Maximum Mass and Radial modes of hybrid star in presence of Magnetic field

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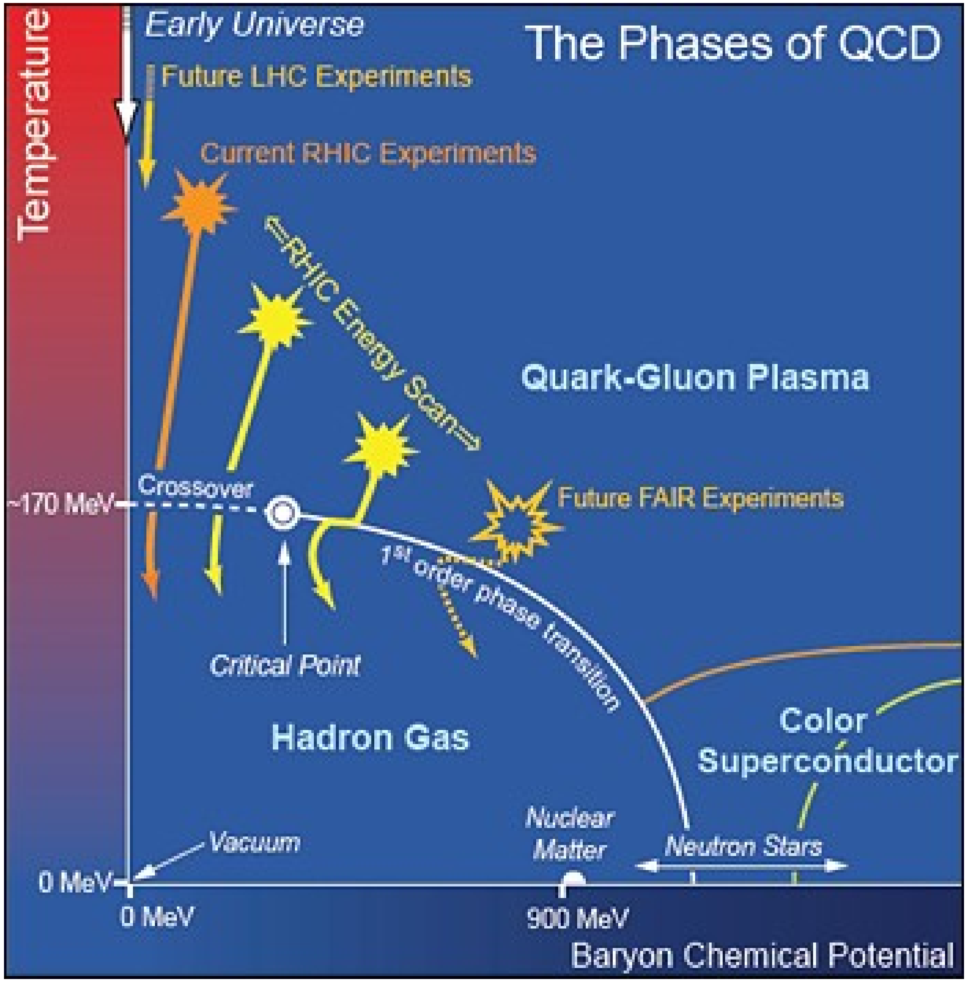
Talk Outline

- Motivation
- Introduction
- Hadronic EOS
- Quark EOS
- Inclusion of Magnetic field
- Magnetic field in hadronic phase and quark phase
- Mixed phase
- Radial modes
- Summary

3rd International FAIRNESS 2014 workshop,Italy on 21-28 September 2014.

Motivation

- Formation of QGP at very large temperatures or densities.
- QGP plays an important role in the development of the early universe and basic structures of some astrophysical objects like neutronstars.
- A simple phase diagram would show confined quarks in a region of low temperature and chemical potential, surrounded by a phase boundary and QGP outside.
- Ambiguities regarding the physics and the existence of the critical point (CP) on the QCD phase boundary still exist and the mist regarding the conjectured QCD phase boundary has not yet cleared.
- Intensive search for a proper and realistic equations of state (EOS) is still continued for studying the phase diagram existing between quark gluon plasma (QGP) and hadron gas (HG) phases.



The conjectured phase diagram

Reference:-The Frontiers of Nuclear Science, 2007 NSAC Long Range Plan

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At FAIR energies using baryon meson enteraction in relativistic theoretical model based on QCD approach may be proposed. Applying this model we can :-

* Study phase transition from Hadron phase to Quark phase

* Investigate for the signatures of QGP formation

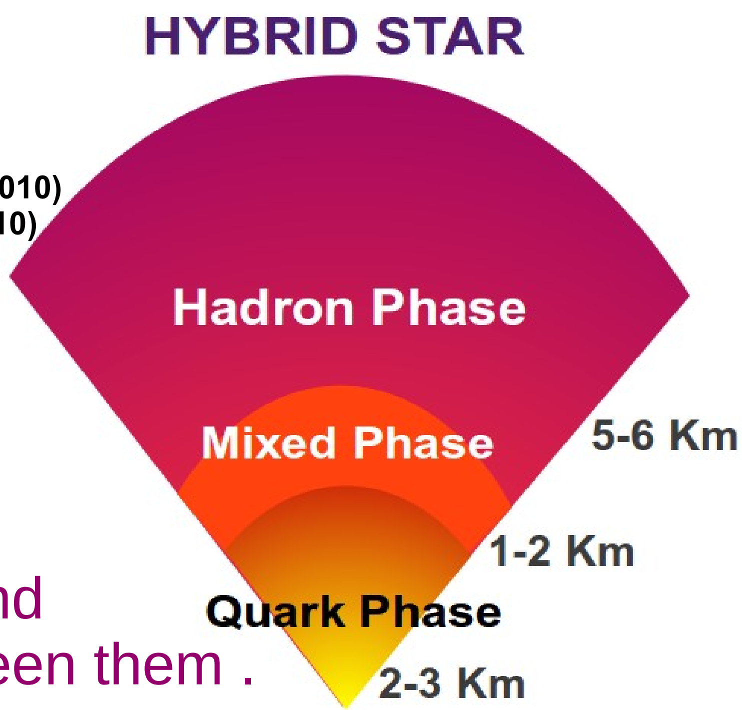
* Verify the ovserved high precession mass of neutron star and its gross properties.

