Spinodal density enhancements in nuclear collisions at the CBM experiment

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Abstract. We discuss a novel approach to describe the evolution of a fireball, created in a high-energy nuclear collision, experiencing spinodal instabilities due to the first-order deconfinement phase transition of quantum chromo dynamics (QCD). We show that initial density fluctuations in these collisions are enhanced in the mechanically unstable region of the QCD phase diagram. In our study we find that the most favorable energy range for observing these density enhancements is at the lower end of the SIS100 accelerator at FAIR, currently under construction. Furthermore we discuss how one can distinguish and constrain different types of QCD phase transitions, one of hadron-quark type and one of liquid-gas type, leading to strong differences in the dynamical evolution of the QCD medium.

1. Introduction

Strongly interacting matter, in particular compressed baryonic matter, is expected to posses a rich phase structure and may exhibit a first-order phase transition that persists up to a certain critical temperature [1]. At vanishing baryon chemical potential, lattice QCD calculations predict a smooth transformation from confined to deconfined matter at a cross-over temperature of $T_\times \approx 150$ - 160 MeV [2, 3]. Unfortunately, the lattice calculations cannot be extended to finite chemical potential due to the so-called sign problem. It is therefore necessary to experimentally determine the phase structure of baryon-rich strongly interacting matter. For such experimental endeavors [4, 5, 6] to be successful, it is important to identify observables that may serve as signals of the expected phase transition. Because the colliding system is relatively small, non-uniform, far from global equilibrium, and rapidly evolving, the connection between experimental observables and the idealized uniform equilibrium matter described by the equation of state are obscured. Consequently it is important to carry out dynamical simulations of the collisions with suitable transport models.

In a novel approach we perform numerical simulations with finite-density fluid dynamics, using a previously developed two-phase equation of state and incorporating a gradient term in the local pressure [7]. This latter refinement emulates the finite-range effects for a proper description of the phase transition physics [8]. Thus we employ a transport model that treats the associated physical instabilities in a numerically reliable manner.

2. A model for the phase transition

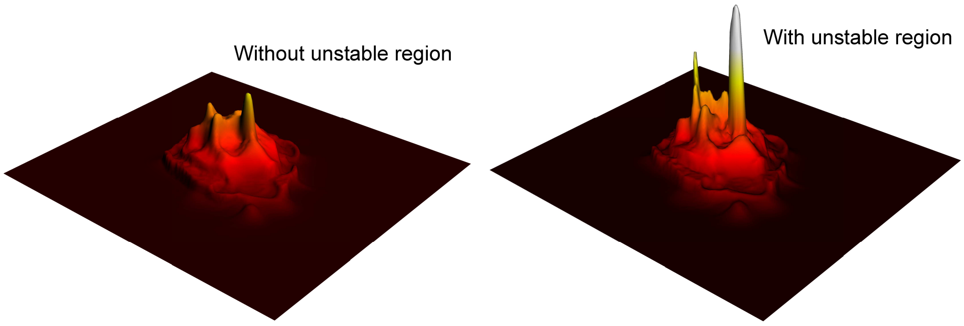

Figure 1. Density profile after 4 fm/c with a single UrQMD initial condition for $E_{\rm lab} = 3A$ GeV. While the initial fluctuations are not amplified in the case without an unstable region (left figure), they are strongly amplified when an unstable region in the EoS is included (right figure).

For our present investigations, we describe the evolution of the colliding system by ideal fluid dynamics. The basic equation of motion in ideal fluid dynamics expresses four-momentum conservation, $\partial_\mu T^\mu = 0$, where the stress tensor is given by

$$T^{\mu\nu}(x) = [p(x) + \varepsilon(x)]u^\mu(x)u^\nu(x) - p(x)g^{\mu\nu} \ , \tag{1}$$

where $u^\mu(x)$ is the four-velocity of the fluid. When taking account of the baryon current density,$N^\mu(x) = \rho(x)u^\mu(x)$, the basic equation of motion is supplemented by the continuity equation, $\partial_\mu N^\mu = 0$. These equations of motion are solved by means of the code SHASTA [10] in which the propagation in the three spatial dimensions is carried out consecutively.

In order to obtain a suitable equation of state, used as input to the fluid dynamical simulations, we employ the method developed in Ref. [9]. For a given T, we obtain the free energy density $f_T(\rho)$ in the phase coexistence region by performing a suitable spline between two idealized systems (either a gas of pions and interacting nucleons or a bag of gluons and quarks) held at that temperature. For each T the spline points were adjusted so the resulting $f_T(\rho)$ would exhibit a concave anomaly, *i.e.* there would be two densities, $\rho_1(T)$ and $\rho_2(T)$, for which the tangent of $f_T(\rho)$ would be common. This ensures phase coexistence because then the chemical potentials $\mu_T = \partial_\rho f_T(\rho)$ match, $\mu_T(\rho_1) = \mu_T(\rho_2)$, as do the pressures, $p_T(\rho_1) = p_T(\rho_2)$. The resulting two-phase EoS (HQ-EoS) therefore exhibits a transition from a hadronic to a deconfined quark phase.

