Track fit with time information

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Track fit with time information

Why, Tracking with time information?

·Advantages:

- More efficient at high frequency: avoids trigger dead time pileup
- No external event trigger detectors needed
- No external time needed in detectors in time-based detectors, as Drift Chambers
- Take full advantage of modern good timing detectors, as timing RPCs, fast scintillators…

·Disadvantages:

- Including one (time) or two (+velocity) parameters more in the particle reconstruction process increases the dimension of the parameter space (from, typically, 4 to 5 or 6) and makes more difficult to find the minima: the curse of dimensionality (the volume of the space grows with the number of parameters and the available data may become sparse).

·Several approaches under development:

- 4D (CBM-FAIR): maximize the number of data, using many very accurate stations + Kalman Filter
- TimTrack (TRASGO-USC): use raw non-reduced data + constraints between the fitted parameters

TimTrack particle reconstruction framework:

1. State vector contains: track parameters (positions, slopes) + time + velocity (slowness)

2. Factorized version of the LSE method

3. Algorithm works directly with non-reduced measured data

4. Unknown parameters and calibration constants may be estimated at the same time

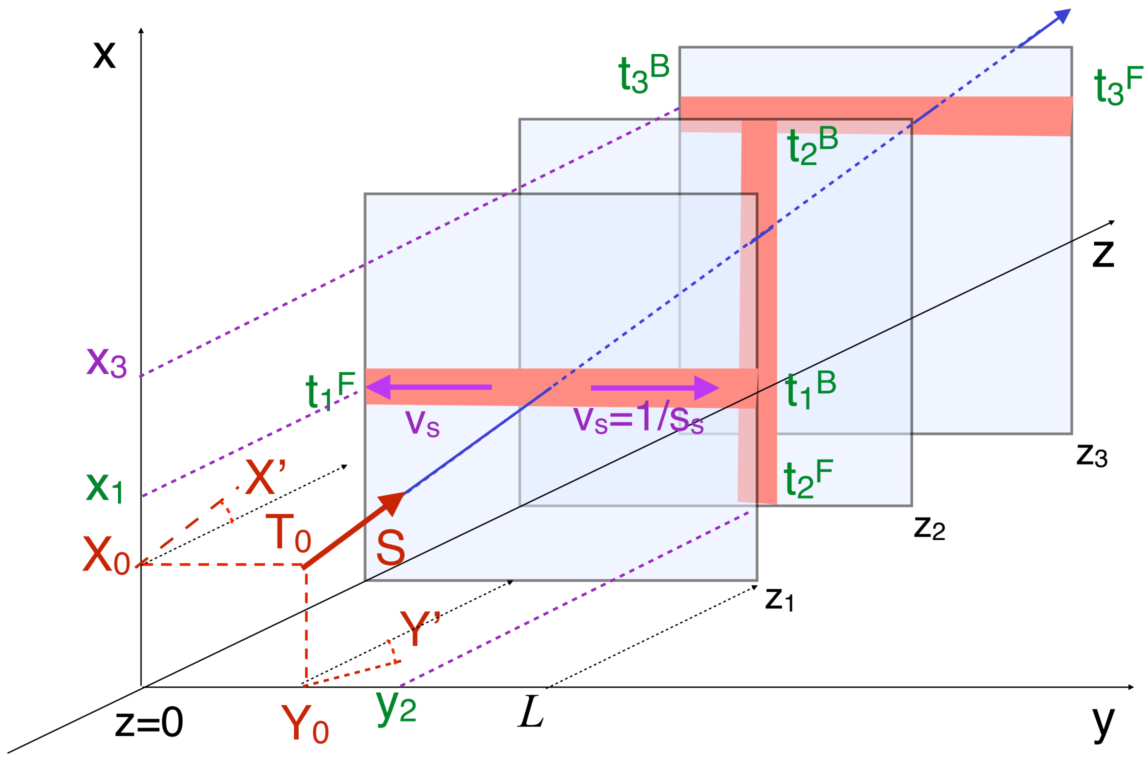
5. Any measured data (momentum, deposited charge,…) can be included in the minimization

6. Known constraints can be included “a priori” in the fit

7. The same state vector is good for any kind of detector and hybrid layouts

TimTrack (static approach):

1. State vector contains: track parameters (positions, slopes) + time + velocity (slowness)



$\mathbf{s} = (X_0, X', Y_0, Y', T_0, S)$; with $S = 1/V$ State vector

$\mathbf{s}_M = (X_0, X', Y_0, Y', T_0, S; M)$; with mass hypothesis Extended state vector

TimTrack (static approach):

2. Factorized version of the LSE method

Linear Least Squares Method:

The Least Squares Estimator: $S(\mathbf{s}) = [\mathbf{d}\text{-}\mathbf{m}(\mathbf{s})]' \cdot V^{-1} \cdot [\mathbf{d}\text{-}\mathbf{m}(\mathbf{s})]'$

- **d**: set of n_m measured data
- V: covariance $n_m \cdot n_m$ matrix of measured data $\Rightarrow$ $W = V^{-1}$ (weight matrix)
- **s**: set of n_s parameters

Linear Measurement model:

- $\mathbf{m}(\mathbf{s}) = \mathbf{g}_0 + G\,\mathbf{s}$: ($n_m$ equations)

The Linear Least Squares Estimator: $S(\mathbf{s}) = (G' \cdot W \cdot G)^{-1} \cdot G' \cdot W \cdot (\mathbf{d}\text{-}\mathbf{g}_0)$

with

$K = G' \cdot W \cdot G$ [dim: n_s x n_s] (Configuration matrix)

$\mathbf{a} = G' \cdot W \cdot (\mathbf{d}\text{-}\mathbf{g}_0)$ [dim: n_s x 1] (Vector of reduced data)

Then $\mathbf{a} = K \cdot \mathbf{s}$ (Normal equations)

Solution: $\boxed{\mathbf{s} = K^{-1} \cdot \mathbf{a} = \mathcal{E} \cdot \mathbf{a}}$ $\mathcal{E}$: error matrix

2. Factorized version of the LSE method

Non Linear Least Squares Method (iterative Newton-Raphson method):

- $\mathbf{m}(\mathbf{s}) = \mathbf{g}(\mathbf{s}) + G(\mathbf{s})\cdot\mathbf{s}$: Measurement model ($n_m$ equations), with:

  $G(\mathbf{s}) = \partial\, \mathbf{m}(\mathbf{s}) / \partial \mathbf{s}$

- Near the minimum, at $\mathbf{s_0} \simeq \mathbf{s}$:

  $\mathbf{m}(\mathbf{s}) \simeq \mathbf{g}(\mathbf{s}_0) + G(\mathbf{s}_0)\cdot\mathbf{s}$

The Least Squares Estimator: $S(\mathbf{s}) \simeq (G(\mathbf{s}_0)'\cdot W\cdot G(\mathbf{s}_0))^{-1}\cdot G(\mathbf{s}_0)'\cdot W\cdot(\mathbf{d}-\mathbf{g}(\mathbf{s}_0))$

with:

$K_0 = G'(\mathbf{s}_0)\cdot W\cdot G(\mathbf{s}_0)$

$\mathbf{a}_0 = G'(\mathbf{s}_0)\cdot W\cdot(\mathbf{d}-\mathbf{g}(\mathbf{s}_0))$

Solution: $\boxed{\mathbf{s}_1 = K_0^{-1}\cdot \mathbf{a}_0}$

Iteratively: $\boxed{\mathbf{s}_i = K_{i-1}^{-1}\cdot \mathbf{a}_{i-1}}$ $\qquad \mathcal{E} = K_i^{-1}$: error matrix

until convergence.