Introduction

Compact stars which has only nuclear matter, basically neutron and proton are called Neutron stars .

$$M_n \approx 1.5 - 2.1 \ M_0, R \approx 10\text{-}15 \, km, \rho \approx 5 - 10 \rho_{nuclear}$$

Reference -[1]P C C Freire, C Bassa, N Wex et al, Mon. Not. R. Astron. Soc. 412, 2763 (2010)
[2] P Demorest, T Pennucci, S Ransom, M Roberts and J Hessels, Nature 467, 1081 (2010)



Since central density of neutron star exceeds nuclear saturation density so it may contain deconfined quark matter.

Hybrid stars are NS having interior quark phase and exterior hadron phase and a mixed phase in between them .

$$\text{Surface magnetic field} = 10^{14} - 10^{15} G, \ \ \text{Central magnetic field} = 10^{17} - 10^{18} G$$

Refernce-D M Palmer, S Barthelmy, N Gehrels et al, Nature 434, 1107 (2005)

The high magnetic field modifies the properties of nuclear and deconfined quark matter of hybrid star

Hadronic EOS

The Lagrangian density is:

$$\mathcal{L}=\sum_b \bar{\psi}_b[\gamma_\nu(i\partial^\mu-g_{\omega b}\omega^\mu-\frac{1}{2}g_{\rho b}\vec{\tau}.\vec{\rho}^\mu)-(m_b-g_{\rho b}\rho)]\psi_b$$
$$+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma-m_\sigma^2\sigma^2)-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu-\frac{1}{4}\vec{\rho}_{\mu\nu}.\vec{\rho}^{\mu\nu}$$
$$+\frac{1}{2}m_\rho^2\vec{\rho}_\mu.\vec{\rho}^\mu-\frac{1}{3}b\,m_n(g_\sigma\sigma)^3-\frac{1}{4}c(g_\sigma\sigma)^4+\frac{1}{4}d(\omega_\mu\omega^\mu)^2$$
$$+\sum_L \bar{\psi}_L[i\gamma_\mu\partial^\mu-m_L]\psi_L$$

*The leptons L are assumed to be non-interacting,

*The baryons are coupled to the meson (scalar σ , isoscalar-vector ω and isovector-vector ρ).

Refernce-PRAMANA journal of physics,Vol. 82, No. 5,May 2014,pp. 797–807

Reference- Nucl. Phy. A 921 (2014) 96-113.

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$$\mathcal{L}_{hy} = \frac{1}{2}\left(\partial_\nu \sigma^* \partial^\nu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}\right) - \frac{1}{4} S_{\mu\nu} S^{\mu\nu}$$
$$+ \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu - \sum_b g_{\sigma^* b} \bar{\Psi}_b \psi_b \sigma^*$$
$$- \sum_b g_{\phi b} \bar{\Psi}_b \gamma_\mu \psi_b \phi^\mu$$

Reference-J. Schaffner, I.N. Mishustin, Phys. Rev. C, Nucl. Phys. 53 (1996) 1416.

Energy density of hadron phase :

$$\varepsilon_{hp} = \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_0^2 + \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{2} m_{\sigma^*}^2 \sigma^{*2} + \frac{1}{2} m_\varphi^2 \varphi_0^2$$
$$+ \frac{3}{4} d\, \omega_0^4 + U(\sigma) + \sum_b \varepsilon_b + \sum_l \varepsilon_l$$

Pressure of hadron phase $P_{hp} = \sum_i \mu_i n_i - \varepsilon$

$U(\sigma)$ stands for scalar self interactions

Quark EOS

Taking density dependance MIT bag model

$$\varepsilon_{qp} = \sum_{i=u,d,s} \frac{g_i}{2\pi^2} \int_0^{P_F^i} dp\, p^2 \sqrt{m_i^2 + p^2} \;+\; B_G$$

$$P_{qp} = \sum_{i=u,d,s} \frac{g_i}{6\pi^2} \int_0^{P_F^i} dp \frac{p^4}{\sqrt{m_i^2 + p^2}} \;-\; B_G$$

$$B_G(n_b) = B_\infty + (B_g - B_\infty) \exp\left[-\beta \left(\frac{n_b}{n_0}\right)^2\right]$$

Inclusion of Magnetic field

For the beta equilibrated matter : $\mu_i = b_i \mu_B + q_i \mu_e$

For charge neutrality condition : $\rho_c = \sum_i q_i n_i$

For the magnetic field along Z-direction : $A^{\mu} \equiv (0, -yB, 0, 0)$, $\vec{B} = B\hat{k}$

$$B = B_s + B_0 \{1 - e^{-\alpha (n_b/n_0)^{\gamma}}\}$$

Single particle energy eigenvalue in presence of magnetic field

$E_i = \sqrt{p_i^2 + m_i^2 + 2\tilde{\nu}|q_i|B}$, where $\tilde{\nu} = 2n + s + 1$

