Chiral dynamics and peripheral partons in the nucleon

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Vietri sul Mare
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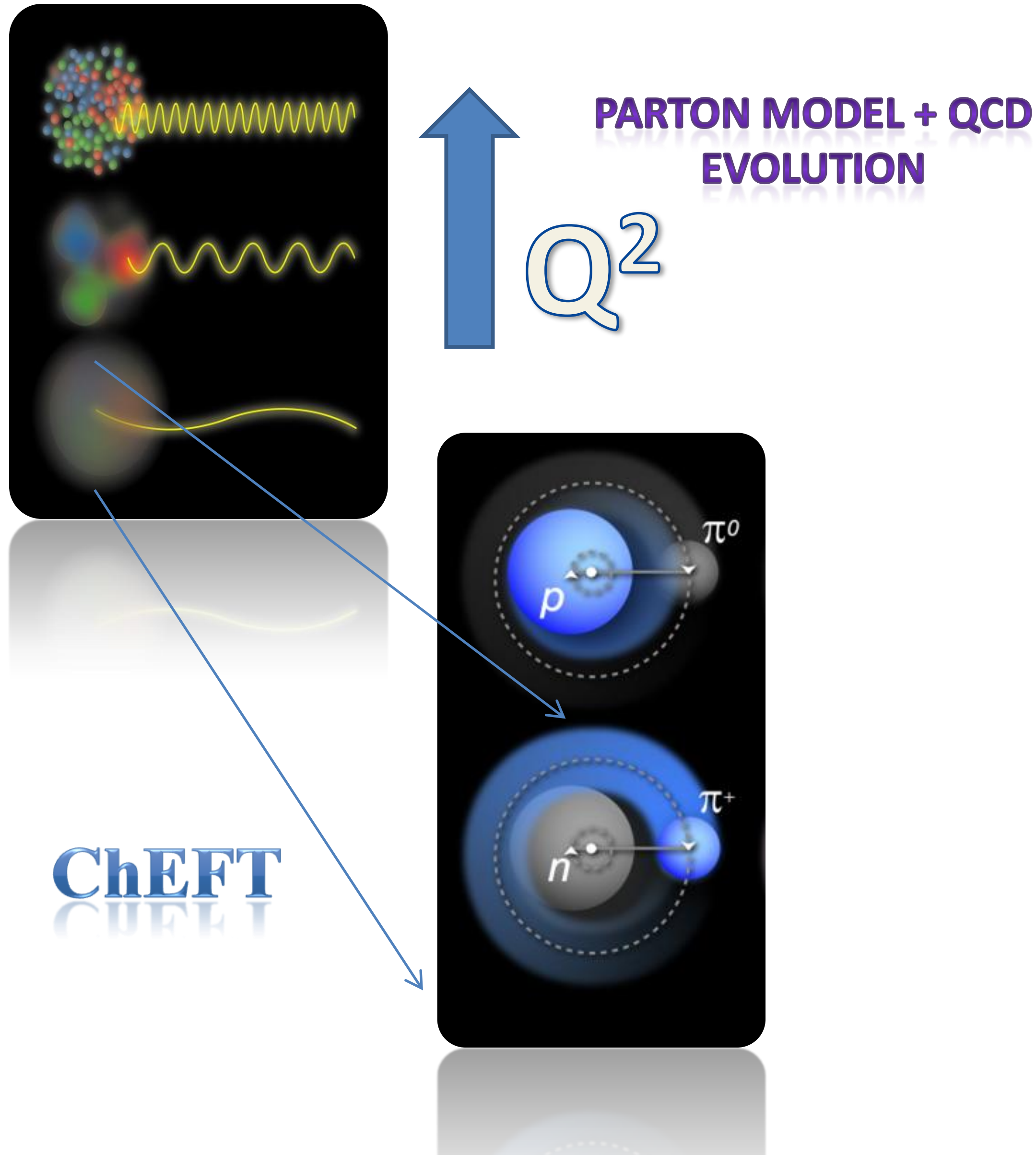
Outline

- Transverse densities:
  - Definition , Context and Motivation
- ChEFT and the covariant framework
- LF formalism and overlap representation
- Peripheral charge and matter distributions

Nucleon Structure



Nucleon Structure

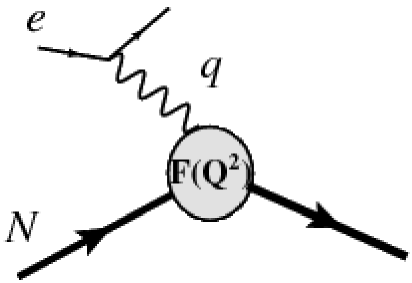
Electromagnetic (EM) Form Factors (FF)

$$\left\langle N_2 \middle| J^\mu \middle| N_1 \right\rangle = \overline{U}_2 \left[\gamma^\mu \mathrm{F}_1\left(\Delta^2\right) + i\sigma^{\mu\nu} \frac{\Delta_\nu}{2M} \mathrm{F}_2\left(\Delta^2\right) \right] U_1$$

Energy-Momentum Tensor (EMT) FF

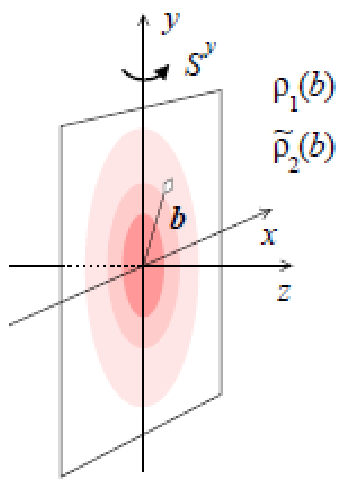
$$\left\langle N_2 \middle| \Theta^{N\pi}_{\mu\nu} \middle| N_1 \right\rangle = \overline{U}_2 \left[\gamma_{(\mu} P_{\nu)} \boldsymbol{A}\left(\Delta^2\right) + P_{(\mu} i\sigma_{\nu)\alpha} \frac{\Delta_\alpha}{2M} \boldsymbol{B}\left(\Delta^2\right) \right.$$
$$\left. + \left(\frac{\Delta_\mu \Delta_\nu - \Delta^2 g_{\mu\nu}}{M} \right) \boldsymbol{C}\left(\Delta^2\right) + M g_{\mu\nu} \tilde{\boldsymbol{C}}\left(\Delta^2\right) \right] U_1$$

Transverse Densities



G.A. Miller (2007)

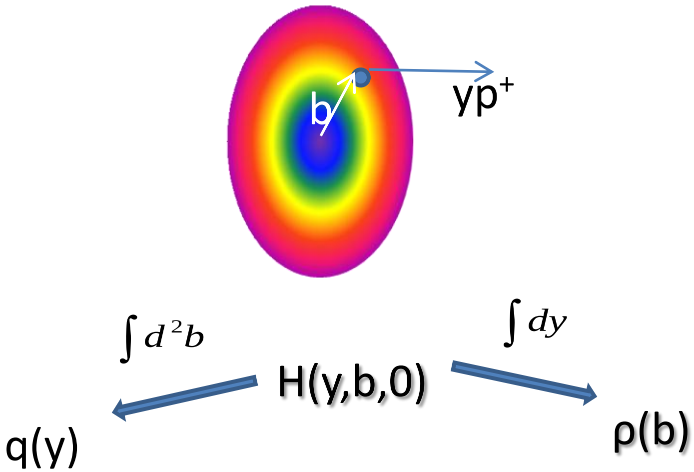
$$\rho(b) \equiv \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp\cdot\mathbf{b}} F(\Delta_\perp)$$

