Investigations of the QCD Phase Diagram with Dyson-Schwinger Equations

Christian A. Welzbacher

Institut für Theoretische Physik - Justus-Liebig-Universität Giessen

Fairness 2014, Vietri sul Mare

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C. S. Fischer, J. Luecker, CAW

Outline

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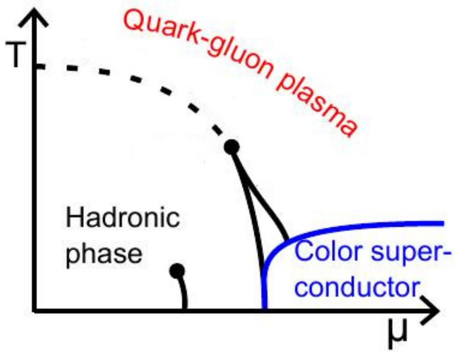
"In the beginning there was nothing, which exploded"

(Sir Terry Pratchett)

Whats the matter? Varieties of the QCD phase diagram:



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- **Various** interesting features (Big Bang, neutron stars, different phases and transitions)
- **Various** attempts to pin down (lattice QCD, models, functional approaches)

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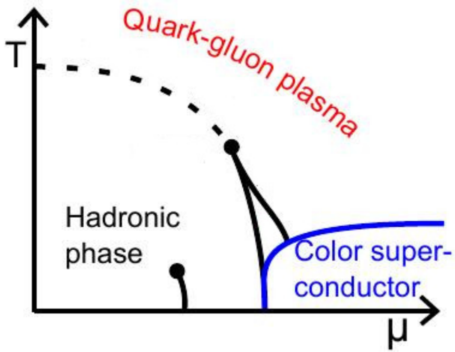

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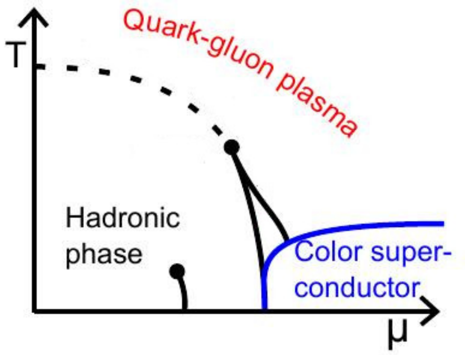

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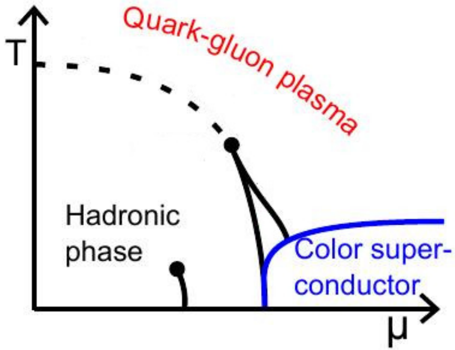

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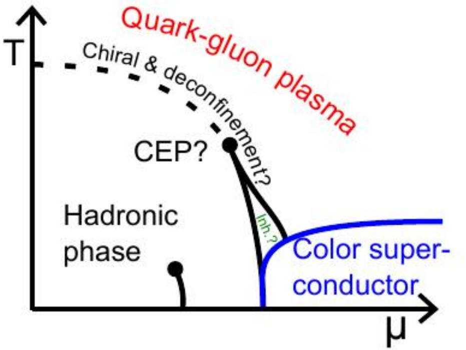

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- Chiral and deconfinement transitions
- Impact of charm quark

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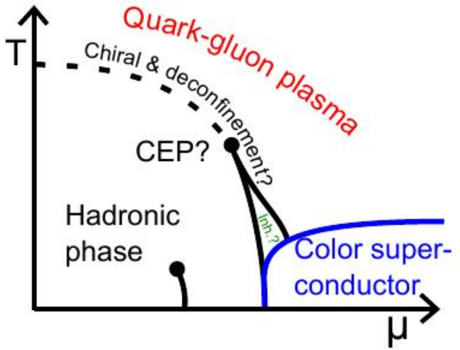

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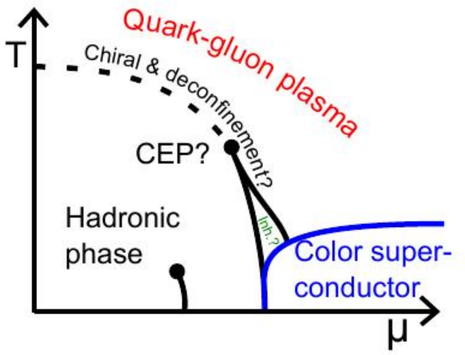

