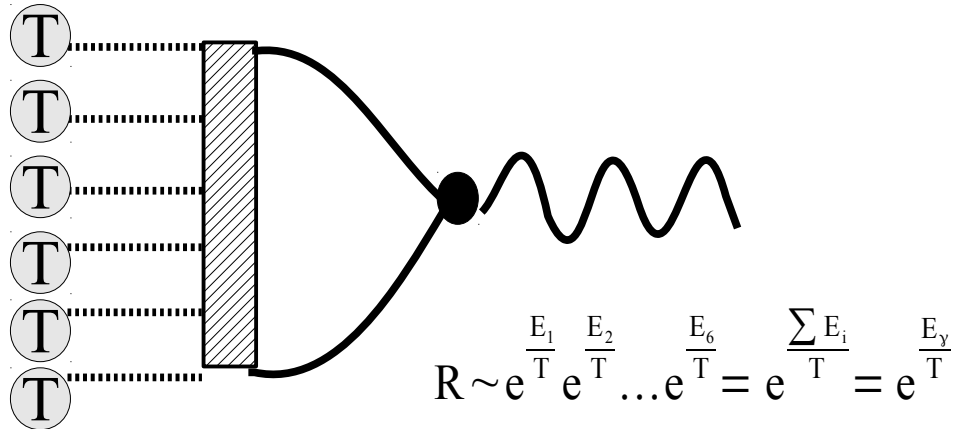
Extract from e^+e^- or
tau-decay data
(Z. Huang PLB 95)



