Determination of freeze-out conditions from fluctuations in the Hadron Resonance Gas model

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Abstract. Fluctuations of conserved charges measured in Heavy-Ion Collisions (HICs) received increasing attention in recent years, because they are good candidates to explore the phase diagram of QCD matter. During the last year, net-electric charge and net-proton moments of multiplicities measured at RHIC have been published by the STAR collaboration, for a range of collision energies which spans a region of the phase diagram at finite chemical potential. Here we present a new freeze-out curve obtained using the Hadron Resonance Gas (HRG) model approach to fit these experimental data. The HRG model is modified in order to have a realistic description of the HICs: kinematic cuts, resonance feed-down and resonance regeneration are taken into account. Our result is in agreement with preliminary studies by the ALICE collaboration, and is supported by a recent lattice analysis of the same quantities.

1. Introduction

Various studies on Quantum ChromoDynamics (QCD) converge to the idea that for arbitrarily high temperature and/or densities the strongly interacting confined matter undergoes a transition to a phase in which the fundamental degrees of freedom, namely quarks and gluons, are deconfined [1]. With joined efforts, physicists all around the world were able to obtain first insight on the formation of such a state by means of Heavy-Ion Collisions (HICs). The quest for the right set of observables in order to get a clean signal of the quark liberation has reached the first step, mainly thanks to the possibility of performing simulations on a lattice (lQCD); on the lattice they are able to exactly solve QCD, and consequently to find probes of the free-quark phase from first principles. From the lattice we know that in the pure gauge sector the Polyakov loop is the order parameter for the confinement first order transition, and that in a system with dynamical quarks the transition is a smooth crossover [2]. Unfortunately, lattice simulations are available only for small chemical potentials, due to the so called sign problem which does not allow to use Monte Carlo techniques. In order to complement lattice simulations many options are available from effective theories, like the quasi-particle approach for the deconfined phase [3, 4, 5, 6, 7]. To study the confined thermal-equilibrated hadron phase, the most used tool is the Hadron-Resonance Gas (HRG) model, which is based on the idea that a system of interacting hadrons in the ground state can be described in terms of a non-interacting gas of hadrons and resonances. This model is very successful in describing the hadron production in HICs for a wide range of collision energies, ranging from AGS to the LHC [8, 9]. In the following we will show how to describe the moments of particle multiplicity distributions measured in experiments, by means of the fluctuations of conserved charges calculated in the HRG model. A first matching with lattice calculations shows a good agreement with the HRG predictions [10] , which potentially can provide the link between experiment and first principle calculations. One has to keep in mind that several sources of non statistical fluctuations might potentially affect the experimental measurements: for a discussion on these issues see e.g. [11, 12]

The paper is organized as follows: in Section 2 we will introduce the main concepts about the HRG model, in Section 3 we will introduce the concept of fluctuations and relate them to the moments of multiplicity distributions, in Section 4 we will describe the main corrections applied to the HRG, in Section 5 we will show the main results, and in the end the conclusions.

2. The Hadron Resonance Gas model

The idea at the basis of the HRG model is that a system of interacting hadrons can be described by a non interacting gas of hadrons and resonances. The resonance formation accounts for the strong interaction among particles.
The thermally equilibrated system created after the hadronization process can be described by the sum of independent particle contributions, e.g. for the pressure:

$$p(T,\{\mu_R\}) = \sum_R (-1)^{B_R+1} \frac{d_R T}{(2\pi)^3} \int d^3p \, \ln \left[1 + (-1)^{B_R+1} e^{-(\sqrt{\vec{p}^2+m_R^2}-\mu_R)/T} \right], \tag{1}$$

where B_R, d_R, m_R are respectively baryon number, degeneracy factor and mass of the particle R. The chemical potentials μ_R are given by a linear combination of all the conserved charges times the correspondent chemical potential, for example in full chemical equilibrium where we consider baryon number, electric charge and strangeness:

$$\mu_R = B_R \, \mu_B + Q_R \, \mu_Q + S_R \, \mu_S \, . \tag{2}$$

The other quantities follow from the usual thermodynamic identities, e.g. the particle number densities:

$$n_R(T,\mu_R) = \left(\frac{\partial p}{\partial \mu_R} \right)_T = \frac{d_R}{(2\pi)^3} \int d^3p \, \frac{1}{1 + (-1)^{B_R+1} e^{-(\sqrt{\vec{p}^2+m_R^2}-\mu_R)/T}} . \tag{3}$$

We use a PDG list [13] with particles with masses up to 2 GeV. Due to its simple ideal-gas nature it is easy in the HRG to modify a selected set of particles in order to fulfill the experimental constraints, as showed below.

