Meson spectroscopy in a study of the Bethe-Salpeter equation

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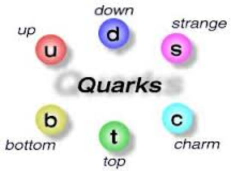
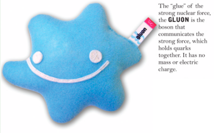
Talk Objectives

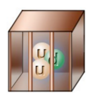
1. **Introduction**
2. Dyson-Schwinger eq. + Bethe-Salpeter eq.
3. Light quark meson spectroscopy
4. Heavy quark meson spectroscopy
5. Rainbow-Ladder variation

Quarks and Gluons

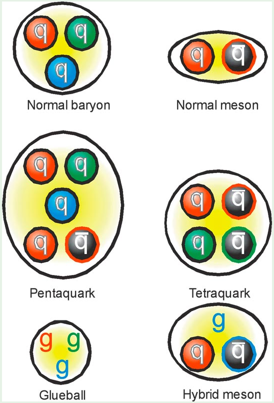




But are confined and not detectable:



Hadrons



Are detectable*, but not a basic objects of the QCD.

Several way to deal with this:

- Effective theories in terms of hadrons
- Nonperturbative QCD: Lattice, Functional methods

Mass generation by weak and strong interaction:

	Quark masses				
	up	down	strange	charm	bottom
M_{weak} $[MeV/c^2]$	3	5	80	1200	4500
M_{strong} $[MeV/c^2]$	350	350	350	350	350
M_{total} $[MeV/c^2]$	350	350	450	1500	4800

The weak masses are input parameters

Nonperturbative QCD: approaches for spectroscopy

Lattice

- Ab initio
- Gauge invariant

Quark models

- Constituent quarks
- Quark-quark potential

Functional method: Bethe-Salpeter equation

- Infinite volume and continuum limit
- Poincare invariance

QCD in Functional approach

QCD generating functional:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, \mathcal{A}] exp \left\{ - \int d^4x \left(\bar{\Psi}(i\not{D} - m)\Psi - \frac{1}{4}(F^a_{\mu\nu})^2 + ... \right) \right\}$$

Quark and Gluon propagators in momentum space:

$$\mathcal{D}^{Gluon}_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}$$

$$\mathcal{S}^{Quark} = \left[-i\not{p} A(p^2) + B(p^2) \right]^{-1}$$

- Euclidean space
- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$
- $D_\mu = \partial_\mu + igt^a A^a_\mu$
- Landau gauge $\partial_\mu A^a_\mu = 0$

The idea is to extract the gauge independent information from the gauge dependent approach

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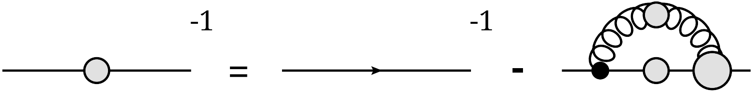
The Dyson-Schwinger equations



$$S^{-1}(p) = Z_2 S_0^{-1} + g^2 Z_{1f} \int_k \gamma^\mu S(k) \Gamma^\nu(k,p) D_{\mu\nu}$$

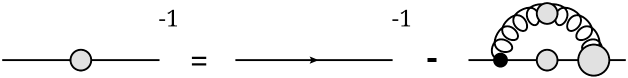
Pieces are still needed:

- Dressed Quark-Gluon vertex.
- Dressed Gluon propagator.

Two approaches:

1. calculate gluon and vertex from their DSE
   Strauss, Kellermann, Fischer PRL 109 (2012), R. Williams et al. PRD 89 (2014)
2. use rainbow-ladder model (Maris-Tandy model) for quark-gluon kernel
   Maris, Tandy PRC 78 1999

The Rainbow-Ladder model



$$\alpha_{\text{eff}}(p^2) = \pi \eta^7 \frac{p^2}{\Lambda^2} e^{-p^2 \frac{\eta^2}{\Lambda^2}} + \alpha_{UV}(p^2)$$

P. Maris, R. Tandy, PRC 78 1999

$$Z_{1f} C_F \frac{g^2}{4\pi} D_{\mu\nu}(q) \Gamma^\nu(k,p) = Z_2^2 C_F T^q_{\mu\nu} \frac{\alpha_{\text{eff}}(q^2)}{q^2} \gamma^\nu$$

$$K^{(2)}_{rs;tu}(p,k;P) = 4\pi Z_2^2 C_F \frac{\alpha_{\text{eff}}(q^2)}{q^2} T^q_{\mu\nu} [\gamma^\mu]_{rt} [\gamma^\nu]_{su}$$

- two parameters η and Λ fixed by m_π and f_π
- $\alpha_{UV}(p^2)$ from perturbation theory
- Renormalizable and momentum dependent

The Maris-Tandy model: how good is it?

Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle\bar{q}q\rangle^0_\mu$	$(0.236\text{ GeV})^3$	$(0.241^\dagger)^3$
m_π	0.1385 GeV	$0.138^\dagger$
f_π	0.0924 GeV	$0.093^\dagger$
m_K	0.496 GeV	$0.497^\dagger$
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	$7.6 \cdot 10^6\ \mathrm{s}^{-1}$	7.38
$\Gamma(K_{\mu 3})$	$5.2 \cdot 10^6\ \mathrm{s}^{-1}$	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^\star K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^\star K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^\star K\gamma}/m_K)^0$	1.28	1.19

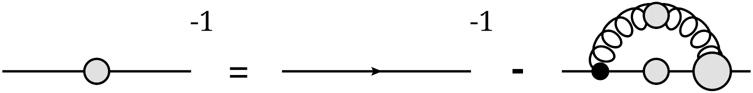
Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

$M_\rho, M_\phi, M_{K^\star}$ good to 5%, $f_\rho, f_\phi, f_{K^\star}$ good to 10%

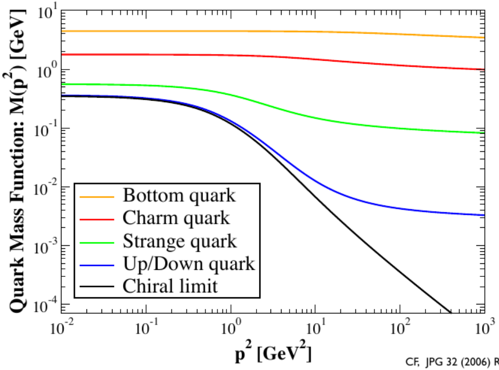
Slide from Pieter Maris

The Dyson-Schwinger equations



$$\mathcal{S} = \left[-i\not{p}\, A(p^2) + B(p^2)\right]^{-1}$$

The dynamically generated quark mass function $M(p^2) = B(p^2)/A(p^2)$:

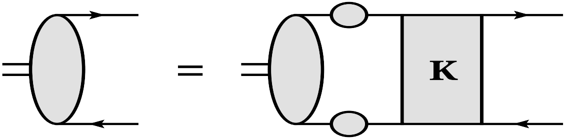


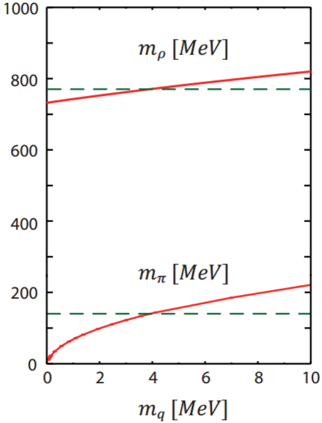
CF, JPG 32 (2006) R253

- $M(p^2)$ momentum dependent
- $M_{Strong} \approx 350\ MeV$
- Flavor dependence

The Bethe-Salpeter equation

Bethe-Salpeter equation $\rightarrow$ meson masses and wave functions





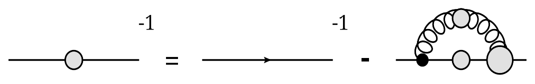
$$[\Psi(p;P)]_{tu} = \lambda \int_k K^{(2)}_{rs;tu} \left[S(k+\frac{P}{2})\Psi(k;P)S(k-\frac{P}{2}) \right]_{sr}$$

Kernel K is uniquely related to quark-DSE via AxWTI

- Pion is bound state and **Goldstone** boson
- $f_\pi^2 m_\pi^2 = 2m_n \langle q\bar{q} \rangle$ GMOR is fulfilled

