^{8}B in Fermionic Molecular Dynamics

K. R. Henninger, H. Feldmeier, T. Neff

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GSI Helmholtzzentrum für Schwerionenforschung GmbH

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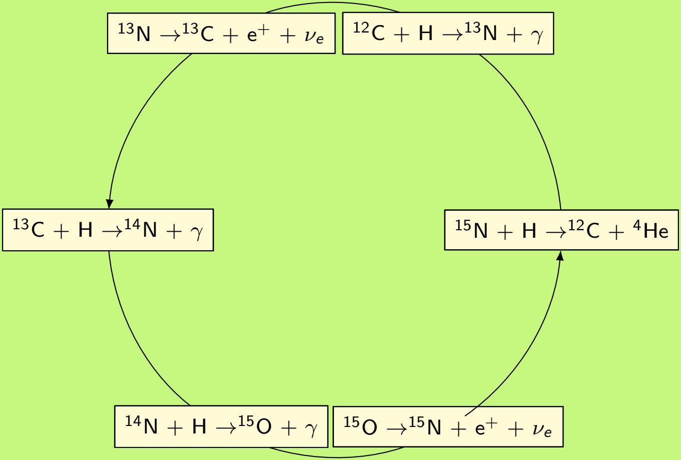
Helmholtz Graduate School for Hadron and Ion Research

Motivation

The creation of stable nuclei takes place via nuclei far from the stability line; so a knowledge of their properties is needed in order to understand the relevant nuclear processes.
Ingemar Bergström, Lysekil 1966

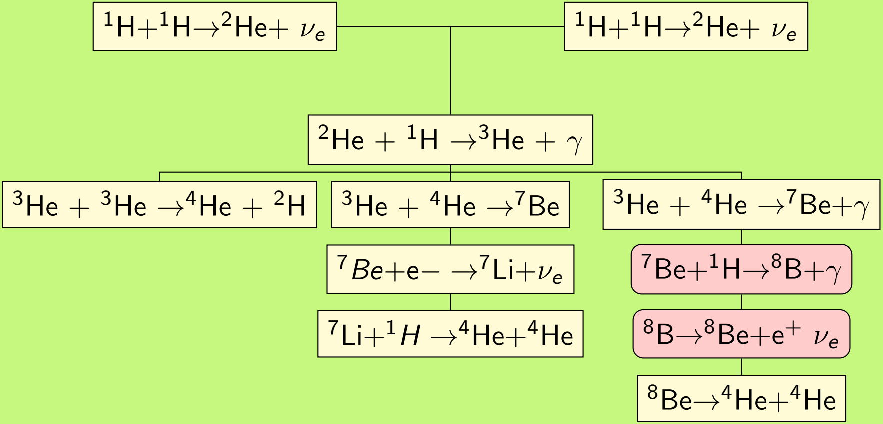
Motivation

The CNO cycle



Motivation

The pp chain



Motivation

Reaction rates

Reaction rate for a particle-induced nonresonant reaction:

$$\sigma(E) = S(E) \times E^{-1} \times e^{-2\pi\eta}$$

$$\eta = (F) Z_1 Z_2 (\tfrac{\mu}{E})^{1/2}$$

- Reaction rate for ^{8}B production and decay: solar core temperature.
- At the low energies relevant for some astrophysical processes, microscopic theoretical calculations may give more accessible information than experimental results.

Motivation

Understanding the effect on structure of neutron extremes is important: *r*-process!

- Occurs via nuclei at the extremes of neutron-richness.
- For abundances; need Q-values; thus masses and thus knowledge of interaction!
- Density-dependence of symmetry term; maybe systematics with changing p:n ratio.

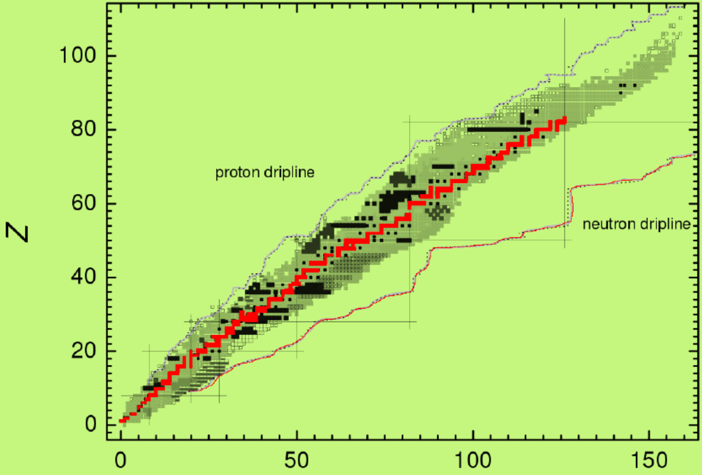
Exotic nuclei:

Definition

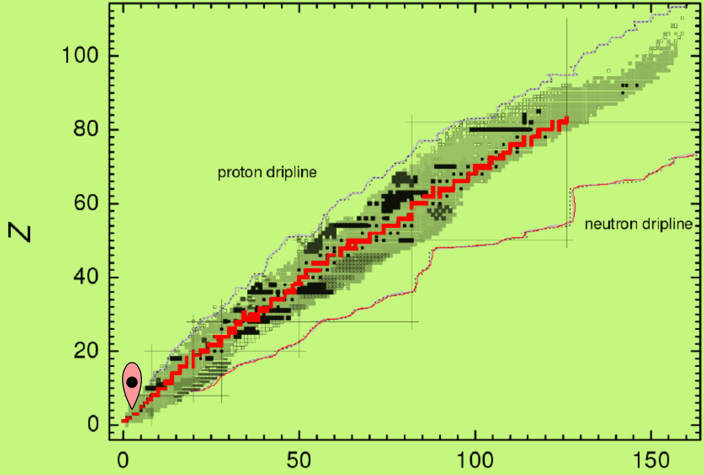
Nuclei with proton:neutron (p:n) ratios that cause them to lie far out of the valley of stability are exotic nuclei.

- Changing p:n ratio $\rightarrow$ some terms become emphasised; others de-emphasised (extreme case $\rightarrow$ neutron matter).
- New structures possible - *e.g.* haloes.

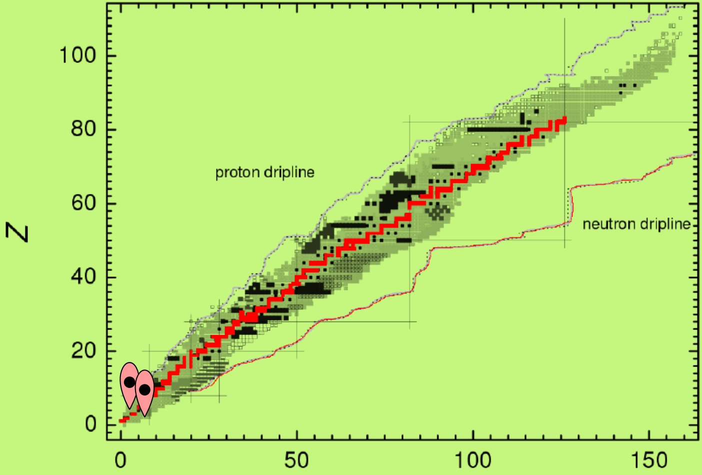
Exotic Nuclei



Exotic Nuclei



Exotic Nuclei



Haloes

Definition

Halo nucleon: A loosely-bound nucleon with more than 50% of its probability density outside the core.

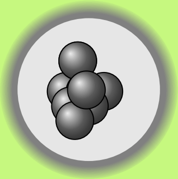


Figure: Artist's impression of a 1-nucleon halo *e.g.* (^{8}B)

- Short-range correlations (scattering to high-k states), reduced *centrifugal barrier* at low l.

Clusters

Definition

A spatially-localised subsystem of strongly-correlated nucleons.

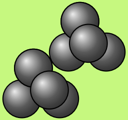


Figure: Two alpha-clusters (^{8}Be)

- Clustering happens near a threshold [Ikeda, 1968].
- Clusters are often resonances.
- "Cluster structure is developed and stabilized in some neutron-rich nuclei" [Hagino, 2012]

Bottom line: Haloes and clusters involve coupling to the continuum!

- Cannot model haloes or clusters in a closed quantum system!
- Clustering: access deformed configurations.
- Halo: Correct asymptotics.

FMD Model

Picture for a moment what one is trying to model:

- System of nucleons with ≈ 30 MeV/nucleon.
- Nucleons are the degrees of freedom.
- Structure of system dictated by nucleon distributions.
- Nucleon distribution dictated by interaction with other nucleons in the system.