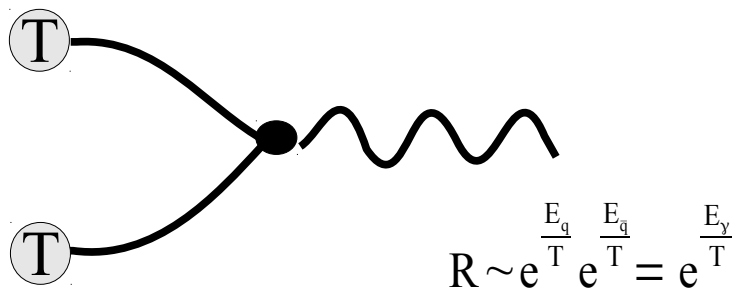
Location of break
in p_t -slopes

Duality

n-pion

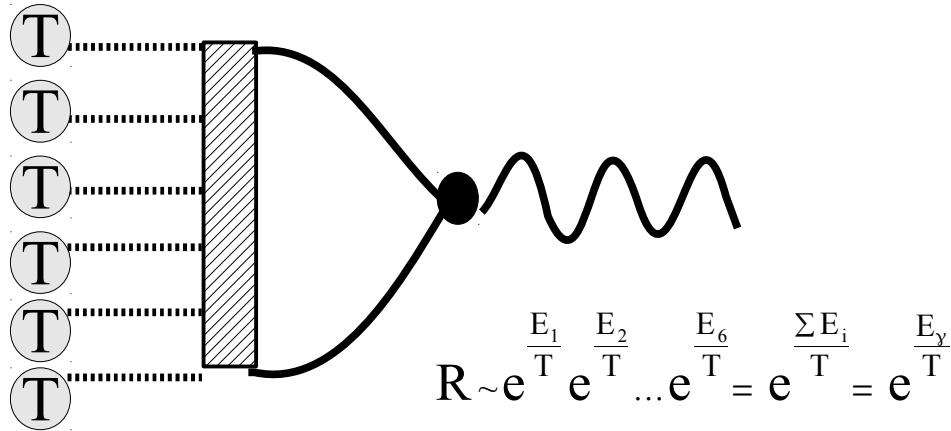


q-qbar



Duality

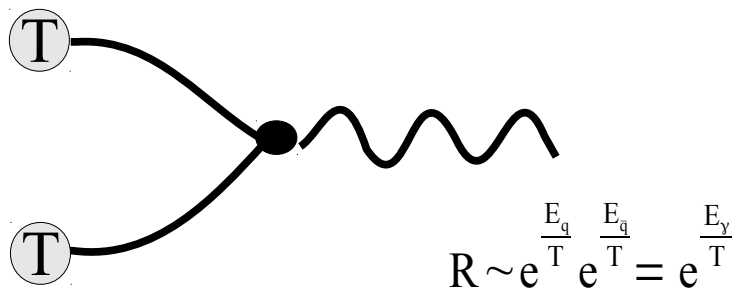
n-pion



$$v_2(e^+e^-) = 2 \mathbf{n} v_2(\text{quark})$$

$$\mathbf{n} > 2$$

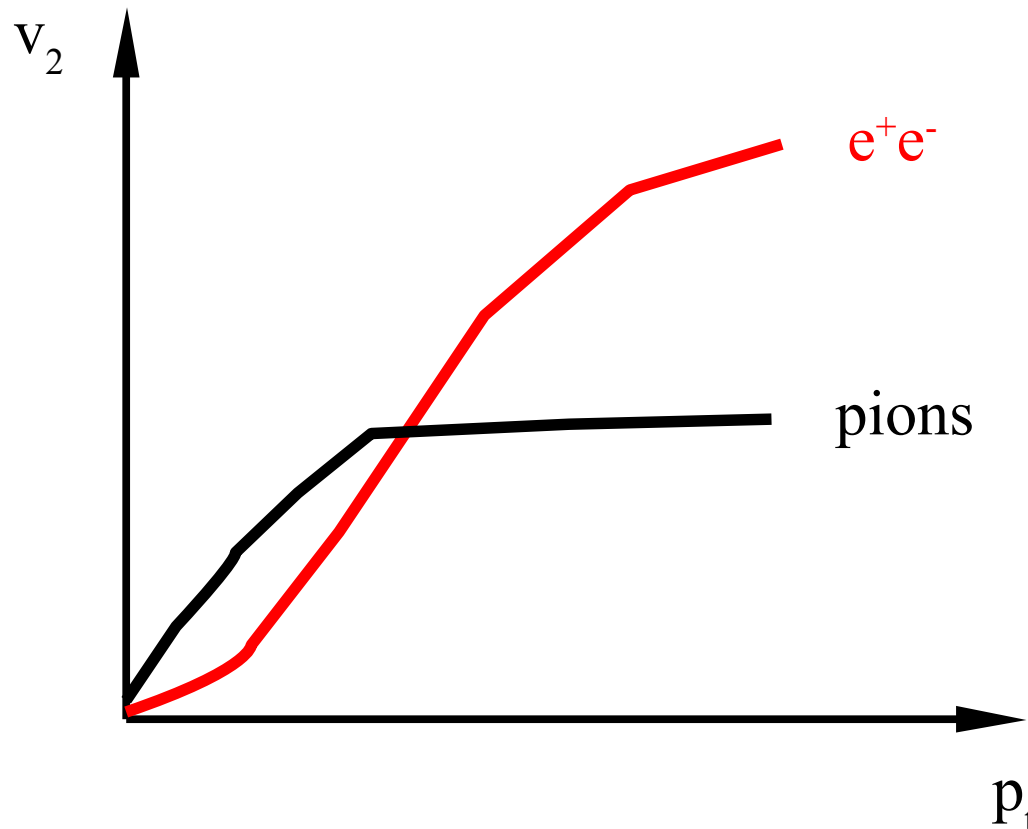
q-qbar



$$v_2(e^+e^-) = 2 v_2(\text{quark})$$

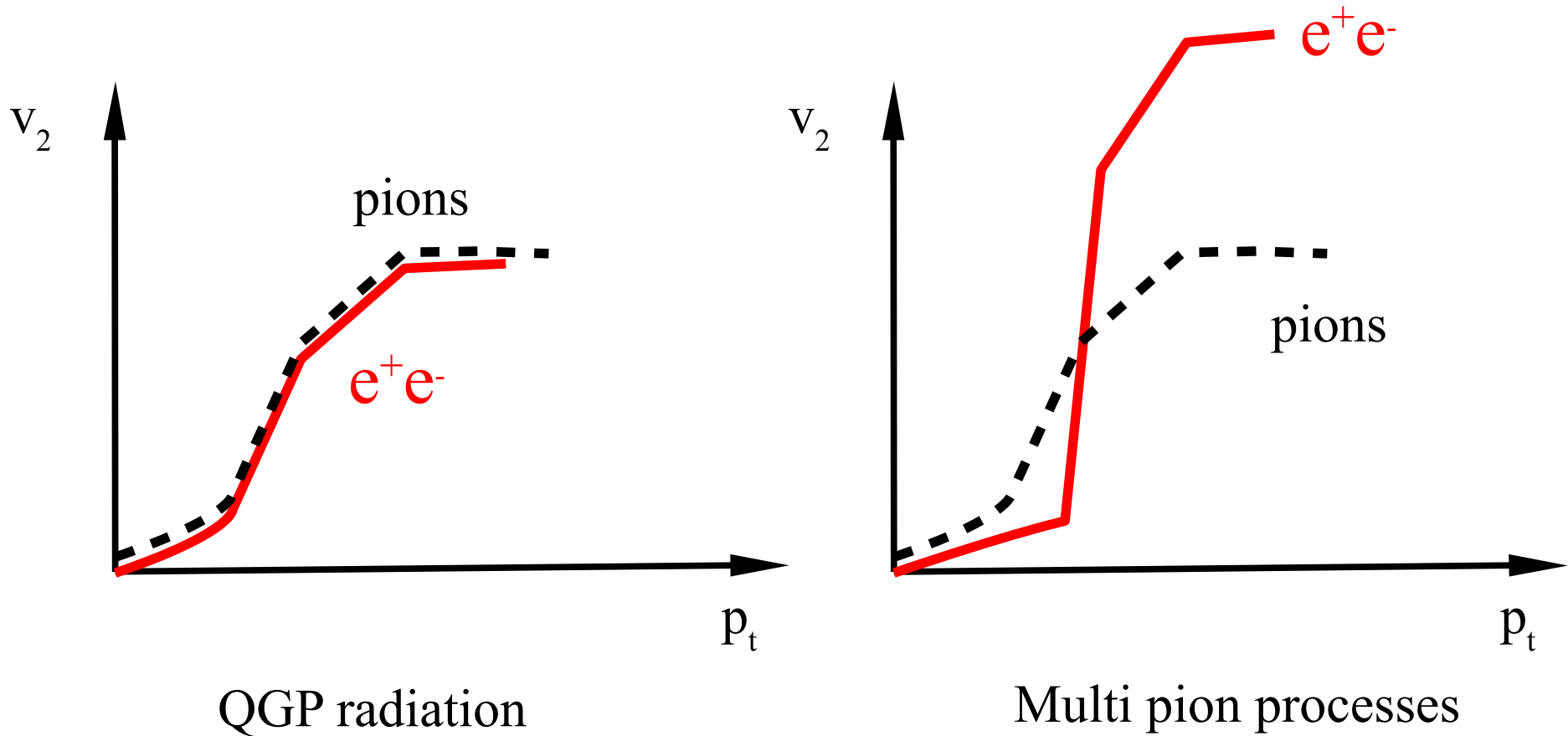
Test the low mass enhancement

$$M_{e^+e^-} \approx m_{\rho}$$



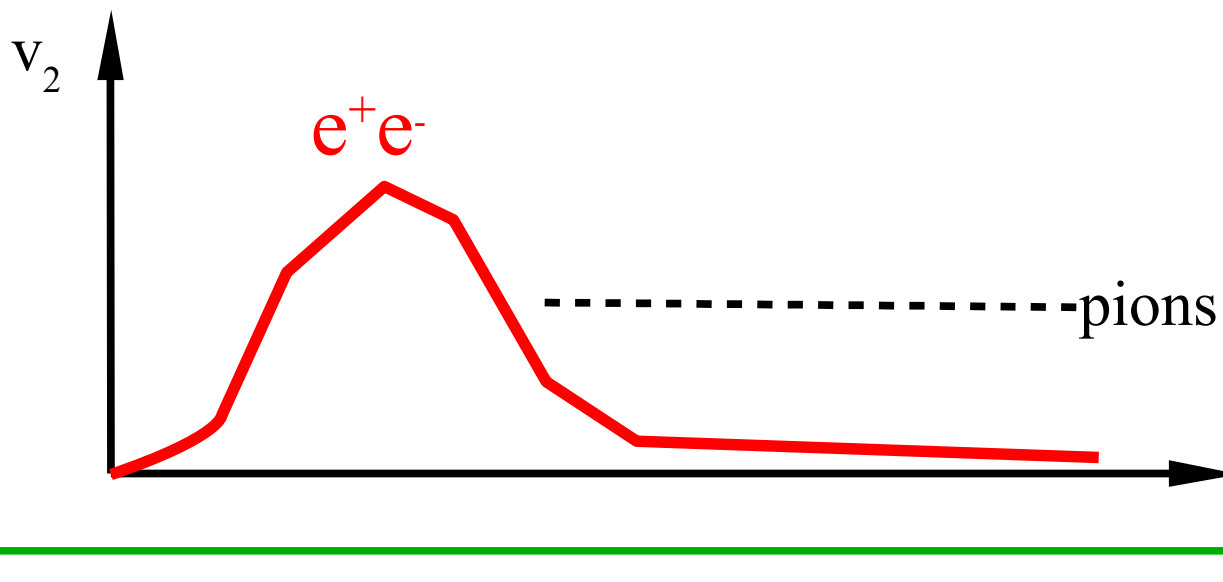
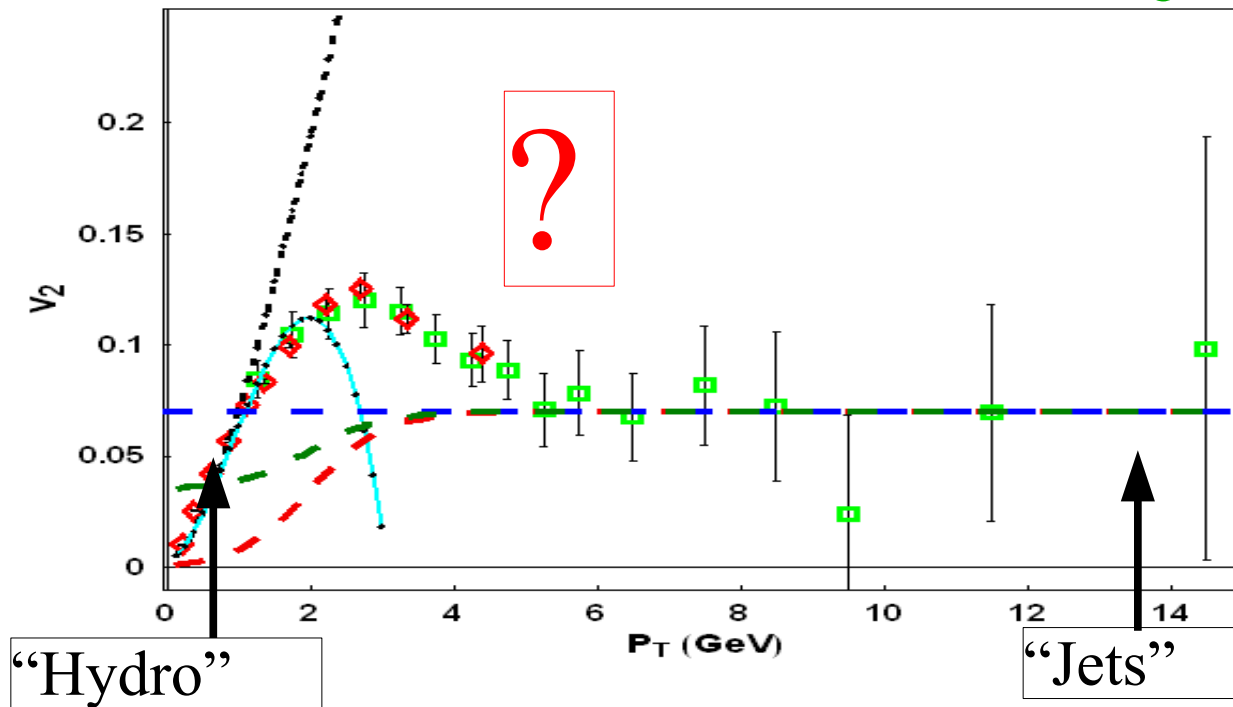
$v_2(e^+e^-) \approx 2 v_2(\text{pion})$
if pion annihilation
is dominant

