# Chiral phase transition in the vector meson extended linear sigma model

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**Abstract.** In the framework of an SU(3) (axial)vector meson extended linear sigma model with additional constituent quarks and Polyakov loops, we investigate the effects of (axial)vector mesons on the chiral phase transition. The parameters of the Lagrangian are set at zero temperature and we use a hybrid approach where in the effective potential the constituent quarks are treated at one-loop level and all the mesons at tree-level. We have four order parameters, two scalar condensates and two Polyakov loop variables and their temperature and baryochemical potential dependence are determined from the corresponding field equations. We also investigate the changes of the tree-level scalar meson masses in the hot and dense medium.

## 1. Introduction

Forthcoming heavy ion experiments, such as the planned CBM experiment at FAIR will explore the QCD phase diagram in the high density and moderately high temperature region. This region is very interesting since it is believed that the critical endpoint (CEP) – which separates the crossover and first order phase transition regions along the phase boundary – if it exists, should be found there.

Earlier experiments such as RHIC, SPS, or LHC investigated the low density, high temperature region. No sign of the chiral phase transition has been observed experimentally, which is not so surprising considering that the phase transition is of crossover type there, which is very hard to observe experimentally. Theoretically, if the transition is of first order – and the system is infinite – certain quantities have discontinuities crossing the phase boundary, which gives a much higher chance to observe it. Although, since in real physical experiments the system is always finite, there will not be any observable discontinuities, but just peaks.

Due to the difficulties of the observation it is very important to gather as much theoretical information as possible from the phase boundary, in order to assist future experiments. According to that, we would like to investigate the chiral phase transition in the framework of a (axial)vector meson extended linear sigma model with additional constituent quarks and Polyakov loops.

The paper is organized as follows. The model is briefly presented in Sec. 2 and its parametrization is discussed in Sec. 3. The four field equations which provide the temperature and chemical potential dependence of the order parameters are given in Sec. 4. Finally, the results are discussed in Sec. 5, where we also conclude.

## 2. The Model
In terms of matrix valued (pseudo)scalar and (axial)vector meson fields the Lagrangian is,

$$\mathcal{L} = \mathrm{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \, \mathrm{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \, \mathrm{Tr}(\Phi^\dagger \Phi)^2$$
$$- \frac{1}{4} \, \mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)]$$
$$+ c_1 (\det \Phi + \det \Phi^\dagger) + i \frac{g_2}{2} (\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$
$$+ \frac{h_1}{2} \, \mathrm{Tr}(\Phi^\dagger \Phi) \, \mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2 \, \mathrm{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2 h_3 \, \mathrm{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \qquad (1)$$
$$+ g_3 [\mathrm{Tr}(L_\mu L_\nu L^\mu L^\nu) + \mathrm{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4 [\mathrm{Tr}(L_\mu L^\mu L_\nu L^\nu)$$
$$+ \mathrm{Tr}(R_\mu R^\mu R_\nu R^\nu)] + g_5 \, \mathrm{Tr}(L_\mu L^\mu) \, \mathrm{Tr}(R_\nu R^\nu) + g_6 [\mathrm{Tr}(L_\mu L^\mu) \, \mathrm{Tr}(L_\nu L^\nu)$$
$$+ \mathrm{Tr}(R_\mu R^\mu) \, \mathrm{Tr}(R_\nu R^\nu)] + \bar{\Psi} i \gamma_\mu D^\mu \Psi - g_F \bar{\Psi}(\Phi_S + i \gamma_5 \Phi_{PS}) \Psi,$$

where $D^\mu \Phi = \partial^\mu \Phi - i g_1 (L^\mu \Phi - \Phi R^\mu) - i e A_e^\mu [T_3, \Phi]$, $L^{\mu\nu} = \partial^\mu L^\nu - i e A_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - i e A_e^\nu [T_3, L^\mu]\}$, $R^{\mu\nu} = \partial^\mu R^\nu - i e A_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - i e A_e^\nu [T_3, R^\mu]\}$, and $D^\mu = \partial^\mu - i G^\mu$. The field content of the Lagrangian is as follows, $\Phi$ is the scalar/pseudoscalar field, $L^\mu$ and $R^\mu$ are the left and right handed vector fields, $\Psi = (u, d, s)^{\mathrm{T}}$ stands for the constituent quark fields, $G^\mu$ is the gluon field, while $H$ is the external field. As usual, the nonstrange and strange scalar fields are shifted by their expectation values $\phi_N$ and $\phi_S$ (scalar condensates). A previous version of this model, without the constituent quarks and Polyakov loops and with a different anomaly term was soundly analyzed at zero temperature in [1].