2. Factorized version of the LSE method

Non Linear Least Squares Method with Constraints:

- $\mathbf{f}(\mathbf{s}) = 0$: set of n_c constraints (let suppose, $n_c=1$).
- $R = \partial\, \mathbf{m}(\mathbf{s}) / \partial \mathbf{s}$: Jacobian matrix of constraint functions
- $L(\mathbf{s}) = [\mathbf{d}-\mathbf{m}(\mathbf{s})]' \cdot W \cdot [\mathbf{d}-\mathbf{m}(\mathbf{s})]' + 2\lambda \cdot \mathbf{f}(\mathbf{s})$
- Near the minimum, at $\mathbf{s_0} \simeq \mathbf{s}$, the iterative process becomes:

$$\begin{pmatrix} \delta \mathbf{s}_1 \\ \lambda_1 \end{pmatrix} = \begin{pmatrix} K_0 & R'_0 \\ R_0 & 0 \end{pmatrix}^{-1} \cdot \begin{pmatrix} \mathbf{a}_0(\mathbf{s}_0) - K_0 \cdot \mathbf{s}_0 \\ -\,\mathbf{f}_0 \end{pmatrix}$$

- The next step solution and Lagrange multiplier are:

$$\mathbf{s}_1 = \mathbf{s}_0 + \delta \mathbf{s}_0$$

$$\lambda_1$$

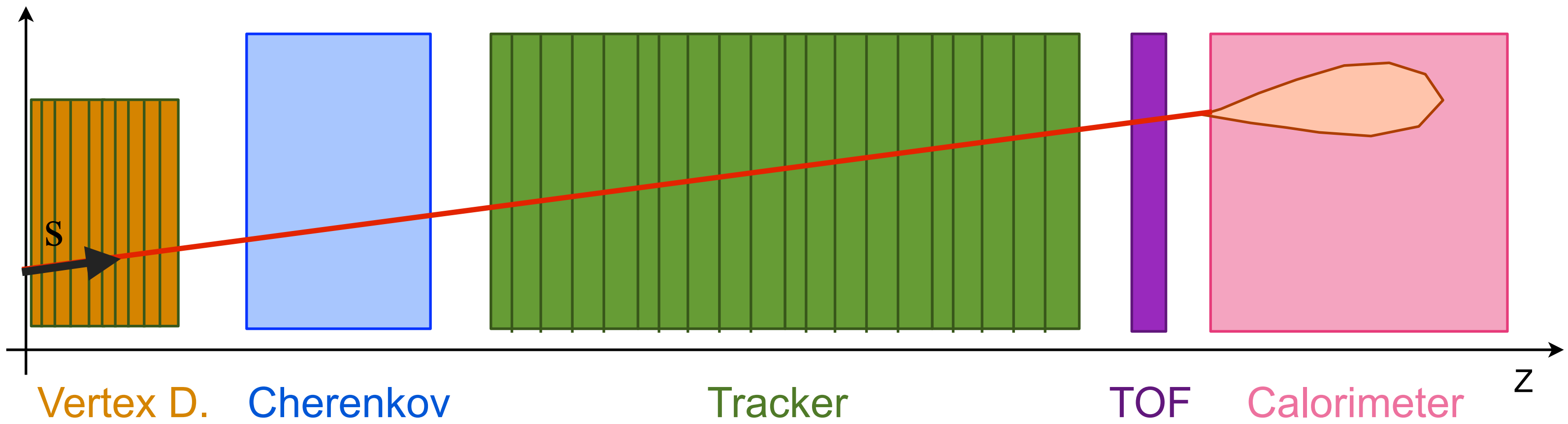
and so on, until convergence:

$$\boxed{\begin{array}{l} \mathbf{s}_i = \mathbf{s}_{i-1} + \delta \mathbf{s}_{i-1} \\ \lambda_i \simeq 0 \end{array}}$$

2. Factorized version of the LSE method

Hybrid set-ups:

If we have a set of different detectors (different models):



models:	$\mathbf{m}_1$	$\mathbf{m}_2$	$\mathbf{m}_3$	$\mathbf{m}_4$	$\mathbf{m}_5$
data (f.ex):	(x,y)	(S=1/V)	(x, x',y, y')	(t)	(E)
K matrices:	K_1	K_2	K_3	K_4	K_5
a vectors:	$\mathbf{a}_1$	$\mathbf{a}_2$	$\mathbf{a}_3$	$\mathbf{a}_4$	$\mathbf{a}_5$

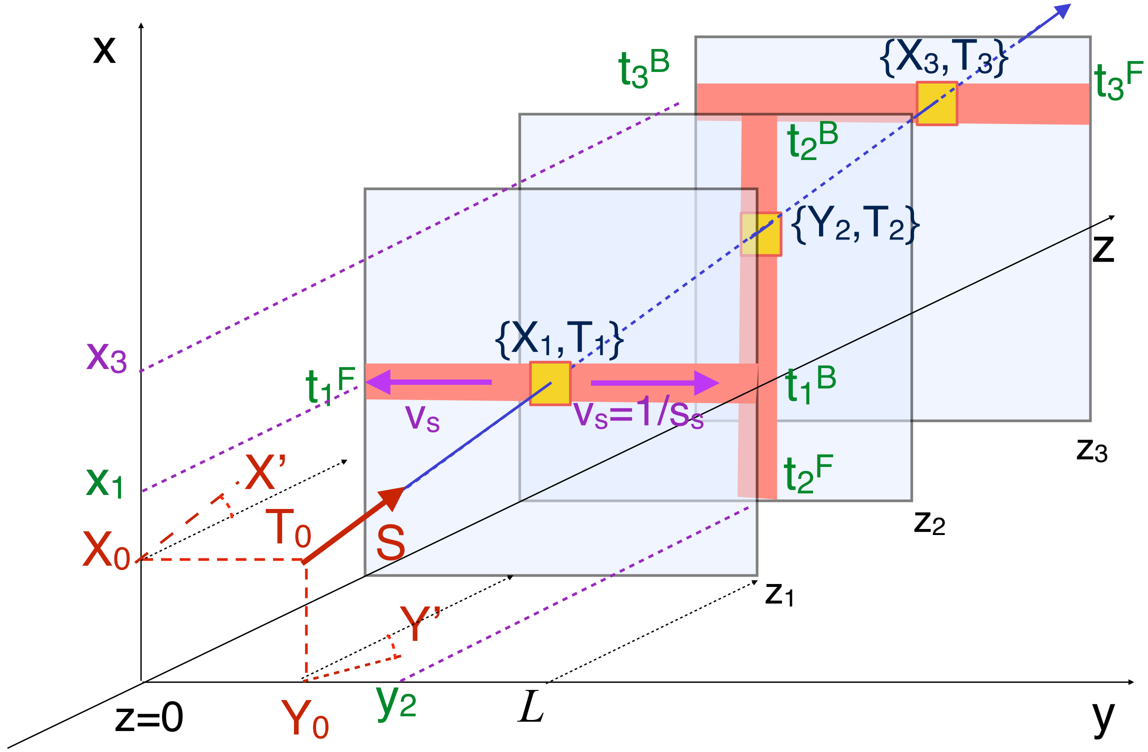
and:

$$\boxed{\begin{array}{l} K = \sum K_i \\ \mathbf{a} = \sum \mathbf{a}_i \end{array} \rightarrow \mathcal{E} = K^{-1} \rightarrow \mathbf{s} = \mathcal{E}\cdot\mathbf{a}}$$

(M. scattering effects can be introduced in the corresponding K_i matrices)

