Two-body scattering processes for low energies with chiral perturbation theory

(H.-F. Fuhrmann (GSI, Theory), July 2014)

- We investigate hadronic interactions in <u>two-body scattering processes</u> for momentum transfers Q ~ 1 GeV.
- While strong interaction processes can be treated perturbatively in QCD for high energies (→<u>Asymptotic freedom</u>), the <u>confinement of quarks of gluons</u> prohibits us to examine our scattering reactions via QCD for low energies.
- We rather estimate <u>hadrons</u> instead of quarks and gluons as <u>effective degrees of freedom</u>. Chiral perturbation theory serves as a complementary theory to QCD.
- Our group's calculations are based on the <u>chiral Lagrangian</u> with a <u>small hadron mass or momentum</u> Q_χ^n instead of the strong coupling constant $\alpha_s(Q^2) = g_s^2(Q^2)/4\pi$ <u>as an expansion parameter</u>.
- The scattering processes examined by the members of our group deal with hadrons built of up (<u>u-</u>), down (<u>d-</u>), strange (<u>s-</u>), charm (<u>c-</u>) <u>quarks and gluons</u>.

Hadron multiplets & photon field:

- Pseudoscalar mesons: ($J^P = 0^-$)

$$\Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

- Vector mesons: ($J^P = 1^-$)

$$V_{\mu\nu} = \begin{pmatrix} \rho^0_{\mu\nu} + \omega_{\mu\nu} & \sqrt{2}\rho^+_{\mu\nu} & \sqrt{2}K^+_{\mu\nu} \\ \sqrt{2}\rho^-_{\mu\nu} & -\rho^0_{\mu\nu} + \omega_{\mu\nu} & \sqrt{2}K^0_{\mu\nu} \\ \sqrt{2}K^-_{\mu\nu} & \sqrt{2}\bar{K}^0_{\mu\nu} & \sqrt{2}\phi^0_{\mu\nu} \end{pmatrix}$$

- Baryon octet: ($J^P = \frac{1}{2}^+$)

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ -\Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}$$

- Baryon decuplet: ($J^P = \frac{3}{2}^+$)

$$\Delta^{111} = \Delta^{++}, \quad \Delta^{113} = \Sigma^{++}/\sqrt{3}, \quad \Delta^{133} = \Xi^0/\sqrt{3}, \quad \Delta^{333} = \Omega^-,$$
$$\Delta^{112} = \Delta^+/\sqrt{3}, \quad \Delta^{123} = \Sigma^0/\sqrt{6}, \quad \Delta^{233} = \Xi^-/\sqrt{3},$$
$$\Delta^{122} = \Delta^0/\sqrt{3}, \quad \Delta^{233} = \Sigma^-/\sqrt{3},$$
$$\Delta^{222} = \Delta^-.$$

- External photon field:
  - ➔ Approach of external fields [1], [2]
  - ➔ ($J^P = 1^-$)-particle ⇒ vector current
  - ➔ EM-4-potential A_μ, charge matrix Q (u-, d-, s-quark charge)

$$v_\mu = -eQA_\mu, \quad Q = \text{diag}\,(2/3, -1/3, -1/3).$$

Extension beyond the threshold:

- Non-perturbative approach for scattering amplitude [4]:

$$T^{(JP)}_{ab} = U^{(JP)}_{ab} + \int_{\mu_{\text{thr.}}}^{\infty} \frac{\mathrm{d}w}{\pi} \frac{\sqrt{s} - \mu_M}{w - \mu_M} \frac{\Delta T^{(JP)}_{ab}(w)}{w - \sqrt{s} - i\epsilon}$$

- Phase-space matrix:

$$\Delta T^{(JP)}_{ab}(\sqrt{s}) = T^{(JP)}_{ac}(\sqrt{s} + i\epsilon)\rho^{(JP)}_{cd}(\sqrt{s})T^{*,(JP)}_{db}(\sqrt{s} + i\epsilon)$$

- Non-linear integral equation:
  - ✔ Microcausality
  - ✔ Unitarity

$$T^{(JP)}_{ab}(\sqrt{s}) = U^{(JP)}_{ab}(\sqrt{s}) + \sum_{c,d} \int_{\mu_{\text{thr.}}}^{\infty} \frac{\mathrm{d}w}{\pi} \frac{\sqrt{s} - \mu_M}{w - \mu_M} \frac{T^{(JP)}_{ac}(w)\rho^{(JP)}_{cd}(w)T^{*,(JP)}_{db}(w)}{w - \sqrt{s} - i\epsilon}$$

➔ Numerator/Denominator (N/D) ansatz: $T^{(JP)}_{ab} = \sum_c N^{(JP)}_{cb}(\sqrt{s}) \left(D^{(JP)}_{ac}\right)^{-1}(\sqrt{s})$