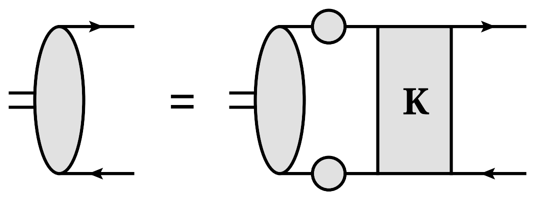
The Recipe

Quark DSE



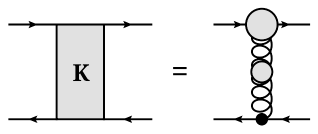
$$S^{-1}(p) = Z_2 S_0^{-1} + g^2 Z_{1f} \int_k \gamma^\mu S(k) \Gamma^\nu(k,p) D_{\mu\nu}$$

Meson BSE



$$[\Psi(p;P)]_{tu} = \lambda \int_k K^{(2)}_{rs;tu}(p,k;P)\,[S\Psi(k;P)S]_{sr}$$

Kernel as an effective one-gluon exchange



$$\alpha_{\text{eff}}(p^2) = \pi \eta^7 \frac{p^2}{\Lambda^2} e^{-p^2 \frac{\eta^2}{\Lambda^2}} + pQCD$$

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General $J = N$ Dirac basis construction

Tensor Structure

$$D^{(1)} = \begin{pmatrix} 1 & t_\mu \gamma^\mu & r_\mu \gamma^\mu & r_\mu t_\nu \frac{1}{2}[\gamma^\mu, \gamma^\nu] \end{pmatrix}$$

$$D^{(5)} = \begin{pmatrix} \gamma_5 & \gamma_5 t_\mu \gamma^\mu & \gamma_5 r_\mu \gamma^\mu & \gamma_5 r_\mu t_\nu \frac{1}{2}[\gamma^\mu, \gamma^\nu] \end{pmatrix}$$

$$\Psi^{(P)}_{\mu_1 \ldots \mu_J}(r, t) = \sum_{i=1}^{4} \left[\lambda_i Q_{\mu_1 \ldots \mu_J} \bar{D}_i^{(P)} + \lambda_{i+4} T_{\mu_1 \ldots \mu_J} \bar{D}_i^{(P)} \right]$$

J=1

$$Q_\mu = \tau^{(t)}_{\mu\nu}\, r^\nu \,, \quad T_\mu = \tau^{(t)}_{\mu\alpha}\, \tau^{(Q)}_{\alpha\nu} \gamma^\nu$$

J=2

$$Q_{\mu_1\mu_2} = Q_{\mu_1} Q_{\mu_2} - \frac{1}{3} Q^2 \tau_{\mu_1\mu_2} \,,$$

$$T_{\mu_1\mu_2} = T_{(\mu_1} Q_{\mu_2)} - \frac{2}{3} \not{Q} \tau_{\mu_1\mu_2}$$

General $J = N$ Dirac basis construction (continuation)

J=3

$$Q_{\mu_1\mu_2\mu_3} = Q_{\mu_1} Q_{\mu_2} Q_{\mu_3} - \frac{1}{5} Q^2 \tau_{(\mu_1\mu_2} Q_{\mu_3)}$$

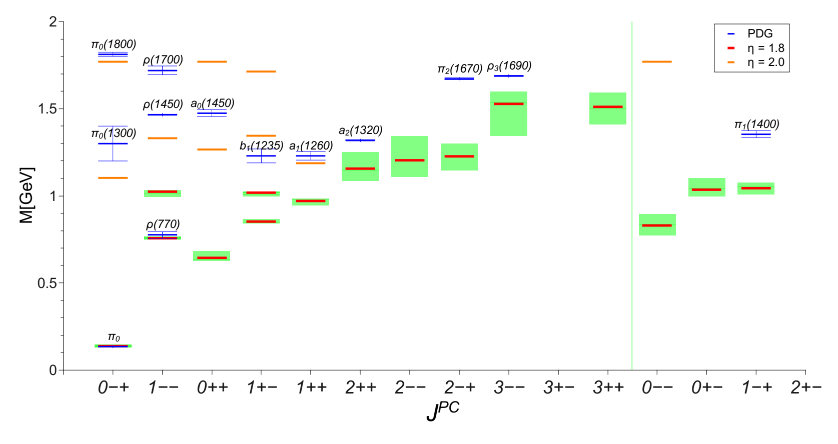
$$T_{\mu_1\mu_2\mu_3} = T_{(\mu_1} Q_{\mu_2} Q_{\mu_3)} - \frac{1}{5} 2\not{Q} \tau_{(\mu_1\mu_2} Q_{\mu_3)} - \frac{1}{5} Q^2 \tau_{(\mu_1\mu_2} T_{\mu_3)}$$