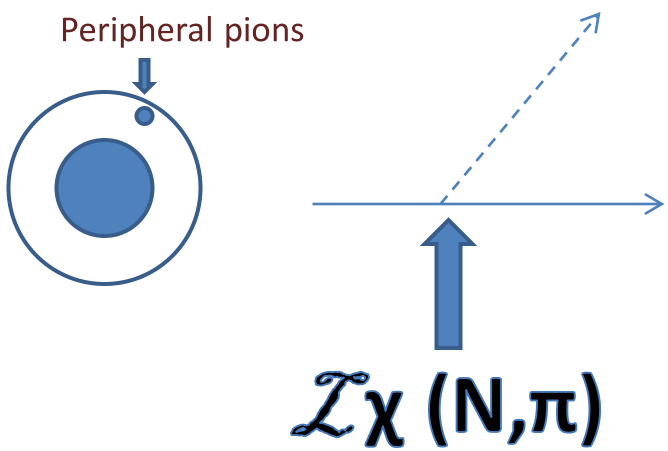


- **Connect Form factors (observables) to spatial structure of nucleon**
- Quantify description of a relativistic multi particle system
- Decompose spin structure of current density of the nucleon

$$\langle J^+(\boldsymbol{b})\rangle_{y-\mathrm{pol}} = \rho_1(b) + (2S^y)\cos\phi \underbrace{\frac{d}{db}\left[\frac{\rho_2(b)}{2M_N}\right]}_{\tilde{\rho}_2(b)}$$

Context





- Structure of hadrons as relativistic composite systems.

  PDF, GPDs, Tranverse densities, etc.

- Universality of large distance dynamics
  - Chiral symmetry Breaking
  - Effective field theory (pions, nucleons d.o.f)

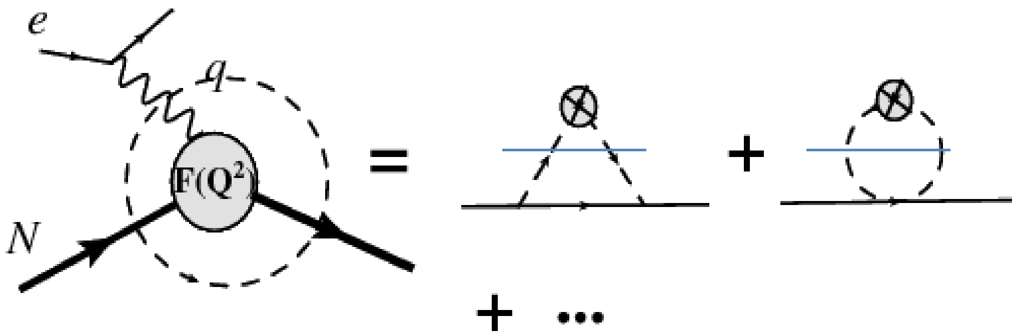
Aim

- Model Independent description

  Chiral Dynamics and Large distance profile : space parameterization of nucleon structure .
- Experiment
  - Form factors measurements in the low Q^2 region

    (JLab E12-11-106 Q2 ~ 10^{-2} - 10^{-4} GeV^2)
  - Connect chiral dynamics with Peripheral Processes in High Energy ep and baryon baryon Reactions.

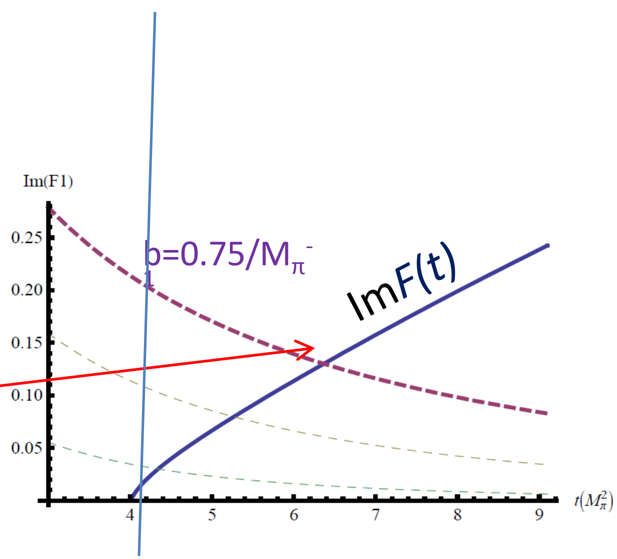
Methodology



Spectral functions from covariant Feynman diagrams

$$\rho(b) = \int_{4M_\pi^2}^{\infty} \frac{dt}{2\pi} K_0\left(\sqrt{t}b\right) \frac{1}{\pi} \operatorname{Im} F\left(t + i0\right)$$

$$K_0\left(\sqrt{t}b\right) \approx \sqrt{\frac{\pi}{2}} \frac{e^{-\sqrt{t}b}}{\left(\sqrt{t}b\right)^{\frac{1}{2}}}$$



Spatial parametrization of internal dynamics

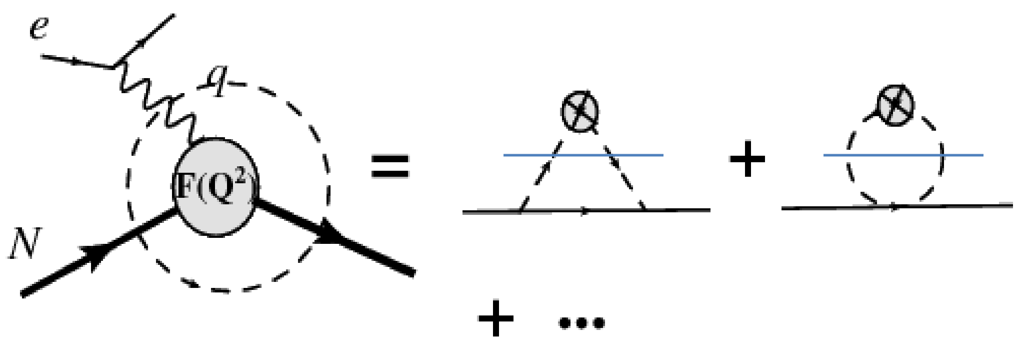
Methodology

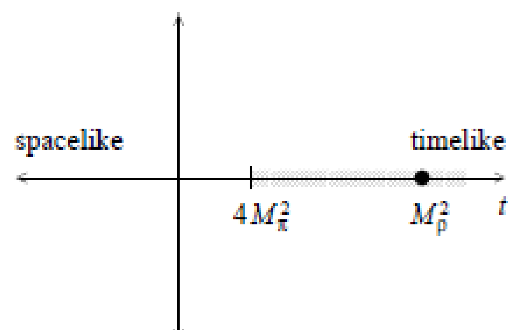
$$\mathcal{L}_{N\pi} = \overline{N}\left[i\hat{\partial} - M_N - \frac{1}{4f_\pi^2}\tau\cdot\pi\times\hat{\partial}\,\pi - \frac{g_A}{f_\pi}\gamma_5\tau\cdot\hat{\partial}\,\pi + \ldots\right]N$$

