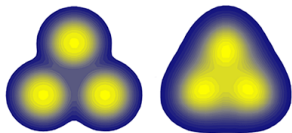


# Throwing triangles against a wall: ground state of $^{12}\text{C}$ from highest-energy collisions

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Inst. of Nuclear Physics PAN, Cracow, and  
Jan Kochanowski U., Kielce



Physics Symposium, 24th CBM Week  
10 September 2014, Cracow

[research with Enrique Ruiz Arriola, Piotr Bożek, Maciej Rybczyński]

# Instead of outline

Two phenomena are related:

$\alpha$  clustering in light nuclei



harmonic flow in ultra-relativistic nuclear collisions

Surprising link:

lowest-energy ground-state structure  $\longleftrightarrow$  highest energy reactions

- New method of investigating many-particle nuclear correlations
- Another test of collective dynamics/harmonic flow

$\alpha$  clusters

# Some history

David Brink: After Gamow's theory of  $\alpha$ -decay it was natural to investigate a model in which nuclei are composed of  $\alpha$ -particles. Gamow developed a rather detailed theory of properties in his book "Constitution of Nuclei" published in 1931 before the discovery of the neutron in 1932. He supposed that  $4n$ -nuclei like  ${}^8\text{Be}$ ,  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$  ... were composed of  $\alpha$ -particles

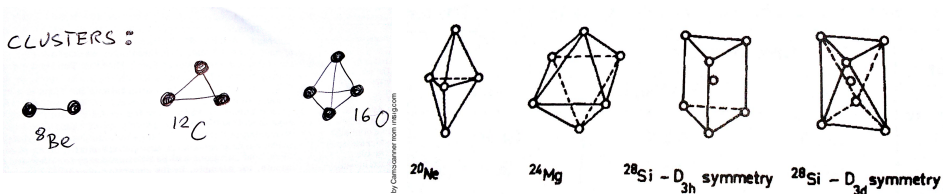
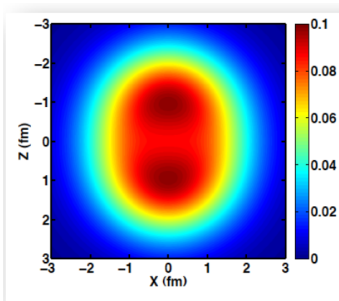


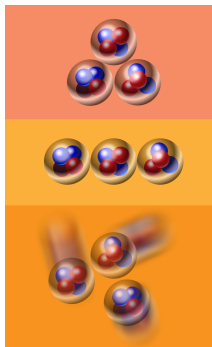
Fig. 1. Alpha-particle configuration for some  $4N$  nuclei.

Generated by CamScanner from intsig.com

# $\alpha$ clusters in light nuclei



${}^9\text{Be}$



ground

Hoyle  $0^+$

other excited,  $2^+$  ...

${}^{12}\text{C}$

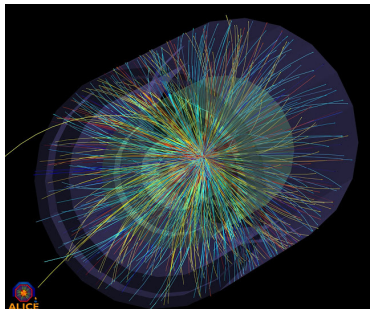
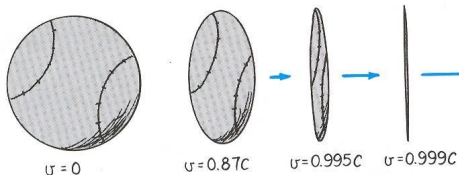
How can we detect the  $\alpha$  clusters in the ground state?  
What is their spatial arrangement?  
Assessment of n-body correlations (one-body not enough)

[Recent status: SOTANCP3 Conference, Yokohama, May 2014]

# Flow

# Ultra-relativistic A+A collisions (LHC, RHIC, SPS)

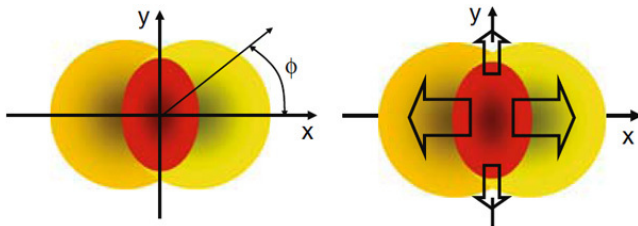
- Lorentz contraction
- Collision: essentially instantaneous passage, frozen configuration
- Reduction of the **ground-state** wave function of the nucleus (like measurement)



- detection of particles in the transverse direction (mid-rapidity)

# Phenomenon of flow

Quark-gluon plasma is formed!