The energy density of charged particles :

$$\varepsilon_i = \frac{|q_i|B}{4\pi^2} \sum_{\tilde{\nu}} [E_F^i p_{F,\tilde{\nu}}^i + \tilde{m}_{\tilde{\nu}}^{i\,2} \ln(\left|\frac{E_F^i + p_{F,\tilde{\nu}}^i}{\tilde{m}_{\tilde{\nu}}^i}\right|)]$$

$$p_{F,\tilde{\nu}}^{i\;2} = E_F^{i\;2} - \tilde{m}_{\tilde{\nu}}^{i\;2} \qquad \tilde{m}_{\tilde{\nu}}^{i\;2} = m_i^2 + 2\tilde{\nu}|q_i|B$$

Magnetic field in hadronic phase and quark phase

Total energy density and pressure of hadronic phase:

$$\varepsilon_{HP} = \frac{1}{2} m_{\omega}^{2} \omega_{0}^{2} + \frac{1}{2} m_{\rho}^{2} \rho_{0}^{2} + \frac{1}{2} m_{\sigma}^{2} \sigma_{0}^{2} + \frac{1}{2} m_{\sigma^*}^{2} \sigma^{*2} + \frac{1}{2} m_{\varphi}^{2} \varphi_{0}^{2}$$

$$+ \frac{3}{4} d\, \omega_{0}^{4} + U(\sigma) + \sum_{b} \varepsilon_{b} + \sum_{l} \varepsilon_{l} + \frac{B^{2}}{8\pi^{2}}$$

$$P_{HP} = \sum_{i} \mu_{i} n_{i} - \varepsilon$$

Thermodynamic potential in presence of strong magnetic field at zero temperature:

$$\Omega_{i} = -\frac{2 g_{i} |q_{i}| B}{4\pi^{2}} \sum_{\tilde{\nu}} \int_{\sqrt{m_{i}^{2} + 2\nu |q_{i}| B}}^{\mu} dE_{i} \sqrt{E_{i}^{2} - m_{i}^{2} - 2\tilde{\nu} |q_{i}| B}$$

Energy density and Pressure of quark phase

$$\varepsilon_{QP} = \sum_{i} \Omega_{i} + B_{G} + \sum_{i} n_{i} \mu_{i}, \quad P_{QP} = -\sum_{i} \Omega_{i} - B_{G}$$

Mixed phase

Performing Glendenning construction and taking the Gibb's condition

$$P_{\mathrm{HP}}(\mu_{e.}\mu_{B}) = P_{\mathrm{QP}}(\mu_{e.}\mu_{B}) = P_{\mathrm{MP}}$$