Gasser, Leutwyler 83; Weinberg 90

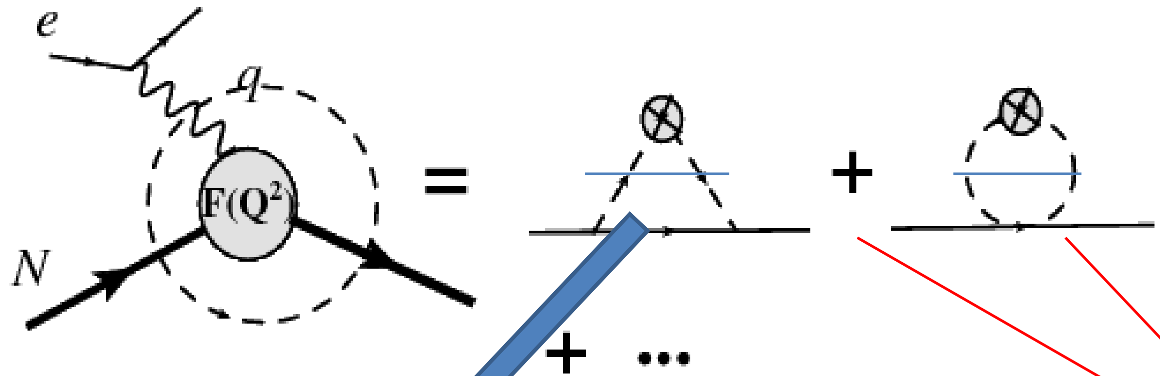
- Covariant approach
  - Invariant ChPT
  - Interaction Lagrangian with AV coupling
  - Dispersion relations and spectral functions

IM





Spectral functions



Contact term~10%

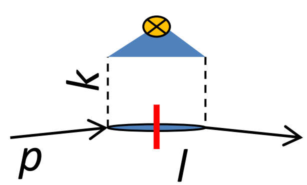
$$\frac{1}{\pi} Im F_1^V(t+i0) = \frac{M_N^2 g_A^2 (t/2 - M_\pi^2)^2}{(4\pi F_\pi)^2 (P^2)^{5/2} \sqrt{t}} \left[-\frac{t}{8} x^2 \arctan x + \left(M_N^2 + \frac{t}{8} \right)(x - \arctan x) \right] + \frac{2(1-g_A^2) k_{cm}^3}{3(4\pi F_\pi)^2 \sqrt{t}}$$

$$\frac{1}{\pi} Im F_2^V(t+i0) = \frac{M_N^4 g_A^2 (t/2 - M_\pi^2)^2}{2(4\pi F_\pi)^2 (P^2)^{5/2} \sqrt{t}} \left[(x^2+3) \arctan x - 3x \right],$$

Strikman, Weiss PRC (2010)

C.G., C. Weiss JHEP (2014)

$$x = \frac{2\sqrt{M_N^2 - \frac{t}{4}}\sqrt{\frac{t}{4} - M_\pi^2}}{\frac{t}{2} - M_\pi^2}$$



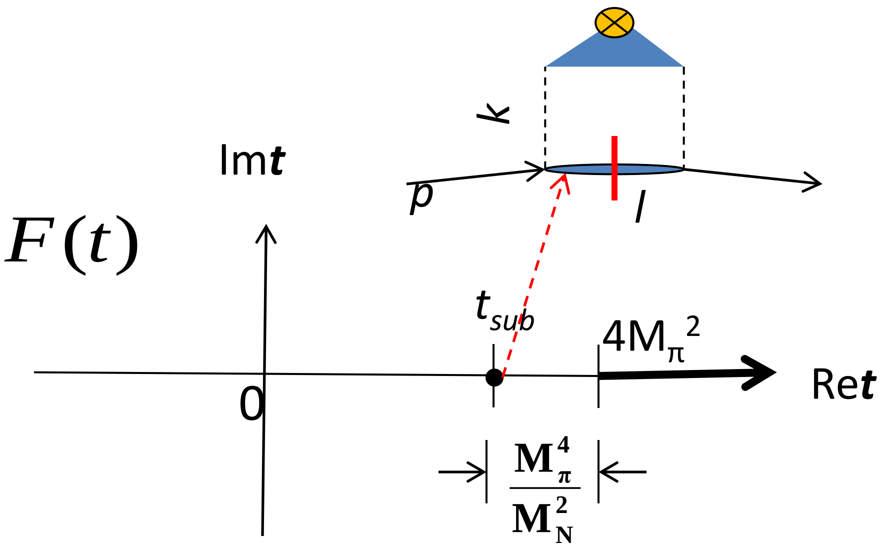
Sub-threshold singularity at

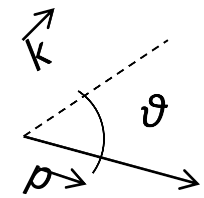
$x(t_{sub})=\pm I$

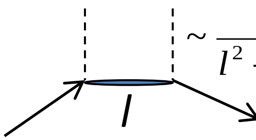
$$t_{sub} = 4M_\pi^2 - \frac{M_\pi^4}{M_N^2}$$

Molecular modes

Analytic Structure Near Threshold







$$\sim \frac{1}{l^2 - M_N^2 + i0} = -\frac{1}{A(t) - iB(t)\cos\theta}$$

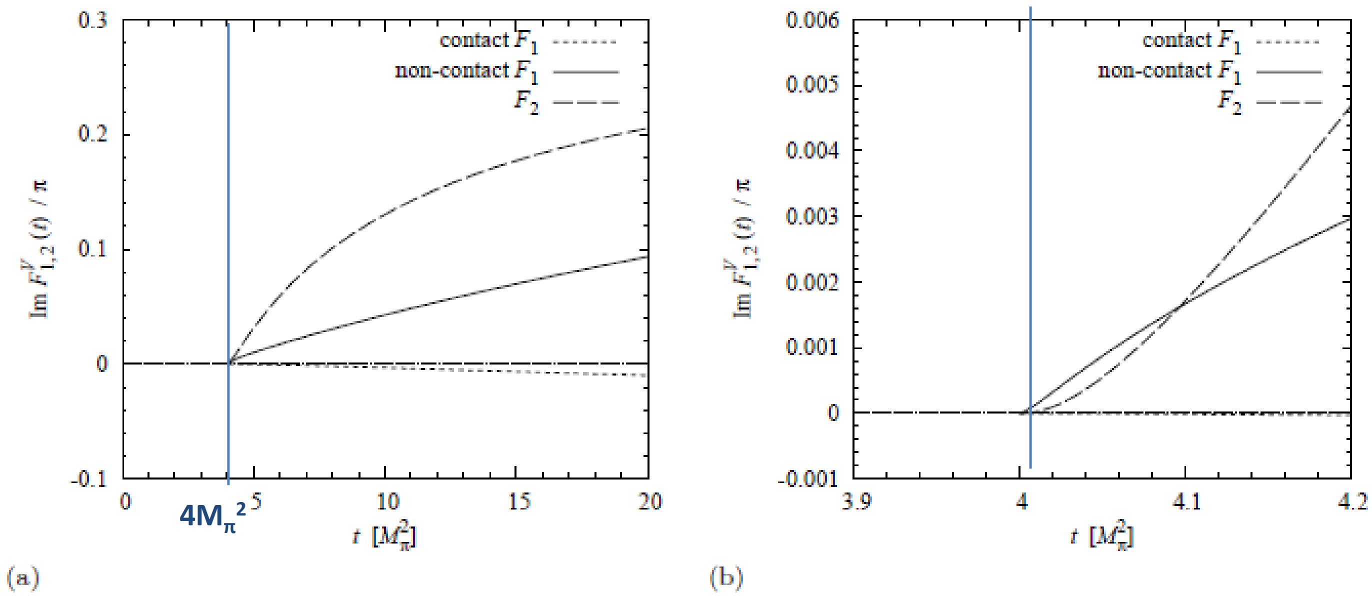
$$\mathrm{Im}\,F = \int_{-1}^{1} \frac{f(t,\cos\theta)}{A(t) - iB(t)\cos\theta} d(\cos\theta)$$

$$A(t_{sub}) = iB(t_{sub}) \Rightarrow t_{sub} = 4M_\pi^2 - \frac{M_\pi^2}{M_N^2}$$