Intermediate masses: QGP radiation?



Increase of $M_{e^+e^-}$

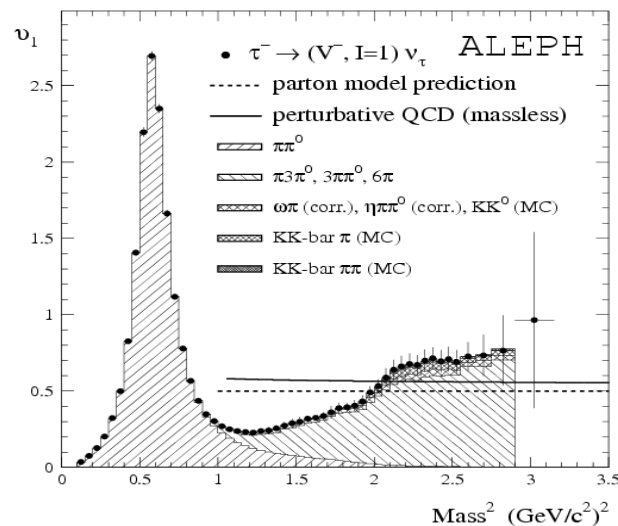
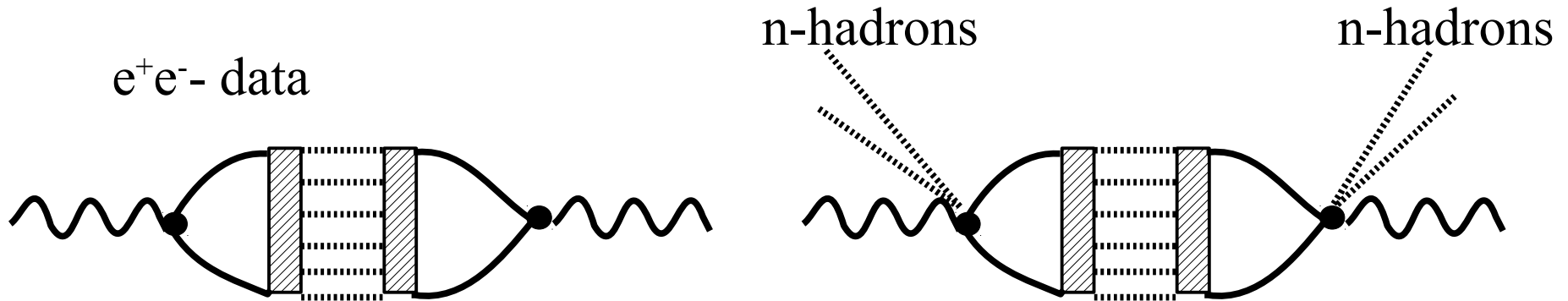
Partonic collectivity?



Dilepton v_2

- Test the present understanding of low mass enhancement
 - $v_2(e^+e^-) \approx 2 v_2(\text{pion})$
- Potential investigate the source for intermediate mass dileptons
- Explore the p_t scale for partonic collectivity

Channels not controlled by e^+e^-



e^+e^- + hadrons in initial/final state are NOT accounted for examples: N^* -Dalitz, a_1 -Dalitz, etc

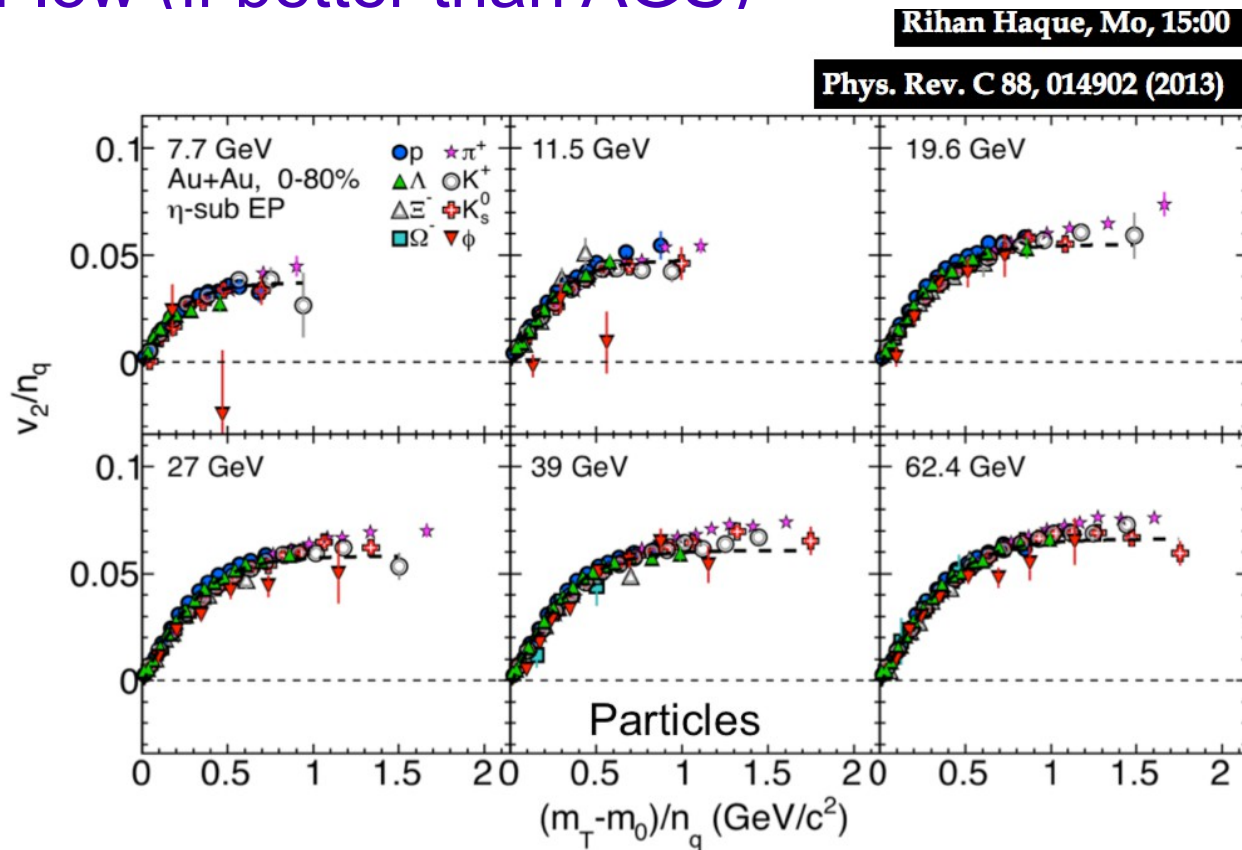
This will be relevant for CBM!
I don't think it will change the argument but needs to be Investigated.

