Radial modes of oscillations of slowly rotating magnetized compact hybrid stars

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Abstract. Stars having a quark core surrounded by a mixed phase followed by hadronic matter are termed as hybrid stars(HS). For the equation of state(EOS) of hadronic matter, we have considered RMF(Relativistic Mean Field) theory, in the presence of strong magnetic fields leading to Landau quantization of the charged particles. For the EOS of quark phase we have used the simple MIT bag model also in presence of strong magnetic field. The bag pressure and magnetic field are parametrized to make density dependant. We have constructed the intermediate mixed phase by using Glendenning conjecture. The EOSs are constructed comprising the hadron, mixed and quark phases. Eigenfrequencies of radial pulsations of slowly rotating magnetised hybrid star are calculated using constructed EOSs in a general relativistic formalism given by Chandrasekhar and Friedman.

1. Introduction

If the central density of neutron stars exceed the nuclear saturation density ($n_0 \sim 0.15\,\mathrm{fm}^{-3}$), then the compact stars might contain deconfined quark matter in them. So along with the hadronic matter, HS have quark matter in their interior. In between the quark and hadronic phases, a quark-hadron mixed phase exists. The size of the core depends on the critical density for the quark-hadron phase transition and the EOS, describing the matter phases.

Recently[1] the mass measurement of millisecond pulsar PSR J1614-2230 and pulsar J1903+0327 have set a new mass limit for compact stars to be $M = 1.97 \pm 0.04\,\mathrm{M_\odot}$ and $M = 1.667 \pm 0.021\,\mathrm{M_\odot}$, respectively[2]. This measurement for the first time has set a very strong limit on parameters of the EOS of the matter under extreme conditions [3, 4]. New observations suggest that in some pulsars, the surface magnetic field can be as high as $10^{14} - 10^{15}$G. It has also been noticed that the observed giant flares, SGR 0526-66, SGR 1900+14 and SGR 1806-20 [5], are the manifestation of such strong surface magnetic field in those stars. If we assume flux conservation from a progenitor star, we can expect the central magnetic field as high as $10^{17} - 10^{18}$G. Such strong fields are bound to effect the properties of HS. It can modify the EOS of matter of the star. The effect of strong magnetic field, both for nuclear matter [6, 7, 8, 9, 10] and quark matter [11, 12, 13] have been studied earlier in detail.

Cameron [14] had suggested that vibration of neutron stars could excite motions that can have interesting astrophysical applications, there have been several investigations of the vibrational properties of neutron stars. Simple dimensional analysis suggests that the fundamental mode period of a vibrating neutron star would be of the order of milliseconds[15]. Typical neutron star model with mass about one solar mass and radius about 10 km give periods (3–5)ms, and the

period is relatively insensitive to the exact value of the central density [16]. Since the rotation is a general property of all stellar bodies, it is interesting to study the normal modes of rotating HS. The determination of normal mode frequencies of a rotating HS is non trivial.

The value of period of oscillation strictly depends on the EOS along with some constraints on the parameters in both hadron and quark phase. Here we have investigated the effect of magnetic field on EOSs of both the matter phases (quark phase and hadron phase). Then we have constructed the mixed phase EOS in presence of strong magnetic field. Eigenfrequencies of radial pulsations of slowly rotating magnetised hybrid star are calculated in a general relativistic formalism given by Chandrasekhar & Friedman [17]. We have studied the square of the frequencies of the slowly rotating HS in the presence of strong magnetic field for various central densities.

2. Formalism

We have considered charge neutral, beta equilibrated matter. For the magnetic field we choose the gauge to be, $A^\mu \equiv (0, -yB, 0, 0)$, B being the magnitude of magnetic field. The momentum of the charged particles in x-y plane is quantized and hence the energy of the nth Landau level is [18] given by

$$E_i = \sqrt{p_i{}^2 + m_i{}^2 + |q_i|B(2n + s + 1)}, \tag{1}$$

where n=0, 1, 2, ..., being the principal quantum numbers for allowed Landau levels, $s = \pm 1$ refers to spin up(+) and down(-) and p_i is the component of particle(species i) momentum along the field direction. Setting $2n + s + 1 = 2\tilde{\nu}$, where $\tilde{\nu} = 0, 1, 2...$, we can rewrite the single particle energy eigenvalue in the following form

$$E_i = \sqrt{p_i{}^2 + m_i{}^2 + 2\tilde{\nu}|q_i|B} = \sqrt{p_i{}^2 + \widetilde{m}_{i,\tilde{\nu}}^2}, \tag{2}$$

where the $\tilde{\nu} = 0$ state is singly degenerate. The total energy density and pressure of the hadronic matter in presence of strong magnetic field can be written as, [19]

$$\begin{aligned}
\varepsilon_{\text{HP}} &= \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_0^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_{\sigma*}^2\sigma^{*2} + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{3}{4}d\omega_0^4 + U(\sigma) \\
&\quad + \sum_b \varepsilon_b + \sum_l \varepsilon_l + \frac{B^2}{8\pi^2}, \\
\text{P}_{\text{HP}} &= \sum_i \mu_i n_i - \varepsilon_{\text{HP}},
\end{aligned} \tag{3}$$

where the last term in ε_{HP} is the contribution from the magnetic field.

