Prospects for thermal dilepton rates from Lattice QCD: A quick excursion

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FAIRNESS 2014
21.09. - 28.09.2014

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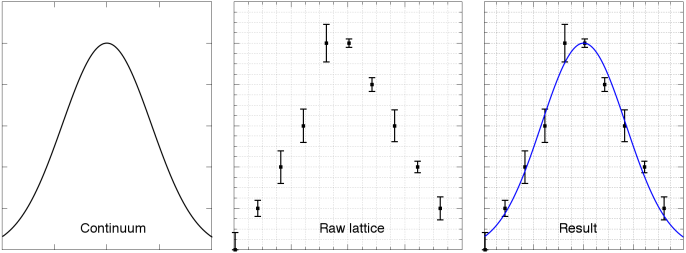
Outline

Lattice QCD: A primer

"Lattice QCD is not a simulation of QCD, it is a realization."
- Analogous citation Frank Wilczek Quark Matter 2014

The method is to numerically integrate the Langrangian of QCD:

- Numerical techniques cannot handle "continuous" $\rightarrow$ "discretize"
- This means: finite volume, finite cut-off = lattice spacing, finite d.o.f
- Thanks to asymptotic freedom:
  Increase cut-off = decrease lattice spacing
  $\rightarrow$ smoothly remove lattice and reach continuum limit



Lattice QCD: The gauge average

"A path integral can be seen as the (weighted) sum over all possible classical paths"

A physically meaningful observable $\langle \mathcal{O} \rangle$ can be **loosely** transcribed via:

$$\langle \mathcal{O} \rangle = \frac{\int dA d\bar{\Psi} d\Psi \;\; \mathcal{O} \;\; e^{\mathcal{S}_{QCD}(A,\bar{\Psi},\Psi)}}{\int dA d\bar{\Psi} d\Psi \;\; e^{\mathcal{S}_{QCD}(A,\bar{\Psi},\Psi)}}$$

The "$\langle ... \rangle$" operation is often referred to as "taking the gauge average" :

- A number of possible gauge configurations are generated according to the (discretized) QCD action via Monte-Carlo methods
  - Need interpretation in terms of probabilities!
  - No imaginary components allowed!
    $\Rightarrow$ (1) Euclidean spacetime (2) No baryon chemical potential
- The observable is "measured" in these gauge backgrounds
- Only the average completes the path integral and has a meaning!

Lattice QCD: Spectroscopy

The thermal information is encoded in the partition function of QCD

$$\mathcal{Z}_{QCD}(A,\bar{\Psi},\Psi)=\int_0^{1/T} dAd\bar{\Psi}d\Psi \quad e^{\mathcal{S}_{QCD}(A,\bar{\Psi},\Psi)}$$

There are many interesting observables $\langle\mathcal{O}\rangle$, all of them are:

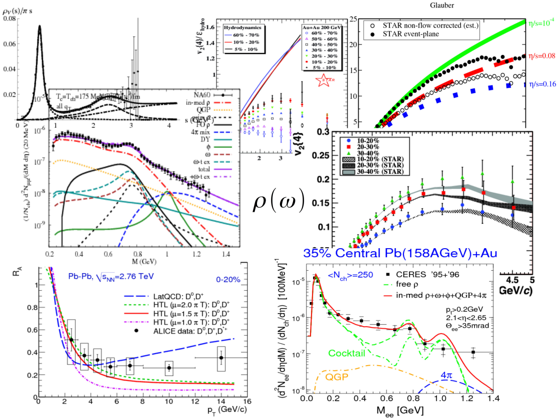
- In Euclidean spacetime
- In thermal equilibrium

Here, the spectrum of vector mesons:
$\Rightarrow$ Correlation of a meson operator $\mathcal{O}_{source}$ at point $(\tau=0,\vec{x}=\vec{0})$ with another meson operator $\mathcal{O}_{sink}$ at points $(\tau,\vec{x})$,

$$\langle\mathcal{O}\rangle=\langle\mathcal{O}_{sink}(\tau,\vec{x}),\mathcal{O}_{source}(0,\vec{0})\rangle=G(\tau,\vec{x})$$

Note, both operators have the quantum numbers of a vector meson particle - **any** vector meson particle
$\Rightarrow$ This correlator contains the full QCD spectrum of vector mesons.

