

# The $g$ -factor of highly charged ions

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**Abstract.** Highly charged ions provide a unique opportunity to test our understanding of atomic properties under extreme conditions: The electric field strength seen by an electron bound to a nucleus at the distance of the Bohr radius ranges from  $10^{10}$  V/cm in hydrogen to  $10^{16}$  V/cm in hydrogenlike uranium. The theory of quantum electrodynamics (QED) allows for calculation *e.g.* of binding energies, transition probabilities or magnetic moments. While at low fields QED is tested to very high precision, new, hypothetical nonlinear effects like photon-photon interaction or a violation of Lorentz symmetry may occur in strong fields which then would lead to an extension of the Standard Model. The ultra-high precision determination of the magnetic moment of a bound electron in a highly charged ion provides a unique possibility to probe the validity of the current Standard Model in extreme conditions.

## 1. Introduction

Calculation of atomic properties reaches the smallest uncertainties for systems containing few or even only one electron. In this contribution we consider the magnetic moment of the electron bound in a hydrogenlike ion as test case for QED calculation. The magnetic moment  $\mu$  is in general expressed by the dimensionless  $g$ -factor

$$\vec{\mu} = -g\mu_B \frac{\vec{s}}{\hbar} \quad (1)$$

with  $\mu_B$  the Bohr magneton and  $s$  the electron's spin. Calculations of the electron's  $g$ -factor in the ground state of hydrogenlike ions as function of the nuclear charge  $Z$  have been performed by different groups [1 and references therein]. They include the relativistic effects of binding (Breit term), of virtual photon exchange in different orders (1- and 2- loop QED), and of nuclear properties and can be written as

$$g = 2 (1 + a_{\text{Breit}} + a_{\text{1loop}} + a_{\text{2loop}} + a_{\text{NuclearSize}} + a_{\text{Recoil}} + \dots). \quad (2)$$

Fig. 1 shows the result of the calculations for low and medium values of  $Z$ . In order to match the theoretical precision and to perform a significant test of the calculated terms the experiment requires

an uncertainty of  $10^{-10}$  or below. In this contribution we describe our experimental approach to reach this goal and our most recent results.

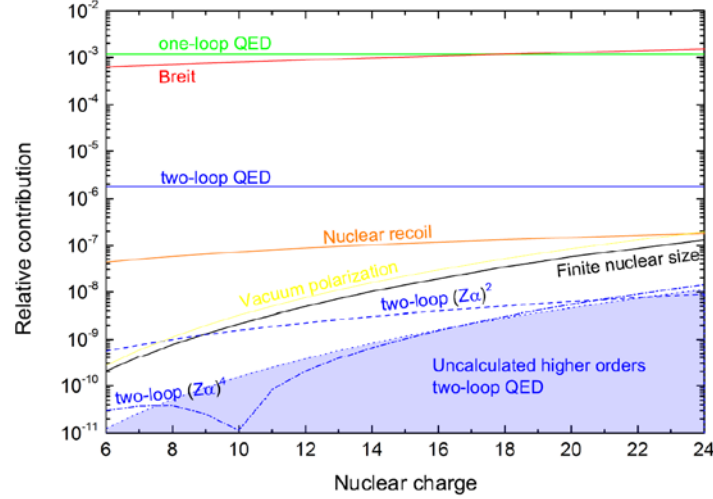


Figure 1: Contributions to the bound electron's  $g$  factor in H-like ions for low and medium nuclear charges (courtesy Z. Harman)

## 2. Experiment

We use a cylindrical Penning trap [2] to confine a single hydrogenlike ion. Measuring the cyclotron frequency of the ion of mass  $M$  and charge  $q$  in the magnetic field  $B$  of the trap as well as the spin precession (Larmor) frequency with  $m$  the electrons mass, we obtain the  $g$ -factor of the electron through the ratio of these frequencies

$$g = 2 \frac{v_L}{v_c} \frac{m}{M} \frac{q}{e} = 2\Gamma \frac{m}{M} \frac{q}{e} . \quad (3)$$

The masses of the electron and of the ion need to be known sufficiently well.