FMD Model

Most intuitive way to model such a thing in its ground state is via a variational method:

- Make an ansatz many-body wavefunction

$$|Q\rangle = \hat{\mathcal{A}}\left\{|q_1^{\mu_1}\rangle \otimes \ldots \otimes |q_A^{\mu_n}\rangle\right\}.$$

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$$\min_{\{q_\nu^{\mu_i}\}} \quad \frac{\langle Q|\hat{H} - \hat{T}_{cm}|Q\rangle}{\langle Q|Q\rangle}, \tag{1}$$

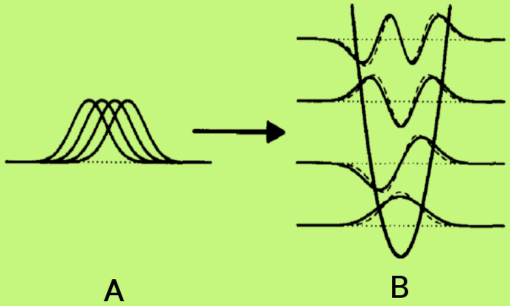
Clustering may happen, but is not imposed *a priori*.

FMD Model

Fermionic Molecular Dynamics:

- One Slater determinant is not enough!
- Make a basis set by minimising subject to constraints.
- Each SD represents a state that contributes to the structure.
- The Hamiltonian is diagonalised in this space to give the overall nuclear states $|\Psi\rangle$.
- The nucleus *is* by nature a superposition of many configurations. Have to include all such, as is done here.

The states $|Q\rangle$ are able to incorporate shell-model states; because antisymmetrised products of Gaussians can give shell-model states.



[H. Feldmeier *et al.* NPA 586 493 1995]

- There is also the possibility of deformed states (Nilsson-type states).

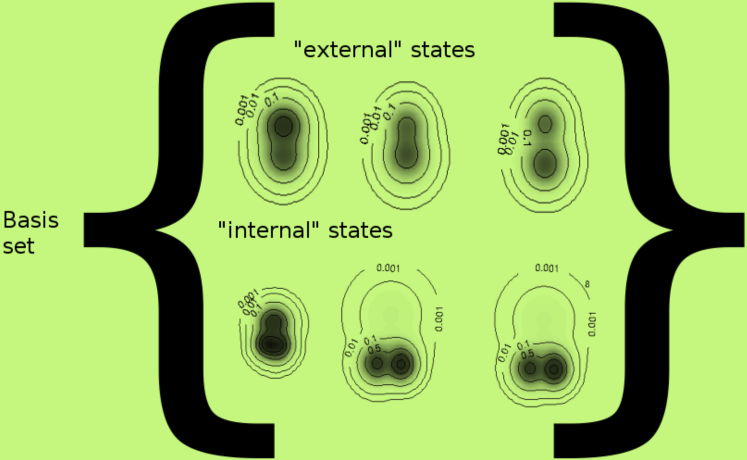
What about very polarised configurations?

- Put clustering in explicitly.
- These "frozen" cluster states are antisymmetrised products of the cluster wavefunctions (RGM-type states). That is:

$$|Q(r); [l]^{J^\pi}_M\rangle = \mathcal{A}\left\{|r, l\rangle \otimes |Q_{c1}\rangle \otimes |Q_{c2}\rangle\right\}$$

- Such configurations approximate scattering states.

A complete basis.



^{8}B proton halo

- Confirmed by longitudinal momentum measurements [Smedburg *et al.*, PLB **452** 1 1999].
- Large quadrupole moment (68.3 $\pm$2.1 mb) [Minamisono *et al.* PRL **69**, 2058, 1992].
- Proton-separation energy of 0.137 MeV.
- Can reproduce large quadrupole moment without halo [*e.g.* NPA 567, 341 1994].
- No measurements for radii yet.

Basis set:

- 58 Basis states.
- Constraint on proton and neutron radius.
- Constraints access proton halo and distorted core.
- Proton radius: 2.1 fm to 3.9 fm.
- Neutron radius: 1.8 fm to 2.3 fm.

Results

Excited states:

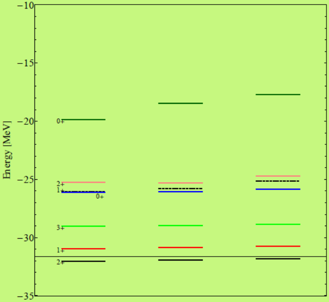


Figure: Level-scheme for various Hilbert spaces compared to experiment. Threshold is calculated.

