Lattice Investigation of Heavy Meson Interactions

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List of contents

Introduction

Motivation

Why do we study tetraquarks?

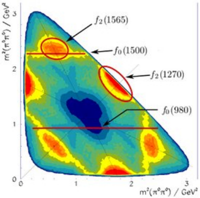
- QCD allows more complex systems than mesons and baryons $\rightarrow$ tetraquark $\bar{q}q\bar{q}q$ (R. L. Jaffe, Phys. Rev. D 15, 267 [1977])
- several hadronic resonances which are tetraquark candidates: σ, κ, D^*_{s0}, ... (J. Beringer, Phys. Rev. D86, 010001 [2012])
- observation of $B^*\bar{B}$ and $B^*\bar{B}^*$ tetraquark candidates at BELLE (A. Bondar, arXiv:1110.2251 [hep-ex])

Home | News | Meetings | Collaboration | Computing | Detector | Physics | Documents | Communications

Overview | Spectroscopy | Hadrons in Matter | Nucleon Structure | Hypernuclei | Theory Advisory Grp

Physics - Hadron Spectroscopy

Search for Gluonic Excitations



One of the main challenges of hadron physics is the search for gluonic excitations, i.e. hadrons in which the gluons can act as principal components. These gluonic hadrons fall into two main categories: glueballs, i.e. states where only gluons contribute to the overall quantum numbers, and hybrids, which consist of valence quarks and antiquarks as hadrons plus one or more excited gluons which contribute to the overall quantum numbers.

The additional degrees of freedom carried by gluons allow these hybrids and glueballs to have J^{PC} exotic quantum numbers. In this case mixing effects with nearby $q\bar{q}$ states are excluded and this makes their experimental identification easier. The properties of glueballs and hybrids are determined by the long-distance features of QCD and their study will yield fundamental insight into the structure of the QCD vacuum. Antiproton-proton annihilations provide a very favourable environment to search for gluonic hadrons.

Charmonium Spectroscopy

The charmonium spectrum can be calculated within the framework of non-relativistic potential models, EFT and LQCD. All 8 charmonium states below open charm threshold are known, but the measurements of their parameters and decays is far from complete (e.g. width and decay modes of h_c and $\eta_c(2S)$). Above threshold very little is known: on one hand the expected D- and F- wave states have not been identified (with the possible exception of the $\psi(3770)$, mostly $3D_1$), on the other hand the nature of the recently discovered X, Y, Z states is not known.

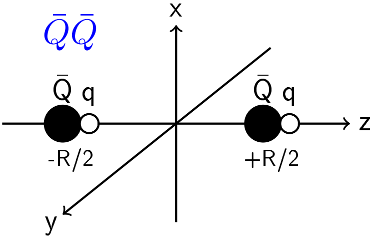
At full luminosity PANDA will collect several thousand $c\bar{c}$ states per day. By means of fine scans it will be possible to measure masses with an accuracy of the order of 100 keV and widths to 10% or better. PANDA will explore the entire energy region below and above the open charm threshold, to find the missing D- and F- wave states and unravel the nature of the newly discovered X, Y, Z states.

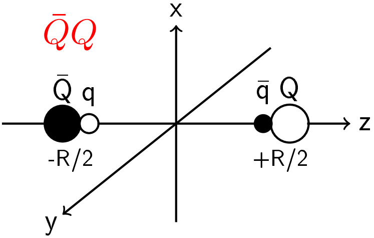
D Meson Spectroscopy

The recent discoveries of new open charm mesons at the BaBar, Belle and CLEO has attracted much interest both in the theoretical and experimental community, since the new states do not fit well into the quark model predictions for heavy-light systems in contrast to the previously known D states. An important quantity which allows to distinguish between the possible different theoretical pictures is the decay width of these states (in particular the D_s states).

Our approach

- investigate heavy-heavy-light-light tetraquark candidates
- static approximation for the heavy quarks Q
- most suitable for $Q \approx b$
- could also provide information for $Q \approx c$
- compute the potential: static-light $\leftrightarrow$ static-light
  $\rightarrow$ insert it into the Schrödinger Eq. $\rightarrow$ bound state?





