Lattice study of hybrid static potentials

Philipp Wolf
in collaboration with Marc Wagner

Goethe-Universität Frankfurt, Institut für Theoretische Physik

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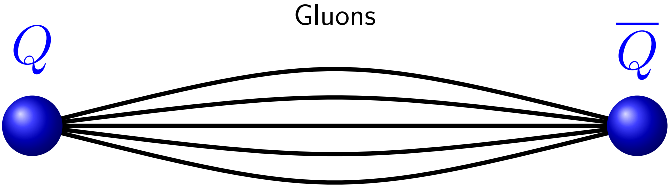
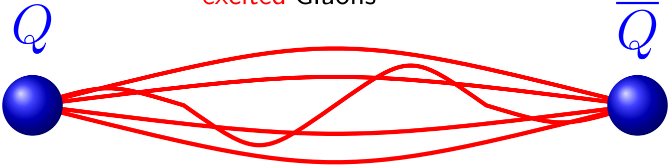
The hybrid static quark antiquark potential



Hybrid Potential

excited Gluons



allow gluons to carry quantum numbers:
parity - charge conjugation, angular momentum

Motivation

- quark model restricts $q\bar{q}$ mesons states to certain quantum numbers
  - $S = 0, 1$ $L = 0, 1, 2, ...,$ $P = (-1)^{L+1}$, $C = (-1)^{L+S}$
  - exotic mesons: not realised states with J^{PC} like $0^{+-}, 0^{--}, 1^{-+}$...
  - excited gluon fields can realise these quantum numbers
- hybrid mesons possible candidates for exotic mesons
  - some candidates for exotic state 1^{-+}: $\pi_1(1400)$, $\pi_1(1600)$, could be hybrid or tetra quark states [W.-M Yao *et al.* 1. Phys. **G33**, 1 (2006)]
- interest in the short distance behaviour of hybrid potentials
  - attractive/repulsive potential or just cut-off effects?

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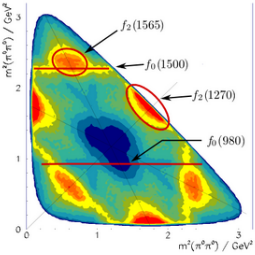
Physics - Hadron Spectroscopy

Search for Gluonic Excitations

One of the main challenges of hadron physics is the search for gluonic excitations, i.e. hadrons in which the gluons can act as principal components. These gluonic hadrons fall into two main categories: glueballs, i.e. states where only gluons contribute to the overall quantum numbers, and hybrids, which consist of valence quarks and antiquarks as hadrons plus one or more excited gluons which contribute to the overall quantum numbers.

The additional degrees of freedom carried by gluons allow these hybrids and glueballs to have J^{PC} exotic quantum numbers. In this case mixing effects with nearby $q\bar{q}$ states are excluded and this makes their experimental identification easier. The properties of glueballs and hybrids are determined by the long-distance features of QCD and their study will yield fundamental insight into the structure of the QCD vacuum. Antiproton-proton annihilations provide a very favourable environment to search for gluonic hadrons.



Correlation function

- euclidian correlation function $\langle\Omega|\,\mathcal{O}^\dagger(t)\mathcal{O}(0)\,|\Omega\rangle$ used to extract masses

$$\langle\Omega|\,\mathcal{O}^\dagger(t)\mathcal{O}(0)\,|\Omega\rangle = \sum_n |\,\langle n|\,\mathcal{O}\,|\Omega\rangle\,|^2 \;\; e^{-t(E_n - E_\Omega)}$$

$$= |\,\langle 0|\,\mathcal{O}\,|\Omega\rangle\,|^2 \;\; e^{-t(E_0 - E_\Omega)} + |\,\langle 1|\,\mathcal{O}\,|\Omega\rangle\,|^2 \;\; e^{-t(E_1 - E_\Omega)} + \ldots$$

$$\xrightarrow{t\to\infty} |\,\langle 0|\,\mathcal{O}\,|\Omega\rangle\,|^2 \;\; e^{-tm}$$

- observe exponential decay for large time separations to receive the mass

effective mass

$$m_{eff} = \ln\left(\frac{C(t)}{C(t+1)}\right) \quad \xrightarrow{t\to\infty} \quad m$$