“Initial shape – final flow” transmutation detectable in the asymmetry of the momentum distribution of detected particles – follows from collectivity

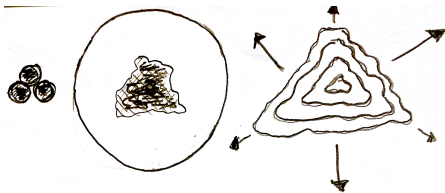


Merge the two ideas ( $\alpha$ 's and flow)  $\rightarrow$

[WB & ERA, PRL 112 (2014) 112501]

# From $\alpha$ clusters to flow in relativistic collisions

$\alpha$  clusters  $\rightarrow$  asymmetry of shape  $\rightarrow$  asymmetry of initial fireball  $\rightarrow$   
 $\rightarrow$  hydro or transport  $\rightarrow$  collective harmonic flow



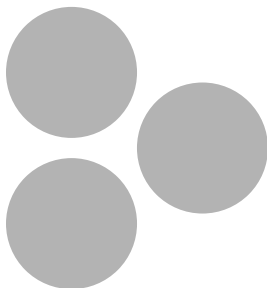
nuclear triangular geometry  $\rightarrow$  fireball triangular geometry  $\rightarrow$  triangular flow

What are the signatures, chances of detection?  
(some blurring by fluctuations)  
“Easy snap-shot but difficult development”

Described later:  $^3\text{He}$ -Au at RHIC [Sickles et al. (PHENIX) 2013]  
The case of  $^{12}\text{C}$  is more promising, as it leads to more abundant fireballs.

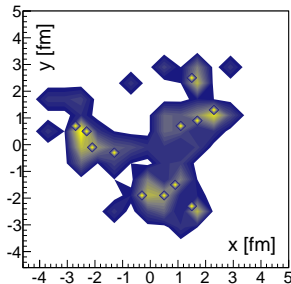
# Our modeling $^{12}\text{C}$

Three  $\alpha$ 's in a triangular arrangement, generate nucleon positions with Monte Carlo, parameters (size of the cluster, distance between clusters) properly adjusted (fit one-body radial distributions from other calculations, fit EM form factor)



## Why ultra-relativistic?

Reaction time is much shorter than time scales of the structure  
→ a frozen “snapshot” of the nuclear configuration



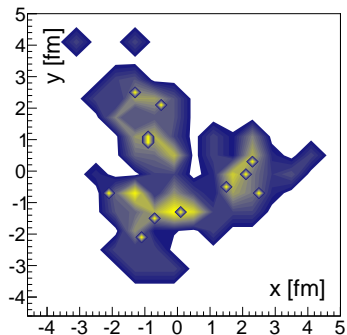
wounding range determined by  $\sigma_{\text{NN}}^{\text{inel}}$

( $N_w > 70$  - flat-on orientation)

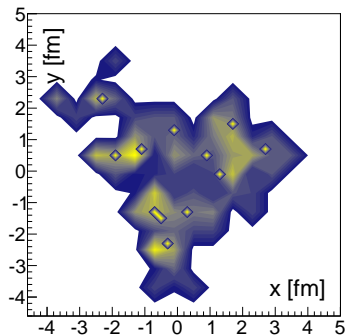
Imprints of the three  $\alpha$  clusters clearly visible

