On-line extraction of model parameters

D. Anchishkin, I. Kisel, V. Vovchenko

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Introduction

Main task: Extract parameters of theoretical models on-line from measured data

Usual procedure

Theoretical model $(T, V, \sigma_{NN}, ...) \Rightarrow \omega_p \frac{dN}{d^3p}$ (observable)

Our idea: solve the inverse problem on-line

Inverse problem

Observable $\omega_p \frac{dN}{d^3p} \Rightarrow$ model parameters $(T, V, \sigma_{NN}, ...)$

Thermal momentum distribution

Simplest version of thermal model:

- Static fireball of volume V
- Boltzmann-Gibbs distribution: $f = \exp[(-\omega_p + \mu)/T]$.

Momentum spectrum

$$\omega_p \frac{dN}{d^3p} = \frac{gV}{(2\pi)^3}\,\omega_p\,\exp[(-\omega_p + \mu)/T]$$

Cleymans, Redlich, PRC 60, 054908 (1999)

$$\omega_p \equiv p^0 = \sqrt{m^2 + p^2}$$

Thermal distribution: properties

Momentum spectrum in collider c.m.s.

$$\omega_p \frac{dN}{d^3p} = \frac{gV}{(2\pi)^3}\, \omega_p \exp[(-\omega_p + \mu)/T]$$

In terms of rapidity and transverse mass

Momentum spectrum in y and m_T for $\mu = 0$ (pions)

$$\frac{dN}{m_T dm_T dy d\varphi} = \frac{gV}{(2\pi)^3}\, m_T \cosh y \exp\left[-\frac{m_T \cosh y}{T}\right]$$

m_T scaling is present

Total number of particles

$$N = \frac{gV}{2\pi^2}\, T m^2\, \mathrm{K}_2(m/T)$$

Temperature extraction

How to extract model parameters from experiment?

Relation between $\langle m_T \rangle$ and T

Lab frame, 4π acceptance:

$$\langle m_T \rangle_{4\pi} = \frac{\int dy \int_0^\infty dp_t \, p_t \, m_T \, \cosh(y - y_{c.m.}) \, e^{-m_T \cosh(y - y_{c.m.})/T} \, m_T}{\int dy \int_0^\infty dp_t \, p_t \, m_T \cosh(y - y_{c.m.}) \, e^{-m_T \cosh(y - y_{c.m.})/T}}$$

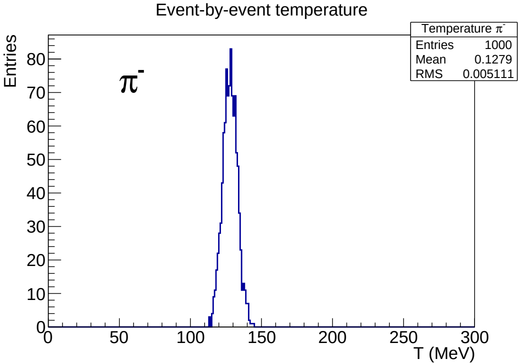
$y_{c.m.}$ – mid-rapidity, $m_T = \sqrt{p_t^2 + m^2}$

Solving the equation for T at given measurable $\langle m_T \rangle$ provides solution for inverse problem

Test calculation

Test calculation for 4π acceptance

- 1000 events with thermal generator of pions (300 π^- per event)
- Use MC tracks with particle identification
- $\langle m_T \rangle$ is determined event-by-event and also from set of events
- Equation for T is solved using bisection method



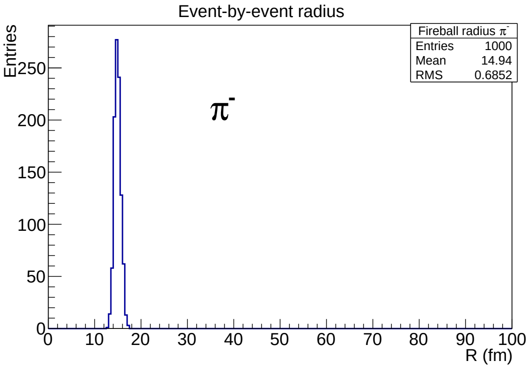
Theory: $T = 128\ MeV$, Extracted: $T = (127.9 \pm 0.2(stat.))\ MeV$