- Sub-threshold singularity
  - End-point singularity
  - Intermediate nucleon on-shell
  - **Limits convergence of expansion near threshold**
  - Controls large $b(\sim M_N^2 M_\pi^{-3})$ behavior of transverse densities

Spectral functions

C.G., C. Weiss JHEP (2014)



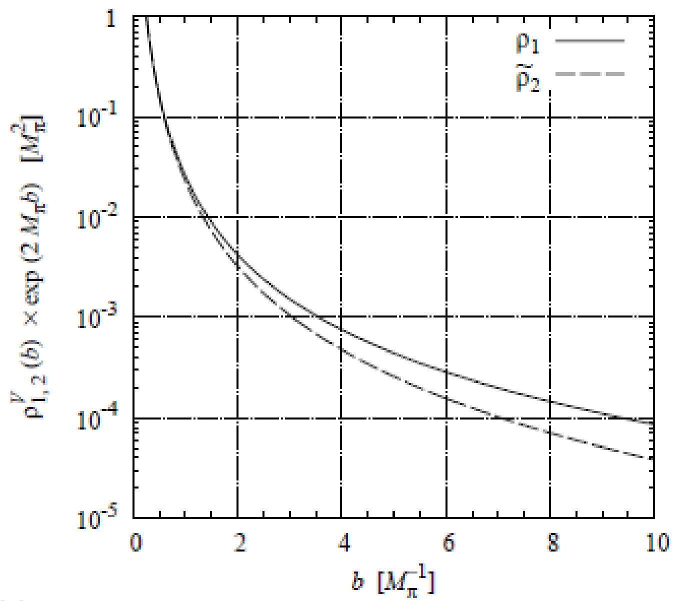
- Chiral modes

$$b \sim O(M_\pi^{-1})$$

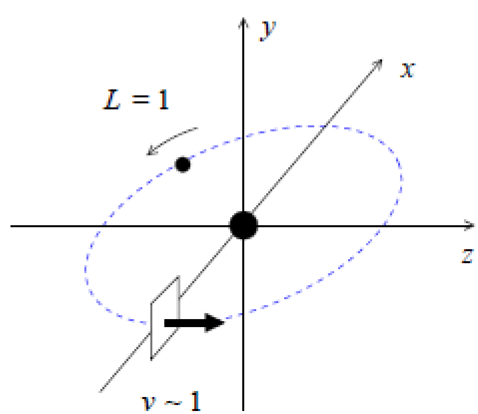
- Molecular modes

$$b \sim O\left(\frac{M_N^2}{M_\pi^3}\right)$$

Transverse Densities



(a)



- Heavy Baryon expansion

$$\rho_1^V(b) = g_A^2\left[\frac{4}{\pi}\sum_{n=0}\epsilon^n R_n(M_\pi b)\right] + (1-g_A^2)\ \frac{4R_{cont}(M_\pi b)}{3\pi},$$

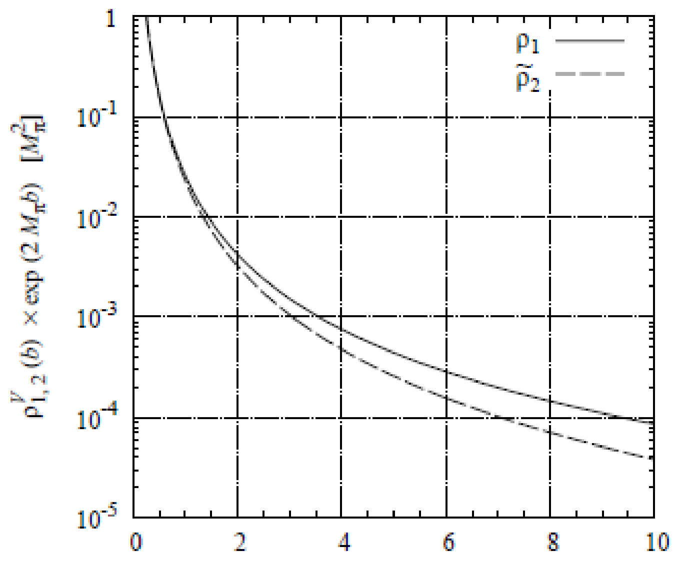
$$\tilde{\rho}_2(b) = g_A^2\left[\frac{4}{\pi}\sum_{n=0}\epsilon^n\,\overline{R}_n(M_\pi b)\right]$$

- Hint a mechanical picture

$$\frac{\tilde{\rho}_2^V(b)}{\rho_1^V(b)} = O\left(\frac{M_\pi^0}{M_N^0}\right)$$

$$\frac{|J^z|}{J^0} = v = O(1)$$

Transverse Densities



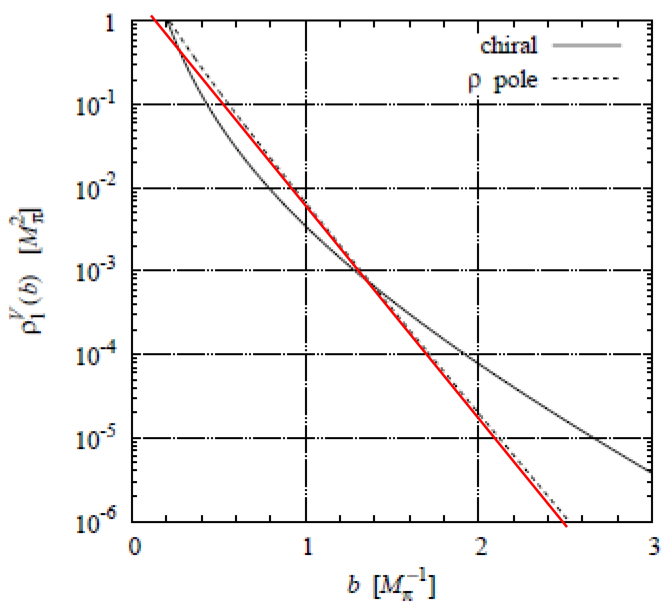
C.G., C. Weiss JHEP (2014)

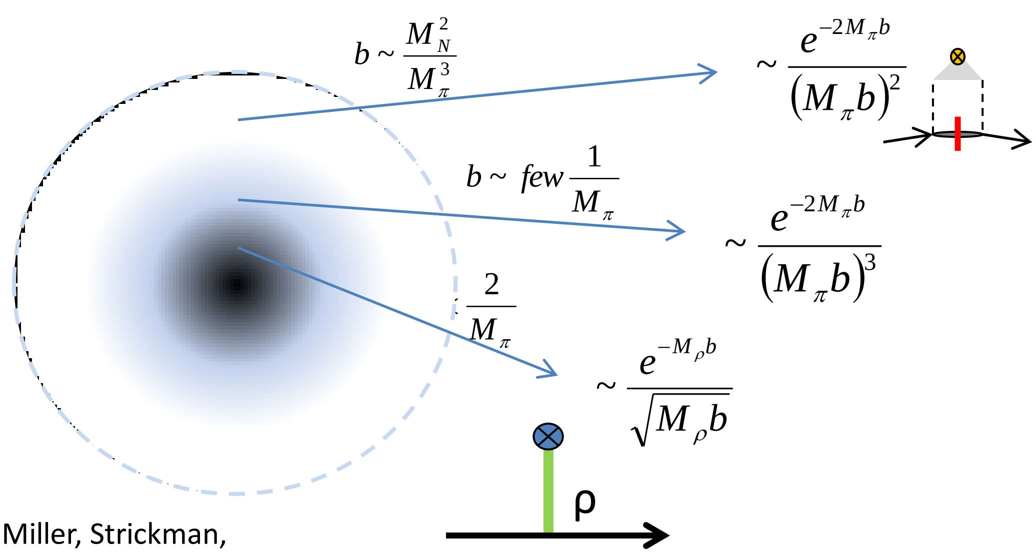
- Heavy Baryon expansion (in chiral region)

