Cooper-Frye Negative Contributions in a Coarse-Grained Transport Approach

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Abstract. Many models of heavy ion collisions employ relativistic hydrodynamics to describe the system evolution at high densities. The Cooper-Frye formula is applied in most of these models to turn the hydrodynamical fields into particles. However, the number of particles obtained from the Cooper-Frye formula is not always positive-definite. Physically negative contributions of the Cooper-Frye formula are particles that stream backwards into the hydrodynamical region.

We quantify the Cooper-Frye negative contributions in a coarse-grained transport approach, which allows to compare them to the actual number of underlying particles crossing the transition hypersurface. It is found that the number of underlying inward crossings is much smaller than the one the Cooper-Frye formula gives under the assumption of equilibrium distribution functions. The magnitude of Cooper-Frye negative contributions is also investigated as a function of hadron mass, collision energy in the range $E_{\text{lab}} = 5 - 160A$ GeV, and collision centrality. The largest negative contributions we find are around 13% for the pion yield at midrapidity at $E_{\text{lab}} = 20A$ GeV collisions.

1. Introduction

In the hot and dense system of strongly interacting matter created in heavy ion collisions the mean free path of the particles is much smaller than the size of the fireball. This fact together with the assumption of fast thermal equilibration allows to apply relativistic hydrodynamics for the dynamical description of heavy ion collisions. At later times of the evolution and at its edges the system is dilute, the mean free path is larger than system size, and hydrodynamics is not applicable. An adequate description of the system in those circumstances is provided by kinetic (transport) equations, such as the Boltzmann equation or its modifications for the quantum case. State of the art simulations of heavy ion collisions couple hydrodynamics for the early stage of the evolution to hadron transport for the late stage. Such approaches are called hybrid approaches [1].

The transition from hydrodynamics to transport (so-called particlization) is non-trivial, because hydrodynamics contains no microscopic information that is needed for transport. Currently, most of the models implement particlization in the following way. Hydrodynamical equations are solved in the whole forward light cone, including those regions, where hydrodynamics is not applicable. Then the particlization hypersurface is found from the hydrodynamical evolution. Usually a hypersurface of constant time, temperature or energy

density is taken. Particle distributions to be fed into the kinetic model are generated on this hypersurface according to the Cooper-Frye formula [2]:

$$p^0 \frac{dN}{d^3p} = p^\mu d\sigma_\mu f(p) \tag{1}$$

Here $\frac{dN}{d^3p}$ is a spectrum of particles emerging from an element of the hypersurface, $d\sigma_\mu$ is the normal four-vector to the hypersurface, its length being equal to the area of hypersurface element. In ideal fluid calculations the distribution function $f(p)$ is taken as an equilibrium Fermi or Bose distribution: $f(p) = \left[exp\left(\frac{p^\mu u_\mu - \mu}{T}\right) \pm 1\right]^{-1}$. An advantage of the Cooper-Frye formula is that it respects conservation laws. The disadvantage is that for space-like elements of the hypersurface (*i.e.* where $d\sigma^\mu d\sigma_\mu < 0$) there exist particle momenta p^μ such that $p^\mu d\sigma_\mu < 0$ and thus $\frac{dN}{d^3p} < 0$ in the Cooper-Frye formula. This is clearly unphysical, because the number of particles must be positive in every element of phase space. However, in all practical cases after integration over the hypersurface one gets positive definite spectra. One can split the integrated Cooper-Frye formula into positive and negative parts:

$$\frac{dN}{d^3p} = \int_\sigma \frac{p^\mu}{p^0} d\sigma_\mu f(p) \Theta(p^\mu d\sigma_\mu) + \int_\sigma \frac{p^\mu}{p^0} d\sigma_\mu f(p) \Theta(-p^\mu d\sigma_\mu). \tag{2}$$

The second term is called Cooper-Frye negative contributions and the first one - positive. For the applicability of the Cooper-Frye formula the negative contributions must be much smaller than the positive ones. This is usually true for simulations of high energy collisions. Negative contributions of around 9% relative to the positive ones were reported for RHIC top energy [4]. At the same time negative contributions at $E_{\text{lab}} = 160 A$GeV were found to be around 13%. This suggests that at lower collision energies negative contributions might become large and Cooper-Frye formula will be inapplicable. Does it really happen? To answer this question we systematically investigate the negative Cooper-Frye contributions differentially in rapidity and transverse momentum against collision energy, centrality and particle species.