Scattering processes on tree-level & Decomposition scheme

- E.g. boson-fermion → boson-fermion
- Decomposition of on-shell scattering amplitudes ***T*** into Lorentz invariant functions $G_n(s,t)$:

$$T_{S_1^{P_1} S_2^{P_2} \to S_3^{P_3} S_4^{P_4}} = \sum_n G_n(s,t)\; T_n(\bar{q}, q, w)$$

- ✔ Mandelstam's integral dispersion relation (→ Analyticity of $G_n(s,t)$):

$$G_n(s,t) = \frac{1}{\pi}\int \mathrm{d}s' \frac{\rho^n_s(s')}{s' - s} + \frac{1}{\pi}\int \mathrm{d}t' \frac{\rho^n_t(t')}{t' - t} + \frac{1}{\pi}\int \mathrm{d}u' \frac{\rho^n_u(u')}{u' - u}$$
$$+ \frac{1}{\pi^2}\int \mathrm{d}u' \int \mathrm{d}s' \frac{\rho^n_{us}(u', s')}{(u' - u)(s' - s)} + \frac{1}{pi^2}\mathrm{d}u' \int \mathrm{d}t' \frac{\rho^n_{ut}(u', t')}{(u' - u)(t' - t)}$$
$$+ \frac{1}{\pi^2}\int \mathrm{d}s' \int \mathrm{d}t' \frac{\rho^n_{st}(s', t')}{(s' - s)(t' - t)}$$

- ✔ Number of $G_n(s,t)$:

$$\frac{1}{2}(2S_q + 1)(2S_p + 1)(2S_{\bar{q}} + 1)(2S_{\bar{p}} + 1)$$

- Step-by-step derivation [3]:
  - $T_{0^- \frac{1}{2}^+ \to 0^- \frac{1}{2}^+}$:

$$T_{0^- \frac{1}{2}^+ \to 0^- \frac{1}{2}^+} = \bar{u}(\bar{p}, \bar{\lambda})(G_1(s,t)\;1 + G_2(s,t)\gamma^\mu w_\mu)u(p, \lambda)$$

  - ✔ Freedom from kinematical constraints
  - ✔ On-shell property of baryons
  - $T_{0^- \frac{1}{2}^+ \to 1^- \frac{1}{2}^+}$:
  - ✔ Results of $T_{0^- \frac{1}{2}^+ \to 0^- \frac{1}{2}^+}$ $(\to \mathbf{1}, \gamma^\mu w_\mu)$
  - ✔ Overcomplete basis set, e. g.

$$\gamma^{\bar{\mu}}\; i\,\gamma_5, w^{\bar{\mu}}\; i\,\gamma_5, q^{\bar{\mu}}\; i\,\gamma_5, \epsilon^{\bar{\mu}\bar{\nu}\rho\sigma}\bar{q}_{\bar{\nu}} w_\rho q_\sigma,$$
$$\gamma^{\bar{\mu}}\gamma^\mu w_\mu\; i\,\gamma_5, w^{\bar{\mu}}\gamma^\mu w_\mu\; i\,\gamma_5, q^{\bar{\mu}}\gamma^\mu w_\mu\; i\,\gamma_5, \epsilon^{\bar{\mu}\bar{\nu}\rho\sigma}\bar{q}_{\bar{\nu}} w_\rho q_\sigma \gamma^\nu w_\nu.$$

  - ✔ Chisholm identity, Schouten identity
  - ✔ Selection principles for basis: Minimal number of momenta & no kinematical singularities
  - $T_{1^- \frac{1}{2}^+ \to 1^- \frac{1}{2}^+}$: $T_{1^- \frac{1}{2}^+ \to 1^- \frac{1}{2}^+} = \sum_n \bar{u}(\bar{p}, \bar{\lambda})\epsilon^*_{\bar{\mu}}(\bar{q}, \bar{\alpha})T_n^{\bar{\mu}\mu}\epsilon_\mu(q, \alpha)u(p, \lambda)$
  - ✔ Basis set: Combinations of $g^{\bar{\mu}\mu}, \gamma^\mu, w^\mu, q^\mu, \bar{q}^\mu, \epsilon^{\mu\nu\rho\sigma}$

[1]: J. Gasser, H. Leutwyler, *Annals of Physics 158* (1984), 142-210
[2]: J. Gasser, H. Leutwyler, *Nucl. Phys. B 250* (1985), 465-516
[3]: Y. Heo, M. F. M. Lutz (2014), arxiv.org/abs/arXiv:1405.1597
[4]: A. Gasparyan, M. F. M. Lutz (2010), arxiv.org/abs/arXiv:1003.3426