TimTrack (static approach):

3. Algorithm works directly with non-reduced measured data



Typical Fitting model:

$x_i = X_0 + X' \cdot z_i$

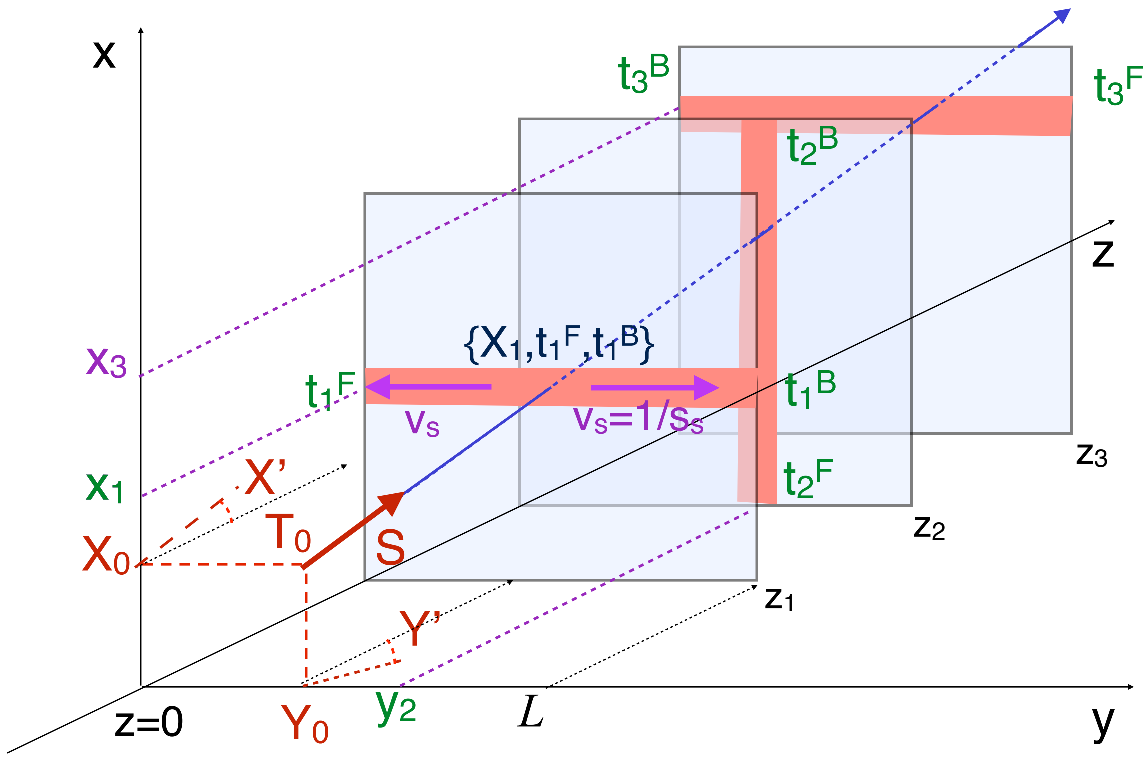
$y_i = Y_0 + Y' \cdot z_i$

$$t_i = T_0 + z_i \cdot \frac{1}{V}\sqrt{1 + X'^2 + Y'^2}$$

3 planes ⇒ 6 measurements ⇒ 6 meas. - 6 param. = 0 d.o.f.

TimTrack (static approach):

3. Algorithm works directly with non-reduced measured data



TT Fitting model (for X-type planes):

$$x_i = X_0 + X' \cdot z_i$$

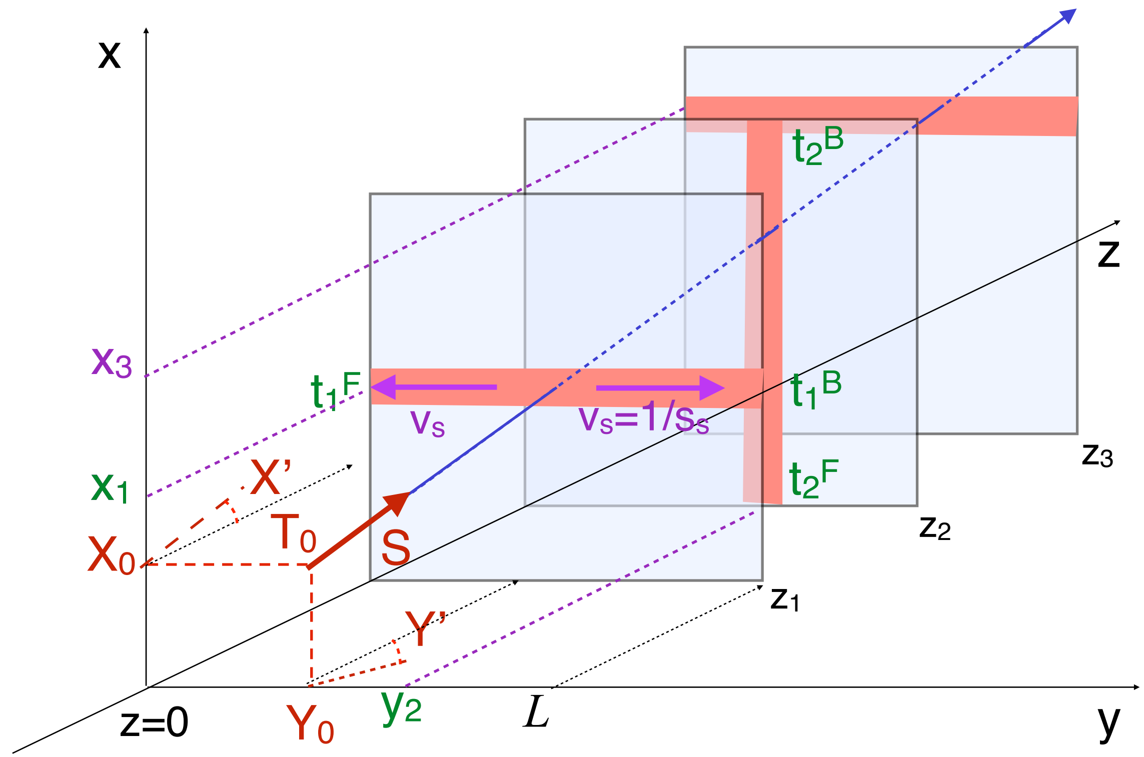
$$t_i^F = T_0 + z_i \cdot \sqrt{1 + X'^2 + Y'^2} \cdot S + (Y_0 + Y' \cdot z_i) \cdot s_s$$

$$t_i^B = T_0 + z_i \cdot \sqrt{1 + X'^2 + Y'^2} \cdot S + (L - (Y_0 + Y' \cdot z_i)) \cdot s_s$$

3 planes ⇒ 9 measurements ⇒ 9 meas. - 6 param. = 3 d.o.f.