$$\rho_1^V(b) = g_A^2 \left[\frac{4}{\pi} \sum_{n=0} \epsilon^n R_n(M_\pi b) \right] + (1-g_A^2)\, \frac{4 R_{cont}(M_\pi b)}{3\pi},$$

$$\tilde{\rho}_2(b) = g_A^2 \left[\frac{4}{\pi} \sum_{n=0} \epsilon^n \overline{R}_n(M_\pi b) \right]$$

- Non-Chiral vs Chiral and Molecular



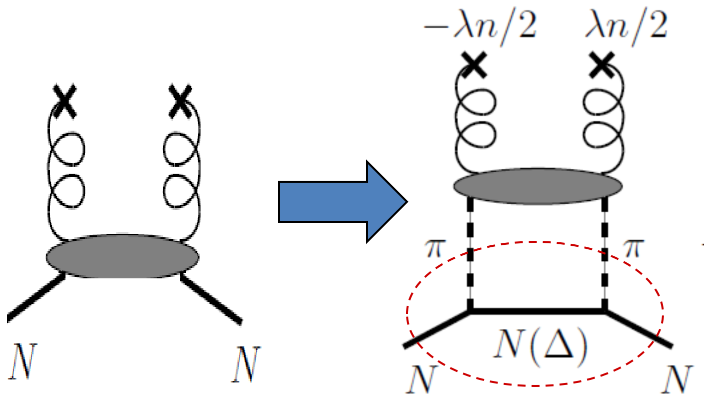


Miller, Strickman, Weiss PRC(2010)

Peripheral Densities in Light-Front χPT

- Develop a partonic formulation of chiral dynamics

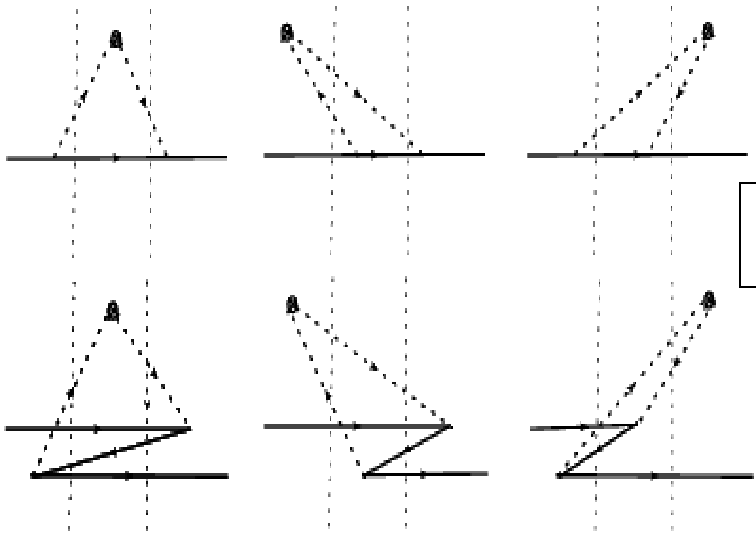
- Connect to GPD formalism

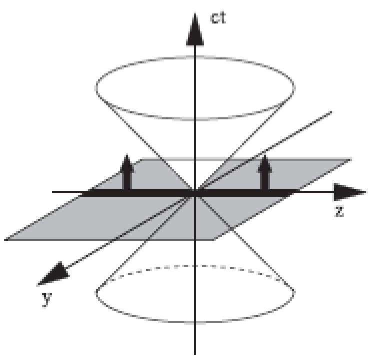


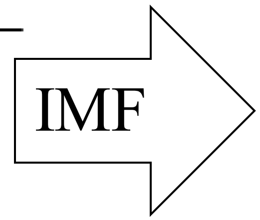
$$H_g(x,t) = \int_x^1 \frac{dy}{y}\, g_\pi(x/y)\, H_\pi(y,t)$$

Peripheral Densities in Light-Front χPT

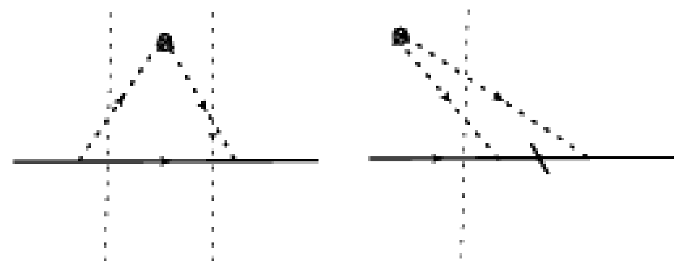
Equal time x_o

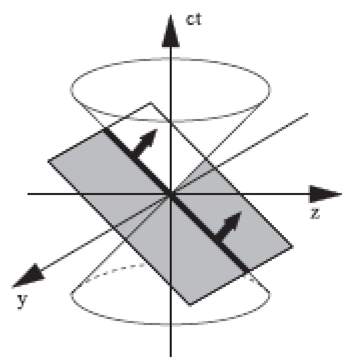


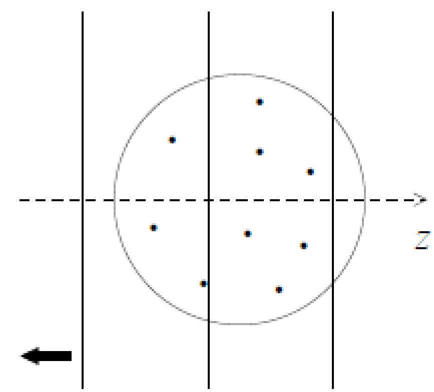




Equal LF time x+

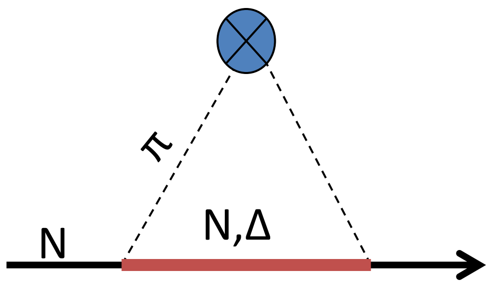




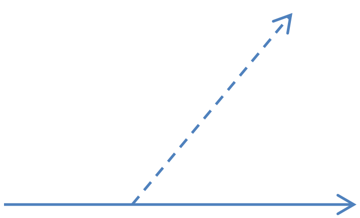


$$x^+ = x^0 + x^3 = \text{const.}$$

Wave Functions in the Light Front



$$\left\langle p_2 \left| \frac{J^{+V}_{\pi N}}{2p^+} \right| p_1 \right\rangle = \int \frac{dy}{2\pi} \int \frac{d^2 k_\perp}{(2\pi)^2} \Psi^\dagger_{\pi N}\left(y, k_\perp + (1-y)\frac{\Delta_\perp}{2}\right) \Psi_{\pi N}\left(y, k_\perp - (1-y)\frac{\Delta_\perp}{2},\right),$$



$$\frac{g_A M_N}{F_\pi} \bar{u}_{s'}(l)\gamma_5 u_s(p_1)$$

$$\Psi_{\pi N}(y, k_\perp, s, s_l) = \frac{1}{\sqrt{y(1-y)}} \frac{i\Gamma(y, k_\perp, s, s_l)}{M_N^2 - M_{\pi N}^2(y, k_\perp^2)}$$

$$\Psi_{\pi N}(y, r_\perp) \equiv \int \frac{d^2 k_\perp}{(2\pi)^2} e^{-i r_\perp \cdot k_\perp} \Psi_{\pi N}(y, k_\perp)$$