Freeze-out volume

Relation between volume and multiplicity

$$N_{4\pi} = \frac{V}{2\pi^2} T m^2 \mathrm{K}_2(m/T) \quad \Rightarrow \quad V = \frac{2\pi^2 N_{4\pi}}{T m^2 \mathrm{K}_2(m/T)}$$

$$R = \left(\frac{3V}{4\pi}\right)^{1/3}$$

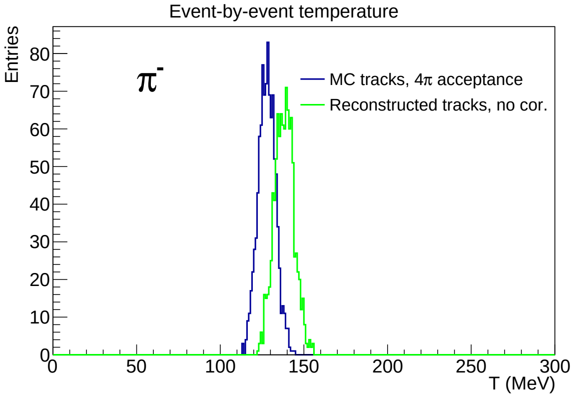


Theory: $R = 14.91$ fm, Extracted: $R = (14.91 \pm 0.02)$ fm

Limitations for reconstructed tracks

We can only work with tracks within acceptance and which have limited momentum accuracy

Using reconstructed STS tracks with $\chi^2_{prim} < 3$ and MC particle ID



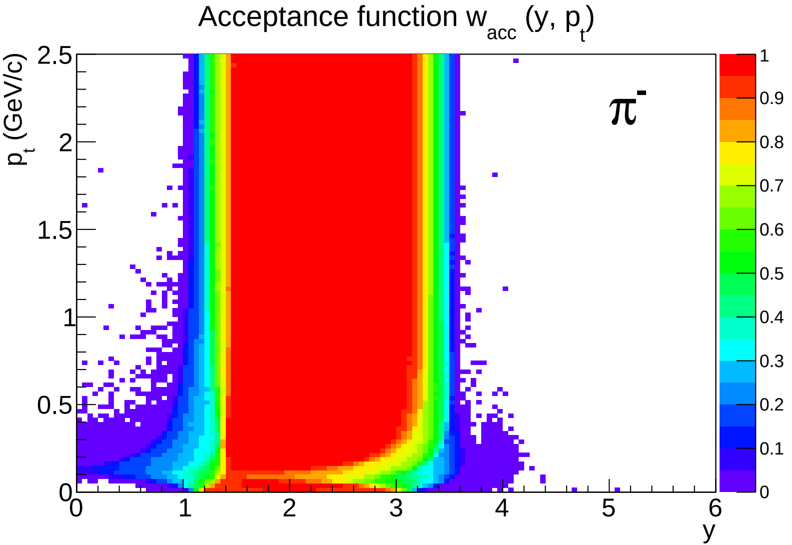
4π acceptance, MC Tracks: $T = (127.9 \pm 0.2)\ MeV$

Reconstructed tracks: $T = (138.2 \pm 0.2)\ MeV$

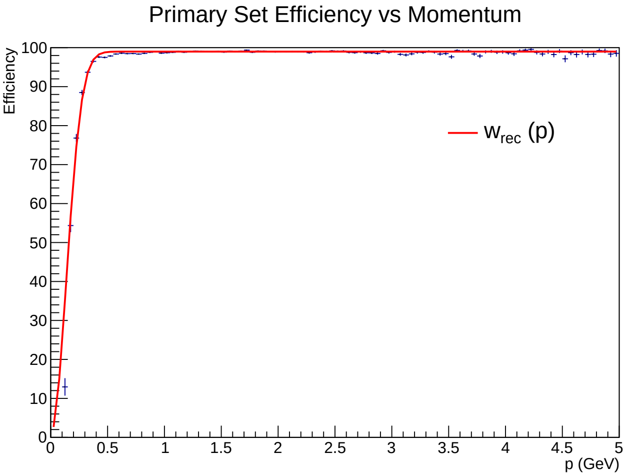
We need to correct inverse problem for acceptance and reconstruction efficiency!