Goals

1. for $\bar{Q}\bar{Q}$: investigate the behaviour of the potential depending on the finite quark mass (s and c quarks)
   $\rightarrow$ bound states?
2. compute the $\bar{Q}Q$-potentials

Correlation function ⤳ hadron masses (1/2)

Correlation function

$$C(t) \equiv \langle\Omega|\mathcal{O}^\dagger(t)\,\mathcal{O}(0)\,|\Omega\rangle \qquad \left(\text{e.g. pion: } \mathcal{O} = \int d^3x\ \bar{u}(\vec{x})\gamma_5 d(\vec{x})\right)$$

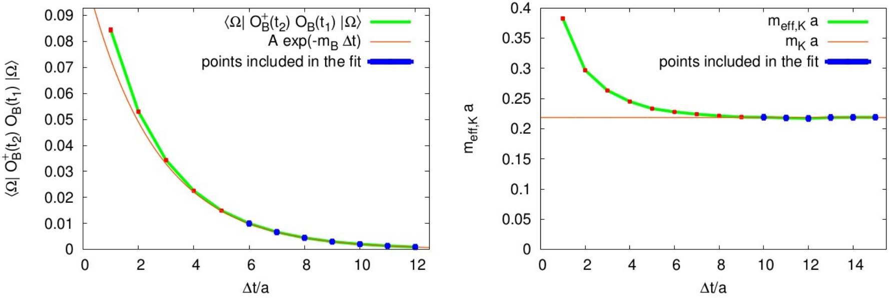
$$= \sum_{n=0}^{\infty} \langle\Omega|\, e^{+Ht}\mathcal{O}^\dagger(0)e^{-Ht}\,|n\rangle\langle n|\,\mathcal{O}(0)\,|\Omega\rangle$$

$$= \sum_{n=0}^{\infty} \underbrace{\left|\langle n|\mathcal{O}|\Omega\rangle\right|^2}_{=|a_n|^2} \exp\left(-\underbrace{(E_n - E_\Omega)}_{=m_n}\, t\right) \overset{t\gg 1}{\approx} |a_0|^2 e^{-m_0\, t}$$

Effective mass

$$m_{\text{eff}}(t) = \frac{1}{a}\log\left(\frac{C(t)}{C(t+a)}\right) \overset{t\gg 1}{\approx} m$$

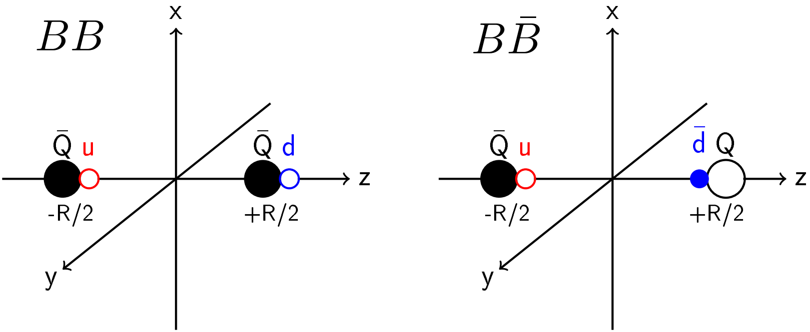
Correlation function $\rightsquigarrow$ hadron masses (2/2)



(J. Weber, S. Diehl, T. Kuske, M. Wagner [arXiv:1310.1760 [hep-lat]])

BB and $B\bar{B}$ potentials

$\bar{Q}\bar{Q} \leadsto BB$ vs. $\bar{Q}Q \leadsto B\bar{B}$



$\underline{BB}$: high indication for a bound state in a specific channel: $I(J^P) = 0(0^+), 0(1^+)$ with a binding energy of $E \approx -50$ MeV (P. Bicudo, M. Wagner [arXiv:1209.6274 [hep-ph]])