Quantum numbers

- J - total spin and P-parity are fixed by given tensor structure
- C-parity detected by $C = \gamma_2\gamma_4$ charge conjugation

$n\bar{n}$ spectrum

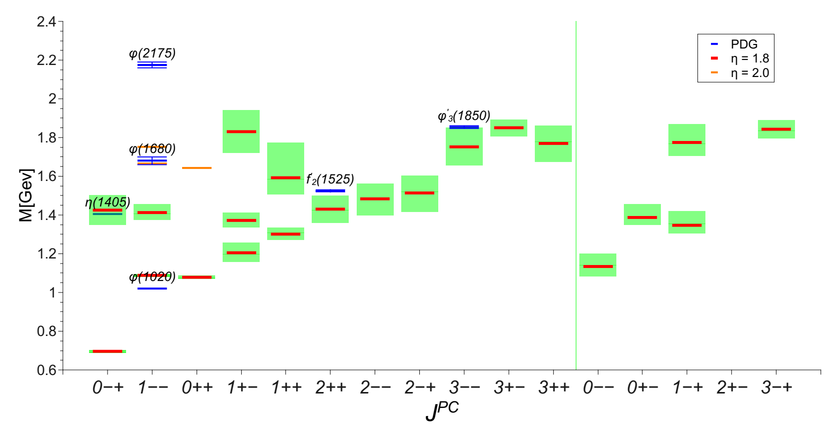
$n\bar{n}$ spectrum



C. Fischer, SK, R. Williams arXiv:1406.4370

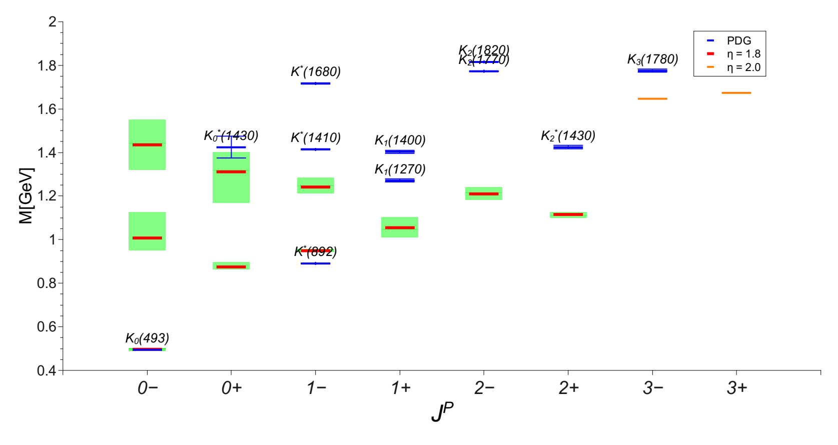
- $J^{PC} = 0^{-+}, 1^{--}$ spiting
- $J^{PC} = 1^{--}, 2^{++}, 3^{--}$ in "good" agreement with experiment
- Exotic channels

$s\bar{s}$ spectrum



- $U_A(1)$ anomaly is missing in the model
- $J^{PC} = 1^{--}, 2^{++}, 3^{--}$ in "good" agreement with experiment
- Exotic channels

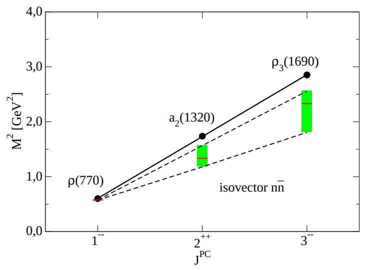
$n\bar{s}$ spectrum



Regge trajectories

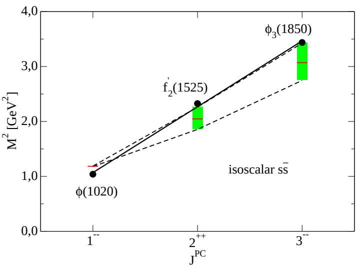
$M_X^2(J) = M_X^2(0) + \beta_X J$ - the Regge slope

$n\bar{n}$ Regge trajectory



$$M_\rho^2(0) = \quad -0.42\ (-0.05)\ \text{GeV}^2$$
$$\beta_\rho = \quad 0.99\ (0.62)\ \text{GeV}^2$$