Several ways to circumvent the problem of negative contributions were suggested (see [5] for a short summary). One practical way to do it in hybrid models would be to simultaneously neglect negative contributions and remove the particles from the transport calculation, if they fly to the hydrodynamic region. If the distributions in hydrodynamics and cascade are the same, then conservation laws will be fulfilled. Such an approach has never been implemented. To check if it is feasible, we compare negative contributions from the Cooper-Frye formula to distributions of backscattered particles from the cascade. We take advantage of the coarse-grained approach, which allows to calculate both within one framework.

2. Methodology
We aim at two goals: systematic estimation of Cooper-Frye negative contributions and comparing them to distributions of transport particles flying inwards to the hydrodynamics region. As a transport model we take Ultra-Relativistic Quantum Molecular Dynamics (UrQMD 3.3p2) [3]. UrQMD allows to simulate heavy ion collisions as a sequence of elementary particle collisions. Included processes are 2 to 2 scattering, resonance formation and decays, string excitation and fragmentation.

We generate an ensemble of UrQMD Au+Au collision events and average them on a rectangular grid to obtain the energy momentum tensor $T^{\mu\nu}$ and the baryon current j^μ in each cell. In each cell we find the Landau rest frame (LRF, a frame, where energy flow is zero: $T^{0i}_{LRF} = 0$). We obtain the energy density $\epsilon_{LRF} = T^{00}_{LRF}$, flow velocity u^μ and baryon density in the LRF $n_{LRF} = j^\mu_B u_\mu$. Knowing ϵ_{LRF} for each grid cell we construct the hypersurface

Σ of constant ϵ_{LRF} and find the normal vectors $d\sigma_\mu$ for each piece of Σ. The latter is done using the Cornelius subroutine [4], that provides a continuous surface without holes and avoids double counting of hypersurface pieces. The hypersurface Σ mimics the transition hypersurface in hybrid models. When Σ is obtained we perform a Cooper-Frye calculation on it and compare to distributions of underlying UrQMD particles that cross Σ.

We perform our calculations with time step $\Delta t = 0.1$ fm/c, grid spacing in the beam direction $\Delta z = 0.3$ fm, and grid spacings in transverse direction $\Delta x = \Delta y = 1$ fm. For collision energy $E_{\text{lab}} = 160A$ GeV we take $\Delta z = 0.1$ fm, and $\Delta x = \Delta y = 0.3$ fm. We have checked that for such a choice of grid spacing conservation laws on the surface are fulfilled with an accuracy better than 1%. In other words $\int_\Sigma T^{\mu0} d\sigma_\mu$ and total energy flowing out of the hypersurface calculated by particles differ by no more than 1%. To create a smooth hypersurface and obtain reproducible results we employ a Gaussian smearing procedure. For the construction of the hypersurface every UrQMD particle with coordinates (t_p, x_p, y_p, z_p) and 4-momentum p^μ is substituted by 300 marker particles with coordinates distributed with the probability density $f(x, y, z) \sim \exp\left(-\frac{(x-x_p)^2}{2\sigma^2} - \frac{(y-y_p)^2}{2\sigma^2} - \gamma_z^2 \frac{(z-z_p)^2}{2\sigma^2}\right)$, where $\gamma_z = (1 - p^z/p^0)^{-1/2}$. In this way every particle contributes to $T^{\mu\nu}$ and j^μ not only of the cell, where it is located, but also to the adjacent cells. We take as number of events $N = 1500$ and the Gaussian width $\sigma = 1$ fm. Choice of N, σ, grid spacing and sensitivity of results to these choices are discussed in [5].