Excited states:

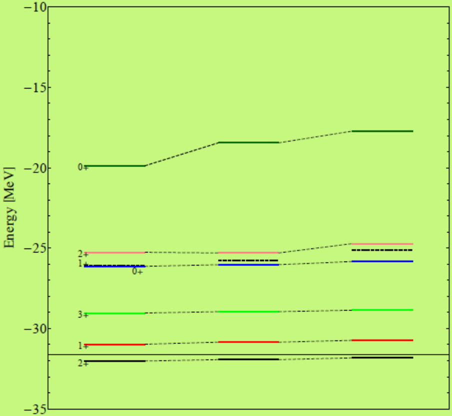


Figure: Level-scheme for various Hilbert spaces compared to experiment. Threshold is calculated.

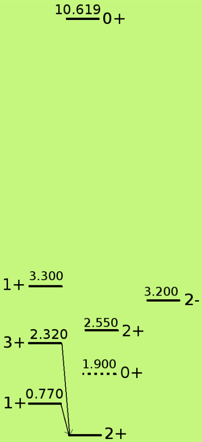
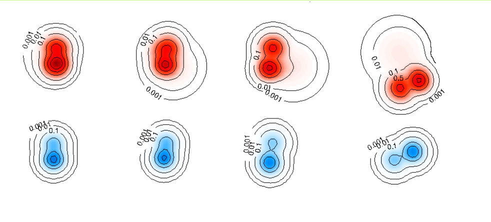
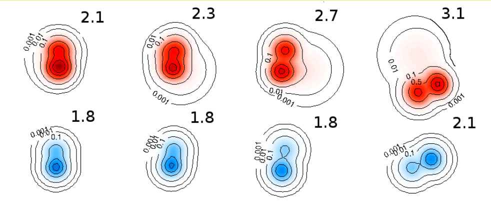


Figure: The levels and known transitions in ^{8}B.



A selection of proton densities (red) and neutron densities (blue) for 2^+ states with lowest projected energy cross-section in the y-z plane).



A selection of proton densities (red) and neutron densities (blue) for 2^+ states with lowest projected energy cross-section in the y-z plane).

Table: Calculated radii for the ^{8}B ground state (full basis set).

R_{matter}	2.258 fm
R_{proton}	2.361 fm
$R_{neutron}$	2.074 fm
R_{charge}	2.511 fm

Table: Calculated transition strengths compared to experiment.

Trans. [MeV]	Type	Basis	$B(\Lambda M)$ (meas.)	(calc.)
0.77	$M1$	R_P-R_N	2.63(12) μ_0^2	4.582 μ_0^2
2.32	$M1$	R_P-R_N	0.38(19) μ_0^2	0.393 μ_0^2

Interaction:

We use a UCOM-modified AV18 interaction.

$$
\begin{aligned}
\hat{V}(\overline{r}_i, \overline{p}_i, \overline{\sigma}_i, \overline{\tau}_i) = & \hat{V}(r) + \hat{V}^{\sigma}(r)\overline{\sigma}_1 \cdot \overline{\sigma}_2 + \hat{V}^{\tau}(r)\overline{\tau}_1 \cdot \overline{\tau}_2 + \\
& \hat{V}^{\sigma,\tau}(r)(\overline{\sigma}_1 \cdot \overline{\sigma}_2)(\overline{\tau}_1 \cdot \overline{\tau}_2) + \hat{V}_{l^2}(r)\overline{L}^2 + \\
& \hat{V}^{\sigma}_{l^2}(r)(\overline{\sigma}_1 \cdot \overline{\sigma}_2)\overline{L}^2 + \hat{V}^{\tau}_{l^2}(r)(\overline{\tau}_1 \cdot \overline{\tau}_2)\overline{L}^2 + \\
& \hat{V}^{\sigma t}_{l^2}(r)(\overline{\sigma}_1 \cdot \overline{\sigma}_2)(\overline{\tau}_1 \cdot \overline{\tau}_2)\overline{L}^2 + \\
& \hat{V}_{ls}(r)(\overline{L} \cdot \overline{S}) + \hat{V}^{\tau}_{ls}(r)(\overline{\tau}_1 \cdot \overline{\tau}_2)(\overline{L} \cdot \overline{S}) + \\
& \hat{V}_{ls^2}(r)(\overline{L} \cdot \overline{S})^2 + \hat{V}^{\tau}_{ls^2}(r)(\overline{\tau}_1 \cdot \overline{\tau}_2)(\overline{L} \cdot \overline{S})^2 + \\
& \hat{V}_t S_{12} + \hat{V}^t_t(r)(\overline{\tau}_t \cdot \overline{\tau}_2) S_{12} + \\
& \hat{V}_i T_{12} + \hat{V}^{\tau}_i(r)(\overline{\sigma}_1 \cdot \overline{\sigma}_2) T_{12} + \\
& \hat{V}_{ti} S_{12} T_{12} + \hat{V}^{\tau}(r)(\overline{\tau}_{1z} \cdot \overline{\tau}_{2z})
\end{aligned}
$$

UCOM method (in brief!)