Summary

- Phase structure requires measurement of fluctuations
- Dynamical treatment of first order phase transition including instabilities within fluid dynamics
 - Good tool for testing observables
 - So far no good observable for instabilities and droplet formation
- Higher order cumulants: Not so easy but important.
 - Need to see ALL thermal particles (“Poissonizer”)
 - Auto correlations
 - Stopping fluctuations.
- Dileptons
 - Low mass are understood
 - Intermediate mass as thermometer

Summary

- Charm: Not clear what to learn from it. Open to suggestions
- Day one physics:
 - Flow (if better than AGS)



Summary

- Day 1: Flow (barometer)
- Day 2: Cumulants (“Phaseometer”)
- Day 3: Dileptons ($M > 1$ GeV, Thermometer)
- Days 4-6: You tell me
- Day 7: Rest

BACKUP

Dilepton and the QCD CP

- Massless “modes” at CP since it is a second order phase transition
- Mode is mixture of “sigma” and “omega”
- However these may likely be space-like modes
 - $M^2 \rightarrow 0^-$

Nambu model p-h excitations

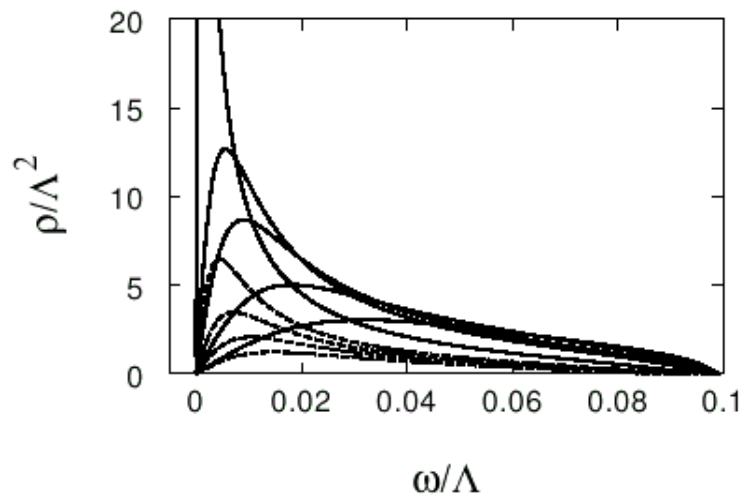
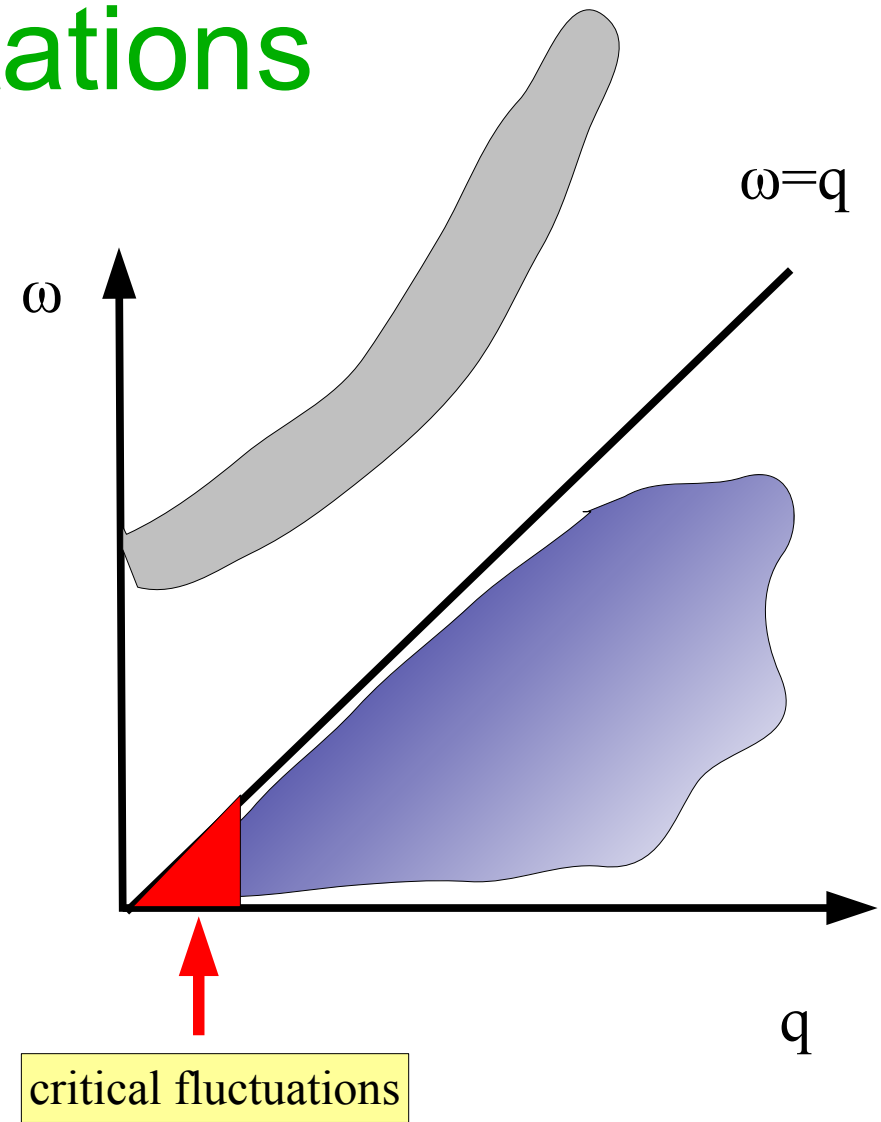


Fig. 3. Spectral function in the spacelike momentum region with $|\mathbf{q}|/\Lambda = 0.1$, $T = T_c$ and $m/\Lambda = 0.01$ for several μ (see text).



Nambu model

(Fuji et al, hep-ph/0401028,0403039)