Trial states

$$BB\text{:}\ \mathcal{O}_j \equiv (C\Gamma_j)_{AB}\, \tilde{\Gamma}_{CD} \left(\bar{Q}_C(\vec{x})\, \psi_A^{(f_1)}(\vec{x})\right) \left(\bar{Q}_D(\vec{y})\, \psi_B^{(f_2)}(\vec{y})\right) |\Omega\rangle$$

$$B\bar{B}\text{:}\ \ \tilde{\mathcal{O}}_j \equiv (\Gamma_j)_{AB}\, \tilde{\Gamma}_{CD} \left(\bar{Q}_C(\vec{x})\, \psi_A^{(f_1)}(\vec{x})\right) \left(\bar{\psi}_B^{(f_2)}(\vec{y})\, Q_D(\vec{y})\right) |\Omega\rangle$$

$$\tilde{\Gamma} \in \{1, \gamma_0, \gamma_3\gamma_5, \gamma_1\gamma_2, \gamma_1\gamma_5, \gamma_2\gamma_5, \gamma_2\gamma_3, \gamma_1\gamma_3\}$$

$$\tilde{\Gamma} \in \{\gamma_5,\ \gamma_0\gamma_5,\ \gamma_3,\ \gamma_0\gamma_3,\ \gamma_1,\ \gamma_2, \gamma_0\gamma_1, \gamma_0\gamma_2\}$$

$$\Rightarrow C_{jk}(t) = \langle\Omega|\mathcal{O}_j^\dagger(t)\, \mathcal{O}_k(0)|\Omega\rangle$$

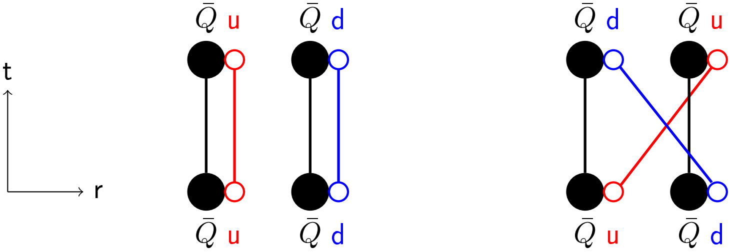
Correlator for $B\bar{B}$

$$C_{jk}^{\text{2meson}}(t) = \langle \Omega | \tilde{\mathcal{O}}_j^\dagger(t)\, \tilde{\mathcal{O}}_k(0) | \Omega \rangle = \ldots$$

$$= -2e^{-2Mt}(\gamma_0 \Gamma_j^* \gamma_0)_{AB}\ (\Gamma_k^T)_{CD} \left\langle \text{Tr}_{\text{col}} \left[U(\vec{x},t;\vec{x},0)\ (D^{-1})_{DA}^{(\text{f}_1)}(\vec{x},0;\vec{x},t) \right] \cdot \text{Tr}_{\text{col}} \left[U(\vec{y},0;\vec{y},t)\ (D^{-1})_{BC}^{(\text{f}_2)}(\vec{y},t;\vec{y},0) \right] \right\rangle$$

- U is the static quark propagator $\rightarrow$ Wilson lines
- D^{-1} is the light quark propagator $\rightarrow$ inverted Dirac operator
- Γ_j,Γ_k and the flavors f_1,f_2 define the quantum numbers