Acceptance function

Reconstructible track – has MC Points on 4 consecutive STS stations
Calculate acceptance probability in (y, p_t) bins.



Reconstruction efficiency correction



Analytical parametrization

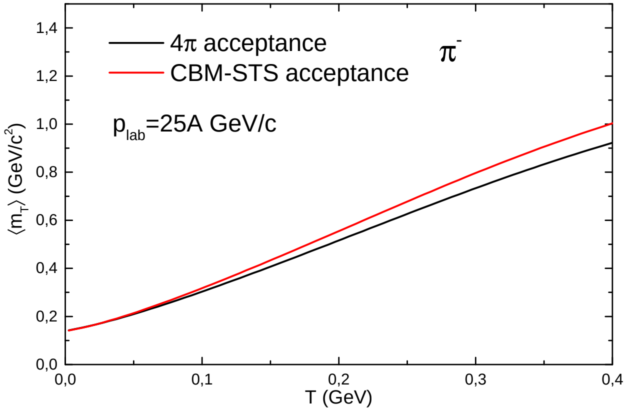
- $w_{rec}(p) = p_0 + p_1 \exp\left(\frac{-p^2}{2p_2^2}\right)$

Acceptance correction

Corrected $\langle m_T \rangle$ for reconstructed tracks from thermal model

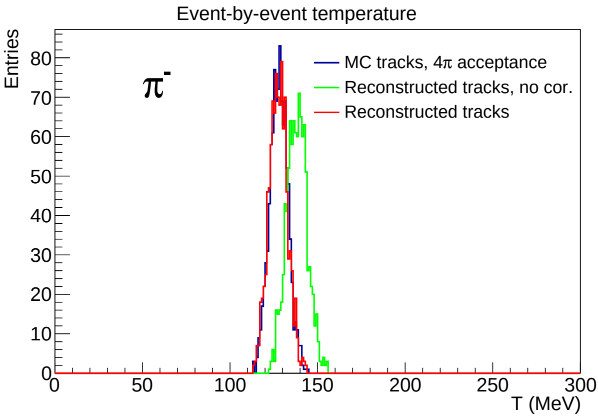
$$\langle m_T \rangle_{rec} = \frac{\int dy \int_0^\infty dp_t\, p_t\, m_T\, \cosh(y - y_{c.m.})\, e^{-m_T \cosh(y - y_{c.m.})/T}\, w_{acc}(y, p_t)\, w_{rec}(p)\, m_T}{\int dy \int_0^\infty dp_t\, p_t\, m_T \cosh(y - y_{c.m.})\, e^{-m_T \cosh(y - y_{c.m.})/T}\, w_{acc}(y, p_t)\, w_{rec}(p)}$$

$p = \sqrt{m^2 \sinh^2(y) + p_t^2 \cosh^2 y}$, $y_{c.m.}$ – mid-rapidity, $m_T = \sqrt{p_t^2 + m^2}$



Thermal source

Thermal generator, 300 π^- per event, $p_{\text{lab}} = 25A\ GeV/c$, 1000 events



Acceptance	T (MeV)
Theory	128.0
4π-MC	127.9 ± 0.2
Reco. uncor.	138.2 ± 0.2
Reco.	127.5 ± 0.2

Errors (1000 events):