Sigma remains massive at CP; CP driven by **spacelike** p-h excitations

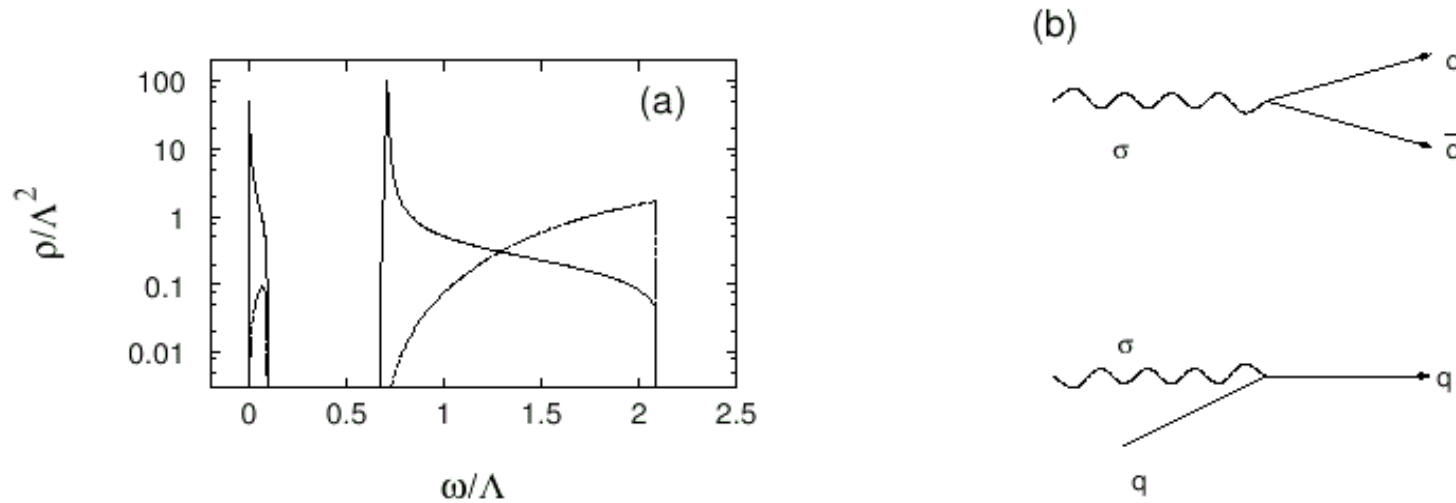
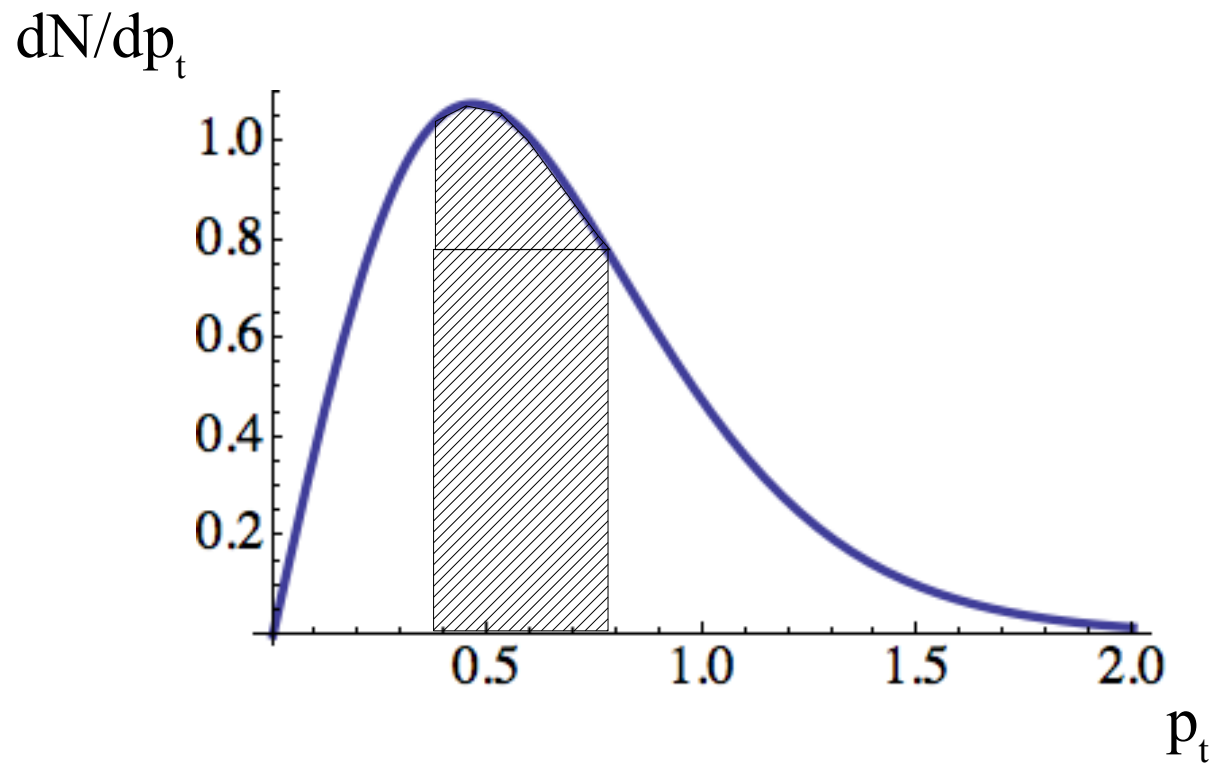
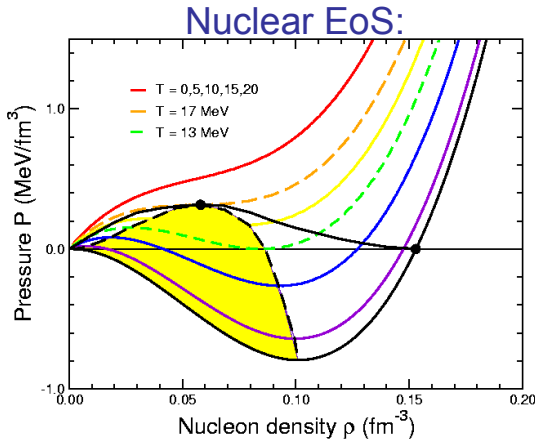


Fig. 2. (a) Spectral function in the scalar channel (solid) with $|q|/\Lambda = 0.1$ at a CEP with $m/\Lambda = 0.01$. The free gas spectrum (dashed) is also shown for reference. (b) Typical processes contributing to the spectrum.



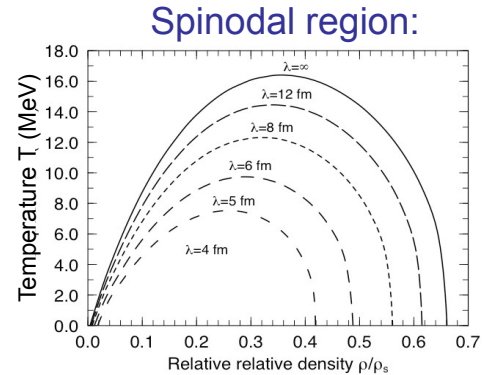
Spinodal Multifragmentation



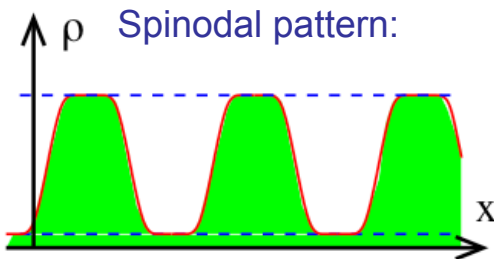
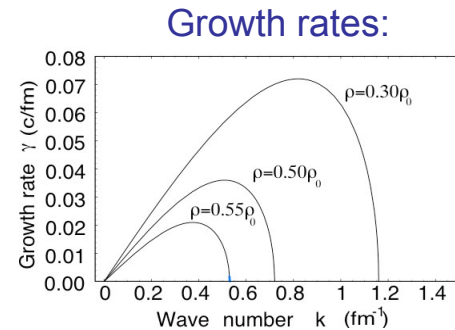
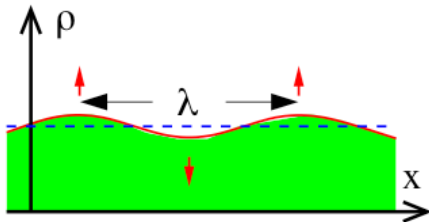
1st order phase transtion



Spinodal instability



Density undulations may be amplified



Ph Chomaz, M Colonna, J Randrup
Nuclear Spinodal Fragmentation
 Physics Reports 389 (2004) 263

⇒ Fragments ≈ equal!