Spectral functions and lattice



- Many (real-time) observables can be linked to spectral functions
- In principle: Accessible via anlytic continuation of lattice data:

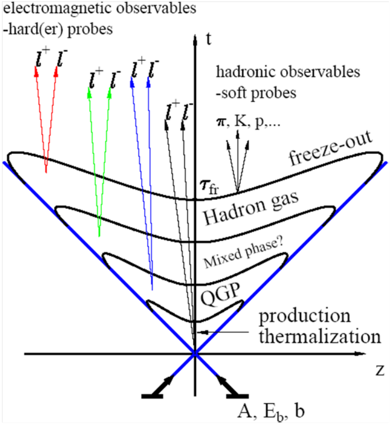
$$G_{latt.}(x) = \int d\omega \rho(\omega) K(\omega, x) \quad \Rightarrow \quad \rho(\omega) = \mathcal{L}^{-1}(G_{latt.}(x))$$

Dileptons and the EM spectral funtion

- Dileptons are produced at every stage of a heavy-ion collision.
- Their production rate $\mathrm{d}N_{l^+l^-}/\mathrm{d}\omega$ is accessible via experiments .
- Their production rate is also available from theory:

$$\frac{\mathrm{d}N_{l^+l^-}}{\mathrm{d}\omega \mathrm{d}^3 p} = \ldots$$

$$\ldots C_{em} \frac{\alpha_{em}^2}{6\pi^3} \frac{\rho(\omega, \vec{p}, T)}{(\omega^2 - \vec{p}^2)(\mathrm{e}^{\omega/T} - 1)}$$



- $\rho(\omega, \vec{p}, T)$ is the SPF of the electromagnetic current

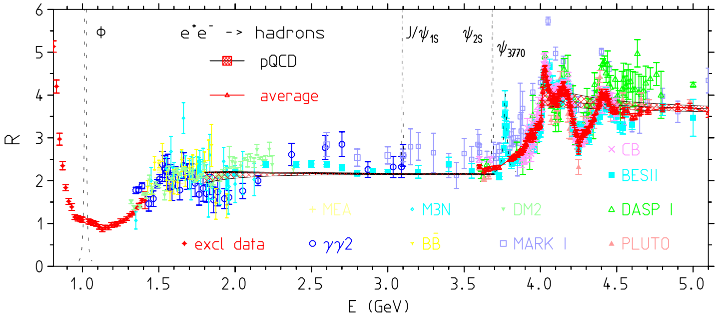
A first look: The vacuum

In vacuum, i.e. in the cold system, the EM SPF can be measured directly in experiment via $R(s) \propto \sigma(e^+e^- \to \mathrm{hadrons})$.

- Optical theorem:

$$\sigma(s) = \frac{R(s)}{12\pi} = \frac{\rho(\omega^2)}{\omega^2}$$

- HICs also include contributions from the hot stage of the event.



[0902.3360]

Cold and hot systems: The lattice perspective

What can we learn about the real-time physics encoded in the spectral functions from the Euclidean formulation of thermal field theory and especially lattice QCD?

How well can we control the reconstruction of the spectral function from non-perturbative lattice input data?

- **Connected observables in the vacuum...**
  - ...LO hadronic contribution to $(g-2)_\mu$ [1306.2532].
  - ...Time-like pion form factors [1105.1892].
- **Connected observables at finite temperature...**
  - ...Quarkonium dissociation [1402.1601], [1204.4945].
  - ...HQ diffusion and electrical conductivity [1311.3759], [1307.6763], [1212-2.4200].
  - ...Dilepton rates [1012.4963].
- **Here, I review a number of recent lattice studies:**

[1012.4963], [1109.3941], [1204.4945], [1212-2.4200], [1301-2.7436], [1307.6763], [1310.7466], [1402.6210]