As mentioned above, a proper description of spinodal decomposition requires that finite-range effects be incorporated [8]. Therefore we write the local pressure as

$$p(\boldsymbol{r}) = p_0(\varepsilon(\boldsymbol{r}), \rho(\boldsymbol{r})) - a^2\varepsilon_s \frac{\rho(\boldsymbol{r})}{\rho_s} \boldsymbol{\nabla}^2 \frac{\rho(\boldsymbol{r})}{\rho_s} \ , \tag{2}$$

where we recall that $p_0(\varepsilon, \rho)$ is the equation of state, the pressure in uniform matter characterized by ε and ρ. With $\rho_s = 0.153/\mathrm{fm}^3$ being the nuclear saturation density and $\varepsilon_s \approx m_N \rho_s$ the associated energy density, the gradient term is normalized such that its strength is conveniently governed by the length parameter a which was chosen to take the value $a = 0.033$ fm [11].

We have shown that our model produces both meaningful interface properties, including the temperature-dependent strength of the associated tension, and reasonable spinodal growth rates, including the emergence of an optimal phase separation scale [7]. The model is therefore suitable for addressing dynamical scenarios involving phase-transition instabilities.

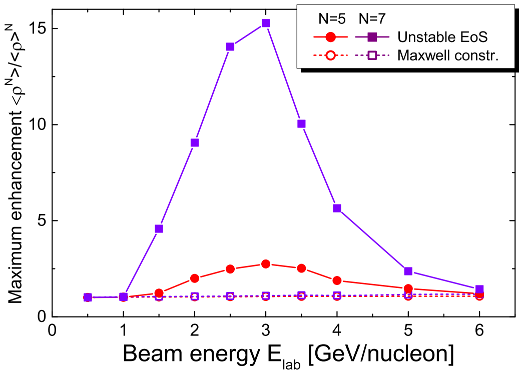

Figure 2. Maximal enhancement of the fifth and seventh moments of the net baryon density distributions as function of the beam energy $E_{\rm lab}$, for central collisions of Pb+Pb. We compare a two phase equation of state including the effects of the spinodal instabilities (solid lines) with a single phase equation of state (dashed lines) obtained via a Maxwell construction.

For a first study, we have considered central collisions of lead nuclei bombarded onto a stationary lead target at various kinetic energies, $E_{\rm lab}$. For each energy, an ensemble of several hundred separate evolutions are generated, each one starting from a different initial configuration generated by (the cascade mode of) the UrQMD model [12, 13, 14] which treats the non-equilibrium dynamics during the very early stage of the collision.

Figure 1 shows the effect of the spinodal instabilities on a single initial density profile, obtained with UrQMD. Here the initial fluctuations are not amplified in the case without an unstable region, they are strongly amplified when an unstable region in the EoS is included. When considering an ensemble of many events, a convenient quantitative measure of the resulting degree of "clumping" in the system is provided by the moments of the baryon density density $\rho(\boldsymbol{r})$,

$$\langle\rho^N\rangle \;\equiv\; \frac{1}{A^N}\int \rho(\boldsymbol{r})^N\rho(\boldsymbol{r})\,d^3\boldsymbol{r}\ , \tag{3}$$

where $A = \int\rho(\boldsymbol{r})d^3\boldsymbol{r} = \langle\rho^0\rangle$ is the total (net) baryon number. The corresponding normalized moments, $\langle\rho^N\rangle/\langle\rho\rangle^N$, are dimensionless and increase with the order N, for a given density distribution $\rho(\boldsymbol{r})$; the normalized moment for $N=1$ is unity.

Figure 2 shows the (ensemble average) maximum enhancement (for $N=5$ and 7) achieved, as a function of the beam energy for the two equations of state. The existence of an optimal collision energy is clearly brought out. While the presently employed equation of state suggests that this optimal range is $E_{\rm lab}\approx 2-4\,A\,\rm GeV$, it should be recognized that others may lead to different results.

3. The effect of the equation of state

As we pointed out, the effective EoS used in the dynamical simulations for the QCD phase transition may have a strong impact on the resulting observable signals, as well as the optimal collisional beam energy needed to see signals of the phase transition. In this section we will discuss possible ways to qualitatively distinguish different types of phase transitions in QCD and how they influence the dynamical evolution of nuclear collisions in the region of large baryon densities, where the corresponding baryon-number chemical potential μ is of the order of the temperature or higher.