$s\bar{s}$ Regge trajectory



$$M_\phi^2(0) = \quad 0.05\ (0.36)\ \text{GeV}^2$$
$$\beta_\phi = \quad 1.12\ (0.78)\ \text{GeV}^2$$

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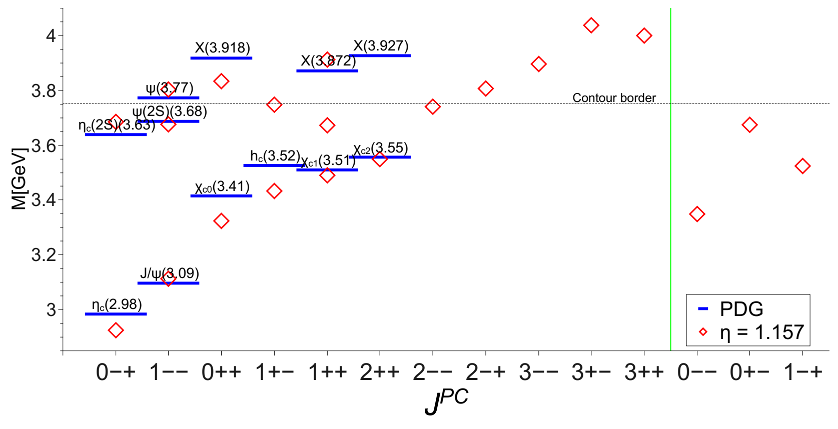
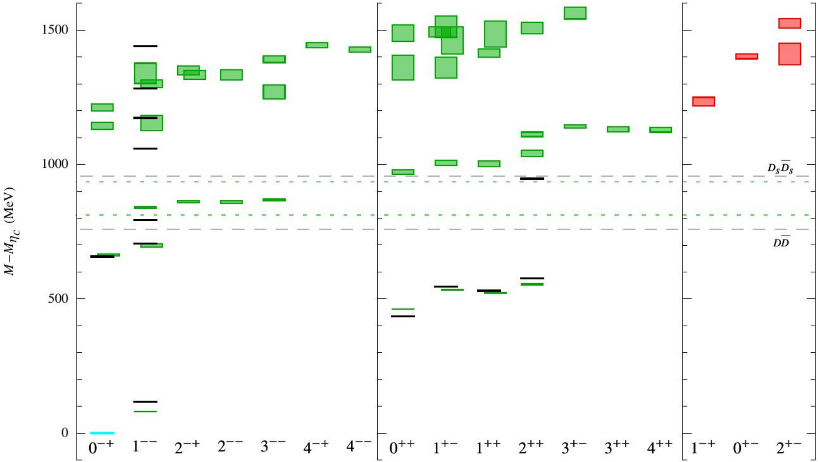
$c\bar{c}$ spectrum