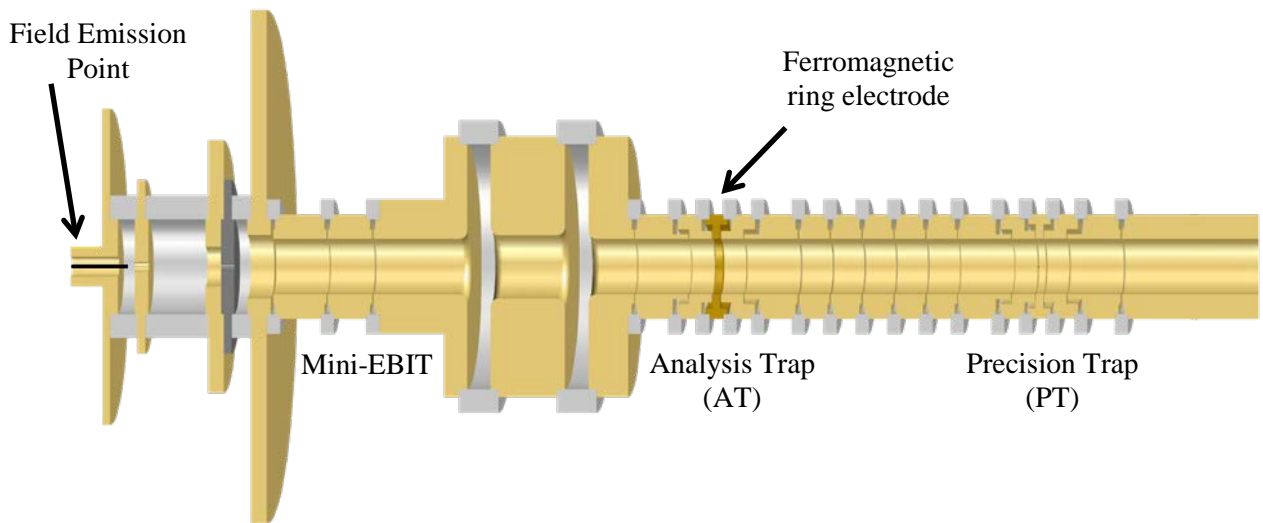


Figure 2: Penning trap arrangement including the Mini-EBIT for the in-situ creation of highly charged ions, the Analysis Trap (AT) for spin-state detection and the Precision Trap (PT) for the determination of the eigenfrequencies.

### 3. Apparatus

The core of our apparatus is a double Penning trap with an attached Mini-EBIT for highly charged ion creation (Fig. 2). Each trap consists of 5 electrodes of 3.5 mm inner radius made from gold plated OFHC copper: a central ring electrode, two endcaps, and two correction electrodes placed between endcaps and ring. A negative voltage applied to the ring electrode (nominally  $\sim 7V$  in our experiment) serves for ion confinement in the axial direction. Radial confinement is provided by a magnetic field of 3.7 T directed along the trap axis. Proper voltages on the correction electrodes make the axial potential harmonic. While one of the traps (precision trap, PT) is made as perfect as possible, in the second one (analysis trap, AT) the central copper ring is replaced by a ferromagnetic material which makes the magnetic field inhomogeneous. This is required, as shown below, for detection of the electron's spin direction.

The trap tower is placed in a hermetically sealed vacuum vessel which is held at liquid helium temperature. Cryopumping serves for ultrahigh vacuum to avoid ion-neutral collisions. We achieved ion storage times of many months, from which an upper limit for the rest-gas pressure of  $p < 10^{-16}$  mbar can be deduced.

### 4. Ion creation and detection

Ions are released from a surface by electron ablation. They are stored in a simple 3-electrode trap arrangement and further ionized in an EBIT-like manner. The ion cloud is transferred to the PT, where unwanted species are removed by selective excitation of their oscillation and the ion number is reduced to the single particle by a careful reduction of the trapping potential. The ion motion is a superposition of three harmonic oscillations with frequencies  $\omega_z/2\pi \approx 670$  kHz (axial oscillation),  $\omega_+/2\pi \approx 27$  MHz (reduced cyclotron oscillation), and  $\omega_-/2\pi \approx 9$  kHz (magnetron oscillation)

For single ion detection a high- $Q$  resonator is attached to a correction electrode. The small (a few fA) image current which the ion induces by its axial oscillation in the trap is amplified by cryogenic electronics and appears as a peak signal in the Fourier transformed noise spectrum of the amplifier. When the ion's axial oscillation is kept continuously in resonance with the detection circuit it will be damped by energy dissipation into the environment until thermal equilibrium at 4.2 K is reached. The ion signal is then a minimum in the noise spectrum as shown in Fig. 3