TimTrack (static approach):

3. Algorithm works directly with non-reduced measured data

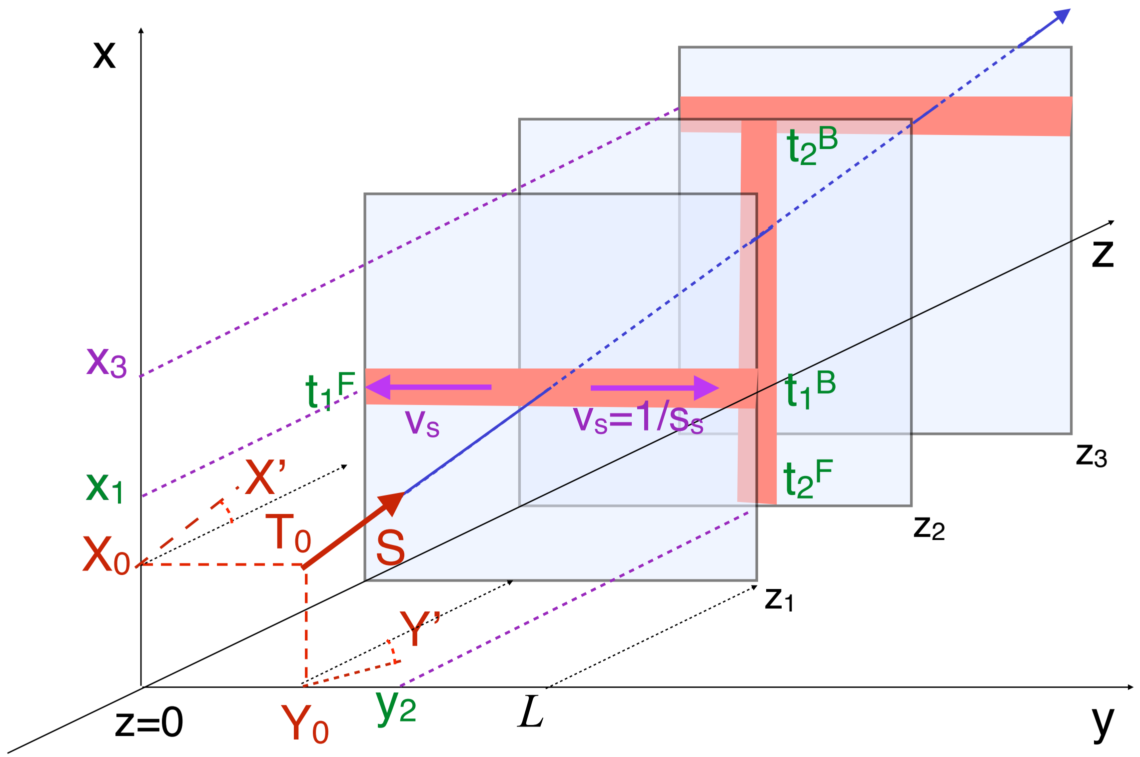


TT Fitting model (for X-type planes):

$$\begin{pmatrix} x_i \\ t_i^F \\ t_i^B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \mathrm{L} \end{pmatrix} + \begin{pmatrix} 1 & z_i & 0 & 0 & 0 & 0 \\ 0 & 0 & s_s & z_i s_s & 1 & \sqrt{1+X'^2+Y'^2} \\ 0 & 0 & \text{-}s_s & \text{-}z_i s_s & 1 & \sqrt{1+X'^2+Y'^2} \end{pmatrix} \cdot (X_0, X', Y_0, Y', T_0, S)$$

TimTrack (static approach):

4. Unknown parameters and calibration constant may be estimated at the same time



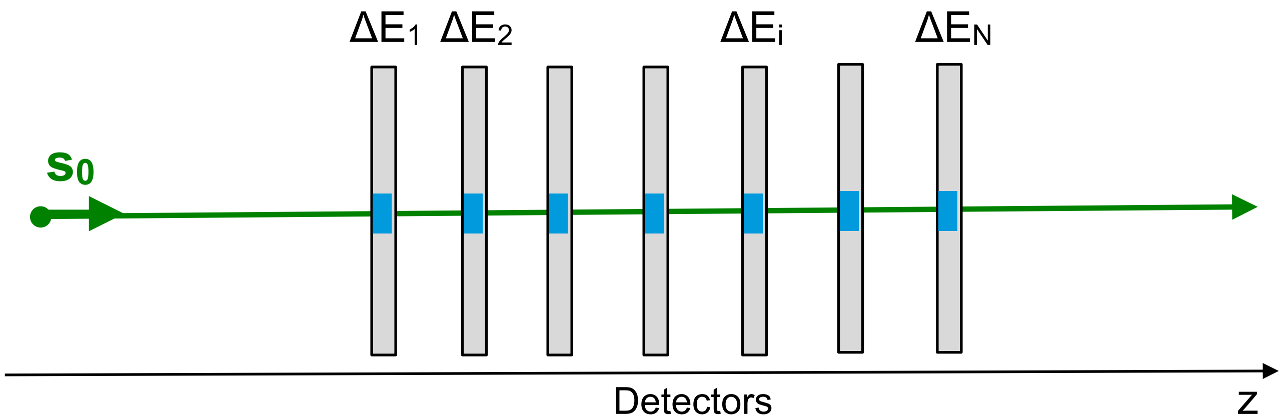
TT Fitting model with free calibration constants:

$$\begin{pmatrix} x_i \\ t_i^F \\ t_i^B \end{pmatrix} = \begin{pmatrix} 0 \\ T_0 + \sqrt{1+X'^2+Y'^2}\cdot S \\ T_0 + \sqrt{1+X'^2+Y'^2}\cdot S \end{pmatrix} + \begin{pmatrix} 1 & z_i & 0 & 0 & 0 & 0 \\ 0 & 0 & s_s & z_i s_s & 0 & 0 \\ 0 & 0 & -s_s & -z_i s_s & 1 & 0 \end{pmatrix} \cdot (X_0, X', Y_0, Y'; \boxed{L, s_s})$$

Here we assume that T_0 and S are known and we want to estimate L and s_S

5. Any measured data (momentum, deposited charge…) can be included in the minimization

Example: Bethe Bloch energy loss



Energy loss is related to the velocity β of a particle through the Bethe- Bloch formula:

$$-\frac{dE}{dx} \simeq k \cdot \frac{1}{\beta^2}\left(\frac{1}{2}\ln\frac{2m_e c^2\beta^2\gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right)$$

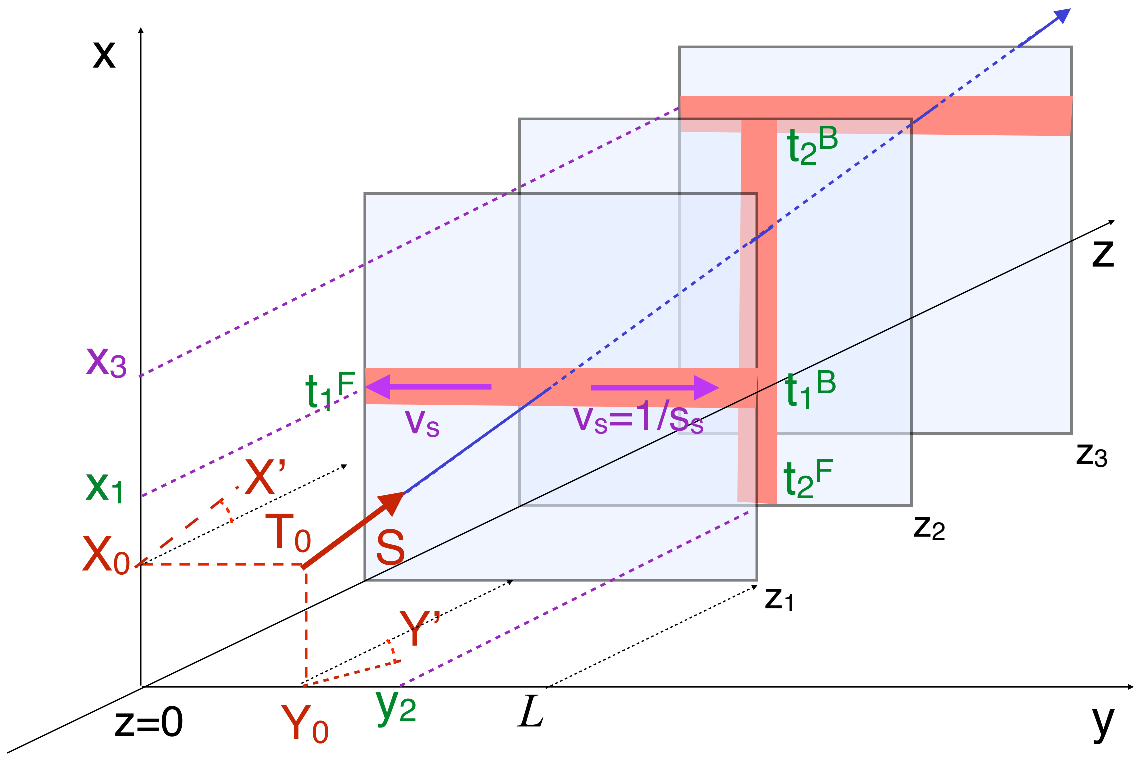
that can be written, as a function of $S = 1/\beta c$, in a simplified form as:

