Cooper-Frye negative contributions at FAIR energies

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September 22, 2014
Hydro: local thermal equilibrium, mean free path $\ll$ system size
\[ \partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j_\mu = 0, \quad \text{EoS, boundary conditions} \]
Transport: Monte-Carlo solution of Boltzmann equation
Hydrodynamics and transport are solved independently
Transition - on a predefined hypersurface
Transition in hybrid models

- Criterion for transition surface: "hydro equivalent to transport"
  - Constant energy density surface $\epsilon(t, x, y, z) = \epsilon_0 = 0.3 - 0.6 \text{ GeV/fm}^3$
  - Constant temperature surface $T = 150 - 170 \text{ MeV}$
Particlization and negative contributions

- Particlization
  - know $\epsilon$, $p$, $u_\mu$ on the surface
  - from EoS - $T$, $\mu$
  - want particles

- "Cooper-Frye formula"
  \[ d^3N(p) = f(p) \frac{d^3p}{(2\pi \hbar)^3} \frac{p_\mu}{p^0} d\sigma_\mu \]
  \[ \frac{p_\mu}{p^0} \cdot d\sigma_\mu \] - analog of $n \cdot V$
  e.g. ideal hydro $f(p) = \left( e^{\frac{p_\mu u_\mu - \mu}{T}} \pm 1 \right)^{-1}$

- Negative contribution
  - $p_\mu d\sigma_\mu > 0$: positive contribution, particles fly out
  - $p_\mu d\sigma_\mu < 0$: negative contribution, particles fly in

$d\sigma_\mu$ - normal 4-vector
$u_\mu = (\gamma, \gamma \vec{V})$ - 4-velocity
$T$ - temperature
$\mu$ - chemical potential
Negative contributions: options

- Account feedback to hydro - great increase in complexity

- Account effectively - artificial constructions
  S. Pratt, 2014, nucl-th1401.0136

- Neglect - violate conservation laws

  How large are negative contributions?
  What changes if we neglect them and how much?
  How much does the choice of transition surface influence results?
  Is hydro equivalent to cascade in the transition region?
Negative contributions: possible solution

- Hybrid model
- Assume: transport equivalent to hydrodynamics
- Neglect negative Cooper-Frye contributions + remove particles from cascade if they fly to hydrodynamical region

Is it possible to compensate negative Cooper-Frye contributions? That would solve problem with conservation laws.
Coarse-grained microscopic transport approach

Hypersurface of constant Landau rest frame energy density: mimic hybrid model transition surface

- Generate many UrQMD events
- On a \((t,x,y,z)\) grid calculate \(T^\mu\nu = \left\langle \frac{1}{V_{\text{cell}}} \sum_{i \in \text{cell}} \frac{p_i^\mu p_i^\nu}{p_i^0} \right\rangle\) event average
- In each cell go to Landau frame: \(T_L^{0\nu} = (\epsilon_L, 0, 0, 0)\)
- Construct surface \(\epsilon_L(t, x, y, z) = \epsilon_0\)

Example: \(E = 160\) AGeV, Au+Au central collision, \(\epsilon_0 = 0.3\) GeV/fm\(^3\)
Definitions for negative contributions

- Hypersurface $\Sigma$: $\epsilon_L(t, x, y, z) = \text{const}$
  - A) Cooper-Frye formula on $\Sigma$
  - B) count UrQMD particles crossing $\Sigma$
- $A \equiv B$ if particle distribution from UrQMD is exactly equilibrated

A) Cooper-Frye

\[
\frac{p^0 d^3 N^+}{dp^3} = \frac{p^\mu d\sigma_\mu}{\exp(p^\nu u_\nu / T) \pm 1} \theta(p^\nu d\sigma_\nu)
\]

\[
\frac{p^0 d^3 N^-}{dp^3} = \frac{p^\mu d\sigma_\mu}{\exp(p^\nu u_\nu / T) \pm 1} \theta(-p^\nu d\sigma_\nu)
\]

B) “by particles”
Gaussian smearing and statistics

Results are very sensitive to surface lumpiness

To get a smooth surface gaussian smearing with $\sigma = 1$ fm was used.
What changes if we neglect negative contributions

- Conservation laws will be violated
- Spectra will change depending on
  - collision energy
  - centrality
  - particle sort
  - transition surface
- We further investigate \( \frac{dN^-}{dy} / \frac{dN^+}{dy} \) in %

Example:

\[
\begin{array}{c|c|c|c|c|c}
    y & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
    dN_{\pi}^-/dy & & & & & & & \\
    dN_{\pi}^+/dy & & & & & & & \\
\end{array}
\]

\( E = 10 \text{ AGeV}, b=0 \text{ fm} \)
Negative contributions: particle mass dependence

E = 40 AGeV, b = 0, $\epsilon_0 = 0.3$ GeV/fm$^3$, $dN/dy$ distributions

**π ($m_\pi = 139$ MeV)**  

**$K^+$ ($m_K = 495$ MeV)**  

**N ($m_N = 938$ MeV)**

Smaller mass - larger negative contribution
Negative contributions: energy dependence

Pions, $\epsilon_0 = 0.3$ GeV/fm$^3$, $dN/dy$ distributions

Lower collision energy - slower expansion - larger negative contributions
Negative contributions: dependence on surface $\epsilon_0$

$E = 40$ AGeV, $b = 0$

Larger $\epsilon_0$ - slower surface expansion - larger negative contributions
Negative contributions: dependence on centrality

More peripheral collision - smaller negative contributions
Summary

- Hydro-transport transition ("particlization") was studied
  - Coarse-grained UrQMD was used to construct $\epsilon(t, x, y, z) = \epsilon_0$ isosurface
  - Negative contributions on this surface are calculated in two ways: from Cooper-Frye formula and explicitly counting particles

- Negative contributions are larger
  - for smaller collision energies
  - for central collisions than for peripheral
  - for smaller particle masses
  - for larger $\epsilon_0$
  - for lumpy transition surface

- Negative contributions by particles are smaller than Cooper-Frye ones
  - no compensation by accident
Backup: surface quality check

Surface - no holes or double counting: Cornelius routine
Conservation laws on hypersurface: accuracy better than 1%

Energy conservation

- $E = 10 \text{ AGeV, } b=0 \text{ fm}$
- $E = 40 \text{ AGeV, } b=0 \text{ fm}$
- $E = 160 \text{ AGeV, } b=0 \text{ fm}$

Discrep., %

- $E = 10 \text{ AGeV, } b=0 \text{ fm}$
- $E = 40 \text{ AGeV, } b=0 \text{ fm}$
- $E = 160 \text{ AGeV, } b=0 \text{ fm}$

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Cooper-Frye negative contributions

Sep. 2014 16 / 19
Backup: statistics matters

E = 160 AGeV, b = 0

Smooth surface

Lumpy surface

Lumpier surface - larger negative contributions
**Spectra: nucleons**

Red lines - by particles, blue lines - Cooper-Frye. $\epsilon_0 = 0.3 \text{ GeV/fm}^3$

<table>
<thead>
<tr>
<th>$E_{coll}$</th>
<th>$y$</th>
<th>$p_T$</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 AGeV</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td>Cooper-Frye works well. Spectra are close.</td>
</tr>
<tr>
<td>160 AGeV</td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
<td>At high $</td>
</tr>
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Similar picture for $\Delta$, $\Lambda$, $K^+$, $K^−$, but ...
Spectra: pions

Red lines - by particles, blue lines - Cooper-Frye. $\epsilon_0 = 0.3$ GeV/fm$^3$

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