## Simulations with GLISSANDO 2

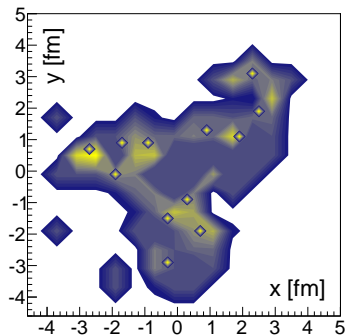
... more events



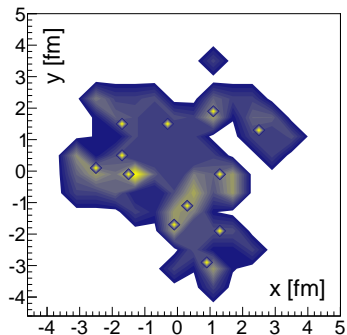
... more events



... more events

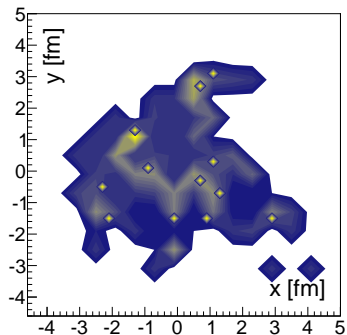


... more events

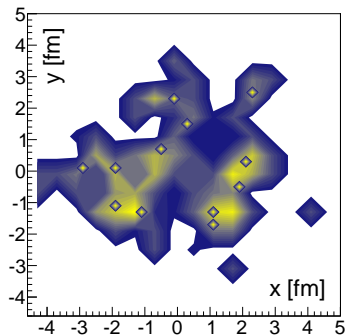




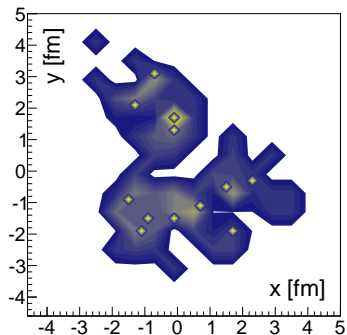
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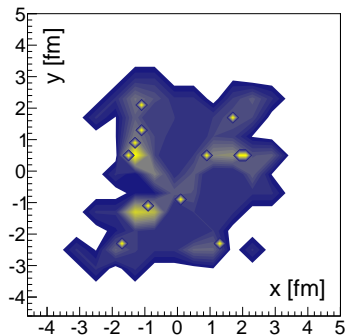
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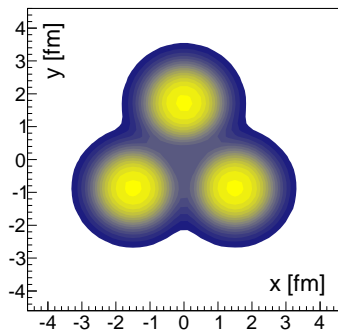
... more events



... more events

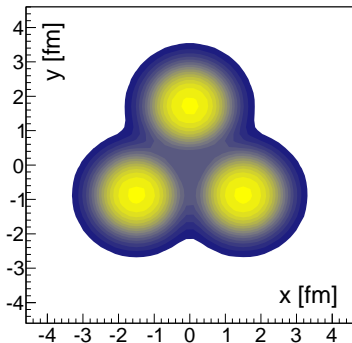


Our intrinsic distributions in  $^{12}\text{C}$ : three  $\alpha$ 's in a triangular arrangement



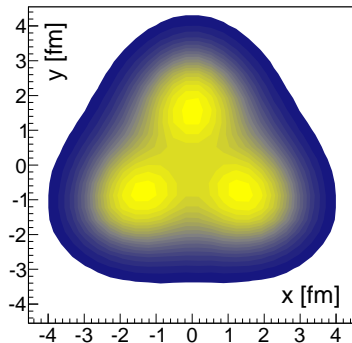
# Geometry of nucleus $\rightarrow$ geometry of fireball

Triangular nucleus causes triangular "damage"!



intrinsic density of  $^{12}\text{C}$

$\rightarrow$



geometry of the fireball  
(flat-on collision)

# Eccentricity parameters

We need some quantitative measures of deformation (heavily used in heavy-ion analyses)

Eccentricity parameters  $\epsilon_n$  (Fourier analysis)

$$\epsilon_n e^{in\Phi_n} = \frac{\sum_j \rho_j^n e^{in\phi_j}}{\sum_j \rho_j^n}$$

describe the shape of each event ( $j$  labels the sources in the event,  $n$ =rank,  $\Phi_n$  is the principal axis angle)

$n = 2$  – ellipticity,  $n = 3$  – triangularity, ...