The total energy density and pressure of the quark matter is given by [19],

$$\varepsilon_{\text{QP}} = \sum_i \Omega_i + B_G + \sum_i n_i\mu_i, \qquad \text{P}_{\text{QP}} = -\sum_i \Omega_i - B_G, \tag{4}$$

where B_G is the bag constant. With the above given hadronic and quark EOS, we now perform the Glendenning construction [20] for the mixed phase, which determines the range of baryon density, where both phases coexist. Allowing both the hadron and quark phases to be separately charged, and still preserving the total charge neutrality as a whole in the mixed phase. Thus the matter can be treated as a two-component system, and can be parametrized by two chemical potentials, usually the pair (μ_e, μ_n), i.e., electron and baryon chemical potential. To maintain mechanical equilibrium, the pressure of the two phases are equal. Satisfying the chemical and beta equilibrium the chemical potential of different species are connected to each other. The
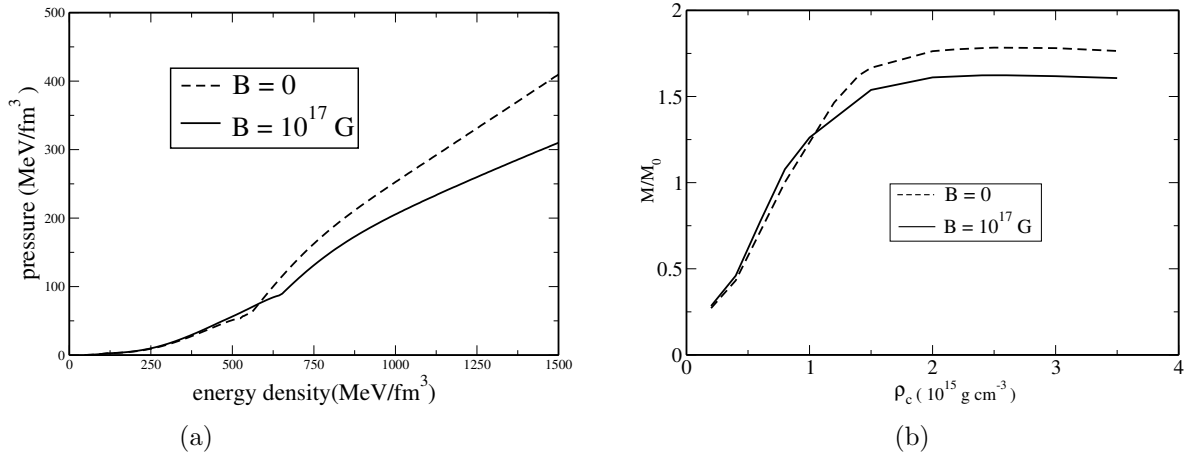
Figure 1: (a)Pressure (MeV/fm^3) against energy density (MeV/fm^3) for zero and non zero value of magnetic field. B=0 (dotted line) and B=$10^{17}G$ (solid line). (b) Gravitational mass in solar mass unit against central density ($g\,cm^{-3}$) for zero and non zero value of magnetic field. B=0 (dotted line) and B=$10^{17}G$ (solid line).

Gibbs condition for mechanical and chemical equilibrium at zero temperature between both phases are given by

$$P_{HP}(\mu_e, \mu_n) = P_{QP}(\mu_e, \mu_n) = P_{MP}. \qquad (5)$$

This equation gives the equilibrium chemical potentials of the mixed phase corresponding to the intersection of the two phases. At lower densities below the mixed phase, the system is in the charge neutral hadronic phase, and for higher densities above the mixed phase, the system is in the charge neutral quark phase. As the two surfaces intersect, one can calculate the charge densities ρ_c^{HP} and ρ_c^{QP} separately in the mixed phase. If χ is the volume fraction occupied by quark matter in the mixed phase, we have

$$\chi \rho_c^{QP} + (1 - \chi)\rho_c^{HP} = 0. \qquad (6)$$

Therefore the energy density ϵ_{MP} and the baryon density n_{MP} of the mixed phase can be obtained as

$$\epsilon_{MP} = \chi \epsilon_{QP} + (1 - \chi)\epsilon_{HP}, \qquad n_{MP} = \chi n_{QP} + (1 - \chi)n_{HP}. \qquad (7)$$

We parametrized the bag constant and magnetic field strength as given in Ref.[19].