## 3. Determination of the parameters of the Lagrangian
In Eq. (1), not considering the unused $g_3, g_4, g_5$, and $g_6$, there are 14 unknown parameters, namely $m_0$ – the bare (pseudo)scalar mass, $\lambda_1$, and $\lambda_2$ – the (pseudo)scalar self-couplings, $c_1$ – the $U_A(1)$ anomaly coupling, $m_1$ – the bare (axial)vector mass, $h_1$, $h_2$ and $h_3$ – the (axial)vector–(pseudo)scalar couplings, $\delta_S$ – the (axial)vector explicit break coupling, $\phi_N$ and $\phi_S$ – the scalar condensates, $g_F$ – the Yukawa coupling, and $g_1$ and $g_2$ – two (axial)vector couplings. Compared to the parametrization done in [1], in the present case we have two modifications: i) there are two additional equations for the constituent quark masses, $m_{u/d} = g_F \phi_N / 2$ and $m_s = g_F \phi_s / \sqrt{2}$, for which we use the values $m_{u/d} = 330$ MeV and $m_s = 500$ MeV; ii) the inclusion of the fermion vacuum fluctuations modifies at $T = \mu = 0$ the masses and decay widths of the model from which the parameters were calculated in [1]. This is because in the present approach the masses used in the parametrization are the curvature masses which are the second derivatives of the grand canonical potential with respect to the meson fields at the minimum. The curvature masses consist of two parts: a usual tree-level part, which can be found in [1], and the vacuum and thermal contributions of the one-loop fermion potential (bosonic fluctuations are neglected). The later one can be found in [2, 3] with Polyakov loops, and in [4] without Polyakov loops. Scanning through the parameter space, curvature masses and decay widths are calculated and compared using a $\chi^2$ minimization method [5] to the experimental data taken from the PDG [6] and the above values of $m_{u/d}$ and $m_s$, as described in [1].

It is important to note, that in the scalar sector there are more physical particles than we can describe with one $q\bar{q}$ nonet, as in nature there are two $a_0$, two $K_0^\star$ and five $f_0$ particles (see [6]). Since in one scalar nonet there are one $a_0$, one $K_0^\star$ and two $f_0$'s, we have $2 \cdot 2 \cdot \binom{5}{2} = 40$ different possibilities for matching the scalar sector with physical particles. However, we consider here only cases where $a_0$ and $K_0^\star$ correspond to the $a_0(980)$ and $K_0^\star(800)$ physical particles.

We apply two different parametrization scenarios here. In the first one we do not fit the very uncertain isoscalars ($f_0^L$, $f_0^H$), consequently $m_0$ and $\lambda_1$ always appear in the same combination $C_1 = m_0^2 + \lambda_1 (\phi_N^2 + \phi_S^2)$ in all the expressions, thus we can not determine them separately. A

**Table 1.** Parameters determined by $\chi^2$ minimization in the two cases

| Parameter | Value(1b) | Value(2) | Parameter | Value(1b) | Value(2) |
|---|---|---|---|---|---|
| $\phi_N$ [GeV] | 0.1359 | 0.1333 | $h_2$ | 4.8765 | 2.7065 |
| $\phi_S$ [GeV] | 0.1400 | 0.13823 | $h_3$ | 4.6523 | 3.7935 |
| $m_0^2$ [GeV$^2$] | $-0.0103$ | 0.0394 | $\delta_S$ [GeV$^2$] | 0.1114 | 0.1178 |
| $m_1^2$ [GeV$^2$] | 0.5600 | 0.5508 | $c_1$ [GeV] | 1.5293 | 1.600 |
| $\lambda_1$ | 0 (undetermined) | $-1.2200$ | $g_1$ | 5.5737 | 5.5761 |
| $\lambda_2$ | 23.0638 | 23.7957 | $g_2$ | 2.1263 | 1.3889 |
| $h_1$ | 0 (undetermined) | 2.6007 | $g_F$ | 4.3650 | 4.4217 |