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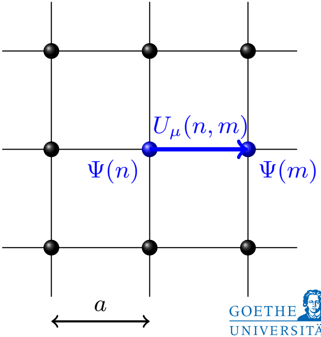
Euclidian path integral

- the correlation function can be written as a path integral

$$\langle\Omega|\, \mathcal{O}^\dagger(t)\mathcal{O}(0)\, |\Omega\rangle = \frac{1}{Z}\int \mathcal{D}[\Psi, A_\mu]\ \mathcal{O}^\dagger(t)\mathcal{O}(0)\ e^{-S_E[\Psi, A_\mu]}$$

- discretizing space-time and using Monte-Carlo methods to evaluate the integral

- fields Ψ live on the discrete lattice points
- while the gluons (U_μ), are connecting two neighbouring fields
- in continuum: $U(x,y) = P\exp\left(i\int_{\mathcal{C}_{xy}} A\cdot \mathrm{d}s\right)$
- closed loops form gauge invariant quantities



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The Static Quark-Antiquark Potential on the Lattice

- suitable operator for a quark antiquark pair acting on the vacuum state:

$$|\Phi\rangle = \overline{Q}(\mathbf{x})U(\mathbf{x},\mathbf{y})Q(\mathbf{y})\,|\Omega\rangle$$

- in the limit of infinite heavy quark masses the quark fields can be integrated out:

$$\xrightarrow{m\to\infty} \frac{1}{Z}\int \mathcal{D}[U]\; W[U]\; e^{-S_E[U]}$$

- leaving a integration over Wilson loops $W[U]$

Wilson Loop: trace of a closed loop of links on the lattice, consists of four parts.

- two *Wilson lines* $S(\mathbf{m},\mathbf{n},n_t), S(\mathbf{m},\mathbf{n},0)$

$$S(\mathbf{m},\mathbf{n},n_t) = \prod_k U_j(k,n_t)$$

- ... and two temporal transporters $T(\mathbf{n},n_t)$

$$T(\mathbf{n},n_t) = \prod_{i=0}^{n_t-1} U_t(\mathbf{n},j)$$

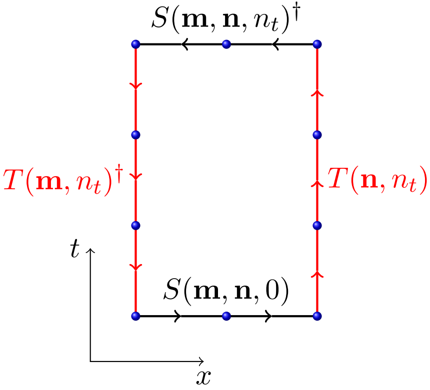


Figure : Wilson-Loop with R=2, T=3 in (x,t)-plane

Wilson Loop: trace of a closed loop of links on the lattice, consists of four parts.

Definition of the Wilson loop

$$W[U] =$$
$$\mathrm{Tr}(S(\mathbf{m},\mathbf{n},0)T(\mathbf{n},n_t)$$
$$\cdot\, S(\mathbf{m},\mathbf{n},_t)^\dagger T(\mathbf{m},n_t)^\dagger)$$