$$\chi \rho_c^{QP} + (1-\chi)\rho_c^{HP} = 0$$

$\chi = 1$: Quark Phase

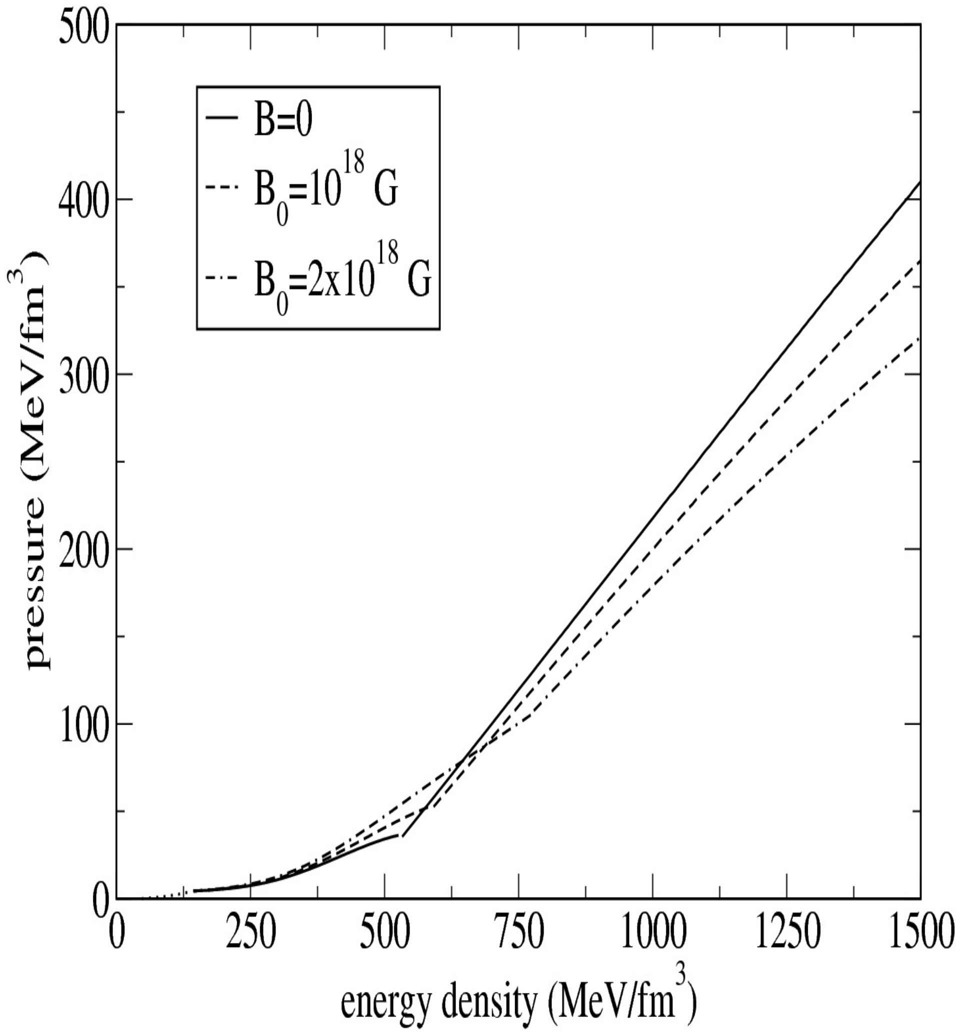
$\chi = 0$: Hadron Phase

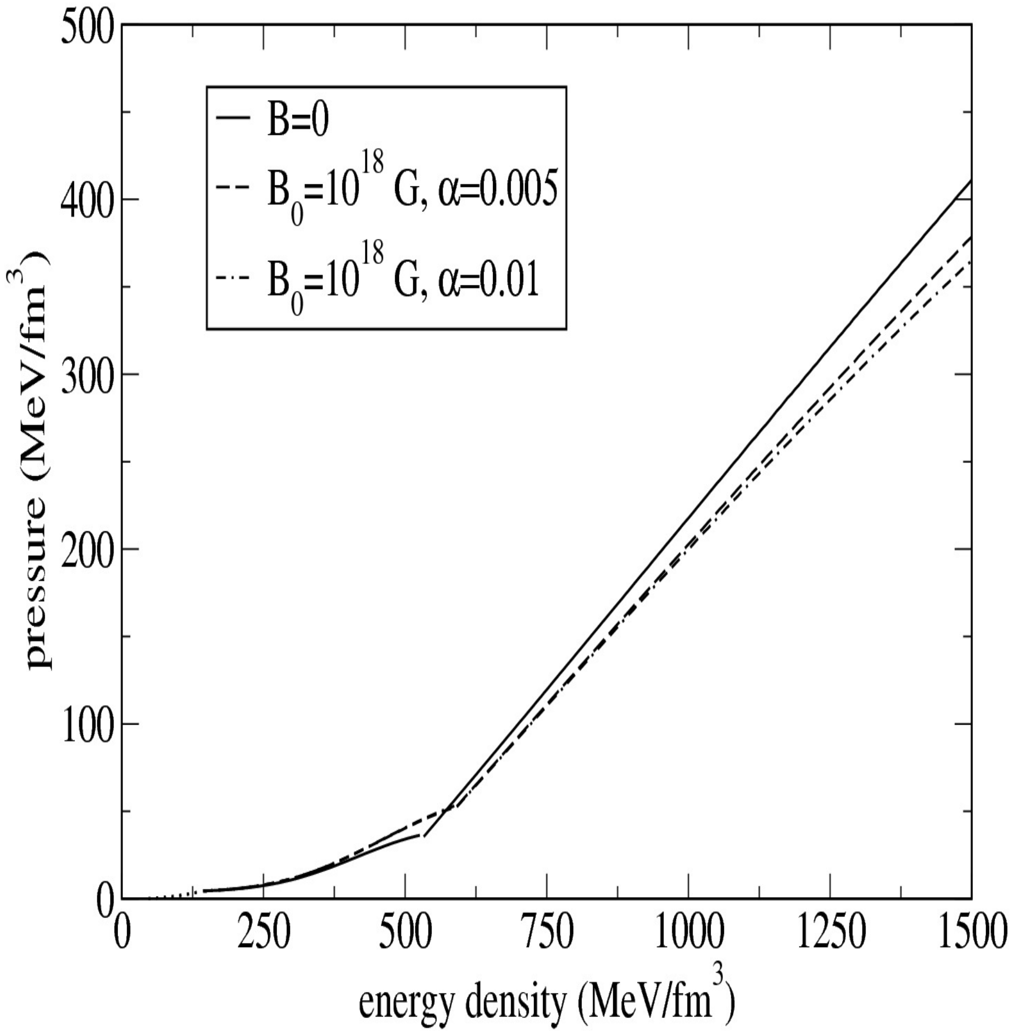
$$\varepsilon_{\mathrm{MP}} = \chi\, \varepsilon_{\mathrm{QP}} + (1-\chi)\, \varepsilon_{\mathrm{HP}}$$