3. Fluctuations and higher order moments

From Eq. 1 it is possible to extract fluctuations of conserved charges as follows:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{\,l+m+n} \left(p/T^4 \right)}{\partial \left(\mu_B/T \right)^l \partial \left(\mu_S/T \right)^m \partial \left(\mu_Q/T \right)^n} . \tag{4}$$

The mean M, the variance σ, the skewness S and the kurtosis κ of the multiplicity distribution of a net-quantity (e.g. electric charge Q) are related to the above fluctuations through the following equations:

$$M = \langle N \rangle = VT^3\chi_1 \quad , \quad \sigma^2 = \langle (\delta N)^2 \rangle = VT^3\chi_2,$$

$$S = \frac{\langle (\delta N)^3 \rangle}{\sigma^3} = \frac{VT^3\chi_3}{(VT^3\chi_2)^{3/2}} \quad , \quad \kappa = \frac{\langle (\delta N)^4 \rangle}{\sigma^4} - 3 = \frac{VT^3\chi_4}{(VT^3\chi_2)^2}. \tag{5}$$

Ratios of them are volume independent (to leading order), and so are in principle easily comparable to lattice simulations and HRG model results:

$$\sigma^2/M = \chi_2/\chi_1 \,, \;\; S\sigma = \chi_3/\chi_2 \,, \;\; S\sigma^2/M = \chi_3/\chi_1 \,, \;\; k\sigma^2 = \chi_4/\chi_2 \;. \tag{6}$$

First experimental data on net-charge and net-proton number have been recently published [15] and compared to lattice QCD results [16].

4. Corrections to match the experimental situation

Many corrections can be applied to the HRG model in order to get closer to the experimental situation. For example, in order to reproduce the initial conditions one usually imposes $N_Q = 0.4\, N_B$ and $N_S = 0$; this connects the temperature T and the three chemical potentials to each other, namely μ_Q and μ_S are functions of T and μ_B.

In the experiment it is not possible to cover the entire phase space, but just a portion of it. This leads to the fluctuations of conserved charges: in the entire phase space, such charges would be strictly conserved and there would be no fluctuations at all.

In accordance with the experimental kinematic setup the formulae modify as follow:

$$n_R(T,\mu_R) = \frac{d_R}{4\pi^2}\int_{-y_{MAX}}^{y_{MAX}} \mathrm{d}y \int_{p_T^{MIN}}^{p_T^{MAX}} \mathrm{d}p_T \frac{p_T\sqrt{p_T^2+m_R^2}\cosh(y)}{(-1)^{B_R+1}+\exp((\cosh(y)\sqrt{p_T^2+m_R^2}-\mu_R)/T)}, \tag{7}$$

$$n_R(T,\mu_R) = \frac{d_R}{4\pi^2}\int_{-\eta_{MAX}}^{\eta_{MAX}} \mathrm{d}\eta \int_{p_T^{MIN}}^{p_T^{MAX}} \mathrm{d}p_T \frac{p_T^2\cosh(\eta)}{(-1)^{B_R+1}+\exp((\sqrt{p_T^2(\cosh(\eta))^2+m_R^2}-\mu_R)/T)}. \tag{8}$$

The higher order cumulants are obtained accordingly. The effects of kinematic cuts on χ_2^Q/χ_1^Q are shown in Fig. 1 (left). For a systematic study see [17].

At a certain point in the evolution of the system it is reasonable to expect that inelastic scatterings between hadrons cease to happen, and hadrons will only interact elastically or quasi-elastically, through the formation and subsequent strong decay of resonances. This is called chemical freeze-out. Only the particles with a lifetime longer than the hadron phase one will be detected; the effective number $\bar{N}_i$ of these *stable* particles, given by their primordial thermal number plus the contribution from the resonance decays (Eq. 9), is thus conserved.

$$\bar{N}_i = N_i + \sum_r d_{r\to i} N_r. \tag{9}$$

This situation is different from the full chemical equilibrium described by the lattice simulations, and so we will call it Partial Chemical Equilibrium (PCE).

As a consequence, after chemical freeze-out, the resonance chemical potentials can be expressed in terms of the stable particle ones, using the branching ratios of the decays.

The contribution to the higher order cumulants for every stable particle in PCE is easily obtained using the partial derivative with respect to its effective chemical potential. See