The formalism given by Chandrasekhar & Friedman [17] is a fully general relativistic one to calculate the effect of rotation (to order Ω^2, where Ω is the angular velocity of rotation) on the eigenfrequencies of radial pulsations of stars. It gives an exact formula to calculate the frequency (σ') of oscillations of a 'slowly' rotating stellar configuration. The Chandrasekhar-Friedman formula is of the form:

$$\sigma'^2 I_1 = I_2 + I_3 + I_4. \qquad (8)$$

The detail mathematical expression of I_1, I_2, I_3, I_4 are given in Ref.[21].

The equations governing infinitesimal radial pulsations of a non-rotating star in general relativity was given by Chandrasekhar [22], and it has the following form :

$$F\frac{d^2\xi}{dr^2} + G\frac{d\xi}{dr} + H\xi = \tilde{\sigma}^2 \xi. \qquad (9)$$

Here $\xi(r)$ is the Lagrangian fluid displacement and $c\tilde{\sigma}$ is the characteristic eigenfrequency (c is the speed of light). The quantities F,G,H depend on the equilibrium profiles of the pressure p and density ρ of the stars. The details are given in Ref.[15].

Table 1: Pulsation frequency for non-rotating and rotating HS as a function of central density ρ_c for n=0, 1 and 2 radial modes in absence and presence of magnetic field.

ρ_c 10^{15} g cm^{-3}	n	(Without magnetic field)			(With magnetic field)		
		M/M_0	$\tilde{\sigma}^2(10^8 s^{-2})$	$\sigma'^2(10^8 s^{-2})$	M/M_0	$\tilde{\sigma}^2(10^8 s^{-2})$	$\sigma'^2(10^8 s^{-2})$
0.25	0	0.32	5.11	6.31	0.32	5.10	5.78
	1		13.30	7.66		13.92	8.20
	2		36.38	2.48		38.12	2.62
0.5	0	0.56	3.38	1.13	0.60	3.92	1.33
	1		25.05	1.90		26.26	2.22
	2		62.04	5.09		64.54	5.92
1	0	1.16	9.53	-2.65	1.26	10.25	1.75
	1		41.53	2.68		47.43	3.08
	2		104.71	8.56		111.98	9.03
1.5	0	1.67	10.18	2.03	1.54	7.61	-5.83
	1		49.58	5.25		49.72	2.75
	2		127.01	1.03		102.64	8.83
2	0	1.77	8.40	-1.52	1.61	6.49	-1.69
	1		34.34	3.42		34.37	2.02
	2		85.88	9.84		83.39	5.49
2.5	0	1.78	6.50	-3.11	1.62	5.40	-2.66
	1		29.04	2.29		26.79	1.58
	2		71.62	8.52		71.39	4.43

For a given EOS $P(\rho)$ and given central density, we solved TolmanOppenheimerVolkoff (TOV)[23] equations to obtain the radius R and mass M = m(R) of the star. Therefore the basic input to solve the structure and pulsation equations is the EOS, $P = P(\rho)$. It has been seen [24] that structure parameters of neutron stars are mainly dominated by the EOS at high densities, specifically around the core. Since the oscillation features are governed by structure profiles of neutron stars, it is expected to possess marked sensitivity on the high density EOS as well.

3. Results and Discussion

The results of our calculation are presented in Fig.1a, Fig.1b and Table 1. In Fig.1a, we have plotted pressure against energy density having density dependent bag pressure 170MeV [19]. It is clear that magnetic field softens the EOS as well as broadens the mixed phase region. In Fig.1b, we have plotted gravitational mass(in solar mass unit) against central density for zero and non zero value of magnetic field. The maximum gravitational mass of HS for non-magnetic case is 1.8 solar mass and for magnetic case is 1.6 solar mass, which are close to observed values [2]. In Table 1 we have presented a calculation of the square of frequencies ($\tilde{\sigma}^2$ and σ'^2) of magnetised and non-magnetised hybrid star as a function of central densities for fundamental mode and for the first two harmonics by using the radial pulsation equations of non-rotating HS, as given by Chandrasekhar [22] as well as for rotating HS as given by Chandrasekhar and Friedman [17] in the general relativity formalism. Due to mixed phase region which is transition from quark to hadron phase, a kink is observed in the frequency with increase in central density. These kinks can be considered as a distinct signature of the mixed and quark matter in neutron stars. In both magnetic and non-magnetic cases for non-rotating HS there is a kink at $\rho_c = 0.5 \times 10^{15} g\ cm^{-3}$ in fundamental mode, then frequency increases and decreases slowly with increasing central density

and for other two higher harmonics there is no remarkable kink but frequency increases and then slowly decreases with central density. In absence of magnetic field for rotating HS there is an oscillating behaviour of square of frequency starting at central density $\rho_c = 0.5 \times 10^{15} g\ cm^{-3}$ for all modes. In presence of magnetic field for the rotating HS there is an oscillating behaviour in square of frequency for first two modes and then slowly decreases but for next mode square of frequency increases and decreases with central density. These behaviour are observed due to presence of mixed and quark matter in HS.

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