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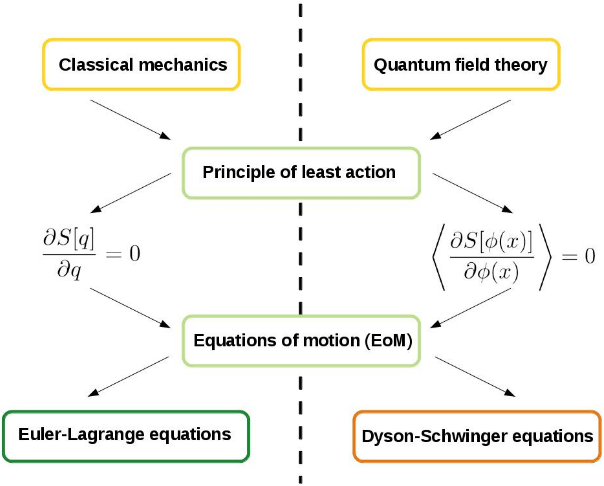
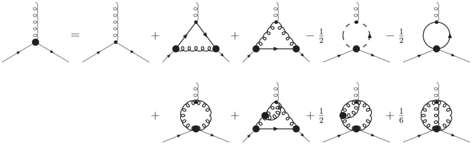
- **Functional approaches:**
  - No sign problem
  - QCD degrees of freedom
  - Truncation needed

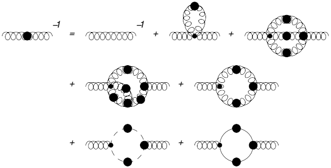
$\rightarrow$ **Dyson-Schwinger equations (DSEs)**

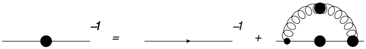
What are Dyson-Schwinger equations?



An infinite tower of coupled integral equations?







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Up and down quark
(2011, 2012)

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N_f=2+1

Strange quark
(2012)

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N_f=2	N_f=2+1	N_f=2+1+1
Up and down quark (2011, 2012)	Strange quark (2012)	Charm quark **(now)**

Outline

Quark propagator in hot and dense matter

Bare quark propagator (vacuum)

$$S_0^{-1}(p) = \quad i p_\mu \gamma^\mu + m_0$$

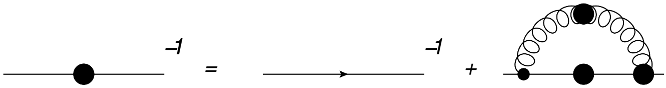
Bare quark propagator (finite T, μ)

$$S_0^{-1}(p) = \quad i\vec{p}\vec{\gamma} + i(\omega_n + i\mu)\gamma_4 + m_0$$

Dressed quark propagator (finite T, μ)

$$S^{-1}(p) = \quad i\vec{p}\vec{\gamma}A(p) + i(\omega_n + i\mu)\gamma_4 C(p) + B(p)$$

Quark DSE in hot and dense matter



- Coupled integral equation
- Base of infinite tower of equations
- Selfconsistently calculate dressing functions A(p), B(p) and C(p)
- Depends (directly) on:
  1. Full quark-gluon vertex
  2. Fully dressed gluon propagator

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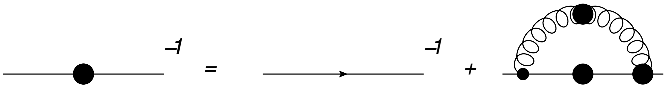

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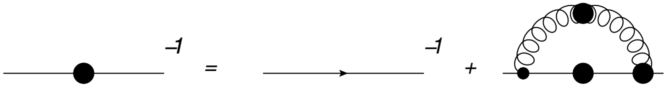

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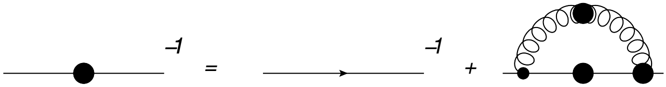

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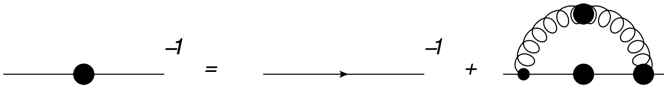

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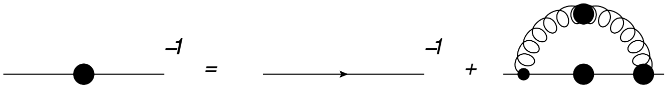

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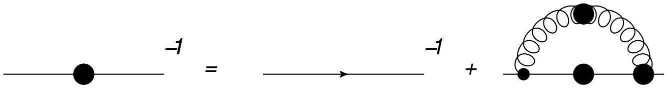

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Gluon propagator

Dressed gluon propagator at finite T (and μ)

$$D_{\mu\nu}(p) \quad = \quad P^L_{\mu\nu}(p)\frac{Z^L(p)}{p^2} + P^T_{\mu\nu}(p)\frac{Z^T(p)}{p^2}$$