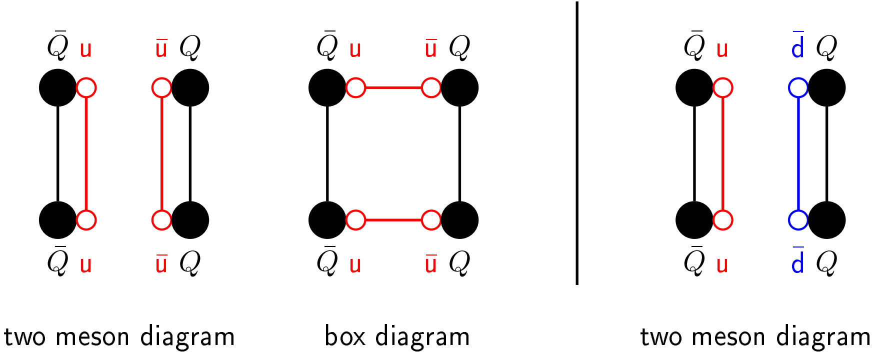
Diagrams for BB



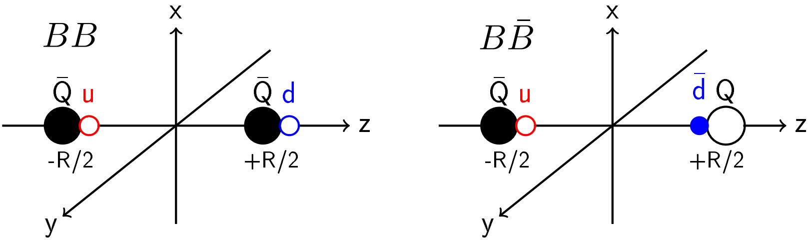
two meson diagram

cross diagram

Diagrams for $B\bar{B}$



Symmetries and quantum numbers



- two light (anti)quarks $\rightarrow$ I and I_z are quantum numbers
- separation along the z-axis $\rightarrow$ only j_z is a quantum number
- for $B\bar{B}$ parity alone is not a symmetry
  ... but $\mathscr{P} \circ \mathscr{C}$ $\rightarrow$ $\mathscr{P} \circ \mathscr{C}$ is a quantum number
- reflection along the x-axis $\rightarrow$ $\mathscr{P}_x$ is a quantum number

$\Rightarrow$ $B\bar{B}$ states can be labelled by $I, I_z, |j_z|, \mathscr{P} \circ \mathscr{C}$ and $\mathscr{P}_x$

Isospin

- two light quarks can be combined to isospin $I = 0$ and $I = 1$:

$$I = 0: \quad ud - du$$
$$I = 1: \quad uu,\ dd,\ ud + du$$

- now we are interested in the coupling of quark and antiquark:

$$\begin{pmatrix} u \\ d \end{pmatrix} \xleftrightarrow{\text{same transformation}} \begin{pmatrix} -\bar{d} \\ +\bar{u} \end{pmatrix}$$

$$I = 0: \quad ud - du \mathrel{\hat{=}} u\bar{u} + d\bar{d}$$
$$I = 1: \quad uu,\ dd,\ ud + du \mathrel{\hat{=}} u\bar{d},\ d\bar{u},\ u\bar{u} - d\bar{d}$$

Meson content (1/3)

Parity projectors

$$P_{\mathscr{P}=+} = \frac{1+\gamma_0}{2} \quad , \quad P_{\mathscr{P}=-} = \frac{1-\gamma_0}{2}$$

Spin projectors

$$P_{j_z=\uparrow} = \frac{1+i\gamma_0\gamma_3\gamma_5}{2} \quad , \quad P_{j_z=\downarrow} = \frac{1-i\gamma_0\gamma_3\gamma_5}{2}$$

- extend the meson content of the tetraquarks
- good cross-check
- can be used for normalization

Meson content (2/3)