Among the many effective models for QCD thermodynamics, those that couple constituent quarks to mesonic fields and the Polyakov loop have become popular because they include chiral-

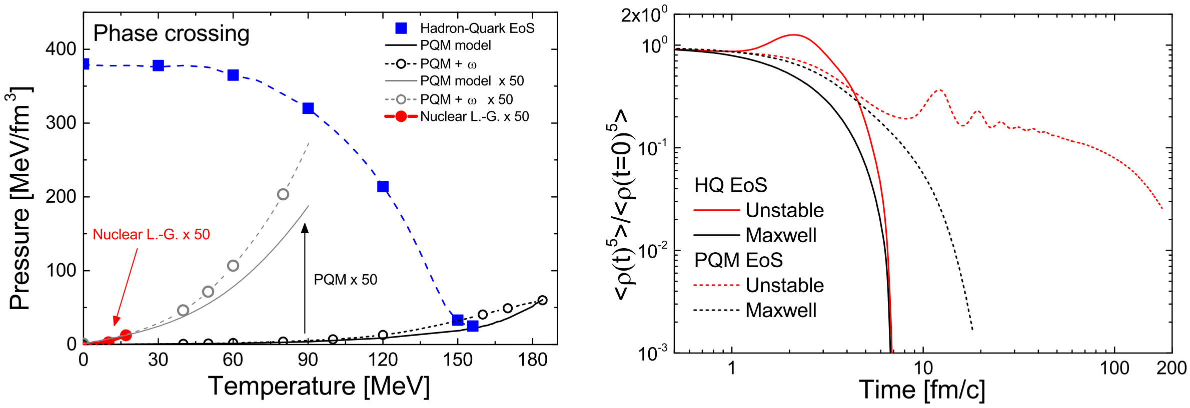

Figure 3. Left: Pressure along the coexistence line for the two different equations of state discussed in the text. Right: Time evolution of the fifth moment of the net baryon density for the two different equations of state discussed in the text, and an initial state that locates the bulk of the system within the unstable region of the phase diagram.

symmetry breaking and a "thermal" confinement of quarks. Such approaches are the Polyakov Nambu–Jona-Lasino (PNJL) [15, 16] and the Polyakov Quark-Meson (PQM) [17] models. These models can be brought into agreement with lattice results at vanishing net baryon density but their results for the finite-density domain depend significantly on the parameterizations used. A particular weakness in most of these models is the absence of explicit hadronic degrees of freedom in the confined phase. There exist only few examples where the equations of state (EoS) includes an approximately realistic description of the hadronic phase. These equations of states are constructed either by matching a purely hadronic model to an MIT-bag EoS or by combining a PNJL/PQM-type model with a chiral hadronic model [18, 19]. Obviously, these models need to be constrained both by lattice QCD results at zero and small chemical potential and by known properties of nuclear matter at small temperatures and densities up to about twice nuclear matter density [20], as well as by astrophysical observations on neutron star properties [21].

It is instructive to consider the temperature dependence of the *pseudo-critical* pressure, $p_{\rm pc}$, the pressure at which the phase transformation occurs for the different types of EoS. Figure 3 (left) shows the temperature dependent pseudo-critical pressure for the two models considered here, the PQM model and the constructed Hadron-Quark EoS, as discussed in section 2. For a specified temperature T, $p_{\rm pc}^{\rm HQ}$ is obtained by increasing the chemical potential μ until the ideal hadron-gas and the quark-gluon bag have the same pressure, while $p_{\rm pc}^{\rm PQM}$ is taken as the pressure along the inflection point of the chiral condensate [22]. While the resulting pseudo-critical pressures are comparable at the highest temperatures (corresponding to $\mu \approx 0$), and consistent with lattice results [2], they deviate strongly at large chemical potentials: As the temperature is reduced, $p_{\rm pc}^{\rm HQ}$ increases steadily, as was already noted in previous studies employing models that describe the hadron-quark transition [23, 24]. By contrast, $p_{\rm pc}^{\rm PQM}$ decreases steadily and vanishes at $T = 0$, a behavior that is a robust feature of the PQM models and persists also in the presence of a repulsive quark interaction [25, 26]. This generic liquid-gas behavior differs qualitatively from the HQ result, for which the pressure *decreases* steadily with temperature.

We find that the PQM model shows a transition that is very similar to that of the liquid-gas transition in nuclear matter and thus this model differs qualitatively from the HQ model with regard to the thermodynamic properties near the phase coexistence line. The qualitative differences between the two equations of state examined in this work lead to significant quantitative differences in the time evolution of fireballs that expand through the respective unstable region of the phase diagram. In dynamical simulations with the PQM model the lifetime of quark clusters is orders of magnitude longer than what is usually expected for the timescales of heavy-ion collisions and it predicts stable dense quark matter droplets at zero temperature, i.e. coexisting with the vacuum [22]. In figure 3 (right) the time evolution of the density moments for the two equations of state discussed above are shown. One can clearly observe, that the moments drop rapidly in the case of the HQ EoS which has a large coexistence pressure, while in the case of the PQM model the density clusters survive for a very long time as they can only slowly decay in the vacuum.

4. Conclusion

In conclusion, we have obtained a transport model that is suitable for simulating nuclear collisions in the presence of a first-order phase transition: it describes both the tension between coexisting phases and the dynamics of the unstable spinodal modes. Applying this model to lead-lead collisions, we have found that the associated instabilities may cause significant amplification of initial density irregularities. Furthermore we have shown that qualitative differences between the PQM and a HQ equations of state lead to considerable differences in the dynamical evolution of the system. We believe that these qualitative differences persist independent of the specific EoS model adopted but are rather characteristic for either a liquid-gas type or a more plausible hadron-quark type transition.

5. Acknowledgments

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