Highly non-statistical ⇒ Good candidate signature

CLUMPING of Baryon Density

J. Randrup

Input required for realistic estimate of conservation effects

Note: This is likely only to work at lower energies where we have baryon stopping

Note: at low energies anti-protons likely to be irrelevant

Need:

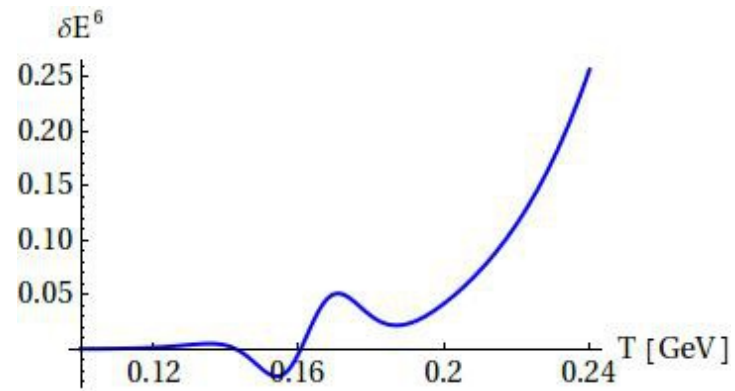
- Total number of protons and (anti-protons) (4 Pi)
- Number of protons and (anti-protons) actually measured
- Total number of charged particles

Big Question: Over what rapidity range are the various charges conserved?

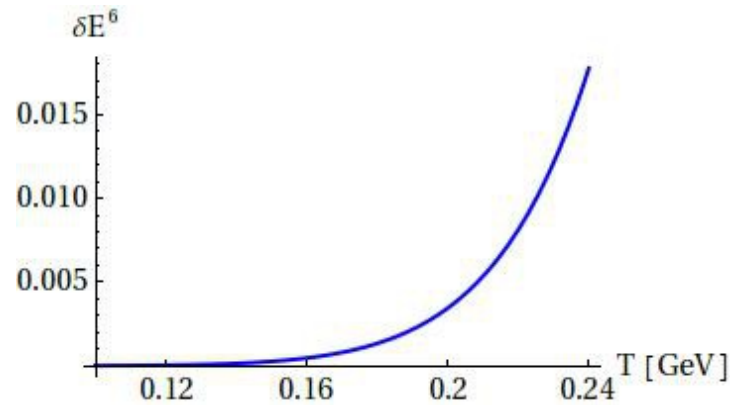
- Balance Functions? Only averages!

QCD vs HRG

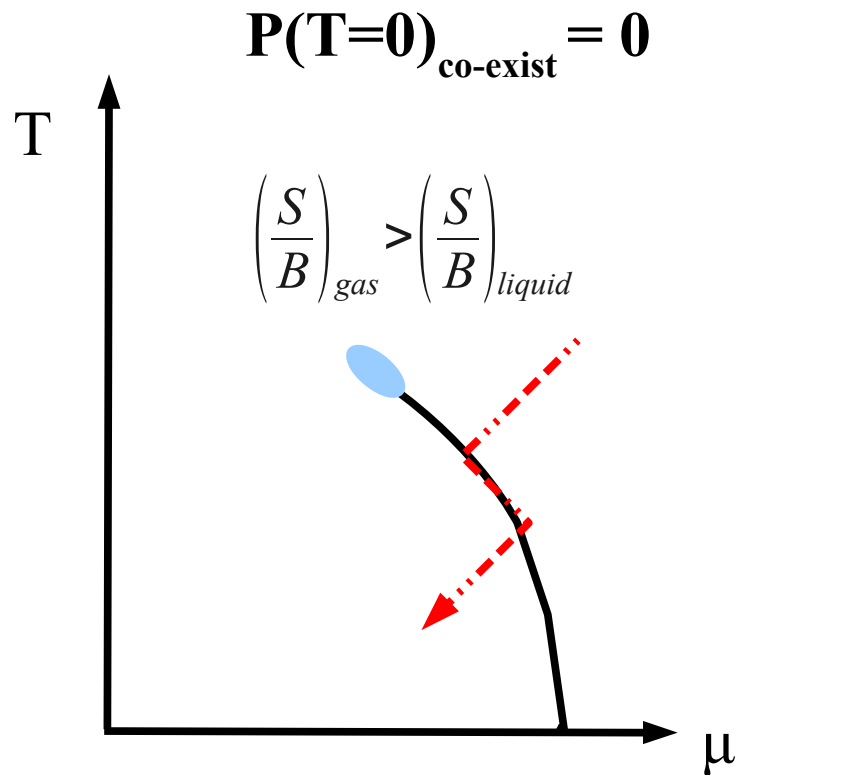
QCD



“HRG”