Γ (pseudo) physical	meson content
γ_5	$+S_\uparrow S_\uparrow + S_\downarrow S_\downarrow + P_\uparrow P_\uparrow + P_\downarrow P_\downarrow$
$\gamma_0\gamma_5$	$-S_\uparrow S_\uparrow - S_\downarrow S_\downarrow + P_\uparrow P_\uparrow + P_\downarrow P_\downarrow$
1	$+S_\uparrow P_\uparrow + S_\downarrow P_\downarrow + P_\uparrow S_\uparrow + P_\downarrow S_\downarrow$
γ_0	$+S_\uparrow P_\uparrow + S_\downarrow P_\downarrow - P_\uparrow S_\uparrow - P_\downarrow S_\downarrow$
γ_3	$+iS_\uparrow S_\uparrow - iS_\downarrow S_\downarrow - iP_\uparrow P_\uparrow + iP_\downarrow P_\downarrow$
$\gamma_0\gamma_3$	$-iS_\uparrow S_\uparrow + iS_\downarrow S_\downarrow - iP_\uparrow P_\uparrow + iP_\downarrow P_\downarrow$
$\gamma_3\gamma_5$	$-iS_\uparrow P_\uparrow + iS_\downarrow P_\downarrow + iP_\uparrow S_\uparrow - iP_\downarrow S_\downarrow$
$\gamma_0\gamma_3\gamma_5$	$-iS_\uparrow P_\uparrow + iS_\downarrow P_\downarrow - iP_\uparrow S_\uparrow + iP_\downarrow S_\downarrow$
γ_1	$+iS_\uparrow S_\downarrow + iS_\downarrow S_\uparrow - iP_\uparrow P_\downarrow - iP_\downarrow P_\uparrow$
$\gamma_0\gamma_1$	$-iS_\uparrow S_\downarrow - iS_\downarrow S_\uparrow - iP_\uparrow P_\downarrow - iP_\downarrow P_\uparrow$
$\gamma_1\gamma_5$	$-iS_\uparrow P_\downarrow - iS_\downarrow P_\uparrow + iP_\uparrow S_\downarrow + iP_\downarrow S_\uparrow$
$\gamma_0\gamma_1\gamma_5$	$-iS_\uparrow P_\downarrow - iS_\downarrow P_\uparrow - iP_\uparrow S_\downarrow - iP_\downarrow S_\uparrow$
γ_2	$-S_\uparrow S_\downarrow + S_\downarrow S_\uparrow + P_\uparrow P_\downarrow - P_\downarrow P_\uparrow$
$\gamma_0\gamma_2$	$+S_\uparrow S_\downarrow - S_\downarrow S_\uparrow + P_\uparrow P_\downarrow - P_\downarrow P_\uparrow$
$\gamma_2\gamma_5$	$+S_\uparrow P_\downarrow - S_\downarrow P_\uparrow - P_\uparrow S_\downarrow + P_\downarrow S_\uparrow$
$\gamma_0\gamma_2\gamma_5$	$+S_\uparrow P_\downarrow - S_\downarrow P_\uparrow + P_\uparrow S_\downarrow - P_\downarrow S_\uparrow$

S: $(\mathscr{P} = -)$ lightest static-light meson $(\rightsquigarrow B/B^*)$

P: $(\mathscr{P} = +)$ first excitation $(\rightsquigarrow B_0^*/B_1^*)$

$\uparrow\downarrow$: light cloud angular momentum

Meson content (3/3)

Γ (pseudo) physical	meson content
γ_5	$+S_\uparrow S_\uparrow + S_\downarrow S_\downarrow + P_\uparrow P_\uparrow + P_\downarrow P_\downarrow$
$\gamma_0\gamma_5$	$-S_\uparrow S_\uparrow - S_\downarrow S_\downarrow + P_\uparrow P_\uparrow + P_\downarrow P_\downarrow$
1	$+S_\uparrow P_\uparrow + S_\downarrow P_\downarrow + P_\uparrow S_\uparrow + P_\downarrow S_\downarrow$
γ_0	$+S_\uparrow P_\uparrow + S_\downarrow P_\downarrow - P_\uparrow S_\uparrow - P_\downarrow S_\downarrow$
γ_3	$+iS_\uparrow S_\uparrow - iS_\downarrow S_\downarrow - iP_\uparrow P_\uparrow + iP_\downarrow P_\downarrow$
$\gamma_0\gamma_3$	$-iS_\uparrow S_\uparrow + iS_\downarrow S_\downarrow - iP_\uparrow P_\uparrow + iP_\downarrow P_\downarrow$
$\vdots$	$\vdots$

$$\gamma_5 - \gamma_0\gamma_5 \quad \rightsquigarrow \quad +S_\uparrow S_\uparrow + S_\downarrow S_\downarrow + S_\uparrow S_\uparrow + S_\downarrow S_\downarrow$$

$$\gamma_5 + \gamma_0\gamma_5 \quad \rightsquigarrow \quad +P_\uparrow P_\uparrow + P_\downarrow S_\downarrow + P_\uparrow P_\uparrow + P_\downarrow P_\downarrow$$

$$\vdots$$

S: $(\mathscr{P} = -)$ lightest static-light meson ($\rightsquigarrow B/B^*$)