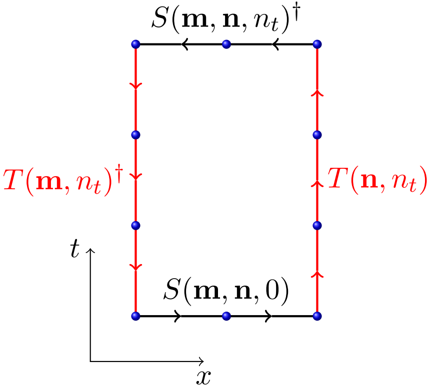
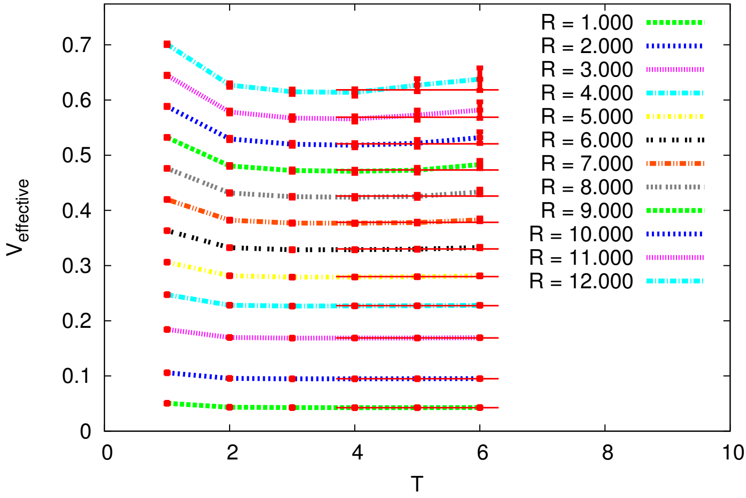
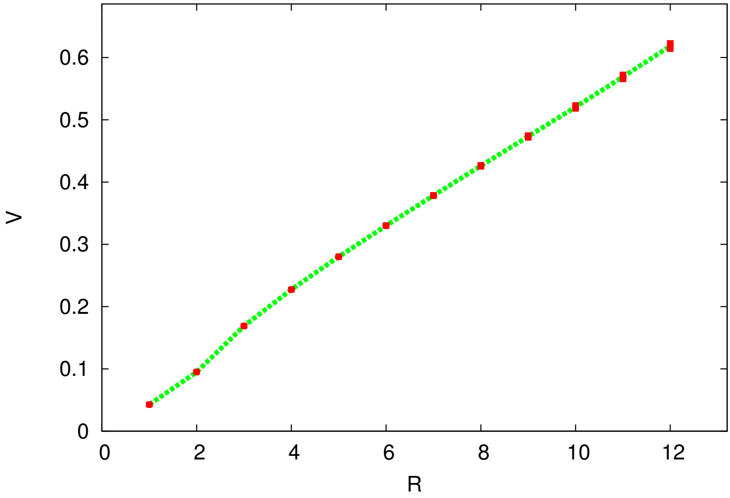


Figure : Wilson-Loop with R=2, T=3 in (x,t)-plane

Static effective $Q\bar{Q}$ potential



Static $Q\bar{Q}$ potential



Generalized Wilson loops

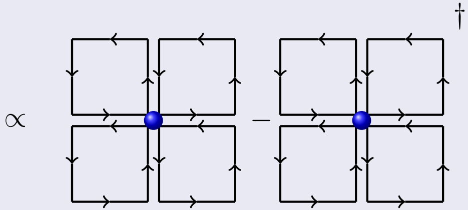
- insert gluonic excitations in the middle of the quark antiquark separation

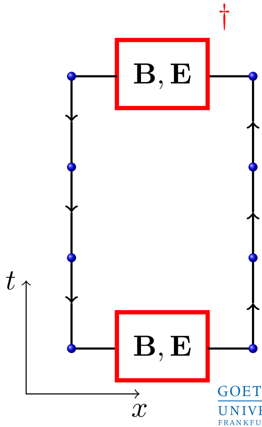
- use $\mathbf{B}, \mathbf{E}$ fields defined by field strength tensor:

$\mathbf{B}, \mathbf{E}$ fields

$$E^i = F^{i0}, \quad B^l = -\frac{1}{2}\varepsilon^{lij} F^{ij}$$

on the lattice





Quantum numbers

- the hybrid potentials can be characterized by 3 quantum numbers
- rotation around the separation axis: $J = 0, 1, 2, ...$ expressed by $\Sigma, \Pi, \Delta, ...$

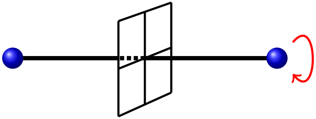


Figure : Σ_u^- State, invariant under rotation around the separation-axis

- combination of charge conjugation and spatial inversion about the midpoint between quark and antiquark: CP $+$ $(-)$ labelled with g (u) in subscript
- Σ states have an additional quantum number: $+$ $(-)$ depending on the reflection in a plane containing the separation axis

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List of operators

gluon state	operator	
Σ_g^+	$\mathbf{R} \cdot \mathbf{E}$,	$\mathbf{R} \cdot (\mathbf{D} \times \mathbf{B})$
Π_g	$\mathbf{R} \times \mathbf{E}$,	$\mathbf{R} \times (\mathbf{D} \times \mathbf{B})$
Σ_u^-	$\mathbf{R} \cdot \mathbf{B}$,	$\mathbf{R} \cdot (\mathbf{D} \times \mathbf{E})$
Π_u	$\mathbf{R} \times \mathbf{B}$,	$\mathbf{R} \times (\mathbf{D} \times \mathbf{E})$
Σ_g^-	$(\mathbf{R} \cdot \mathbf{D})(\mathbf{R} \cdot \mathbf{B})$	