- Light Front Formulation
  - LC perturbation theory from $\mathcal{L}_\chi$ (N,π)
  - Solve LC-Hamiltonian with pseudo-scalar int. + ~~cont. term~~. Pion-Nucleon comp.
  - Relativistic Wave functions in k_T and b space

Wave Functions in the Light Front

$$\Psi_{\pi N}(y,k_\perp,s,s_l)=-i\frac{g_A M_N}{F_\pi}2\sqrt{y}\,\frac{yM_N S_3(s,s_l)+k_\perp\cdot S_\perp(s,s_l)}{k_\perp^2+\widetilde{M}_\pi^2(y)}$$

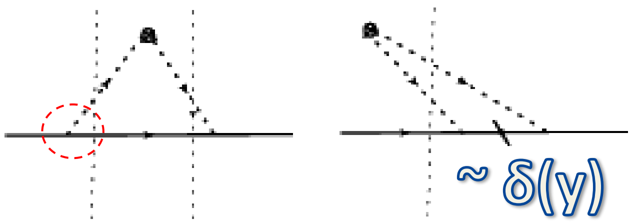
$$\Psi_{\pi N}(y,r_\perp,s,s_l)=-2i\psi_0(y,r_\perp)S_3(s,s_l)-2\psi_1(y,r_\perp))\,\hat{r}_\perp\cdot S_\perp(s,s_l)$$

$$\psi_0(y,r_\perp)=\frac{g_A}{F_\pi}\frac{y\sqrt{y}}{2\pi}M_N^2 K_0(\widetilde{M}_\pi r_\perp)$$

$$\psi_1(y,r_\perp)=\frac{g_A}{F_\pi}\frac{\sqrt{y}\,M_N}{2\pi}\widetilde{M}_\pi K_1(\widetilde{M}_\pi r_\perp),$$

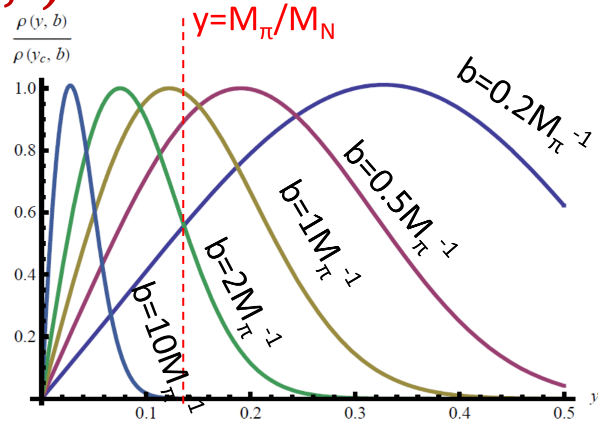
Hints to OAM expansion in impact parameter space

Transverse Densities



$$\rho(b)=\int dy(1-y)^{-2}|\psi(y,b)|^2$$

$$\rho(b)=\int dy\ \rho(y,b)$$



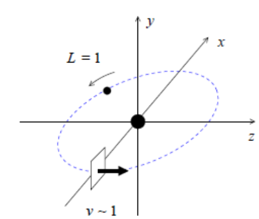
$$\rho_1(b)=\int \frac{dy}{y(1-y)^2}\left[|\psi_0(y,b')|^2+|\psi_1(y,b')|^2\right]$$

$$\tilde{\rho}_2(b)=\int \frac{dy}{y(1-y)^2}\left[\psi_0^\dagger(y,b')\psi_1(y,b')+\psi_1^\dagger(y,b')\psi_0(y,b')\right]$$

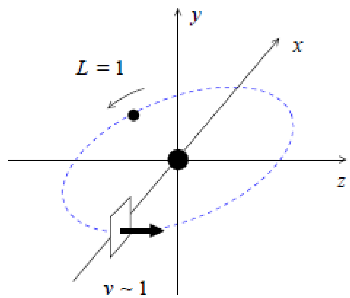
- Partonic (GPDs) and zero mode contributions
- Integrated GPD= Non contact term from covariant AV interaction
- Periphery populated by slower but relativistic pions
- Positive definitiveness of LC current:

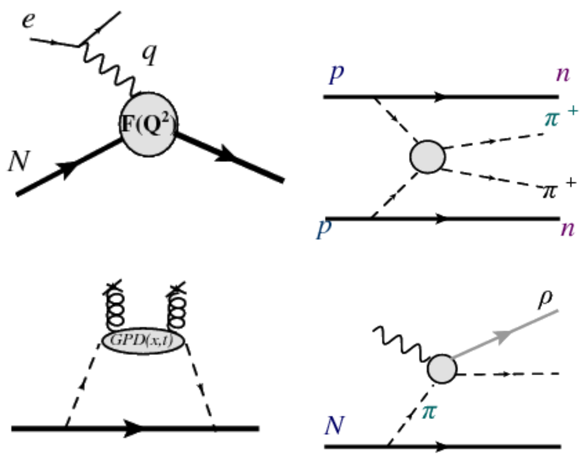
  Quasi free peripheral pions

? $\Rightarrow$ $\left\langle J^+(b)\right\rangle > 0$



Motivation

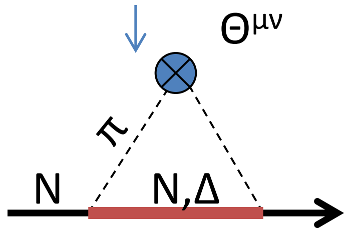




ρ(y,b)
and other
distributions

- Mechanical Picture of the nucleon
  - Relativistic wave functions,
  - Properly defined densities
  - Matter distributions, Angular momentum
- Experiment
  - Explore Universality

Energy Momentum Tensor



$$\Theta^{N\pi}_{\mu\nu}(0)=\overline{u}_2\Big[A(\Delta^2)\gamma_{(\mu}P_{\nu)}+B(\Delta^2)P_{(\mu}i\sigma_{\nu)\alpha}\frac{\Delta^\alpha}{2M_N}$$

$$+C(\Delta^2)\left(\frac{\Delta_\mu\Delta_\nu-\Delta^2 g_{\mu\nu}}{M_N}\right)+\tilde{C}(\Delta^2)M_N g_{\mu\nu}\Big]u_1,$$

$$J_\pi=\frac{1}{2}[A(0)+B(0)]$$

$$\rho_{J_\pi}(b)=\frac{1}{2}[\rho_A(b)+\rho_B(b)]$$

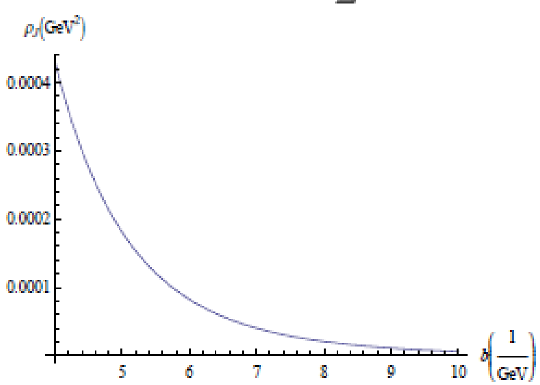
- Probe internal kinematic structure of the nucleon
- Reconstruct nucleon intrinsic properties from parton distributions i.e., FFs as moments of GPDs
  - Spatial parametrization through transverse densities
  - Model independence in chiral periphery
- Compare to QM definitions from wave function
  - Orbital Angular Momentum