$$n_{\mathrm{MP}} = \chi\, n_{\mathrm{QP}} + (1-\chi)\, n_{\mathrm{HP}}$$

Results

Energy density vs Pressure





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Results

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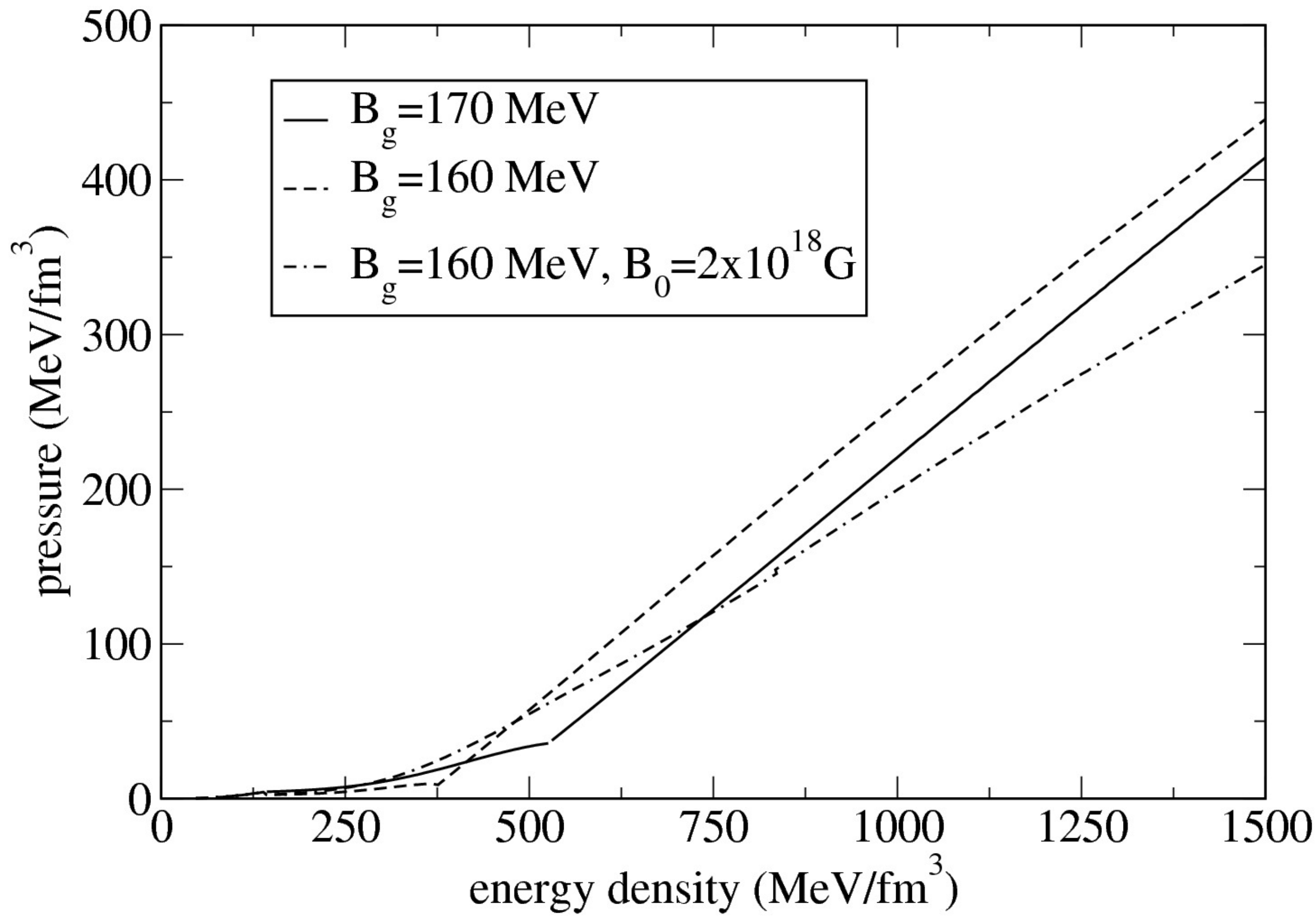
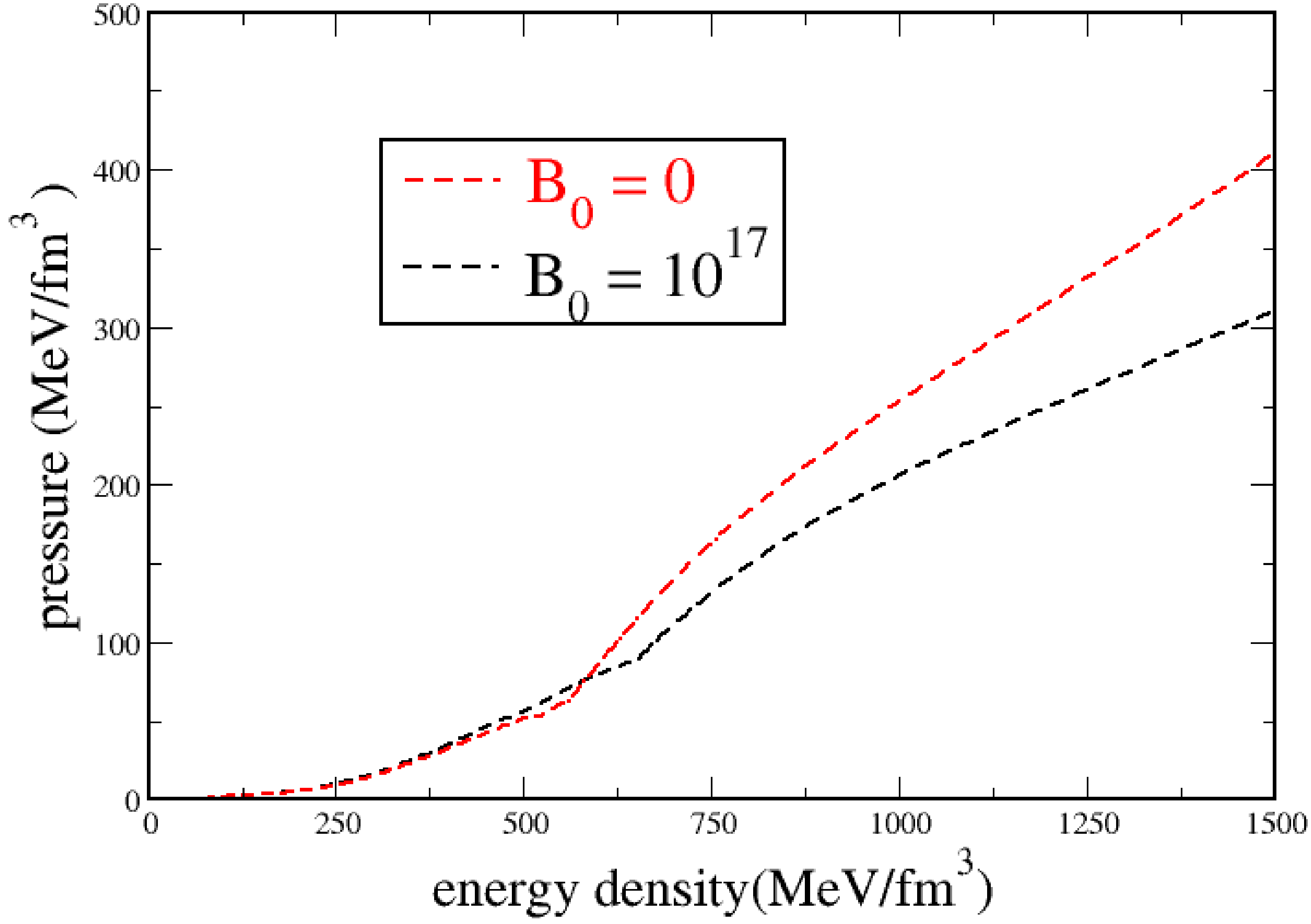