similar combination $C_2 = m_1^2 + \frac{h_1}{2}\left(\phi_N^2 + \phi_S^2\right)$ appears in the vector sector for $m_1$ and $h_1$ [1]. Practically, we choose $\lambda_1 = h_1 = 0$ in this scenario. Even if we do not fit the isoscalars, we still have the possibility to get parametrizations with different values of $m_{f_0^L}$, which has a huge effect, as will be seen immediately, on the finite $T/\mu_B$ behavior. We consider two subcases, labeled by '1a' and '1b': one with a high and one with a low value of $m_{f_0^L}$, that is 1326 MeV and 402 MeV, respectively. In the second scenario, labeled as case '2', we fit the isoscalars to $f_0(500)$ and $f_0(1370)$. The parameter values for case '1b' and case '2' are given in Table 1.

**4. $T/\mu_B$ dependence of the order parameters and curvature masses**

In medium, the values of the order parameters – which are the two scalar condensates $\phi_N$ and $\phi_S$ and the two Polyakov loop variables $\Phi$ and $\bar{\Phi}$ – change with the temperature/chemical potential. The two scalar condensates encodes the effect of both spontaneous and explicit symmetry breakings, while the Polyakov loop variables mimic some properties of the quark confinement, which naturally emerge in mean field approximation, if one calculates free fermion grand canonical potential on a constant gluon background (for more details see [7]).

In order to determine the $T/\mu_B$ dependence of the order parameters and curvature masses, we use four coupled stationarity equations (field equations), which require the vanishing of the first derivatives of the grand canonical potential with respect to the order parameters. We apply here a hybrid approach, where we only consider vacuum and thermal fluctuations for the fermions and not for the bosons. In this case the equations are given by

$$-\frac{d}{d\Phi}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^-(p)}}{g_q^-(p)} + \frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)}\right) = 0, \qquad (2)$$

$$-\frac{d}{d\bar{\Phi}}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right) + \frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^+(p)}}{g_q^+(p)} + \frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)}\right) = 0, \qquad (3)$$

$$m_0^2\phi_N + \left(\lambda_1 + \frac{1}{2}\lambda_2\right)\phi_N^3 + \lambda_1\phi_N\phi_S^2 - h_N + \frac{g_F}{2}N_c\left(\langle u\bar{u}\rangle_T + \langle d\bar{d}\rangle_T\right) = 0, \qquad (4)$$

$$m_0^2\phi_S + (\lambda_1 + \lambda_2)\phi_S^3 + \lambda_1\phi_N^2\phi_S - h_S + \frac{g_F}{\sqrt{2}}N_c\langle s\bar{s}\rangle_T = 0, \qquad (5)$$

where $U(\Phi,\bar{\Phi})$ is the Polyakov loop potential, for which we used a polynomial form with coefficient taken from [8], and

$$g_q^+(p) = 1 + 3\left(\bar{\Phi} + \Phi e^{-\beta E_q^+(p)}\right)e^{-\beta E_q^+(p)} + e^{-3\beta E_q^+(p)},$$