$$\boxed{-\frac{dE}{dx} \simeq k' \cdot \left(S^2 \cdot \ln\frac{1}{I_c \cdot \sqrt{S^2 - S_c^2}} - S_c^2 - \delta'\right)}$$

with: $S_c = \frac{1}{c^2}$ and $I_c = \frac{I}{\sqrt{2 \cdot m_e \cdot T_{max}}}$

TimTrack (static approach):

5. Any measured data (momentum, deposited charge,…) can be included in the minimization



TT Fitting model, including measured deposited charge:

$$x_i = X_0 + X' \cdot z_i$$

$$t_i^F = T_0 + z_i \cdot \sqrt{1 + X'^2 + Y'^2} \cdot S + (Y_0 + Y' \cdot z_i) \cdot s_s$$

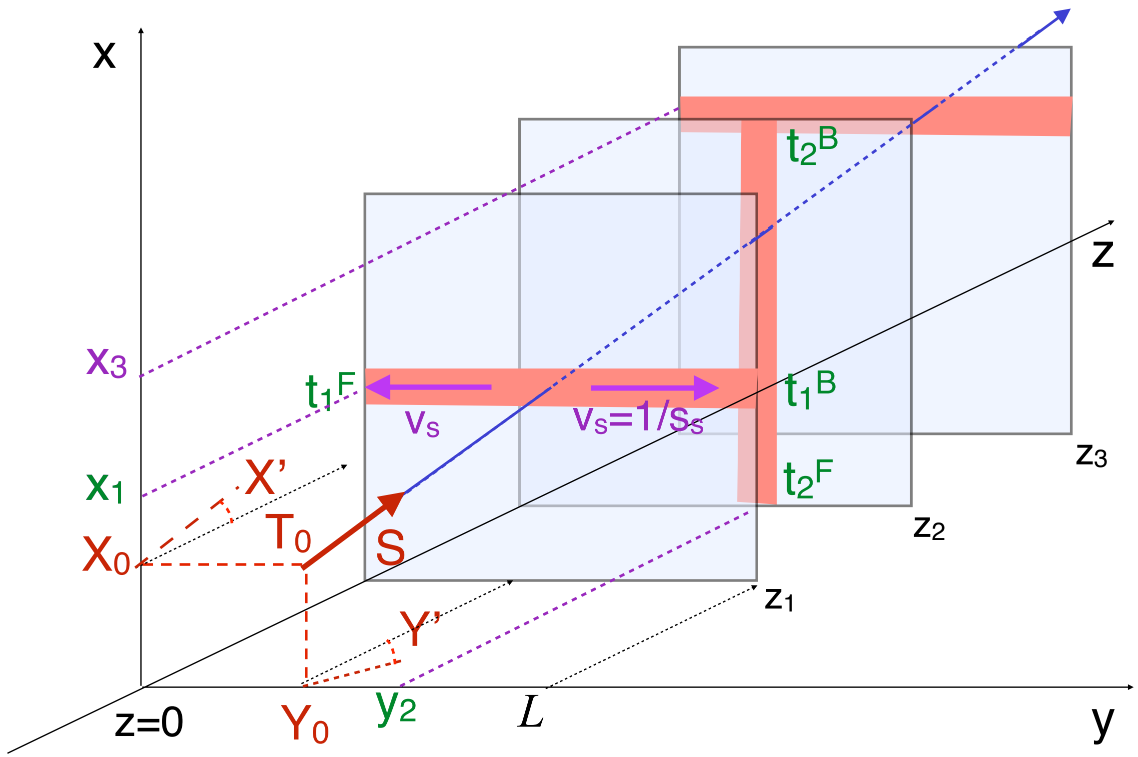
$$t_i^B = T_0 + z_i \cdot \sqrt{1 + X'^2 + Y'^2} \cdot S + (L - (Y_0 + Y' \cdot z_i)) \cdot s_s$$

$$Q_i \simeq k' \cdot \left(S^2 \cdot \ln \frac{1}{I_c \cdot \sqrt{S^2 - S_c^2}} - S_c^2 - \delta' \right) \cdot dz_i$$

3 planes ⇒ 3·4 = 12 measurements ⇒ 12 meas. - 6 param. = 6 d.o.f.

TimTrack (static approach):

6. Known constraints can be included "a priori" in the fit



TT Fitting model, including vertex condition at $z = z_v$:

$$x_i = X_0 + X' \cdot z_i$$

$$t_i^F = T_0 + z_i \cdot \sqrt{1 + X'^2 + Y'^2} \cdot S + (Y_0 + Y' \cdot z_i) \cdot s_s$$

$$t_i^B = T_0 + z_i \cdot \sqrt{1 + X'^2 + Y'^2} \cdot S + (L - (Y_0 + Y' \cdot z_i)) \cdot s_s$$