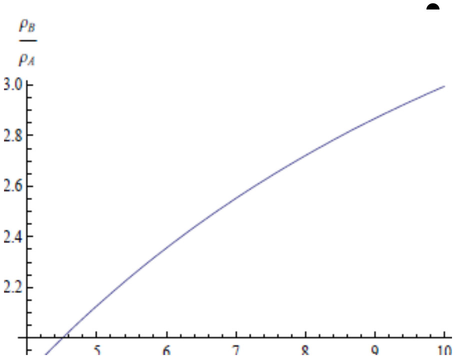
Energy Momentum Tensor

$$\frac{1}{\pi} Im A(t) = \frac{3}{8} \frac{M_N^2 g_A^2 (t/2 - M_\pi^2)^3}{(4\pi F_\pi)^2 \sqrt{P^2}^{\,5} \sqrt{t}} \left(\frac{4}{3}\left(1 - \frac{M_N^2}{P^2}\right) x^3 + 2x - \left(\left(2 - 3\frac{M_N^2}{P^2}\right) x^2 - 3\right) \arctan(x) \right)$$

$$\frac{1}{\pi} Im B(t) = \frac{3}{8} \frac{M_N^2 g_A^2 (t/2 - M_\pi^2)^3}{(4\pi F_\pi)^2 \sqrt{P^2}^{\,5} \sqrt{t}} \frac{M_N^2}{P^2} \left(\frac{4}{3} x^3 + 5x - (3x^2 + 5) \arctan(x) \right)$$

$$\rho_{J_\pi}(b) = \frac{1}{2} \left[\rho_A(b) + \rho_B(b) \right]$$





- **Spectral Functions**

  No contact term contribution, but expected for C(Δ^2). Further insight into composite nature of nucleon

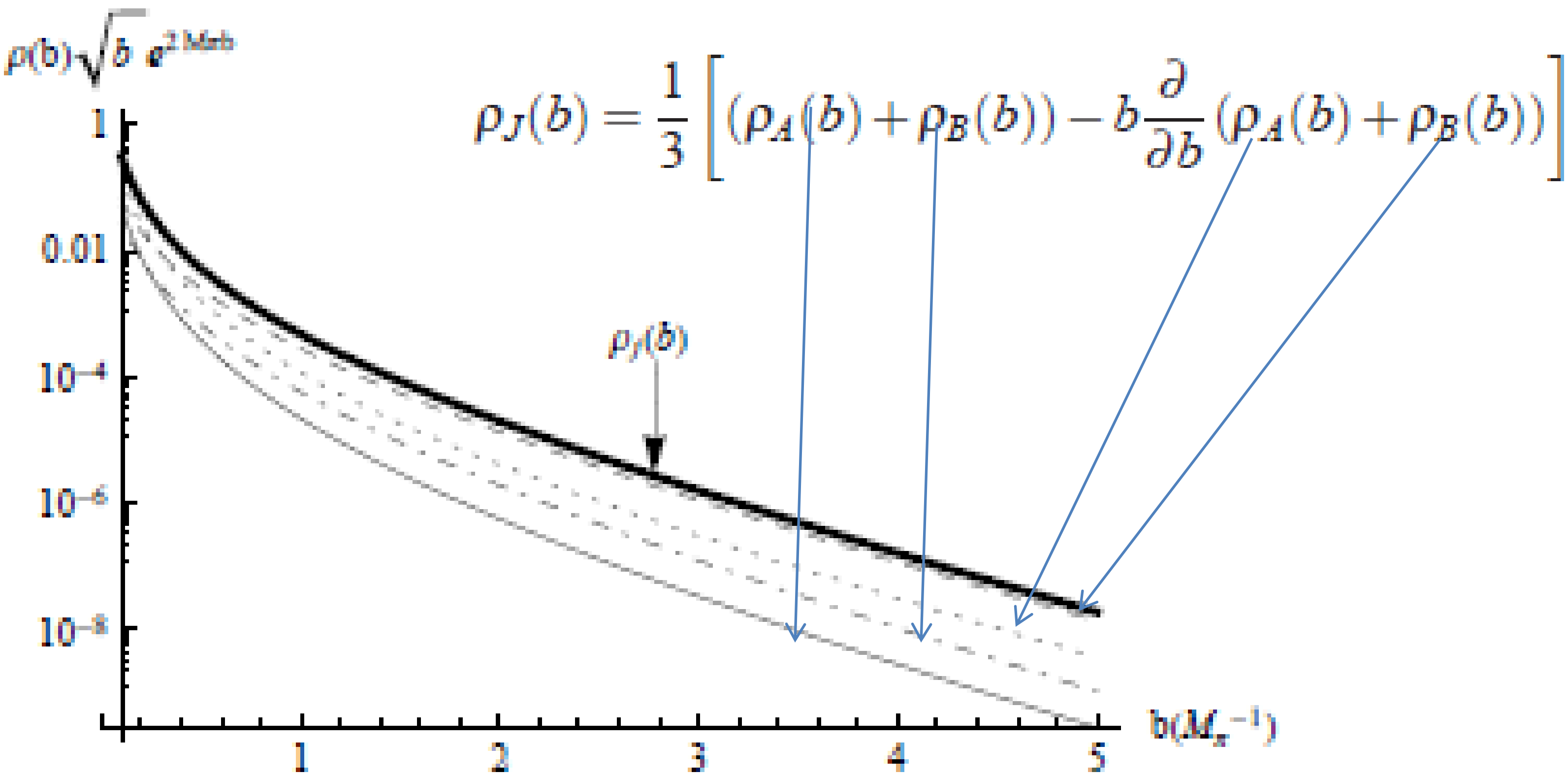
- Transverse densities

$$\frac{\rho_B}{\rho_A} \sim \frac{\rho_2}{\rho_1} \sim O\left(\frac{M_N}{M_\pi}\right)$$

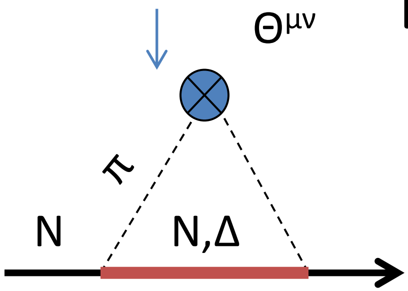
- Peripheral transverse density of Orbital Angular Momentum

Energy Momentum Tensor

$$\rho_J(b) = \frac{1}{3}\left[(\rho_A(b)+\rho_B(b)) - b\frac{\partial}{\partial b}(\rho_A(b)+\rho_B(b))\right]$$