Results

Energy density vs Pressure



Radial Modes

$$X\frac{d^2\xi}{dr^2}+Y\frac{d\xi}{dr}+Z\xi=\tilde{\sigma}^2\xi$$

$\xi(r)$ is the Lagrangian fluid displacement $c\tilde{\sigma}$ is the characteristic eigenfrequency

The function X, Y, Z depend on equilibrium profile of pressure $'P'$ and enrgy density $'\varepsilon'$ and metric functions $\lambda(r)$ and $\nu(r)$

TOV equations :

$$\frac{dm(r)}{dr}=4\pi r^2\epsilon(r)$$

$$\frac{dP(r)}{dr}=\frac{-Gm(r)\epsilon(r)}{r^2}\frac{[1+P(r)/\epsilon(r)][1+4\pi r^3 P(r)/m(r)]}{1\text{-}2Gm(r)/r}$$

$$\lambda(r)=\ln(1-2Gm/rc^2)\qquad \frac{d\nu(r)}{dr}=\frac{2G}{r^2c^2}\frac{(m+4\pi r^3P/c^2)}{1-2Gm/rc^2}$$

Contd....

$$\xi(r=0)=0$$

$$\delta P(r) = -\xi \frac{dP}{dr} - \Gamma P \frac{e^{\nu/2}}{r^2} \frac{\partial}{\partial r}\left(r^2 e^{\nu/2} \xi\right)$$

$$\delta P(r=R)=0$$

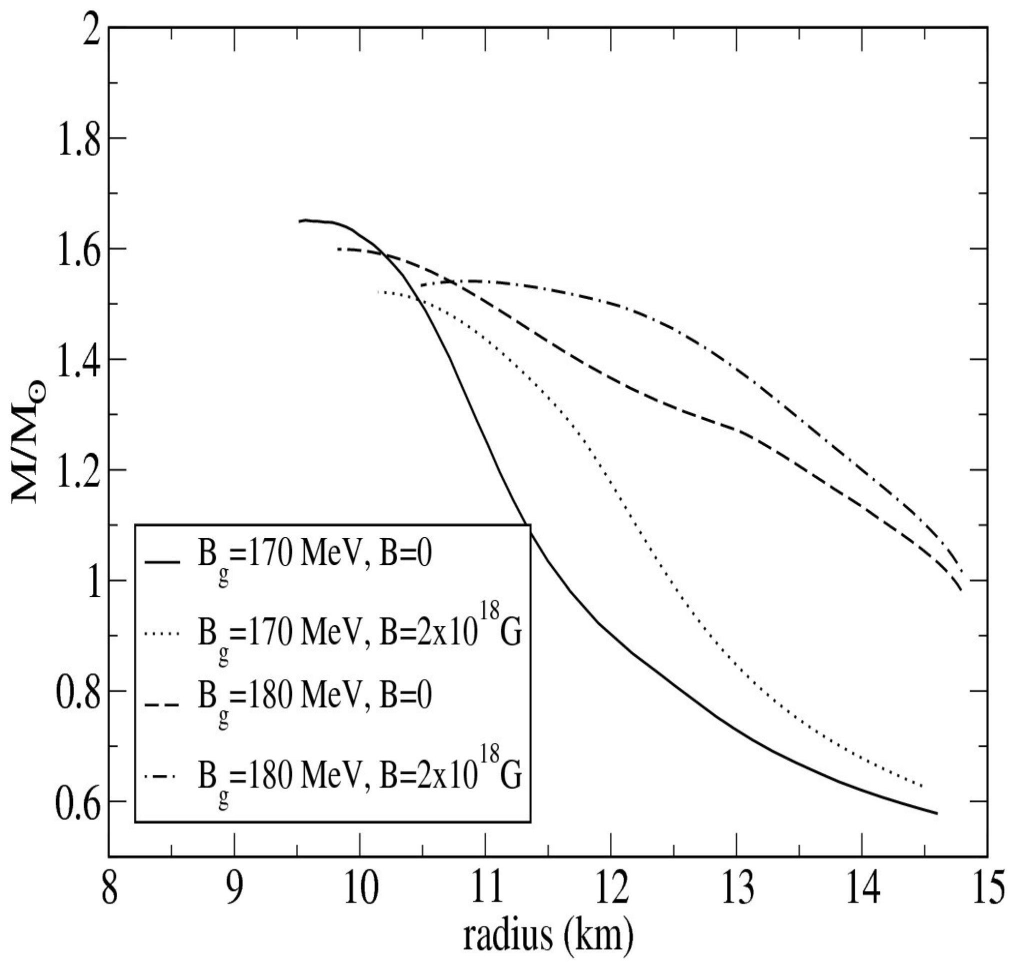
$$\sigma_0^2 < \sigma_1^2 < \sigma_2^2 < ......< \sigma_n^2$$