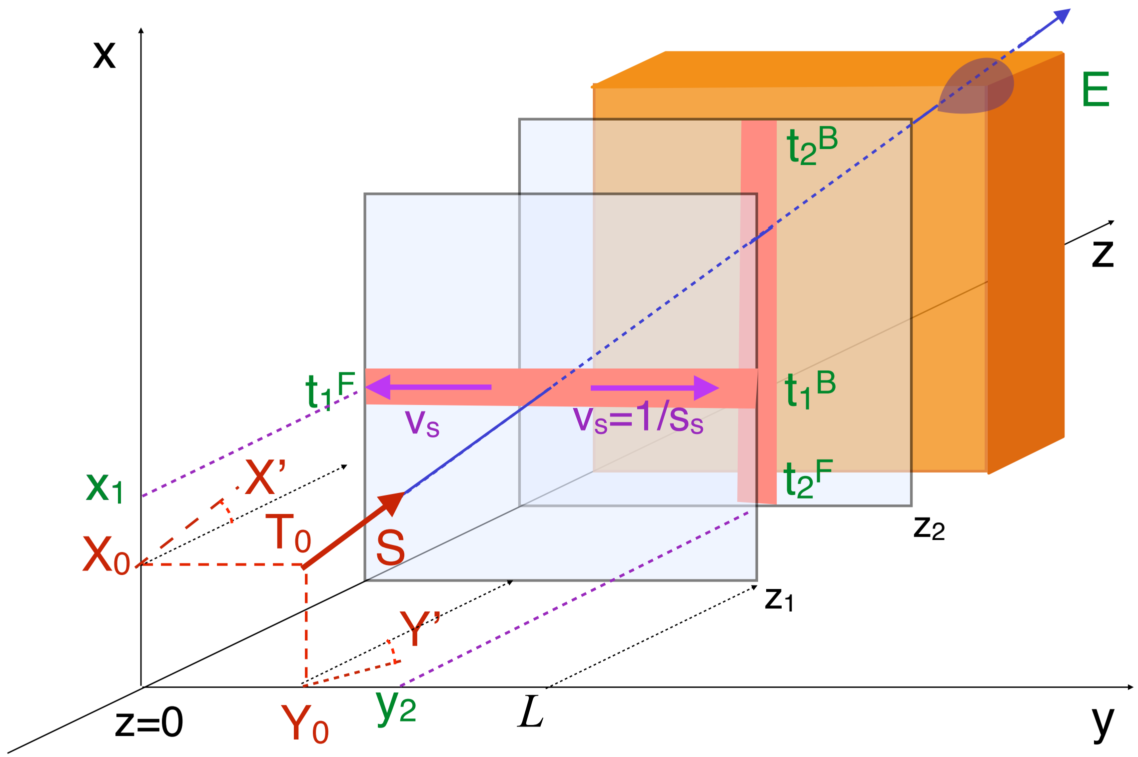
$$X_0 - X' \cdot (z - z_v) = 0$$

$$Y_0 - Y' \cdot (z - z_v) = 0$$

3 planes ⇒ 9 measurements ⇒ 9 meas. - 6 param. - 2 constr. = 1 d.o.f.

TimTrack (static approach):

7. The same state vector is good for any kind of detector and hybrid layouts



Fitting model for 2 strip planes and a calorimeter, assuming a mass hypothesis, M:

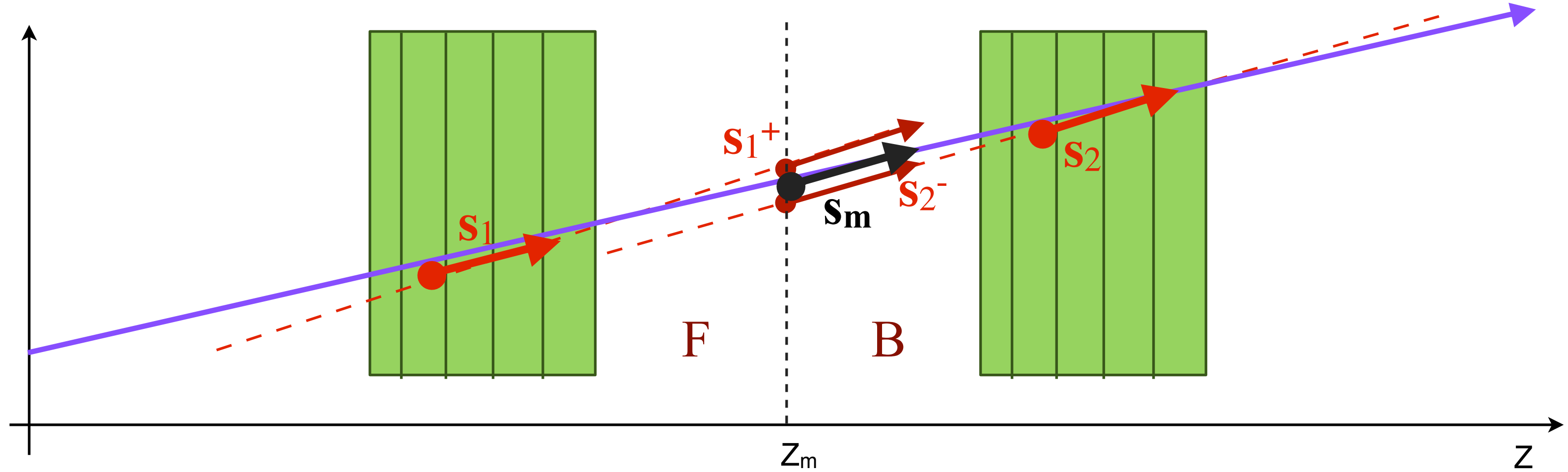
$$x_i = X_0 + X' \cdot z_i$$
$$t_i^F = T_0 + z_i \cdot \sqrt{1 + X'^2 + Y'^2} \cdot S + (Y_0 + Y' \cdot z_i) \cdot s_s$$
$$t_i^B = T_0 + z_i \cdot \sqrt{1 + X'^2 + Y'^2} \cdot S + (L - (Y_0 + Y' \cdot z_i)) \cdot s_s$$
$$E_i = \frac{M \cdot S}{\sqrt{S^2 - S_c^2}}$$

2 planes + Cal. $\Rightarrow$ 7 measurements $\Rightarrow$ 7 meas. - 6 param. = 1 d.o.f.

TimTrack with Transport of the state vector:

Some algebra of state vectors

Algebra basics



- Compatibility (Mahalanobis distance)

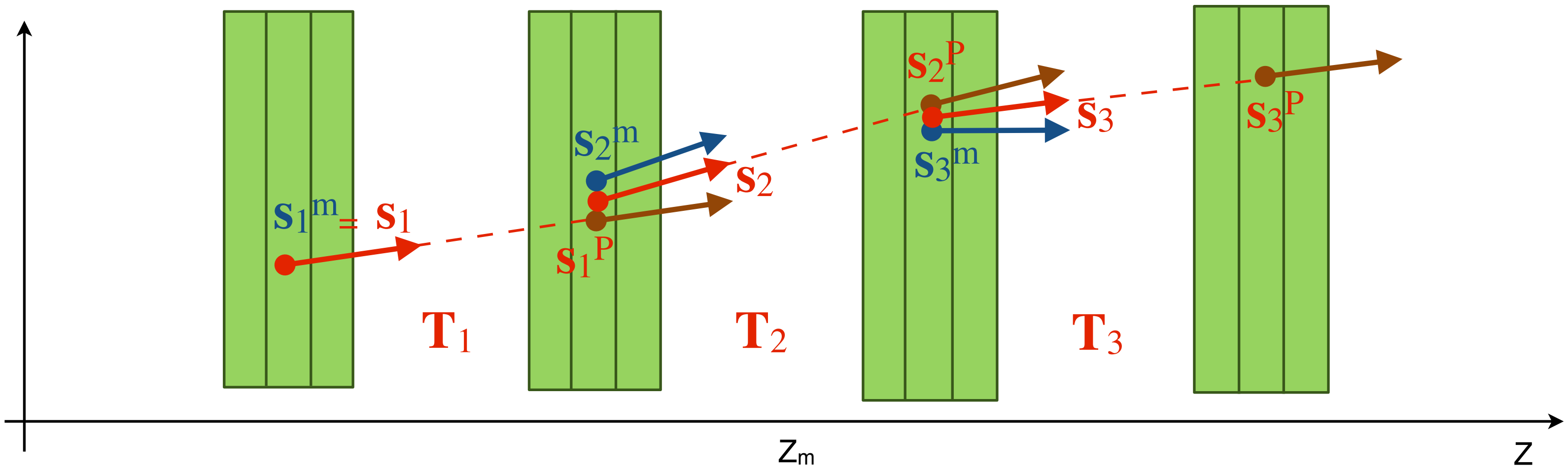
$$(\mathbf{s}_1, \mathcal{E}_1) \rightarrow (\mathbf{s}_1^+ = F\cdot\mathbf{s}_1,\ \mathcal{E}_1^+ = F'\cdot\mathcal{E}_1\cdot F)$$
$$(\mathbf{s}_2, \mathcal{E}_2) \rightarrow (\mathbf{s}_2^- = B\cdot\mathbf{s}_2,\ \mathcal{E}_2^- = B'\cdot\mathcal{E}_2\cdot B)$$

$$\longrightarrow \quad d_M(\mathbf{s}_1^+,\mathbf{s}_2^-) = \sqrt{(\mathbf{s}_1^+-\mathbf{s}_2^-)'\cdot(\mathcal{E}_1^+ + \mathcal{E}_2^-)\cdot(\mathbf{s}_1^+-\mathbf{s}_2^-)}$$