3. Results

If the distribution of UrQMD particles is thermal on some closed hypersurface and the system is in chemical equilibrium, then the Cooper-Frye formula should give results identical to explicit particle counting. This would allow to compensate negative contributions by removing particles from UrQMD, if they cross the hypersurface inwards. In such a treatment conservation laws on the surface would be respected. We check, if this can be done on the hypersurface Σ of constant energy density in Landau frame, $\epsilon = 0.3$ GeV/fm^3.
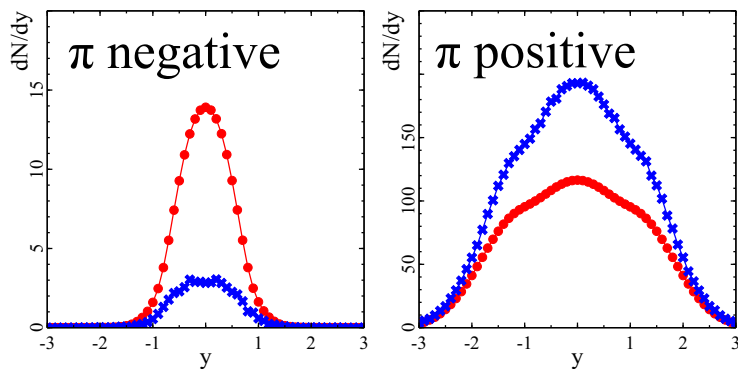

Figure 1. Cooper-Frye rapidity spectra for pions on Σ (red circles) are compared to distribution of UrQMD pions crossing Σ (blue crosses). Left panel: negative contributions and inward crossings. Right panel: positive contributions and outward crossings. Collision energy $E_{\text{lab}} = 40$ AGeV, central collisions. Note the very different scale of negative and positive contributions.
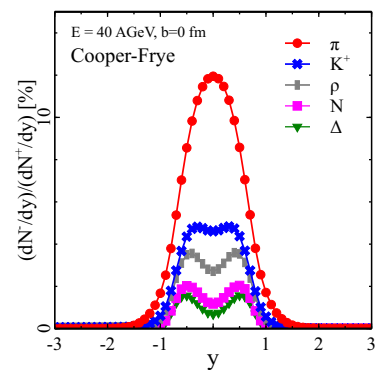
Figure 2. Rapidity distribution of the ratio of negative to positive contributions for different hadrons: pions (red circles), K^+ (blue crosses), ρ (grey bars), nucleons (magenta rectangles) and Δ (green triangles). Cooper-Frye calculation in central Au+Au collisions at $E_{\text{lab}} = 40$ AGeV.

When calculating the net number of pions passing through the surface, one finds that the number of pions in UrQMD is larger than in the equilibrated Cooper-Frye calculation [5]. It

might be possible to explain this as a sign of chemical non-equilibrium in UrQMD, but when one looks at the positive and negative contributions to the pion distributions shown in Fig. 1, one sees that a difference in the pion density only is not sufficient to explain the differences in the contributions. The positive contribution is much larger in UrQMD than in the Cooper-Frye calculation, whereas the negative contribution depicts the opposite behaviour: in UrQMD it is much smaller than in the Cooper-Frye scenario. This kind of distributions may indicate that the collective flow velocity of pions is much larger than the collective velocity of other particles [6, 7], or that the dissipative corrections are very large.

Since neither negative contributions coincide with inward crossings, nor positive contributions coincide with outward crossings, the above mentioned idea of compensating negative contributions will not work in our case. Instead we concentrate on finding when negative contributions play the most prominent role, and evaluate the ratio of negative contributions to positive ones to estimate the error they bring into hybrid calculations. For that we vary hadron sort, collision energy and centrality. As a relevant variable we consider the ratio of negative to positive contributions integrated over the hypersurface, $(dN^-/dy)/(dN^+/dy)$.

From Fig. 2 one can see that negative contributions become smaller for larger particle mass. It is simple to understand this result in the rest frame of the fluid element where the surface is moving. If the hypersurface Σ moves inwards, as is usually the case in fluid-dynamical calculations, a particle must be faster than the surface to cross it inwards. For larger particle mass the average velocities of the thermal motion are smaller and the probability to catch up the hypersurface is also smaller. Therefore, to find maximal negative contributions it is sufficient to consider pions only. For them we find out that if negative contributions are binned according to rapidity, then the largest contribution occurs at midrapidity.