P: $(\mathscr{P} = +)$ first excitation ($\rightsquigarrow B_0^*/B_1^*$)

$\uparrow\downarrow$: light cloud angular momentum

Symmetry checks and averaging

- check the contractions ($\rightsquigarrow$ correlations function) concerning different symmetries
- supports the credibility of the numerical results
- reduces the statistical errors (after averaging)
- available symmetries are:
  - twisted mass time reversal
  - twisted mass parity
  - twisted mass γ_5-hermiticity
  - charge conjugation
  - cubic $\frac{\pi}{2}$-rotations around all spacial axes
  - cubic π-rotations around the x- and y-axis

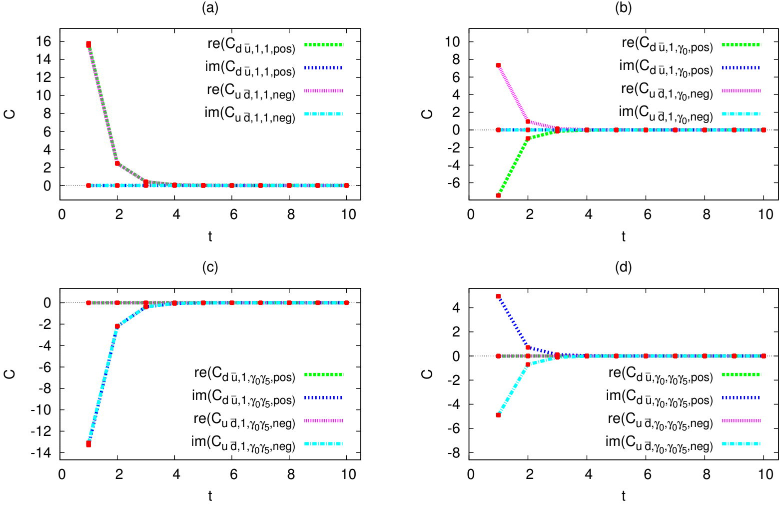
Symmetry checks and averaging (example: tm. time reversal)

Performing the symmetry transformation yields:

- time: positive time direction $\leftrightarrow$ negative time direction
- flavour: both change, i.e. $u\bar{d} \leftrightarrow d\bar{u}$ and $u\bar{u} \leftrightarrow d\bar{d}$
- spin/Γ matrices: extra minus sign, if
  $\Gamma(0)$ or $\Gamma(t) \in \{\gamma_5, \gamma_0, \gamma_0\gamma_3, \gamma_3\gamma_5, \gamma_0\gamma_1, \gamma_1\gamma_5, \gamma_0\gamma_2, \gamma_2\gamma_5\}$

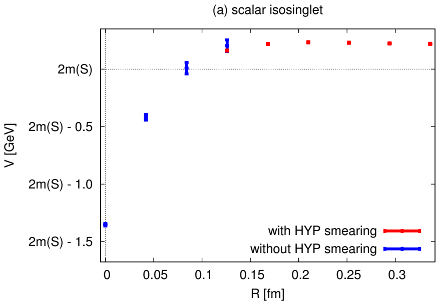
Symmetry checks and averaging (example: tm. time reversal)

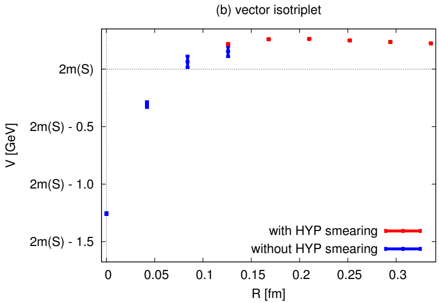
...extra minus sign, if $\Gamma(0)$ or $\Gamma(t) \in \{\gamma_5, \gamma_0, \gamma_0\gamma_3, \gamma_3\gamma_5, \gamma_0\gamma_1, \gamma_1\gamma_5, \gamma_0\gamma_2, \gamma_2\gamma_5\}$