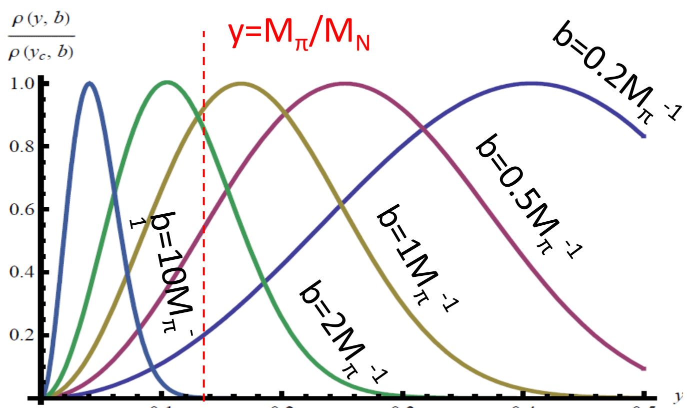
EMT Transverse Densities from LCWFs



- EMT Form factors from LCWF overlap

$$\left\langle p_2 \left| \frac{\Theta^{++V}_{\pi N}}{(2p^+)^2} \right| p_1 \right\rangle = \int \frac{dy}{2\pi} \int \frac{d^2k_\perp}{(2\pi)^2} \, y \Psi^\dagger_{\pi N}\left(y, k_\perp + (1-y)\frac{\Delta_\perp}{2}\right) \Psi_{\pi N}\left(y, k_\perp - (1-y)\frac{\Delta_\perp}{2}\right)$$

$\rho_A(y,b) = y\, \rho_1(y,b)$



$$\rho_A(b) = \int \frac{dy}{(1-y)^2} \left[|\psi_0(y,b')|^2 + |\psi_1(y,b')|^2 \right]$$

$$\tilde{\rho}_B(b) = \int \frac{dy}{(1-y)^2} \left[\psi_0^\dagger(y,b')\psi_1(y,b') + \psi_1^\dagger(y,b')\psi_0(y,b') \right]$$

- Transverse densities as moments of GPDs
- Helicity flip GPD and $\rho_B(y,b)$, dominate at periphery

$$\frac{\rho_B(y,b)}{\rho_A(y,b)} \sim \frac{\rho_2(y,b)}{\rho_1(y,b)} \sim y \sim O\left(\frac{M_N}{M_\pi}\right)$$

Summary

- Explored Nucleonic Structure in a setting that guarantees a model independent analysis of the dynamics governed by χ EFT.
  - Derived EM and EMT transverse peripheral densities from analiticity of corresponding FF
  - **Spatial parametrization of internal kinematics of nucleon**
- Calculated transverse densities from LC-Wave Functions (Connection to GPD formalism)
  - Numerical agreement with covariant formalism
  - First approach to **Orbital angular momentum of peripheral pions**
- **Emerging mechanical picture of nucleon's periphery**
- Universality of transverse densities

Outlook

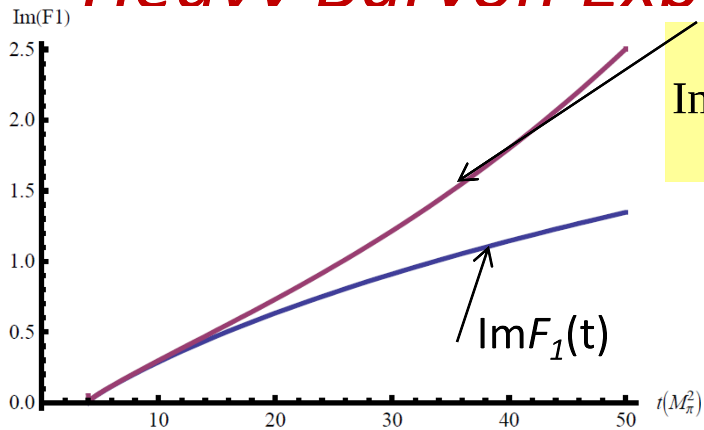
- Extend Light-cone framework to Delta-pion and other non-nucleonic states
- Understand quantitatively origin of contact term (higher mass states, nucleon compositeness)
- Test use of πN-LCWF and transverse densities in experimental studies at Low and High energies

  Peripheral exclusive processes in e-N and N-N reactions in dedicated facilities.
- Extend connections to other distributions

Peripheral Densities from Invariant χPT
Heavy Baryon Expansion

$$\varepsilon \equiv \frac{M_\pi}{M_N}$$

$$\varepsilon << 1, \quad \frac{t}{M_\pi^2} \sim O(\varepsilon^0)$$



$$\mathrm{Im}\, F_1^{HB}(t) = \pi \frac{g_A^2 M_\pi^2}{(4\pi F_\pi)^2}\left[\sum_{i=0}^{\infty} \varepsilon^i C_i\left(\frac{\sqrt{t}}{2M_\pi}\right)\right]$$

$$C_0(\tau) = \frac{1}{\tau\sqrt{\tau^2-1}}\left[\tau^4 - \frac{3}{2}\tau^2 + \frac{1}{2}\right]$$

$$C_1(\tau) = -\frac{\pi}{2\tau^2}\left[3\tau^5 - 3\tau^3 + \frac{1}{2}\tau\right]$$

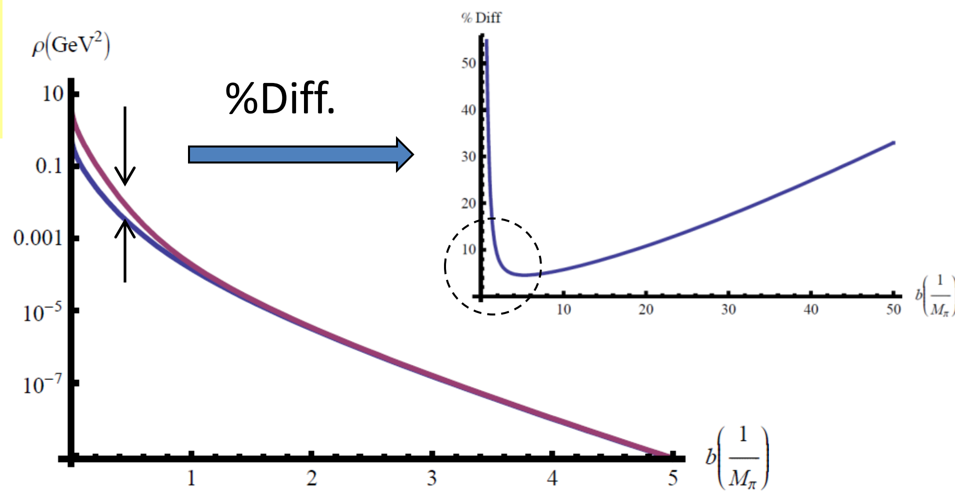
$$C_2(\tau) = \frac{1}{\tau\sqrt{\tau^2-1}}\left[4\tau^6 - 6\tau^4 + \frac{9}{4}\tau^2 - \frac{1}{8}\right]$$

$$\rho_1^{HB}(b) = \frac{1}{2\pi}\frac{4g_A^2 M_\pi^2}{(4\pi F_\pi)^2}\left[\sum_{i=0}^{\infty} \varepsilon^i f_i(2M_\pi b)\right]$$

$$f_0(\beta) = \frac{1}{16}(K_2(\beta))^2 - \frac{1}{8}(K_1(\beta))^2 + \frac{1}{16}(K_0(\beta))^2$$

$$f_1(\beta) \approx -\left(\frac{\pi}{2}\right)^{\frac{3}{2}}\left[3\beta^{-5}\Gamma\left(\frac{9}{2},\beta\right) - 3\beta^{-3}\Gamma\left(\frac{5}{2},\beta\right) + \frac{1}{2}\beta^{-1}\Gamma\left(-\frac{1}{2},\beta\right)\right]$$

$$f_2(\beta) = \frac{1}{2}(K_3(\beta))^2$$



Peripheral Densities from Invariant χPT
Heavy Baryon Expansion