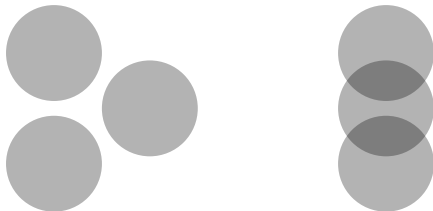
Two components:

- intrinsic (from existent mean deformation of the fireball)
- from fluctuations

# Geometry vs multiplicity correlations in $^{12}\text{C-Pb}$

## Two cases of angular orientation

cluster plane parallel or perpendicular to the transverse plane:

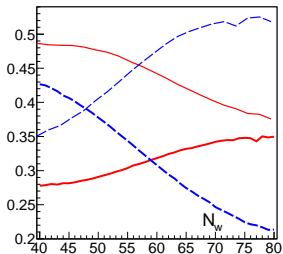


higher multiplicity  
higher triangularity  
lower ellipticity

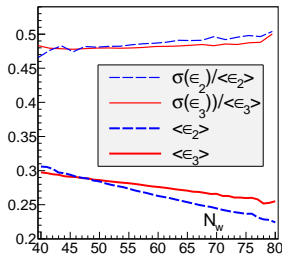
lower multiplicity  
lower triangularity  
higher ellipticity



# Ellipticity and triangularity vs multiplicity



clustered



unclustered

Clusters: (qualitative signal)

When  $N_w \nearrow$  then  $\langle\epsilon_3\rangle \nearrow$  and  $\langle\epsilon_2\rangle \searrow$

and  $\langle\sigma(\epsilon_3)/\epsilon_3\rangle \searrow$ ,  $\langle\sigma(\epsilon_2)/\epsilon_2\rangle \nearrow$

No clusters:

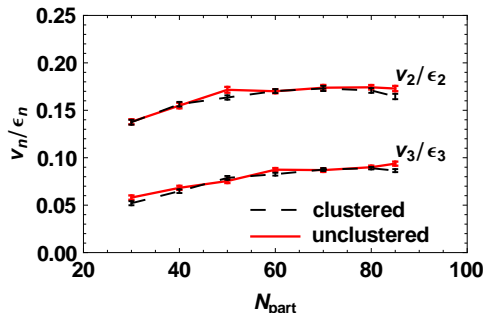
similar behavior for  $n = 2$  and  $n = 3$

# Shape-flow transmutation

The eccentricity parameters are transformed (in all models based on collective dynamics) into asymmetry of the transverse-momentum flow.

**Linear response:**

$v_n$  grows with  $\epsilon_n$



[Božek 3+1 viscous hydro + THERMINATOR]

We have to a very good approximation

$$v_n = \kappa_n \epsilon_n, \quad n = 2, 3, \dots$$

( $\kappa_n$  depends on mutiplicity and hydro details)

Cumulant moments:

$$\epsilon_n \{2\}^2 = \langle \epsilon_n^2 \rangle, \quad \epsilon_n \{4\}^4 = 2 \langle \epsilon_n^2 \rangle - \langle \epsilon_n^4 \rangle$$

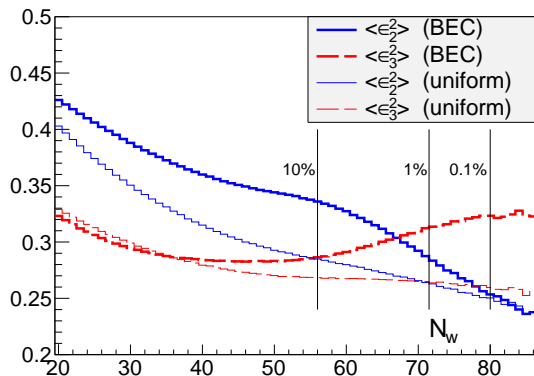
Ratio's insensitive to response:

$$\frac{v_n \{m\}}{v_n \{2\}} = \frac{\epsilon_n \{m\}}{\epsilon_n \{2\}}, \quad m = 4, 6, \dots$$

(infer info on flow from just the eccentricities, no hydro!)