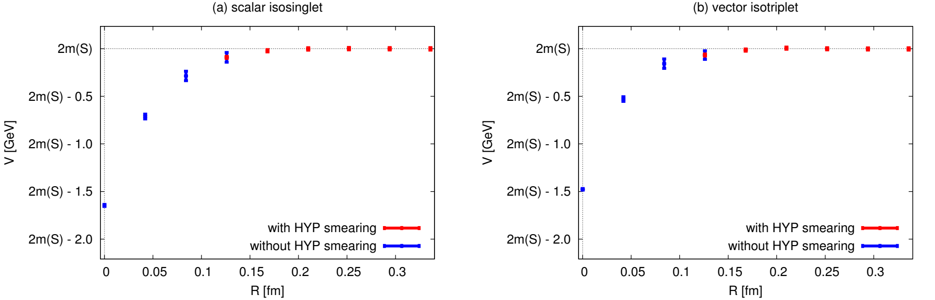
Results for $\bar{Q}\bar{Q}$ potentials

Charm quarks $\rightsquigarrow$ $B_c B_c$ potentials

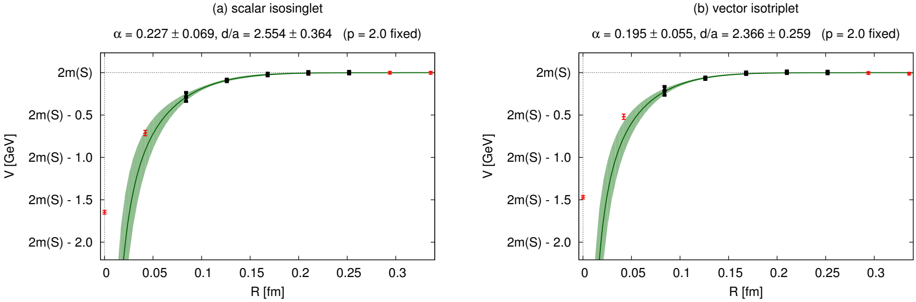




Charm quarks $\rightsquigarrow$ B_cB_c potentials (adjusted)

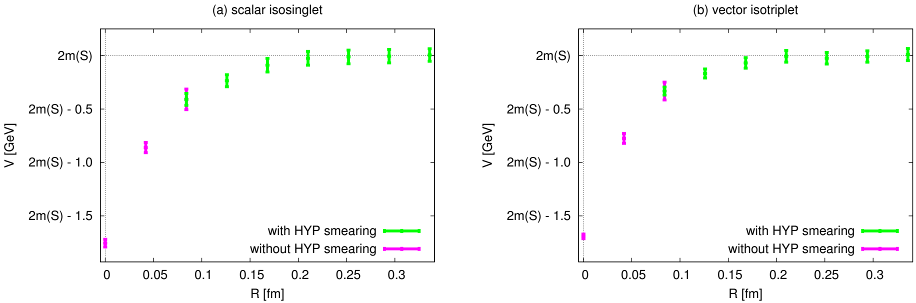


Charm quarks $\rightsquigarrow$ B_cB_c potentials (fitted)

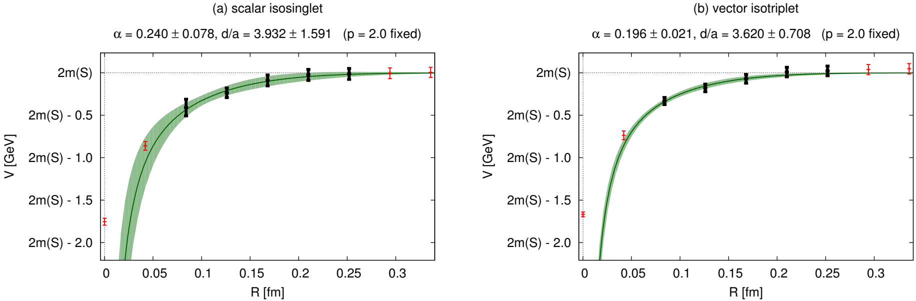


$$V(r) = -\frac{\alpha}{r} \exp\left[-\left(\frac{r}{d}\right)^p\right]$$

Strange quarks $\rightsquigarrow$ B_sB_s potentials (adjusted)