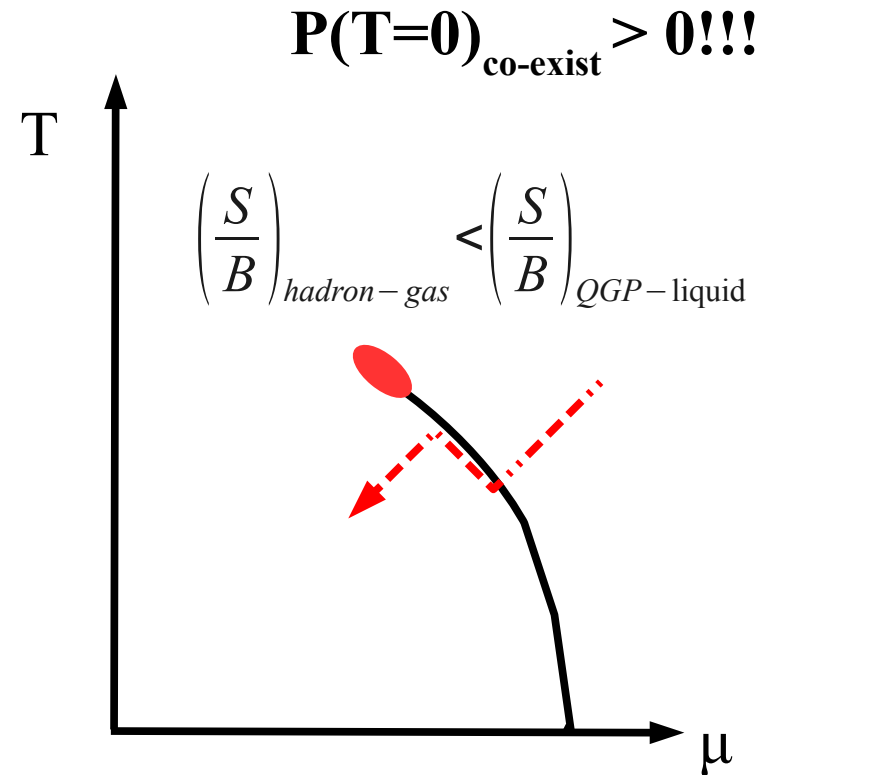


Liquid-gas vs QCD



Droplets are stable in vacuum

$$\frac{dP}{dt} > 0$$

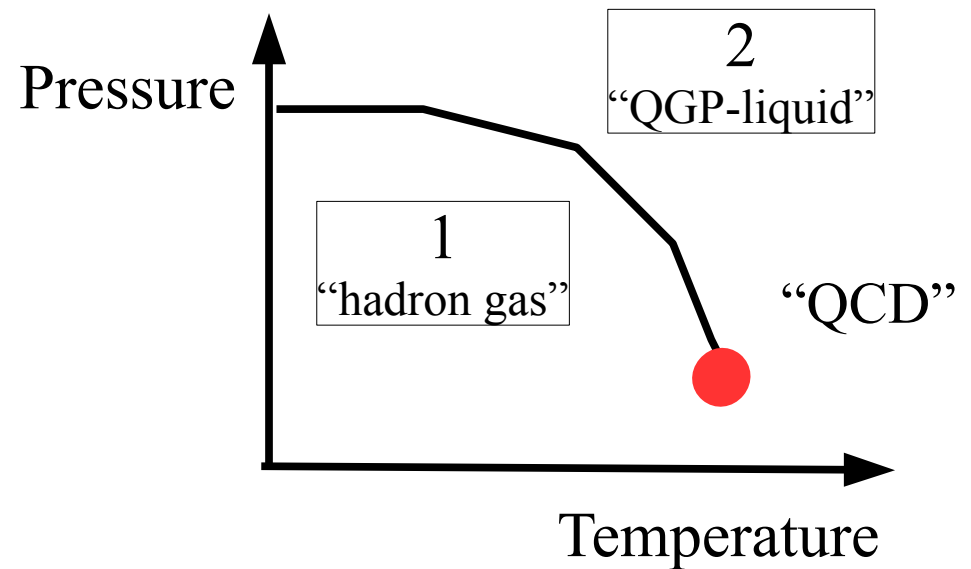
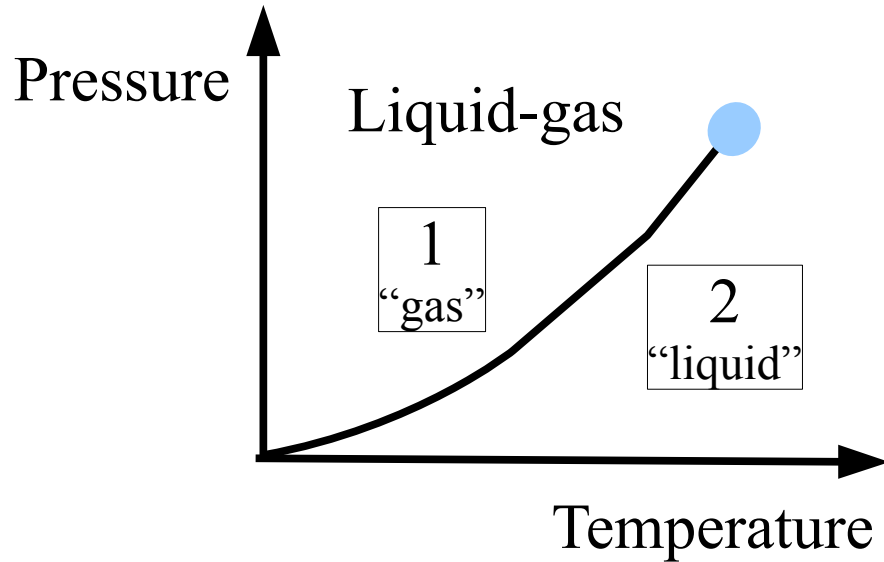


No stable droplets in vacuum

$$\frac{dP}{dt} < 0$$

Difference between Liquid Gas and QCD PT

Dexheimer et al, arXiv:1302.2835



Clausius-Clapeyron: $\frac{dP}{dT} = \frac{S_1/B_1 - S_2/B_2}{1/\rho_1 - 1/\rho_2}$ $\rho_2 > \rho_1 \rightarrow (1/\rho_1 - 1/\rho_2) > 0$

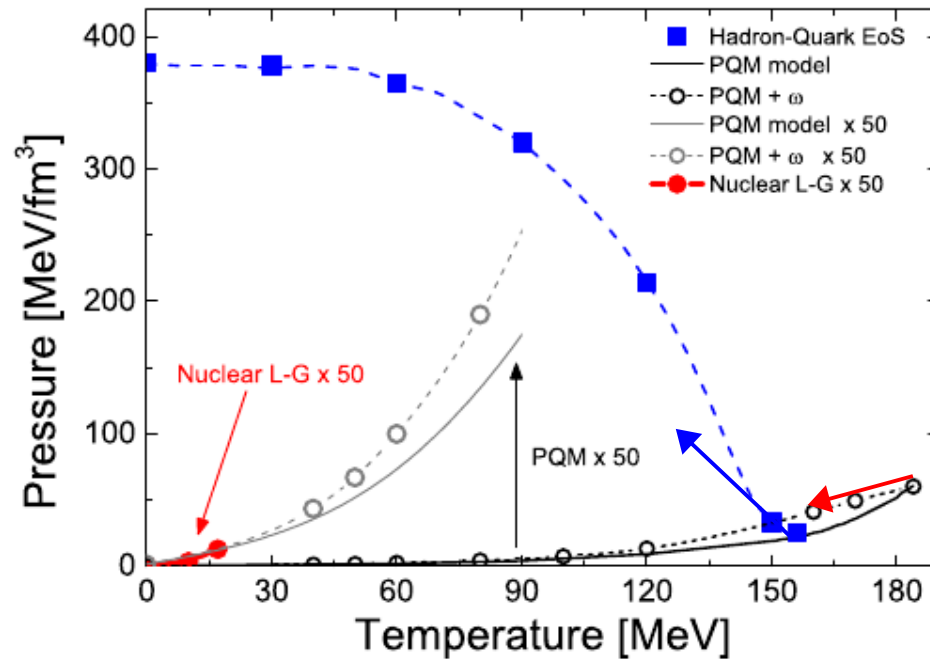
$$\frac{dP}{dT} > 0 \rightarrow S_1/B_1 > S_2/B_2$$

$$\left(\frac{S}{B}\right)_{gas} > \left(\frac{S}{B}\right)_{liquid}$$

$$\frac{dP}{dT} > 0 \rightarrow S_1/B_1 < S_2/B_2$$

$$\left(\frac{S}{B}\right)_{hadron-gas} < \left(\frac{S}{B}\right)_{QGP-liquid}$$

Lattice to the rescue?



Slope of pressure
along pseudo-critical line

$$\frac{\partial}{\partial T} p_{\text{pc}}(T, \mu = 0)|_{T=T_{\times}} = s(T_{\times}, \mu = 0) - \frac{T_{\times}^3}{2\kappa} \chi_2(T_{\times}) . \quad (18)$$

Lattice data from Wuppertal/Budapest:

Sign depends on definition of
pseudo-critical line ☹️

Higher moments (cumulants) and ξ

- Consider probability distribution for the order-parameter field:

$$P[\sigma] \sim \exp \{ -\Omega[\sigma]/T \},$$

$$\Omega = \int d^3x \left[\frac{1}{2} (\nabla \sigma)^2 + \frac{m_\sigma^2}{2} \sigma^2 + \frac{\lambda_3}{3} \sigma^3 + \frac{\lambda_4}{4} \sigma^4 + \dots \right]. \quad \Rightarrow \quad \xi = m_\sigma^{-1}$$

- Moments (connected) of $q = 0$ mode $\sigma_V \equiv \int d^3x \sigma(x)$:

$$\kappa_2 = \langle \sigma_V^2 \rangle = VT \xi^2; \quad \kappa_3 = \langle \sigma_V^3 \rangle = 2VT^2 \lambda_3 \xi^6;$$

$$\kappa_4 = \langle \sigma_V^4 \rangle_c \equiv \langle \sigma_V^4 \rangle - 3 \langle \sigma_V^2 \rangle^2 = 6VT^3 [2(\lambda_3 \xi)^2 - \lambda_4] \xi^8.$$

- Tree graphs. Each propagator gives ξ^2 .