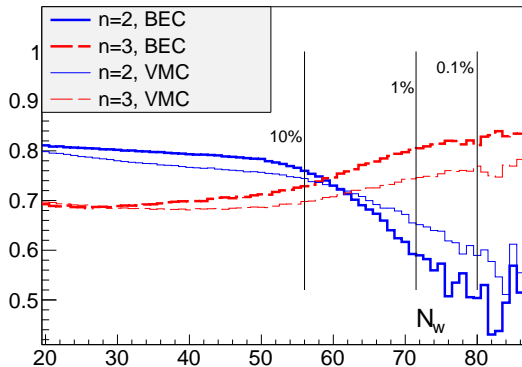
wounded nucleon model



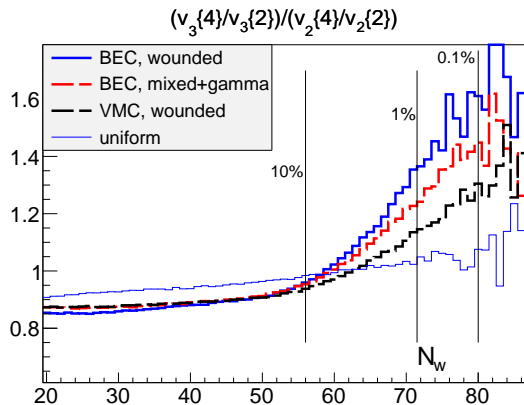
# Ratios of cumulant moments



$v_n\{4\}/v_n\{2\}$  (wounded)



# Double ratio of cumulant moments

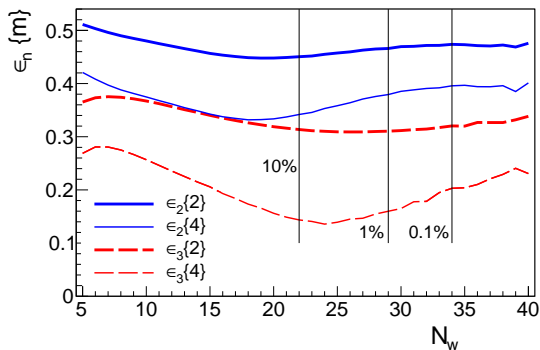


${}^3\text{He-Au}$

(being presently analyzed by PHENIX)

[hydro: J. Nagle et al., arXiv:1312.4565]

[hydro without hydro: Piotr Bożek and WB, arXiv:1409.2160]

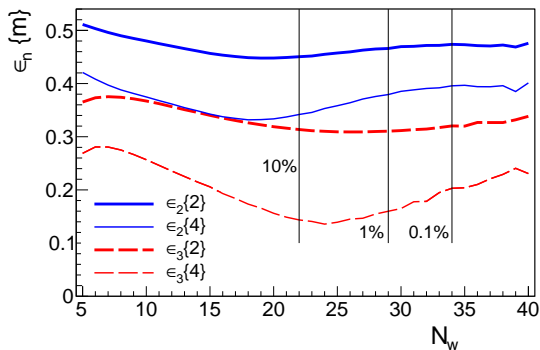




(being presently analyzed by PHENIX)

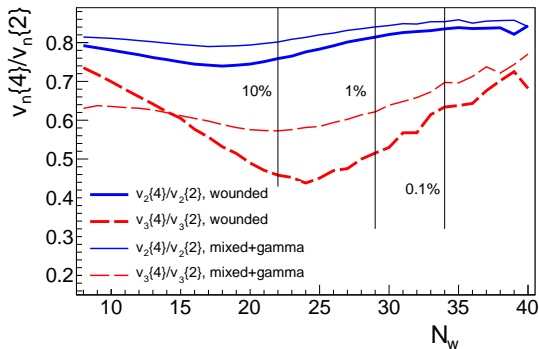
[hydro: J. Nagle et al., arXiv:1312.4565]

[hydro without hydro: Piotr Bożek and WB, arXiv:1409.2160]



(not equilateral)

# Ratio for ${}^3\text{He-Au}$



(to be confirmed by the experiment!)