$\Delta T_{stat} = 0.16$ MeV

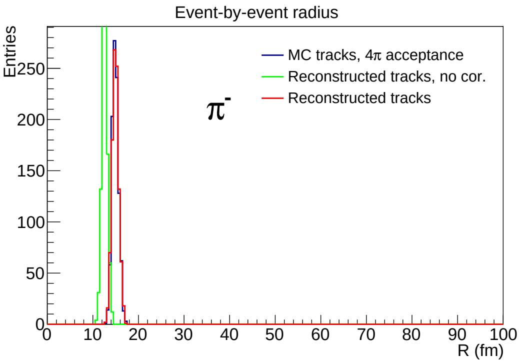
$\Delta T_{mom} = 0.005$ MeV

Thermal source

Corrected multiplicity

$$N_{CBM} = \alpha(T)\frac{V}{2\pi^2} T m^2 \, \mathrm{K}_2(m/T)$$

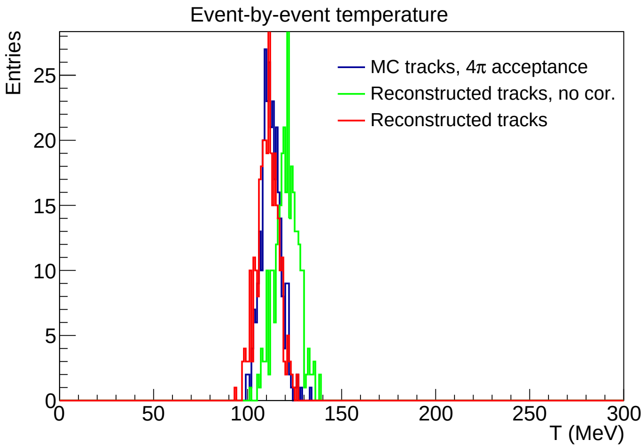
$$\alpha(T) = \frac{\int dy \int_0^\infty dp_t \, p_t \, m_T \, \cosh(y - y_{c.m.}) \, e^{-m_T \cosh(y-y_{c.m.})/T} \, w_{acc}(y, p_t) \, w_{rec}(p)}{\int dy \int_0^\infty dp_t \, p_t \, m_T \, \cosh(y - y_{c.m.}) \, e^{-m_T \cosh(y-y_{c.m.})/T}}$$



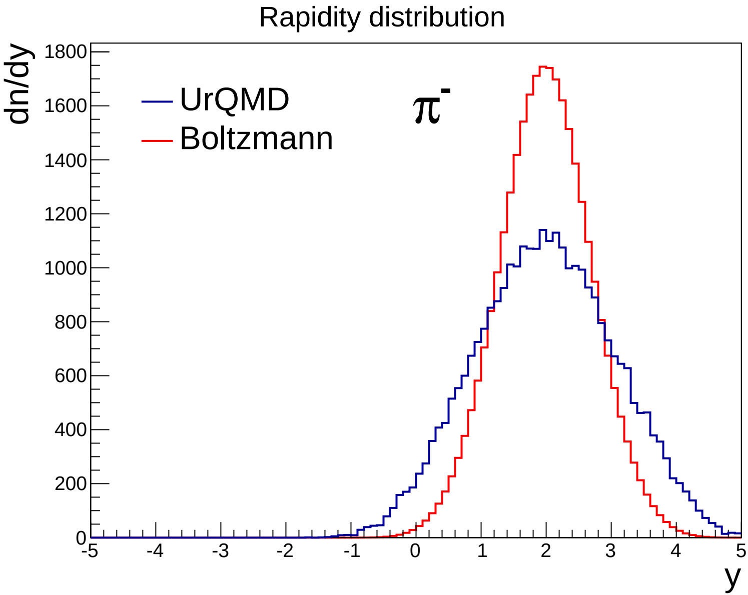
Acceptance	R (fm)
Theory	14.91
4π-MC	14.91 ± 0.02
Reco. uncor.	12.27 ± 0.02
Reco.	14.92 ± 0.02

UrQMD

UrQMD, Au+Au, $p_{\text{lab}} = 25A\ GeV/c$, 1000 central events



Acceptance	T (MeV)	R (fm)
4π-MC	112.3 ± 0.2	18.43 ± 0.03
Reco. uncor.	120.7 ± 0.2	14.40 ± 0.03
Reco.	110.7 ± 0.2	17.40 ± 0.03


Rapidity distribution
dn/dy
1800
1600
1400
1200
1000
800
600
400
200
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
y
UrQMD
Boltzmann
π^-