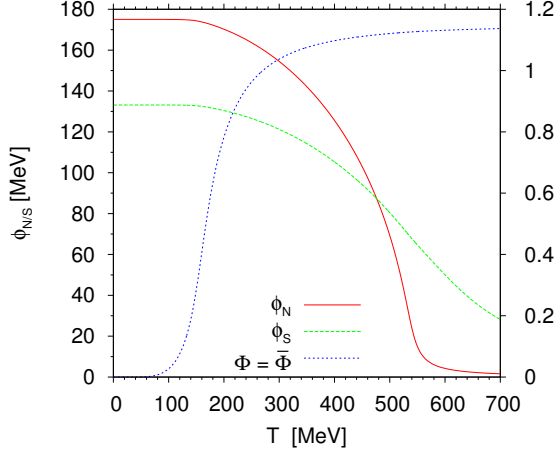$$g_q^-(p) = 1 + 3\left(\Phi + \bar{\Phi} e^{-\beta E_q^-(p)}\right)e^{-\beta E_q^-(p)} + e^{-3\beta E_q^-(p)},$$

**Figure 1.** Temperature dependence of the order parameters with the parametrization '1a' ($m_{f_0^L} = 1326$ MeV).
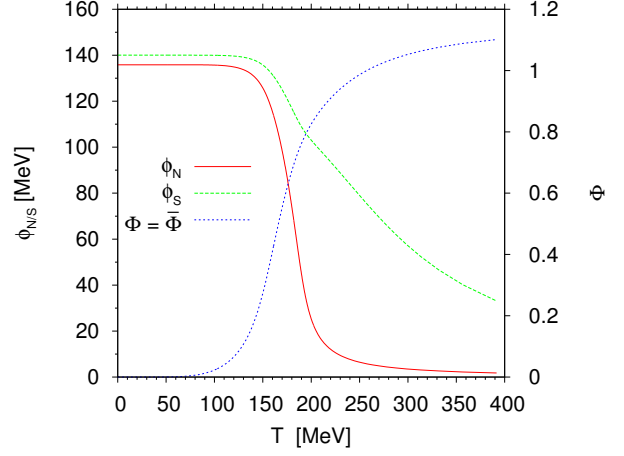


**Figure 2.** Temperature dependence of the order parameters with the parametrization '1b' ($m_{f_0^L} = 402$ MeV).

$$E_q^{\pm}(p) = E_q(p) \mp \mu_B/3, \ E_{u/d}(p) = \sqrt{p^2 + m_{u/d}^2}, \ E_s(p) = \sqrt{p^2 + m_s^2},$$

$$\langle q\bar{q}\rangle_T = -4m_q \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_q(p)} \left(1 - f_\Phi^-(E_q(p)) - f_\Phi^+(E_q(p))\right),$$

with the modified Fermi–Dirac distribution functions

$$f_\Phi^+(E_p) = \frac{\left(\bar{\Phi} + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}{1 + 3\left(\bar{\Phi} + \Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}},$$

$$f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar{\Phi} e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}{1 + 3\left(\Phi + \bar{\Phi} e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}.$$

## 5. Results and Conclusion

By solving the system of Eqs. (2)-(5), the temperature and baryochemical potential dependence of the order parameters and the curvature masses can be determined. In Fig. 1 the order parameters can be seen with the parametrization '1a' at $\mu_B = 0$. Here the pseudocritical temperature ($T_c$) is around 550 MeV, which is much higher than the continuum lattice result [9], which is $T_c = 151$ MeV. In Fig. 2 the order parameters are shown with the parametrization '1b', where $m_{f_0^L} = 402$ MeV. In this case $T_c$ is between $150 - 200$ MeV, which is in the range of the lattice results.

It is a common belief that there is a critical endpoint (CEP) along the phase boundary in the $T - \mu_B$ plane. For this to happen, the phase transition should be of first order as a function of $\mu_B$ along the $T = 0$ axis. However, as can be seen in Fig. 3 (case '1b') the phase transition at $T = 0$ is of crossover type, while in Fig. 4 with parametrization '2' the transition is of first order. In this case the value of the pseudocritical temperature $T_c$ at $\mu_B = 0$ is close to that of case '1b'. The temperature dependence of the pseudo(scalar) curvature masses can be seen in Fig. 5 and Fig. 6 for the parametrization of case '2'.