We show the negative to positive contribution ratio for the pion yield at midrapidity for collision energies $E_{\mathrm{lab}} = 5$–$160A$ GeV in Fig. 3. The ratio in UrQMD calculation is much smaller than in the equilibrated Cooper-Frye calculation at all collision energies, but the maximum lies in the region of $20A$ GeV in both approaches. The value of the maximum is about 13% in Cooper-Frye calculation, and about 4% in UrQMD calculation.
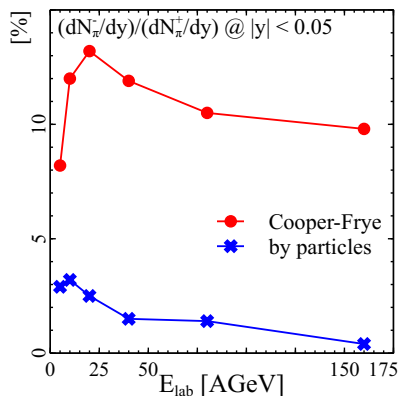

Figure 3. The ratio of negative to positive contributions on the $\epsilon(t, x, y, z) = \epsilon_c = 0.3$ GeV/fm^3 surface for pions at midrapidity. Red circles depict the ratio in a Cooper-Frye calculation assuming thermal equilibrium, and blue crosses in explicit calculation of UrQMD particles.
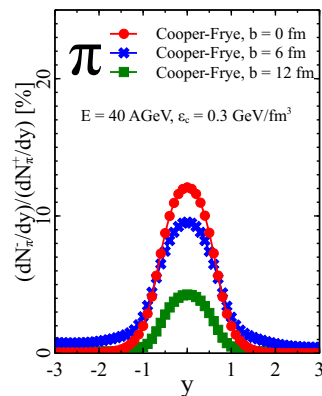
Figure 4. Rapidity distribution of the ratio of negative to positive contributions for pions in Au+Au collisions at $E_{\mathrm{lab}} = 40$ AGeV at various centralities: $b = 0$ (red circles), $b = 6$ fm (blue crosses) and $b = 12$ fm (green rectangles).

The dependency of negative to positive contribution ratio on collision energy is non-monotonous. This is not obvious, because several factors influence the ratio: the temperature on the hypersurface, the relative velocities between the flow and the surface, and the relative

amounts of volume and surface emission, *i.e.* emission from the time- and space-like parts of the surface. Larger temperature results in larger negative contributions because the thermal velocities increase. Larger relative velocity leads to smaller negative contributions. The larger the relative amount of volume-emission, the smaller the negative contributions. With increasing collision energies the temperature saturates and thus the changes in the last two factors make negative contributions fall with increasing collision energy. The decrease of negative contributions at higher energy is predictable, because the relative amount of volume emission and relative velocity between flow and surface increase with collision energy. However, the behaviour of negative contributions at lower energy is caused by the interplay of all three factors.

We plot negative contributions for pions versus collision centrality in Fig. 4, and find that for peripheral collisions negative contributions are smaller than for central collisions. The relative amount of surface and volume emission plays the most prominent role here. For peripheral collisions volume emission dominates due to the short lifetime of the system and negative contributions are small.

After varying the collision energy and the centrality, we can conclude that in the worst scenario negative contributions can hardly exceed 15%. Changing the criterion of hypersurface to $\epsilon_c = 0.6$ GeV/fm^3 does not change this conclusion (see Ref. [5]).

4. Conclusions

We have investigated negative Cooper-Frye contributions and backscattering using a coarse-grained molecular dynamics approach. Au+Au collisions at $E_{\mathrm{lab}} = 5$–160 A GeV energies have been simulated using UrQMD, and a hypersurface Σ of constant Landau rest frame energy density has been constructed. On this surface we have calculated two quantities: The ratio of Cooper-Frye negative to positive contributions (r_{eq}), which assumes local thermal equilibrium, and the ratio of UrQMD particles crossing Σ inward to crossing Σ outward (r_{neq}), which does not assume equilibrium.

We found that at all collision energies $r_{eq} \gg r_{neq}$. We explain this by a deviation of pions in UrQMD simulation from equilibrium. A non-monotonous dependency of r_{eq} and r_{neq} on collision energy was found with a maximum at 10-20 A GeV, maximal r_{eq} being around 13%. The size of the negative contributions is a result of an interplay of several factors: the temperature on the hypersurface, the relative velocities between flow and surface, and the relative amounts of volume and surface emission.

References

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