C. Fischer, SK, R. Williams arXiv:1409.5076

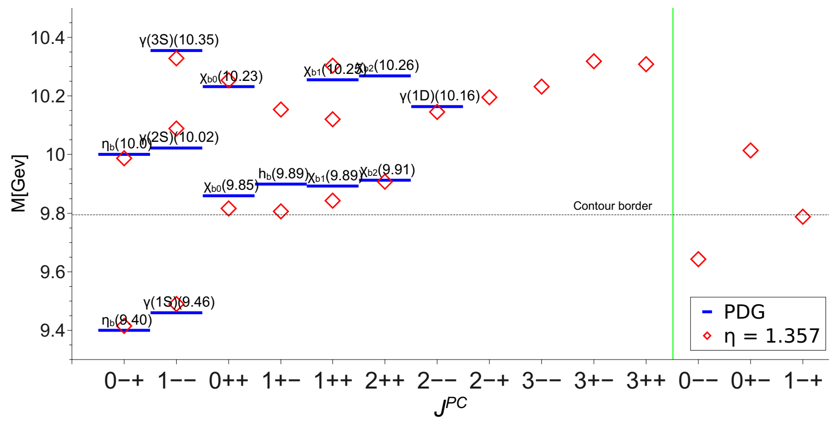
- $M_{3^{--}} = 3896\,\text{MeV}$

Charmonium spectrum on the lattice



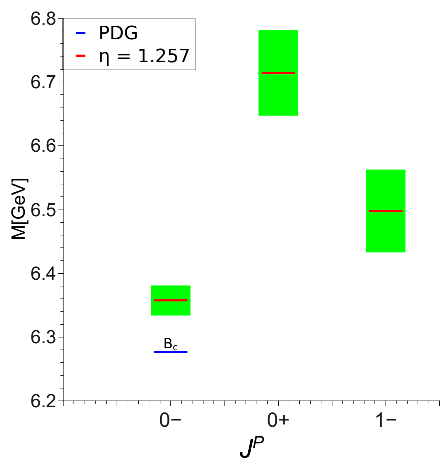
Liuming Liu et al., JHEP 07 (2012) 126

$b\bar{b}$ spectrum



- $M_{3^{--}} = 10232\,\text{MeV}$

$b\bar{c}$ spectrum



- Large numerical error for the extraction
- Quark poles contribution to BSE

Overview

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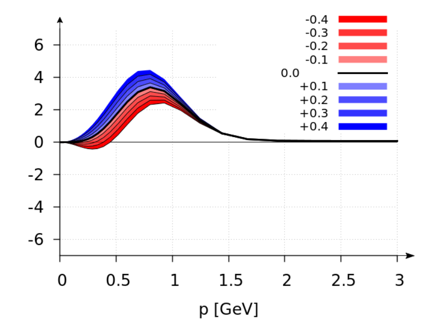
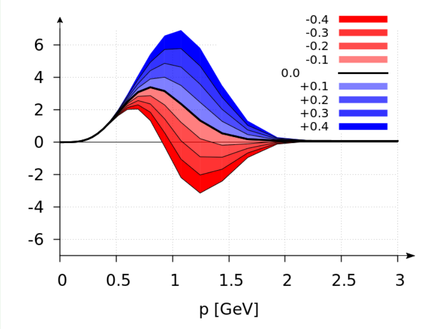
Maris-Tandy model variation

Effective running coupling $\alpha_{\text{eff}}(p^2)$



a_1



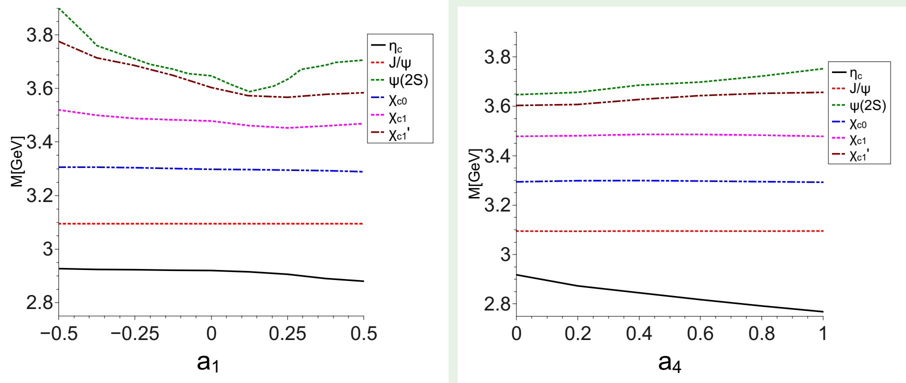
a_4

$$\alpha_{\text{eff}}(x^2) = \pi\eta^7 P_n(x^2) e^{-x^2\eta^2} + pQCD, \quad x^2 = \frac{p^2}{\Lambda^2}$$

$$P_n(x^2) = \sum_{i=0}^{n} a_i x^{2i-2}$$

Maris-Tandy model variation

Masses



- Gradient decent
- Simplex method
- Simulated annealing

No way to fit all spectra within the Rainbow-Ladder approach

Summary

Outlook

- $J \leqslant 3$ Light meson spectrum
  - $J^{PC} = 0^{-+}, 1^{--}$ spiting
  - $J^{PC} = 1^{--}, 2^{++}, 3^{--}$ in "good" agreement with experiment
  - Regge behavior
- $J \leqslant 3$ Heavy meson spectrum
  - Overall good comparison with data
  - $M_{3^{--}}^{Charm} = 3896\,\mathrm{MeV}$
  - $M_{3^{--}}^{Bottom} = 10232\,\mathrm{MeV}$
  - Agreement for $J = 2$ and $J = 3$ with the lattice

Futher Development

- Charmonium decays
- Charm form factors and transition form factors
- Beyond Rainbow-Ladder approach
- Extract the quark-quark potential