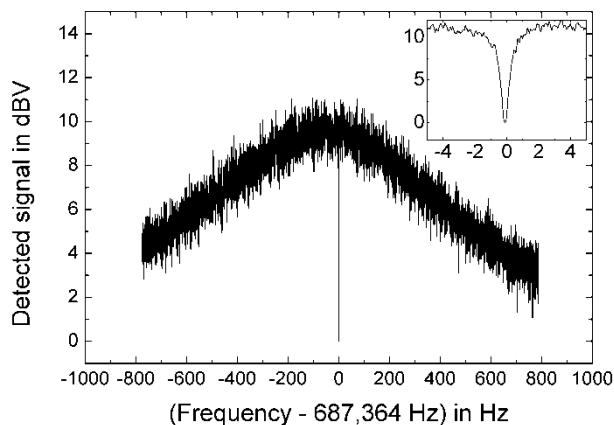


Figure 3: Fourier transform of the amplifier noise with a cooled single ion present in the trap, manifested by a minimum (dip) at the ion's axial oscillation frequency. The insert shows the dip with high resolution.

## 5. Measurement of the ion's oscillation frequencies

Figure 3 indicates that the axial oscillation frequencies can be measured with very high resolution. The modified cyclotron and the magnetron oscillations can be measured by coupling them to the axial mode by an additional radio-frequency field at the frequencies  $\omega_+ - \omega_z$  and  $\omega_+ + \omega_z$ , respectively. In order to reach the required high precision of about  $10^{-10}$  in the frequency measurement, an averaging time of several minutes is required. Magnetic field variations during this time may limit the precision. In order to reduce this time we developed a novel method measuring the phase of the radial motion, called PnA (Phase and Amplify) [3]: After ion cooling of the reduced cyclotron motion a fixed phase is imprinted on the motion by a short pulse at  $\omega_+/2\pi$ . Then a period of free oscillation follows. Finally the reduced cyclotron mode is coupled to the axial one by a strong pulse at  $(\omega_+ + \omega_z)/2\pi$  which allows detection of the phase through the axial motion. Figure 3 shows the timing sequence. The PnA method reduced the time for the determination of the cyclotron frequency to a few seconds, which is essential to reach the required precision. The cyclotron frequency as required for the  $g$ -factor determination (see eqn (3)) is obtained by the so-called invariance relation[4]  $\omega_c^2 = \omega_+^2 + \omega_z^2 + \omega_-^2$ . Possible shifts of the eigenfrequencies caused by trap imperfections cancel in this relation to first order.

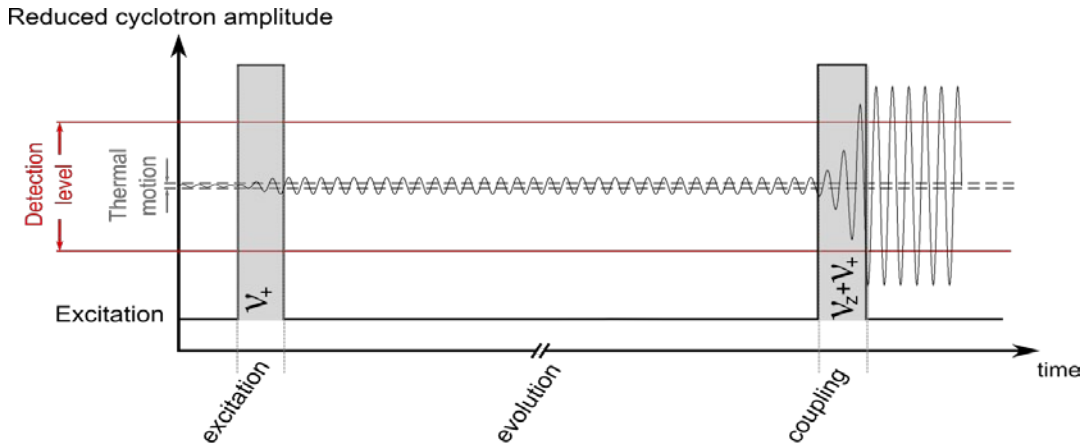


Figure 4: Timing sequence of the PnA method, depicting the low oscillation amplitude during the phase evolution, below the detection threshold of the tank circuit and the large oscillation following the parametric phase-sensitive amplification.

## 6. Measurement of the spin precession frequency

The axial oscillation frequency is the only access to the ion in our experiment. Therefore we have to couple the spin motion to the axial one. This is performed by the inhomogeneity of the magnetic field in the AT [5]. The force of the magnetic field on the magnetic moment associated with the electron's spin adds or subtracts to the electric trapping force, depending on the spin direction, thus changing the axial frequency. The size of this change is given by

$$\Delta \nu_z = \frac{g \mu_B B_2}{4\pi^2 M \nu_z} . \quad (4)$$

where  $B_2$  is the quadratic coefficient in a series expansion of the inhomogeneous  $B$ -field. It amounts for our experimental conditions, depending on the ion's mass, to values between 140 and 600 mHz. Figure 5 shows that a spin flip, induced by a microwave field, can be unambiguously detected.