In conclusion we can say that the parametrization which fulfills the two physical requirements of having a pseudocritical temperature $T_c \approx 150$ MeV at $\mu_B = 0$ and a first order transition in $\mu_B$ at $T = 0$ is realized in case '2', that is when $f_0(500)$ and $f_0(1370)$ are used for parametrization. Similar results were found in [2, 3] without (axial)vector mesons.
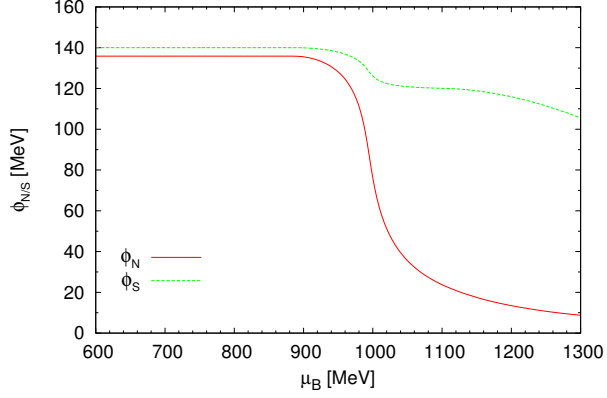
**Figure 3.** $\mu_B$ dependence of the condensates with the parametrization '1b'.
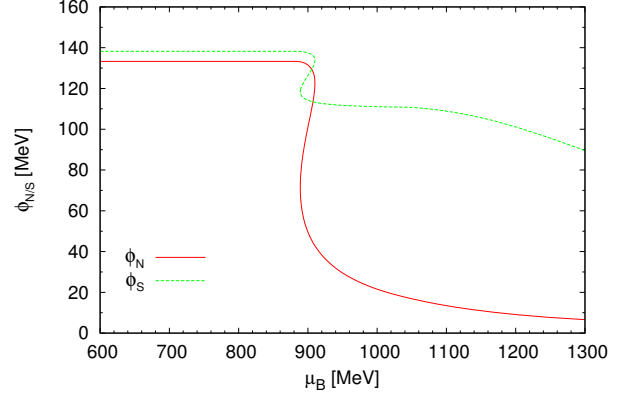


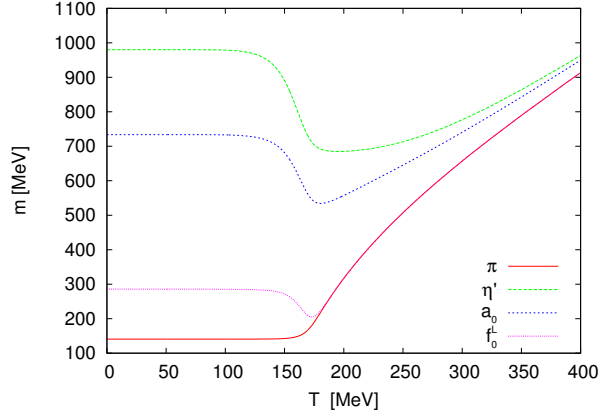**Figure 4.** $\mu_B$ dependence of the condensates with the parametrization '2'.



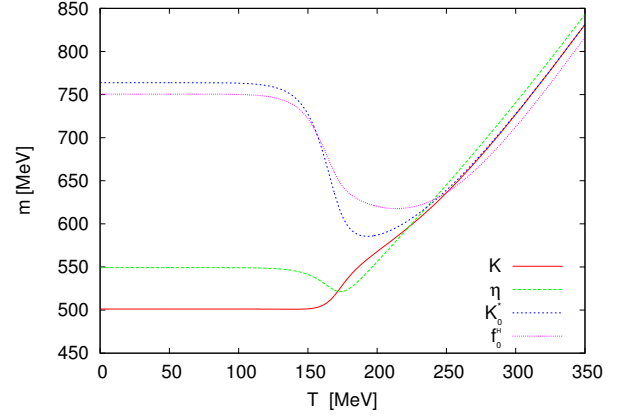**Figure 5.** Temperature dependence of the $\pi$, $\eta'$, $a_0$ and $f_0^L (i.e.\ \sigma)$ particle masses.



**Figure 6.** Temperature dependence of the $K$, $\eta$, $K_0^\star$ and $f_0^H$ particle masses.

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