- Reduction of state vectors:

$$\mathbf{s}_\mathbf{m} = [(\mathcal{E}_1^+)^{-1} + (\mathcal{E}_2^-)^{-1}]^{-1}\cdot[(\mathcal{E}_1^+)^{-1}\cdot\mathbf{s}_1^+ + (\mathcal{E}_2^-)^{-1}\cdot\mathbf{s}_2^-]$$

TimTrack with Transport of the state vector:



Transport of the state vector: $\mathbf{s}_1^P = T_1.s_1$

Transport of the error matrix: $\mathcal{E}_1^P = T_1.\mathcal{E}_1.T_1'$

Transport of the reduced vector of data: $\boldsymbol{\alpha}_1^P = (T_1')^{-1}.\boldsymbol{\alpha}1$

Reduced vector of new measured data: $\boldsymbol{\alpha}_2^m = G_2.W_2.(\mathbf{d}_2\text{-}\mathbf{g}_2)$

New state vector: $\mathbf{s}_2 = [(\mathcal{E}_1^P)^{-1}+ G_2'.W.\ G_2]^{-1}\ (\boldsymbol{\alpha}_1^P + \boldsymbol{\alpha}_2^m)$

Last equation has a, usually, very complicated matrix inversion. Applying the Identity of Woodbury:

$$\boxed{\mathbf{s}_2 = [I-(\mathcal{E}_1^P).G_2'.(V_2+G_2.\mathcal{E}_1^P.G_2')^{-1}.G_2]\ (\mathbf{s}_1^P + (\mathcal{E}_1^P.\boldsymbol{\alpha}_2^m))}$$

This solution is very similar to the one provided by the Kalman Filter!

TimTrack vs Kalman Filter

TimTrack			Kalman Filter		
Parameter space		Measurement space	Parameter space		Measurement space
$\mathbf{s}_p, \mathcal{E}_p$			$\mathbf{r}_p, \mathcal{E}_p$		
	F: Transport			F: Transport	
$\mathbf{s} = F \cdot \mathbf{s}_p$ $\mathcal{E}_s = F \cdot \mathcal{E}_p \cdot F'$		$\mathbf{d}$, $W_d = V_d^{-1}$	$\mathbf{r} = F \cdot \mathbf{r}_p$ $\mathcal{E}_r = F \cdot \mathcal{E}_p \cdot F'$		$\mathbf{d}$, V_d
	G: Measurement $\mathbf{m}(\mathbf{s}) = G \cdot \mathbf{s} + \mathbf{g}(\mathbf{s})$	$V_s = G \cdot \mathcal{E}_s \cdot G'$		H: Measurement $\mathbf{m}(\mathbf{r}) = H \cdot \mathbf{r} + \boldsymbol{\eta}$	$\mathbf{d}_r = H \cdot \mathbf{r}$ $V_r = H \cdot \mathcal{E}_r \cdot H'$
		$\mathbf{d}_c = \mathbf{d}\text{-}\mathbf{g}(\mathbf{s})$ $V_c = V_d + V_s$			$\delta\mathbf{d} = \mathbf{d}\text{-}\mathbf{d}_r$ $V_c = V_r + V_d$
$\mathbf{s}_d = C \cdot \mathbf{d}_c$ $\delta\mathcal{E}_s = \mathcal{E}_s \cdot W_s \cdot \mathcal{E}_s$	$C = \mathcal{E}_s \cdot G' \cdot V_c^{-1}$ $W_s = G' \cdot V_c^{-1} \cdot G$		$\delta\mathbf{r} = K \cdot \delta\mathbf{d}$ $\delta\mathcal{E}_r = \mathcal{E}_r \cdot W_r \cdot \mathcal{E}_r$	$K = \mathcal{E}_r \cdot H' \cdot V_c^{-1}$ $W_r = H' \cdot V_c^{-1} \cdot H$	
$\mathbf{s}_{p+1} = (I - \mathcal{E}_s \cdot W_s) \cdot (\mathbf{s} + \mathbf{s}_d)$ $\mathcal{E}_{p+1} = \mathcal{E}_s - \delta\mathcal{E}_s$			$\mathbf{r}_{p+1} = \mathbf{r} + \delta\mathbf{r}$ $\mathcal{E}_{p+1} = \mathcal{E}_r - \delta\mathcal{E}_r$		

TT works mainly in the Parameter space

KF works in both the Measurement and Parameter spaces

TimTrack: example

HADES MDCs (Mini Drift Chambers)

In a drift chamber, each wire gives:

1. The wire coordinate
2. A time = t.o.f. + drift time + pulse time
3. dE/dx (not used)

Typical fitting:

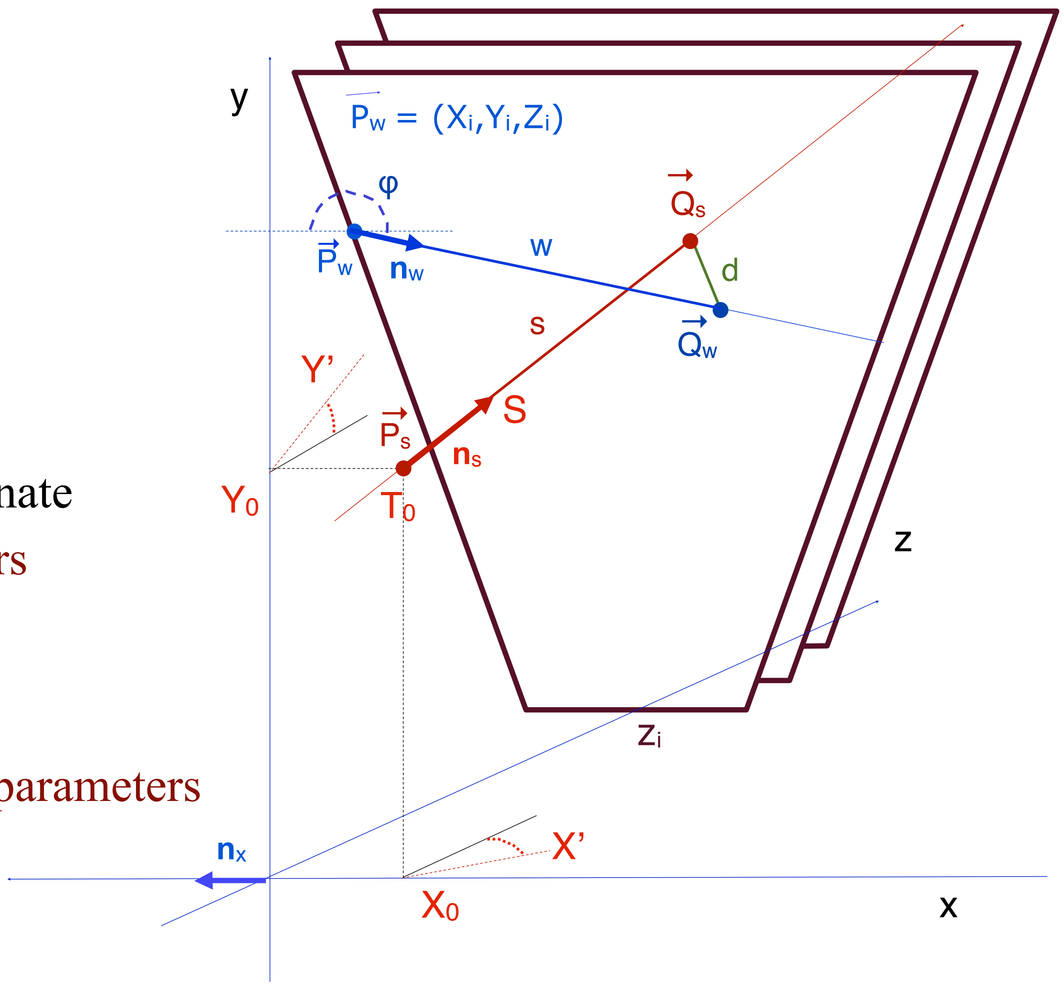
Time + wire coordinate are reduced to one coordinate