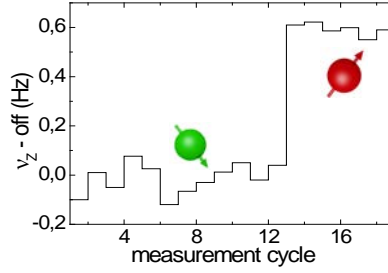


Figure 5: Change of the axial frequency upon an induced spin flip in a single  $^{12}\text{C}^{5+}$  ion.

## 7. $g$ -factor determination

The sequence to perform a  $g$ -factor determination starts by determination of the spin direction in the AT. The resistively cooled ion is adiabatically transferred to the PT, approximately 5 cm apart, where the eigenfrequencies are measured with high precision. Simultaneously, microwaves close to the spin precession frequency are injected into the trap, attempting to flip the spin. The ion is then transferred back to the PT, its spin direction is measured again in order to see whether or not it has changed in the PT. We plot the spin transition probability versus the scanned microwave frequency, divided by the simultaneously measured cyclotron frequency to obtain a resonance curve, from which the  $g$ -factor is obtained. Experiments have been performed on  $^{12}\text{C}^{5+}$  [6],  $^{16}\text{O}^{7+}$  [7], and  $^{28}\text{Si}^{13+}$  [8]. The most precise value has been obtained for  $^{28}\text{Si}^{13+}$ :  $g = 1.995\,348\,958\,7(5)(3)(8)$ . The quoted uncertainties relate to the electron mass, statistical and systematic errors, respectively. The value agrees well with the theoretical prediction  $g = 1.995\,348\,958\,0(17)$  [8], and represents to date the most precise test of QED in strong fields.

## 8. The electron's atomic mass

The experimental accuracy obtained in our experiments is independent of the ion under investigation. Therefore, we repeated the earlier experiment on  $^{12}\text{C}^{5+}$  with now improved experimental precision. Moreover, the various contributions to the  $g$ -factor are for  $^{12}\text{C}^{5+}$  significantly smaller than for  $^{28}\text{Si}^{13+}$  and have been calculated to the  $10^{-11}$  level. The mass of  $^{12}\text{C}^{5+}$  shows only a small uncertainty due to the binding energies of the missing electrons, since the neutral carbon atom is the basis of the definition of the atomic mass scale. The agreement between theory and experiment in the case of  $^{28}\text{Si}^{13+}$  lead to the conclusion that theory is correct at the level of our experiment. The theoretical uncertainty is dominated by the yet uncalculated QED terms of order  $(Z\alpha)^5$  and higher, which in turn allows to even cancel the leading-order contribution by scaling the difference of experimentally measured and calculated  $g$ -factors for hydrogenlike  $^{28}\text{Si}^{13+}$ . Therefore, we can use experimental and theoretical results to obtain a new value of the atomic mass of the electron by rewriting eqn. (3):

$$m = \frac{g_{theo}}{2} \frac{\omega_c}{\omega_L} \frac{e}{q} M \quad (5)$$

Our result of  $m = 0.000\,548\,579\,909\,067(16)$  u [9] improves the presently listed value in the CODATA tables of fundamental constants [10] by more than an order of magnitude.

## 9. $g$ -factor of high- $Z$ ions

For technical reasons the accessible hydrogenlike ions in our experiment are limited to  $\text{Ca}^{19+}$ . Since the QED contributions to the  $g$ -factor scale approximately with  $Z^2$ , higher- $Z$  ions would allow more

stringent tests of QED. To this end, a new setup at the Max-Planck Institute for Nuclear Physics in Heidelberg is currently in the commissioning phase which will allow injecting heavy highly-charged ions from an external source. While in the first phase these ions will be provided by the MPIK Super-EBIT, the system is also capable of accepting ions and possibly antiprotons from the HITRAP cooler trap at the future FLAIR facility.

## 10. The proton's atomic mass

Our method to use phase sensitive detection of the ions oscillation frequencies will be used to improve the present value of the proton's atomic mass. Presently a significant contribution to our error budget is given by the fluctuations of the magnetic field strength during the time of the measurement. Its influence can be reduced by a measurement of the cyclotron frequency of ions simultaneously stored in different potential minima of a multi-electrode trap. We aim for an uncertainty in the proton's mass below  $10^{-11}$ .

## 11. Acknowledgements

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