# Conclusions

**Snapshots** of the ground-state wave function

Spatial correlations in the ground state → harmonic flow

**Signatures** in clustered  $^{12}\text{C}$ - $^{208}\text{Pb}$  collisions

- Increase of triangularity with multiplicity for the highest multiplicity events
- Anticorrelation of ellipticity and triangularity
- Very clear signals from ratios of cumulant moments
- Stronger effect at lower  $\sigma_{NN}^{\text{inel}}$  (i.e., at lower collision energies)
- Even stronger effect on the  $^{12}\text{C}$  side in rapidity
- Ratios depend on the nuclear wave function and the initial-state model, but not on hydro

Possible data (NA61@SPS, RHIC) would allow to place constraints on the spatial structure of the light projectile. Conversely, the knowledge of the nuclear distributions helps to verify the fireball formation models

# Back-up

## Intrinsic distributions

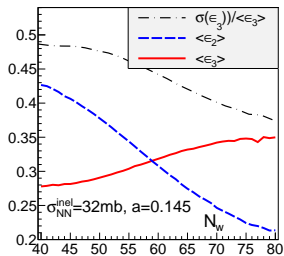
Ground state of  $^{12}\text{C}$  is a  $0^+$  state (rotationally symmetric wave function).  
The meaning of *deformation* concerns **multiparticle correlations** between the nucleons

Superposition over orientations:

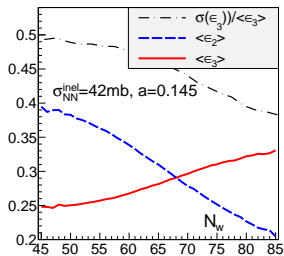
$$|\Psi_{0^+}(x_1, \dots, x_N)\rangle = \frac{1}{4\pi} \int d\Omega \Psi_{\text{intr}}(x_1, \dots, x_N; \Omega)$$

The *intrinsic* density of sources of rank  $n$  is defined as the average over events, where the distributions in each event have aligned principal axes:  
 $f_n^{\text{intr}}(\vec{x}) = \langle f(R(-\Phi_n)\vec{x}) \rangle$ . Brackets indicate averaging over events and  $R(-\Phi_n)$  is the inverse rotation by the principal-axis angle in each event

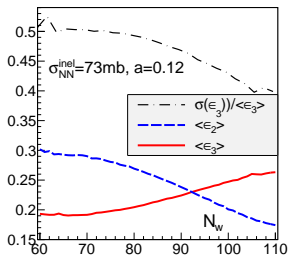
# Dependence on the collision energy



$\sigma_{NN}^{inel} = 32\text{mb}$  (SPS)



42mb (RHIC)

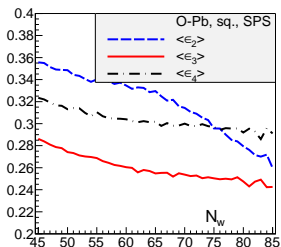
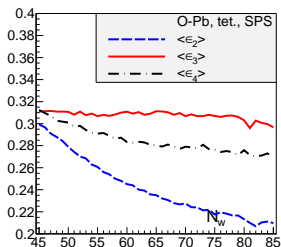
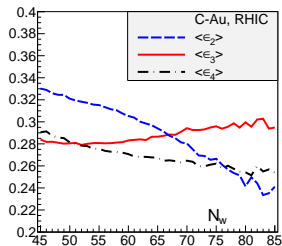
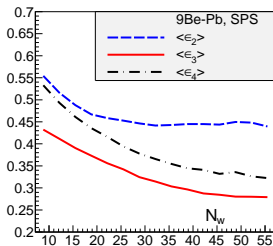
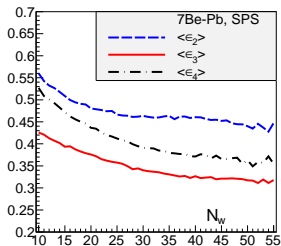


72mb (LHC)

Qualitative conclusions hold from SPS to the LHC

# Other systems

(distributions matched to Wiringa's et al. radial densities)



[work with Maciej Rybczyński]