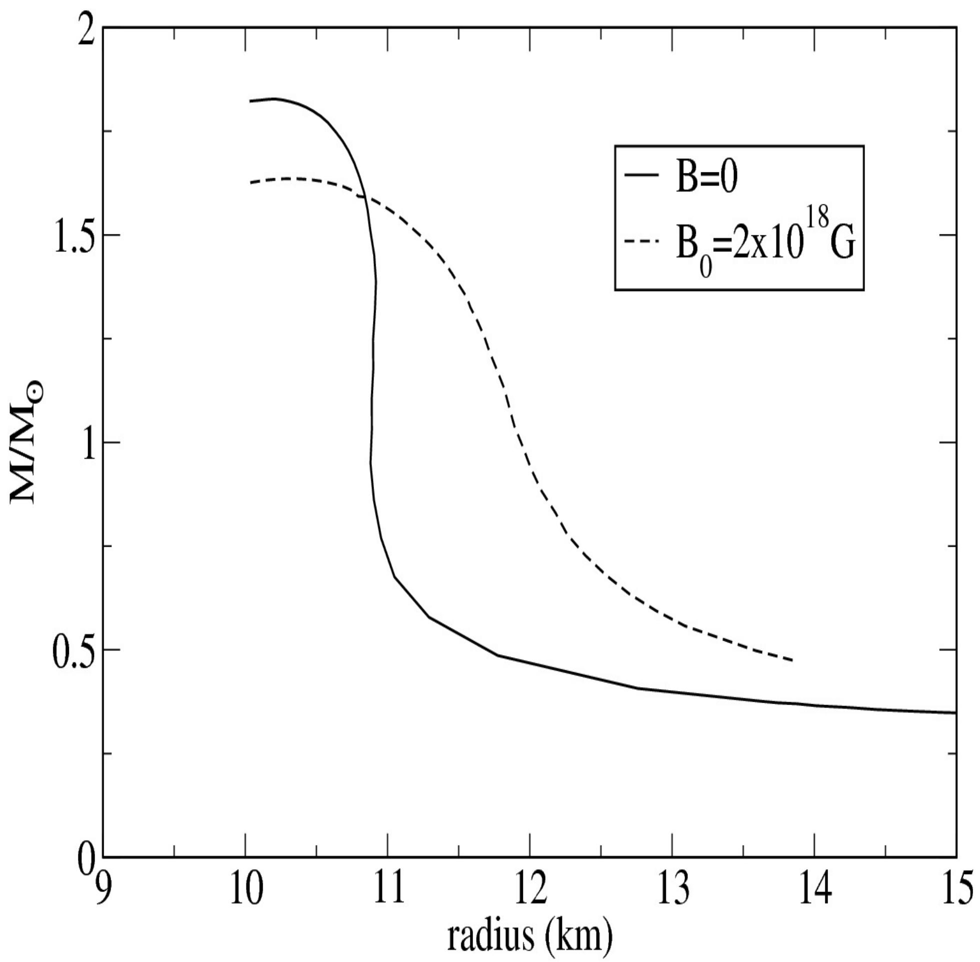
Hence if fundamental radial mode of a star is stable ($\sigma_0^2 > 0$), then all radial modes are stable