Finite temperature gluon fully determined by dressing functions $Z^L(p)$ and $Z^T(p)$

Gluon DSE

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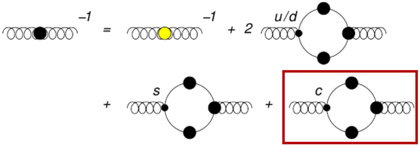
$$D^{-1}_{\mu\nu}(p) = \left[D^{que.}_{\mu\nu}(p)\right]^{-1} + \sum_f \Pi^f_{\mu\nu}(p)$$



- Use input from lattice QCD for quenched gluon propagator $D^{que.}_{\mu\nu}(p)$
- Calculate quark loop for each flavor
- Quenched gluon propagator from *C.F. Fischer et al., Eur. Phys. J.C. 68, 165 (2010)*

Quark-gluon vertex

Quark-gluon vertex ansatz

$$\Gamma_\mu(l,p;q) = \gamma_\mu \cdot \Gamma_{[d_1]}(l^2,p^2,q^2) \cdot \left(\delta_{\mu,4} \frac{C(l)+C(p)}{2} + \delta_{\mu,i} \frac{A(l)+A(p)}{2} \right)$$

- Designed along symmetries and constraints
- Depends on temperature, chemical potential and quark flavor via first term of the Ball-Chiu vertex
- Vertex dressing function depends on parameter d_1 (interaction strength at small momenta) $\rightarrow$ d_1 *becomes important when including the charm quark*

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Chiral condensate

Quark condensate

$$\langle\bar{\psi}\psi\rangle_f \propto \int \mathrm{Tr}_D\left[S^f(p)\right]$$
$$\approx c(T,\mu) + m_0^f \cdot \Lambda^2$$

Regularized condensate

$$\Delta_{l,s} = \langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s$$

- Order paramter for chiral symmetry (exact in chiral limit)
- There are different ways to extract (different) T_C for crossover, we use:
  1. Maximum of chiral susceptibility: $\frac{\partial\langle\bar{\psi}\psi\rangle}{\partial m}$
  2. Inflection point of chiral condensate: $\frac{\partial\langle\bar{\psi}\psi\rangle}{\partial T}$

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Polyakov loop

Polyakov loop

$$L[A_0] = \frac{1}{N_C} \mathrm{Tr}\, P e^{ig \int_0^\beta d\tau A_0(\vec{x},\tau)}$$

$$\langle L[A_0] \rangle \propto e^{-F_q/T} = \begin{cases} 0 & \text{if } F_q = \infty \\ \text{non-zero} & \text{if } F_q < \infty \end{cases}$$

- Order parameter for deconfinement ($F_q = \infty \to$ no free quarks)
- Polyakov loop of minimum of background field potential upper bound for expectation value of full Polyakov loop
- For more details see *C.F. Fischer et al., PLB 732 (2014)* and *Fister and Pawlowski, PRD 88 (2013)*

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Some time passes...

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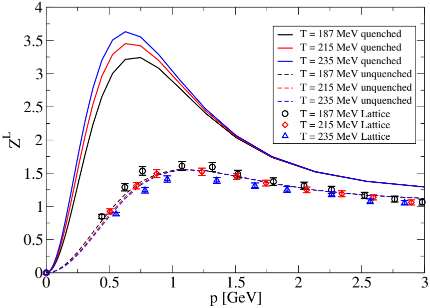
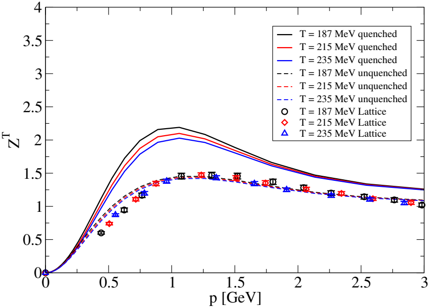
Unquenched gluon propagator N_f=2





- Gluon propagators for N_f=2 with m_π=316 MeV
- DSE results calculated before lattice results
- Lattice results from *R. Aouane et al., Phys.Rev. D87 (2013) 11, 114502*

$\rightarrow$ Procedure works qualitatively (quantitatively on a good level)

Including the charm quark

Remember...