- $\mathbf{R}$ is the unit vector of the separation axis
- $\mathbf{D}$ is the covariant derivative

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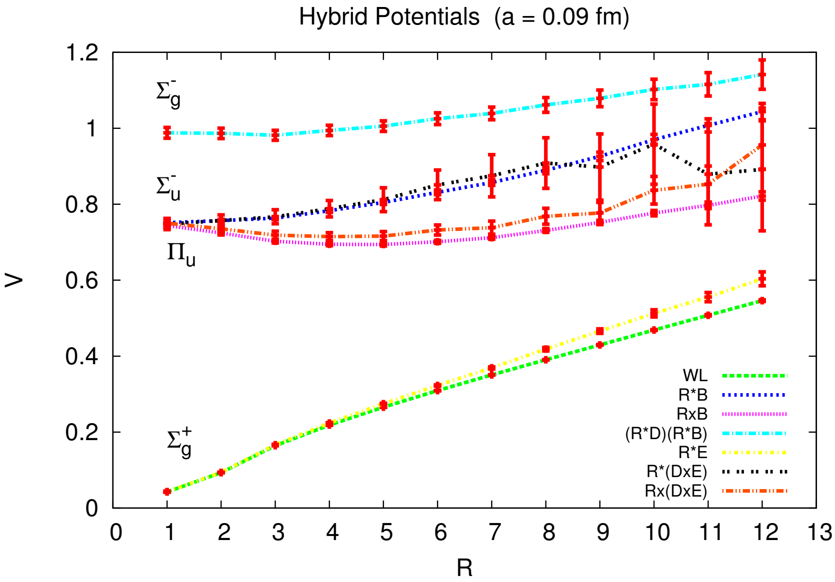
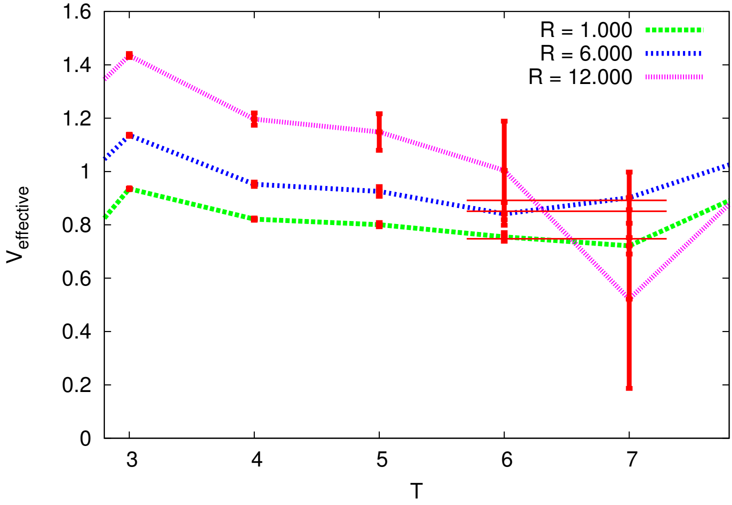
Results



Hybrid Potentials (a = 0.09 fm)
V
R
0
0.2
0.4
0.6
0.8
1
1.2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
Σ_g^-
Σ_u^-
Π_u
Σ_g^+
WL
R*B
RxB
(R*D)(R*B)
R*E
R*(DxE)
Rx(DxE)

Static effective $\mathbf{R} \cdot (\mathbf{D} \times \mathbf{E})$ hybrid potential



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An approach for hybrid meson masses

- leading Born-Oppenheimer approximation to calculate the mass
  - solve a Schrödinger equation, like for diatomic molecules
  - heavy quarks correspond to the nuclei, the gluons to the electrons

$$\left(\frac{\mathbf{p}^2}{2\mu} + V_{Q\bar{Q}}(r)\right)\Psi_{Q\bar{Q}}(r) = E\Psi_{Q\bar{Q}}(r)$$

  - insert computed hybrid potential and solve numerically [K. Jimmy Juge, Julius Kuti, Colin Morningstar [arXiv:nucl-th/0307116]]

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Summary

- first step to analyse hybrid mesons in a non-perturbative way
- LBO treatment provides approximation for hybrid meson masses
- still to do:
  - extend to SU(3) gauge group
  - calculate with different lattice spacings to analyse cut-off effects

Thank you for your attention