The period of oscillation $= \dfrac{2\pi}{c\,\tilde{\sigma}}$

Results

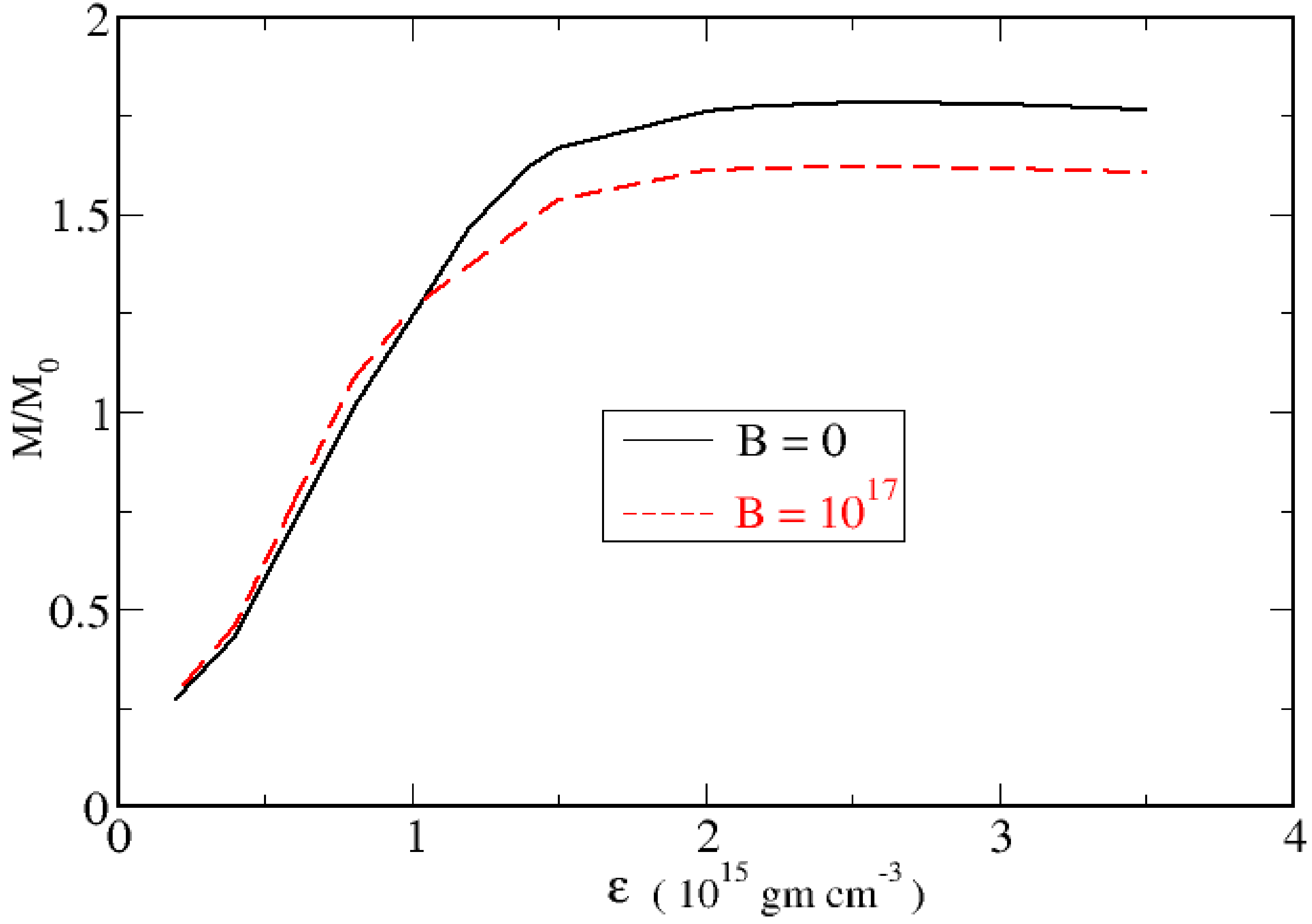
Gravitational mass vs. Radius





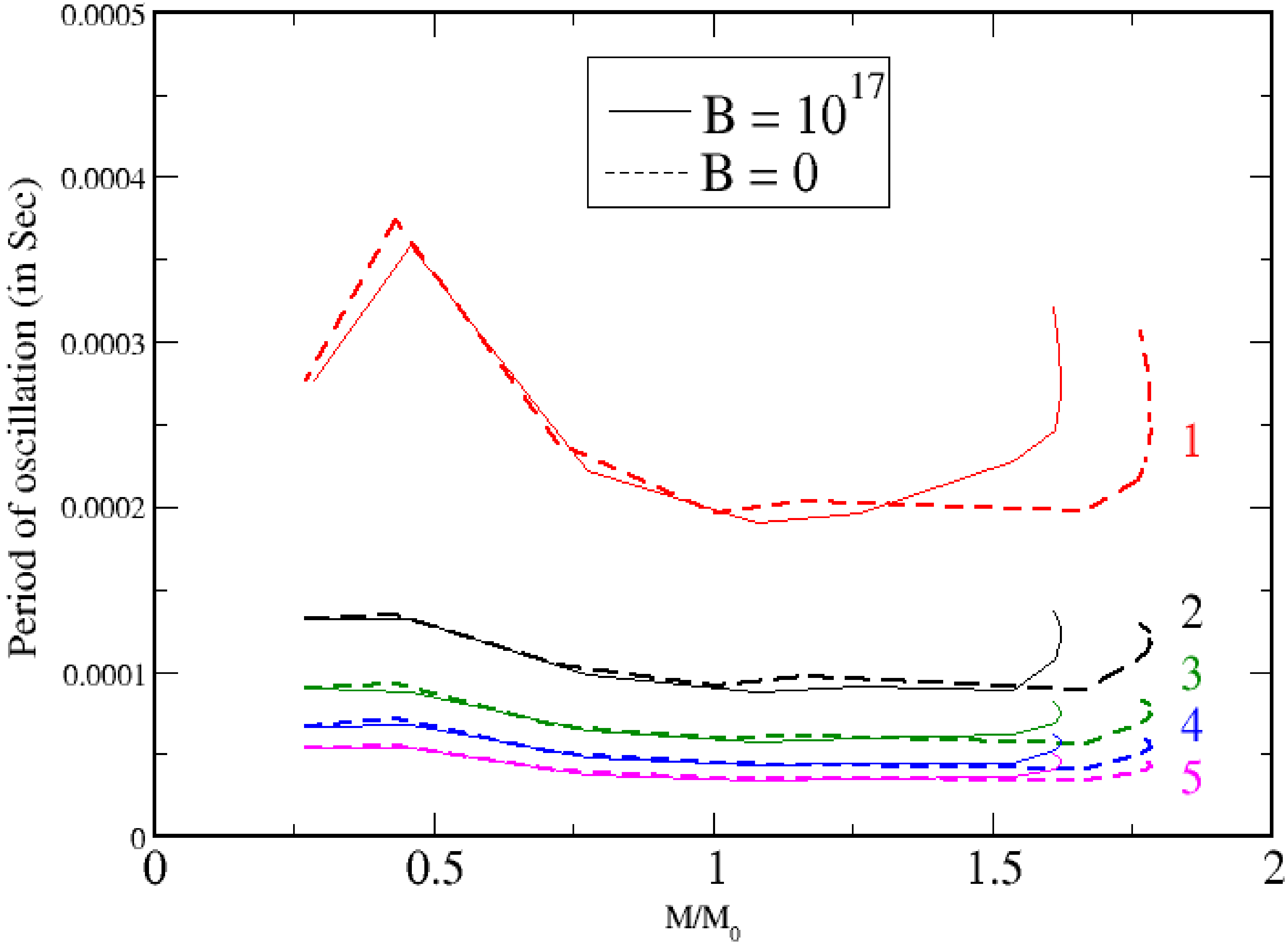
Results

Density vs. Gravitational mass



Results

Gravitational mass vs. Period of osillation



Summary & Remarks

- The effect of magnetic field is much more pronounced in quark matter .
- The period of oscillation shows a kink around the point where mixed phase starts,in primary as well as in the higher modes. Which is the distinct signature of quark matter onset in neutron star.
- The presence of magnetic field increases period of oscillation of fundamental as well as in higer mode at maximum mass but the effect is significant in fundamental mode.
- The presence of magnetic field broadens the mixed phase region .

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Thank You