1. Fix d_1 to reproduce T_c of lattice QCD results for N_f=2+1 at μ=0 MeV and add charm quark
   $\rightarrow$ Sets A
2. Fix d_1 to reproduce the scale from vacuum physics in the same truncation (N_f=2+1 and N_f=2+1+1 seperately via Bethe-Salpeter equation (BSE), see *W. Heupel, T. Goecke, C.F. Fischer, Eur.Phys.J. A50 (2014) 85*)
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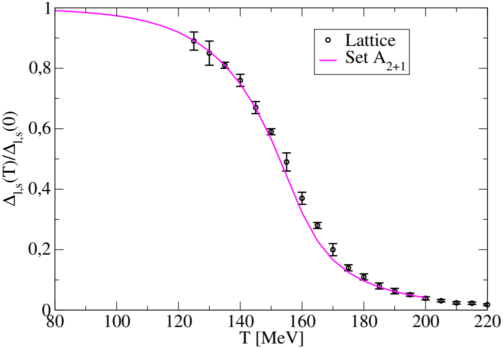
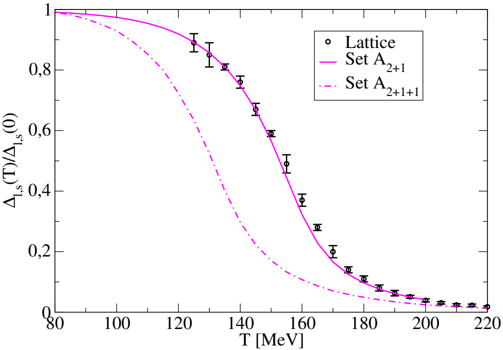
Results at $\mu = 0$ MeV I



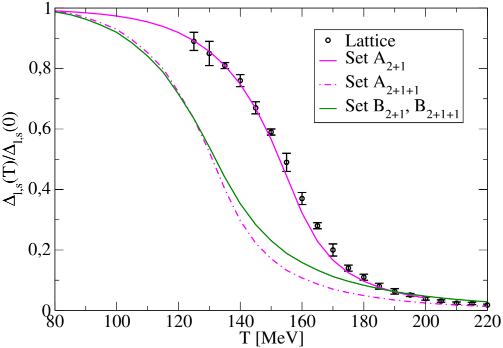
- Tuned d_1 to get good agreement with lattice data *Borsanyi et al. JHEP 1009 073*
- *Nontrivial result: perfect agreement for steepness for Set A_{2+1}*

Results at $\mu = 0$ MeV II



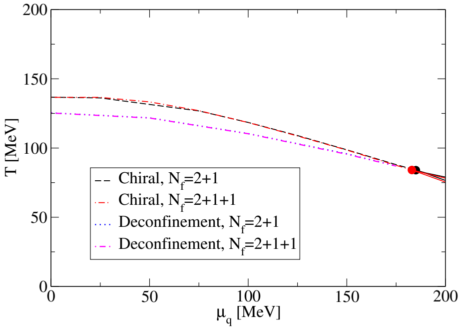
- Steepness is conserved
- *Adding the charm quark without adjustment shifts the curve to lower temperatures for Set A_{2+1+1}* ($\Delta T_c \approx -18$ MeV)

Results at $\mu = 0$ MeV III



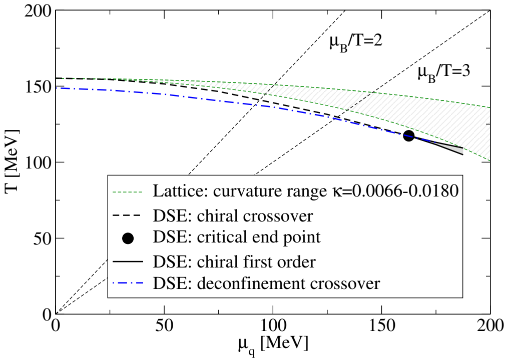
- Shape of the curve changed slightly
- *Chiral condensate for Sets B does not differ for N_f=2+1 and N_f=2+1+1*
- Does that continue for $\mu > 0$?

Phase diagram Sets B



- Difference $\Delta T_c \approx -23$ MeV compared to lattice results at $\mu = 0$ MeV *Borsanyi et al. JHEP 1009 073*
  $\rightarrow$ systematical error for scale set in truncation
- *Physics fixed in vacuum $\rightarrow$ no influence of charm quark within numerical resolution for Sets B*

Phase diagram - prediction



- Chiral crossover defined via inflection point, Set A_{2+1}
- Curvature lattice from *G. Endrodi et al.,JHEP 1104, 001 (2011)*, *O. Kaczmarek et al.,Phys.Rev. D83, 014504 (2011)* and *P. Cea et al.,Phys.Rev. D89, 074512 (2014)*

Outline

Conclusion and outlook

Conclusion

- Used Dyson-Schwinger equations to calculate quark and gluon propagators in medium
- Results quantitatively comparable with lattice QCD
- Prediction for the phase diagram for N_f=2+1 holds also for N_f=2+1+1 due to little influence of charm quark

Outlook

- Baryonic (future: mesonic) effects under investigation
- Quark-gluon vertex?
- Spectral properties of the quark

THANK YOU FOR YOUR ATTENTION!

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