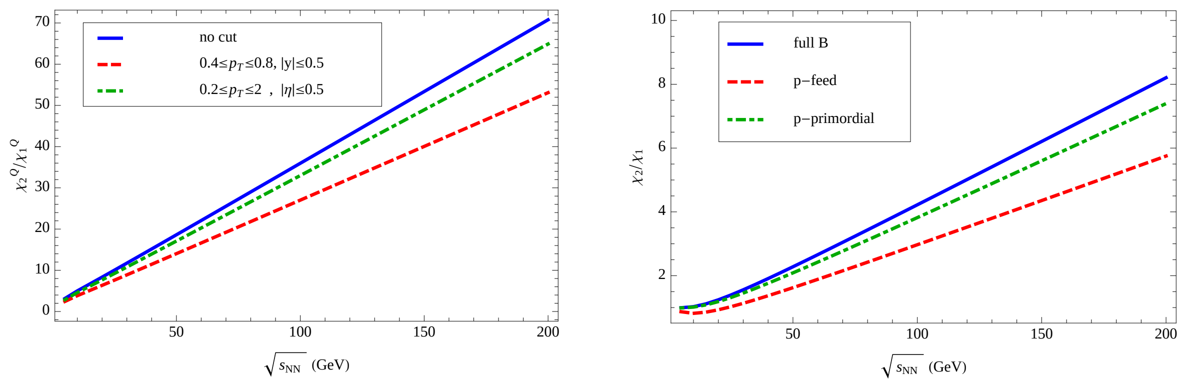

Figure 1. (Color online) Left panel: effects of the kinematic cuts on χ_2/χ_1 for the net-electric charge compared to the situation of no cuts (blue full line); the different cuts read: $0.4 \leq p_T \leq 0.8$ (GeV) and $|y| \leq 0.5$ (red dashed line), $0.2 \leq p_T \leq 2$ (GeV) and $|\eta| \leq 0.5$ (green dot-dashed line). Right panel: feed down correction to χ_2/χ_1 for the net-proton number (red dashed line), compared to the result for the net-baryon (blue full line) and primordial net-proton (green dot-dashed line). The curves are obtained using the freeze-out parametrization from [21].

Fig. 1 (right) to see the effect of the feed down correction. Experimentally, pions, kaons and (anti)protons are the most abundant particles in the energy range of interest, but we extended the set of stable particles in order to exclude the contribution from weak decays [14].

The multiplicity distribution of neutrons has not been measured. Nevertheless, the contribution of neutrons on the distribution of their isospin partner can still be sizable, due to some particular processes. Consider for example the formation cycle of a Delta resonance:

$$p(n) + \pi^0(\pi^+) \to \Delta^+ \to n(p) + \pi^+(\pi^0)\,; \tag{10}$$

this regeneration process during the hadron stage realizes an isospin randomization, namely at the detection it is impossible to distinguish between a primordial proton, or a proton coming from the interaction of a neutron with a pion. The lifetime of a Delta resonance, for the temperature range of interest, is big enough to perform a complete isospin randomization for collision energies $\sqrt{s_{NN}} \geq 10$ GeV. The prescription to include such a process has been defined in [18], and effectively this correction has a well established effect on the proton distribution. See [19] for more details.

5. Results and conclusions

Using the experimental data from [15] on σ^2/M for net-charge and net-protons, we are able to calculate for each collision energy (11.5, 19.6, 27, 39, 62.4 and 200 GeV) the temperature and the μ_B at the chemical freeze-out. For further details on the calculation see [20]. In Fig. 2 (right) we show a comparison to the previous results by [21].

In our approach, we were able to find a common freeze-out surface which describes simultaneously the net-charge and net-proton low order fluctuations. The agreement with previous results is satisfactory for the μ_B, while our temperature is lower by about 20 MeV. This difference may follow from the fact that the observables we are analyzing are dominated by light quarks, namely pions, kaons and protons, while the former result is obtained from a simultaneous fit of particle multiplicities belonging both to the light and strange sectors. Such a lower freeze-out temperature is suggested also from another analysis of lattice calculations [22] and recent LHC data [23], but further studies to solve this issue will be performed. Preliminary

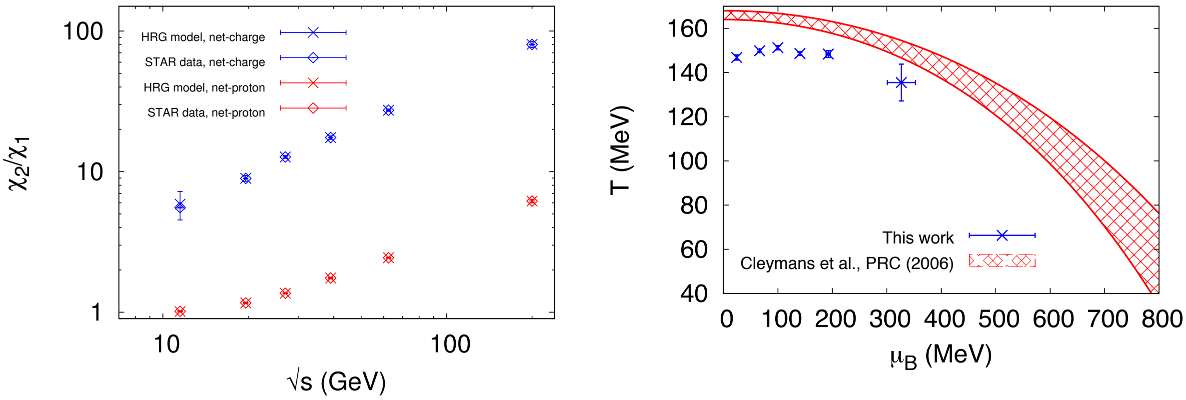

Figure 2. (Color online) Left panel: χ_2/χ_1 calculations using the HRG model with our freeze-out parameters compared to the STAR data [15]; right panel: our freeze-out parameters (blue crosses), compared to the previous result by [21] (red band).

results on the sensitivity of the higher order cumulants are available [24], and confirm our present finding that the lower order fluctuations can be a good candidate to explore the QCD phase diagram.

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