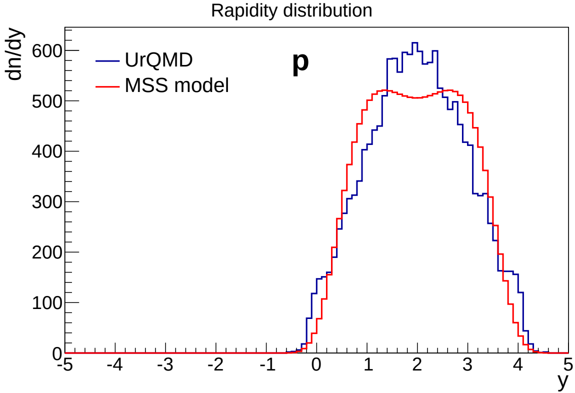
Multiscattering-statistical model

Momentum distribution for protons (simplified version of model)

$$dN/d^3p = Ae^{-\frac{p_\perp^2}{2\sigma_\perp^2}}\left(e^{-\frac{(p_z+\langle Q_z\rangle-p_{in})^2}{2\sigma_z^2}} + e^{-\frac{(p_z-\langle Q_z\rangle+p_{in})^2}{2\sigma_z^2}}\right)$$

Anchishkin, Naboka, Cleymans, arXiv:1303.6047

UrQMD 3.3, Au+Au, $p_{\rm lab} = 25A\ GeV/c$, 100 central events



$\langle\sigma_\perp\rangle = 0.56$ GeV, $\langle\sigma_z\rangle = 1.51$ GeV, $\langle Q_z\rangle = 3.21$ GeV.

Hadron-resonance gas at freeze-out model

Consider all hadrons with $m < 2$ GeV$/c^2$ at chemical freeze-out

Density of thermal hadrons

$$n_i = \frac{g_i}{(2\pi)^3} \int d^3p \left\{\exp[(\omega_p^i - \mu_i)/T] \pm 1\right\}^{-1},$$
$$\mu_i = B^i \mu_B + Q^i \mu_Q + S^i \mu_S,\ \omega_p^i = \sqrt{m_i^2 + p^2}.$$

Total hadron density – thermal + resonance decays

$$n_i^{tot} = n_i + \sum_{j \neq i} Br(j \to i) n_j.$$

Additional conditions

Net strangeness: $\sum_i S^i n_i = 0$

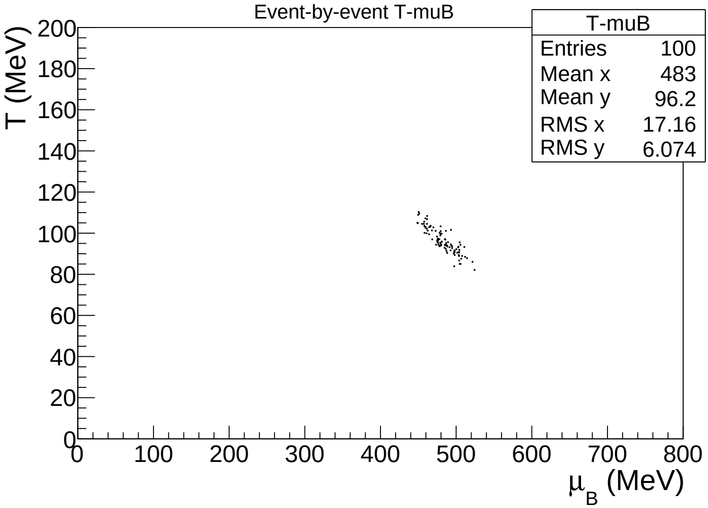
Charge to baryon ratio: $\frac{\sum_i Q^i n_i}{\sum B^i n_i} = Z/A \approx 0.4.$

Two free parameters: T and μ_B

Event-by-event extraction by fitting multiplicity ratios of bulk observables π^+, π^-, K^+, K^-, p.

Very preliminary!

UrQMD 3.3, Au+Au, $p_{\mathrm{lab}} = 25A\ GeV/c$, 100 central events



$\langle \chi^2 / N_{\mathrm{df}} \rangle_{EbE} \approx 0.54$

Summary

1. A package to extract the parameters of theoretical models is implemented in CBMROOT.
2. Inverse problems for the extraction of parameters of three different models are formulated.
3. Correction for acceptance and reconstruction efficiency is performed.
4. Thermal parameters are extracted on event-by-event basis.

Plans

- Optimize work of a package.
- Add other models.