- Scaling requires "running": $\lambda_3 = \tilde{\lambda}_3 T (T\xi)^{-3/2}$ and $\lambda_4 = \tilde{\lambda}_4 (T\xi)^{-1}$, i.e.,

$$\kappa_3 = \langle \sigma_V^3 \rangle = 2VT^{3/2} \tilde{\lambda}_3 \xi^{4.5}; \quad \kappa_4 = 6VT^2 [2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4] \xi^7.$$

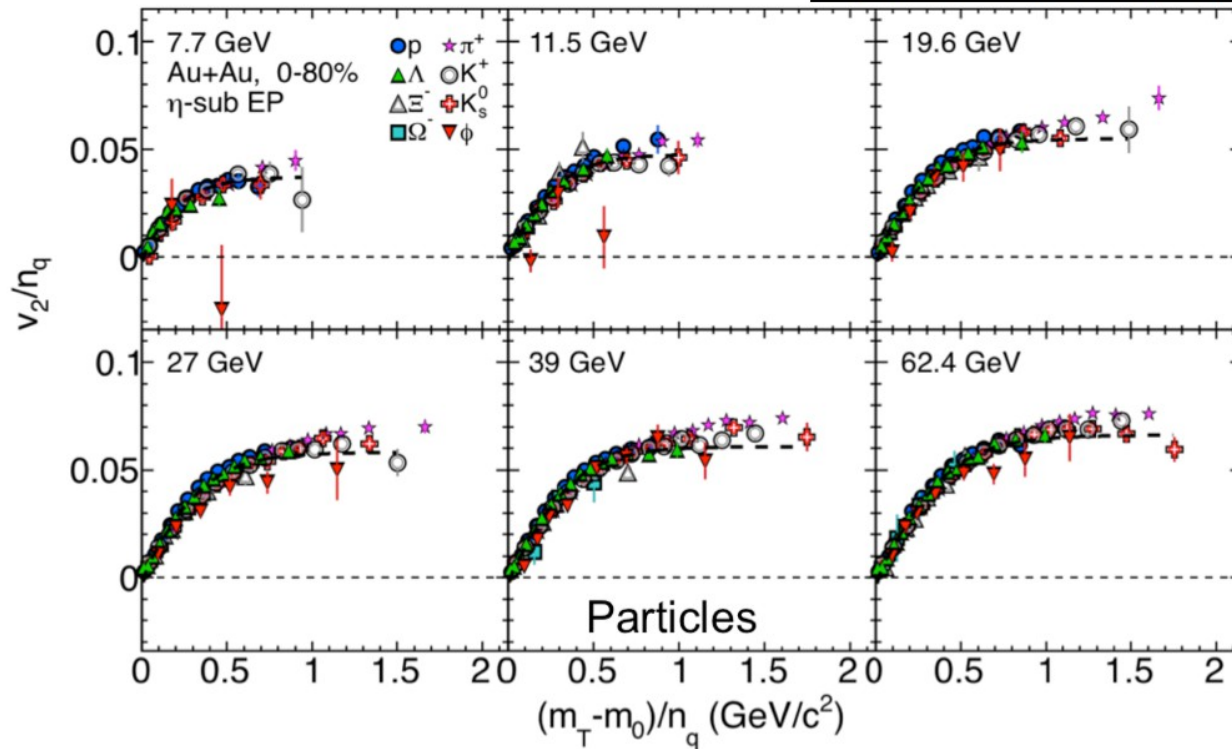
Flow



v_2 NCQ Scaling of Particles

Rihan Haque, Mo, 15:00

Phys. Rev. C 88, 014902 (2013)



- NCQ-scaling holds for particles and anti-particles separately at all energies
→ Partonic degrees of freedom?

- High $m_T - m_0$ not measured at lower energies
- Do ϕ -mesons deviate?

NCQ = Number of Constituent Quark

Particle and Anti-particle flow

Steinheimer et al Phys.Rev. C86 (2012) 044903 :

- Excitation function of v_2
- Centrality dependence of freeze out parameters

Both agree with STAR measurement

Essential: stopping of baryon number
Explains difference in elliptic flow
between protons and anti-protons.

Not yet included:

Stopping of **isospin**.

Qualitatively explains the trend seen for pions

Strangeness conservation: strangeness chemical
Potential same sign as baryon chemical potential:
Flow difference of kaons same sign as protons

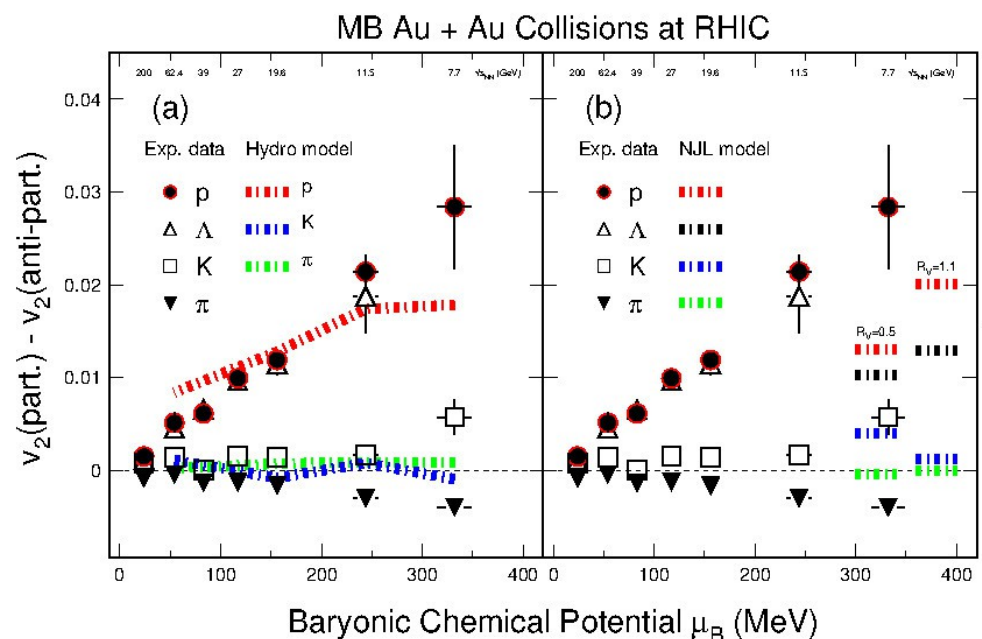
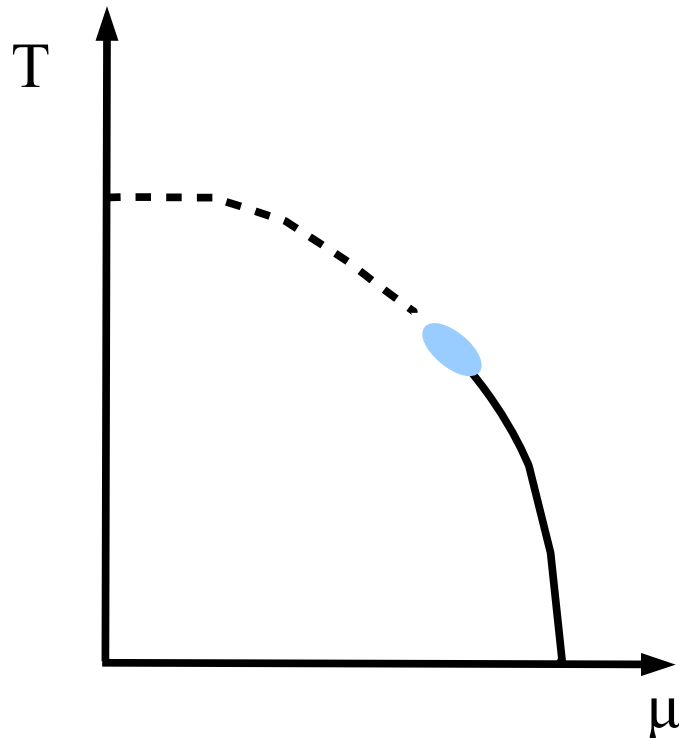


Figure courtesy N. Xu

Another way

$$F = F(r), \quad r = \sqrt{T^2 + a\mu^2}$$

$a \sim$ curvature of critical line



$$\partial_{\mu}^2 F(T, \mu)_{\mu=0} = \frac{a}{T} \partial_T F(T, 0)$$

$$\partial_{\mu}^4 F(T, \mu)_{\mu=0} = 3 \frac{a^2}{T} (T \partial_T^2 - \partial_T) F(T, 0)$$

Baryon number cumulants give same info. Less problem with flow etc.

The sources for v_2