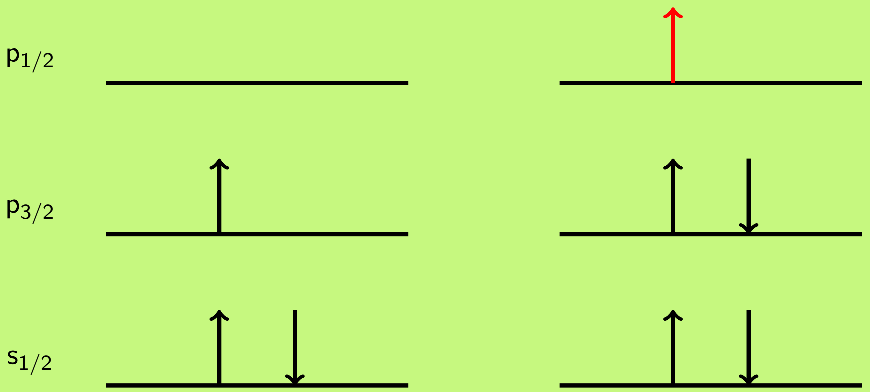
- UCOM is a way to incorporate short-range correlations explicitly, within the model-space.
- The short-range correlations are incorporated by means of a correlation operator $\hat{C}$.
- Either modify $|\Psi\rangle$ or make an effective operator $\hat{C}^\dagger \hat{H} \hat{C}$

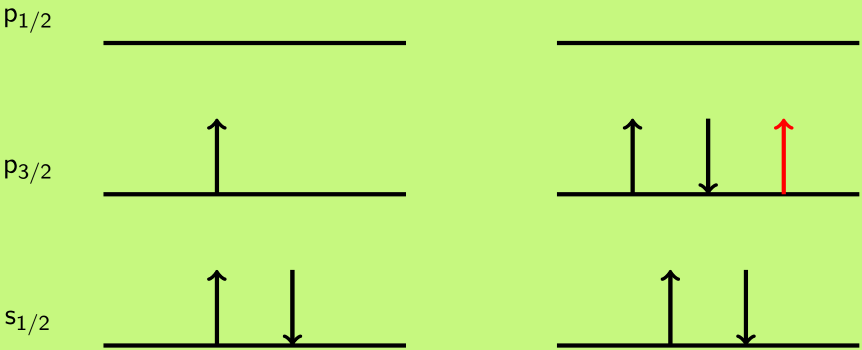
$$\langle\tilde{\Psi}|\hat{H}|\tilde{\Psi}\rangle = \langle\Psi|\hat{C}^\dagger \hat{H} \hat{C}|\Psi\rangle. \quad (2)$$

- $\hat{C}$ has the form $\hat{C} = e^{-i\hat{G}}$, where $\hat{G}$ is a two-body "shift" operator.
- The effective Hamiltonian $\hat{C}^\dagger \hat{H} \hat{C}$ thus has many-body terms.
- We truncate at 2-body, but still need effects of missing many-body terms [NPA **632** 61 1998].
- The $\overline{L} \cdot \overline{S}$ term is associated with these terms and can be weighted to compensate for them.

Table: Calculated transition strengths compared to experiment.

Trans. [MeV]	Type	Basis	$B(\Lambda M)$ (meas.)	(calc.)
0.77	$M1$	R_P-R_N	2.63(12) μ_0^2	4.582 μ_0^2
		R_P-R_N LS 1.5		4.413
2.32	$M1$	R_P-R_N	0.38(19) μ_0^2	0.393 μ_0^2
		R_P-R_N LS 1.5		0.194


$p_{1/2}$
$p_{3/2}$
$s_{1/2}$


$p_{1/2}$
$p_{3/2}$
$s_{1/2}$

Additional constraint:

An idea is to program a constraint on the determinant of quadrupole moment $det|Q|$

One could thus determine whether prolate or oblate configuration of ^{8}B is more favoured.

Conclusion

- The structure of light exotic nuclei constitutes important input for stellar nucleosynthetic reaction rates.
- The FMD, due to its flexible basis, is capable of accessing structure of light exotics.
- The light exotic ^{8}B appears to have a 1-proton halo.
- Work on the role of core-deformation and the placement of the odd proton is continuing.

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