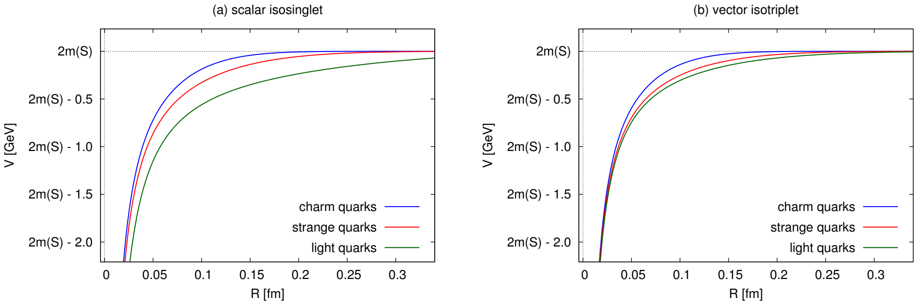


Strange quarks $\rightsquigarrow$ $B_s B_s$ potentials (fitted)



$$V(r) = -\frac{\alpha}{r} \exp\left[-\left(\frac{r}{d}\right)^p\right]$$

Comparison of B_sB_s, B_cB_c and BB



Results for $\bar{Q}\bar{Q}$

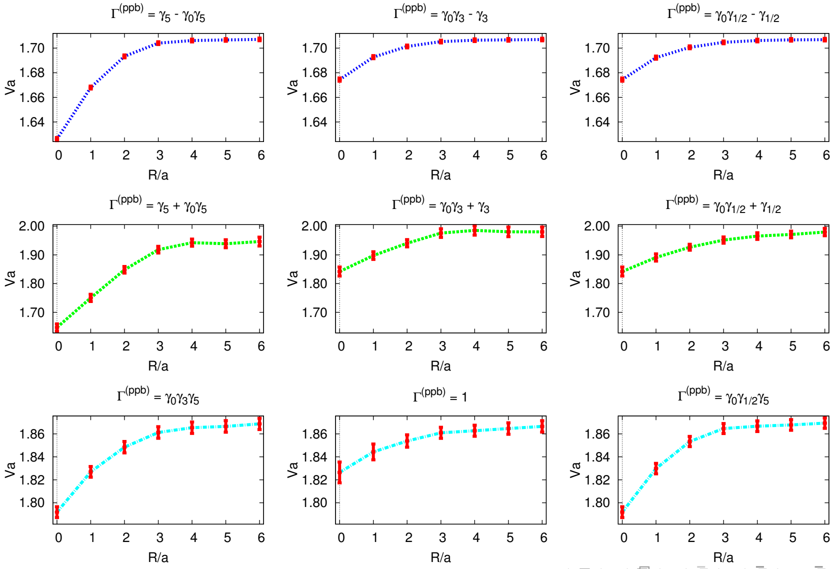
- qualitative behaviour (attractive/repulsive) of the potentials does not depend on the finite quark mass
- potentials get more narrow by increasing the finite quark mass
- no binding for strange and charm quarks (binding for masses $\gtrsim 1.9\, m_{B_s}$ and $\gtrsim 3.1\, m_{B_c}$, respectively)

$\Longrightarrow$ qualitative rule:

Increasing the finite quark mass means lowering the chance of finding a bound $\bar{Q}\bar{Q}$ state.

Results for $\bar{Q}Q$ potentials

Charm quarks



Summary

$\bar{Q}\bar{Q}$ potentials:

- binding only for light (up and down) quarks
- increasing the finite quark mass lowers the binding

$\bar{Q}Q$ potentials:

- only attractive potentials

Outlook

$\bar{Q}\bar{Q}$ potentials:

- continuum limit
- lighter quark masses
- a more refined phenomenological analysis (including several spin channels, i.e. separating B and B*, in the SE)

$\bar{Q}\bar{Q}$ potentials:

- implementation of the box diagrams $\Longrightarrow$ potentials with $I = 0$
- computation on a finer lattice with more statistics $\Longrightarrow$ fit potentials and solve SE $\Longrightarrow$ bound states?
- consideration of strange and light quarks