1 plane ~ 1 coordinate ⇒ 4 planes for 4 parameters

TT fitting:

Time and wire coordinates are kept independent

1 plane ~ 1 coordinate + 1 time ⇒ 3 planes for 6 parameters



HADES MDCs (Mini Drift Chambers)

Measured time model:

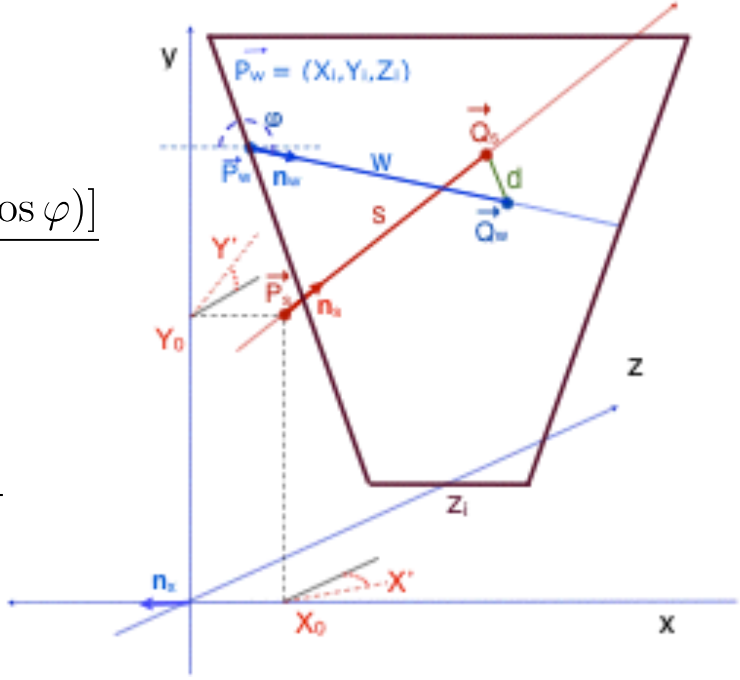
$$s = \frac{z_i \cdot \sqrt{X'^2 + Y'^2 + 1} \cdot [1 - (-X' \sin\varphi + Y' \cos\varphi)(-X'_i \sin\varphi + Y'_i \cos\varphi)]}{1 + (-X' \sin\varphi + Y' \cos\varphi)^2}$$

$$d = \frac{z_i \cdot [-(X' + X'_i) \sin\varphi + (Y' + Y'_i) \cos\varphi]}{\sqrt{1 + (-X' \sin\varphi + Y' \cos\varphi)^2}}$$

$$w = \frac{z_i \cdot [-(X' + X'_i) \cos\varphi + (Y' + Y'_i) \sin\varphi - (-X' \sin\varphi + Y' \cos\varphi)(X' Y'_i - X'_i Y')]}{1 + (-X' \sin\varphi + Y' \cos\varphi)^2}$$

where:

$$X'_i = \frac{X_0 - X_i}{z_i} \qquad Y'_i = \frac{Y_0 - Y_i}{z_i}$$



Finally, the time t measured in a wire is:

$$\boxed{t = T_0 + \frac{s}{V} + \frac{d}{v_d} + \frac{w}{v_w}}$$

$$\boxed{t = T_0 + s \cdot S + d \cdot s_d + w \cdot s_w}$$

(Note: no left-right effect!)

TimTrack: example

HADES MDCs (Mini Drift Chambers)

Standard Runge-Kutta method

TimTrack method

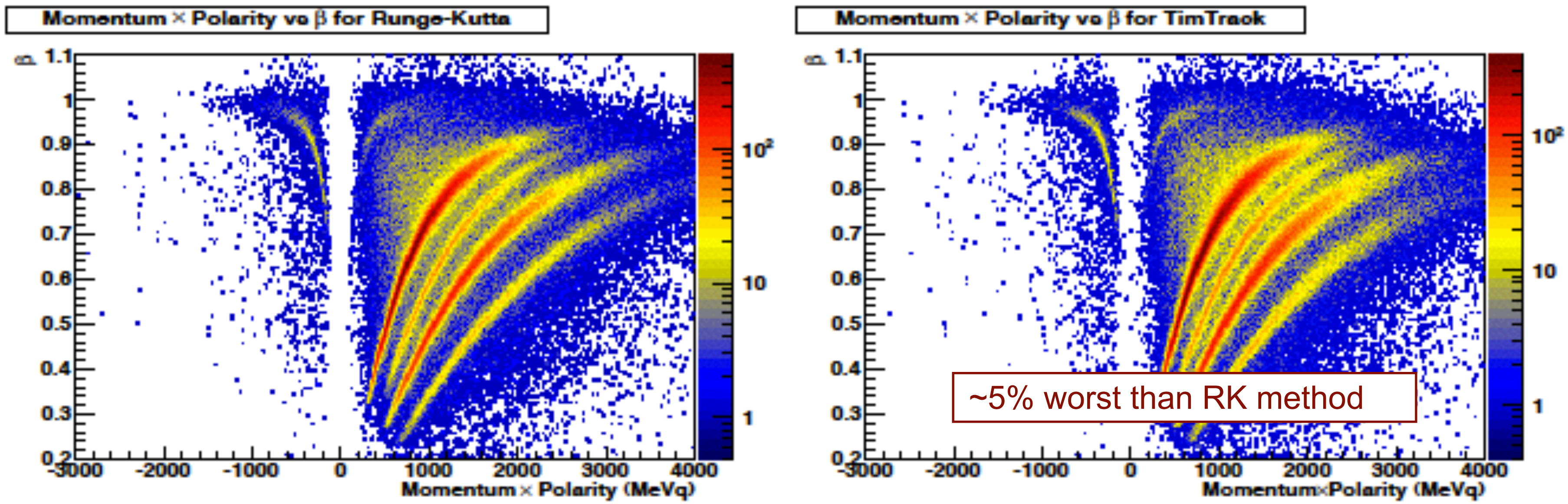


Figure 5.18: Comparative between distributions of β vs momentum for Tim-Track and Runge-Kutta.

[Ph.D. thesis of Georgy Kornakov. U.S. Compostela 2012]

Conclusions

- The time may become a very important tracking parameter.
- Using the time as a fit parameter allows to run detectors independently of external triggers or external conditions allowing to run experiments in a continuous mode.
- Working separately with time and coordinates do offer important advantages mainly in hybrid environments where the same state vector is used for different detectors.
- We have shown a real case, the MDCs of the HADES experiment at GSI, where a tracking with time has been successfully tested providing results comparable with other well tested methods.
- Introducing the time, and the velocity, as tracking parameters increases the dimension of the space of parameters, making more difficult to find the minimum. Special strategies and tools should be developed.

Thanks!

The end