Cold and hot systems: The lattice perspective

- The current-current correlator is given by

$$G_{\mu\nu}(\tau, T) = \int d^3x \langle J_\mu(\tau,\vec{x}) J_\nu(0,\vec{0})^\dagger \rangle$$

with the isospin current[1]:

$$J_\mu(\tau,\vec{x}) = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$$

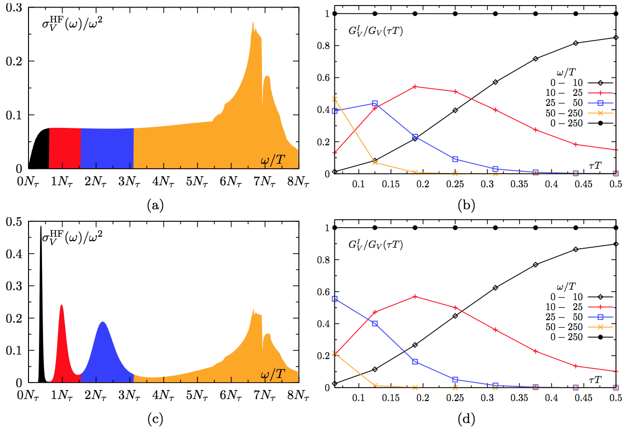
- For the lattice vector correlator $G_{ii}(\tau, \vec{p} = 0, T = 1/\beta)$ the connection to the spf $\rho(\omega, T)$ is:

$$G_{ii}(\tau, T) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho(\omega, T) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh(\omega(\beta/2 - \tau))}{\sinh(\omega\beta/2)} \rho(\omega, T)$$

[1] Isospin $\approx$ EM: The disconnected part is negligible up to $\sim$ 1.5fm, Lat'14

Noise, lattice effects and the convolution integral

[Wissel'06]



- Correlator is only sensitive to area under SPF.
- Contribution of low peak region at midpoint:
  ∼85% for dissociated vs. ∼90% for bound state.

Noise, lattice effects and the convolution integral

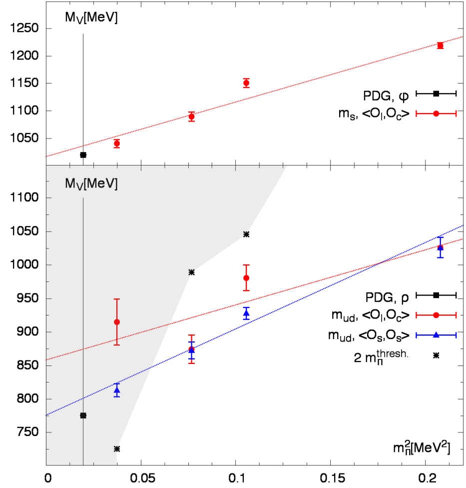


Figure: M_ρ and M_ϕ on $N_f = 2$ ensembles by CLS.

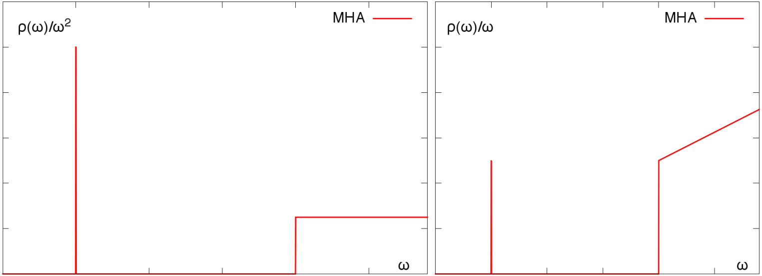
- $T = 0$, the signal is often lost before the ground state dominates
  $\Rightarrow$ Noise increase $\propto \exp(m_\pi)$
  $\Rightarrow$ Signal goes $\propto \exp(m_\rho)$.
- $T > 0$, same as $T = 0$, also: lattice extent is often too short.
- Most lattice setups do not allow for an unstable ρ-meson.

Physics expectations

Expectations: Transport phenomena and bound-state dissociation

- A simple $T = 0$ SPF is the minimal hadronic Ansatz (MHA1)

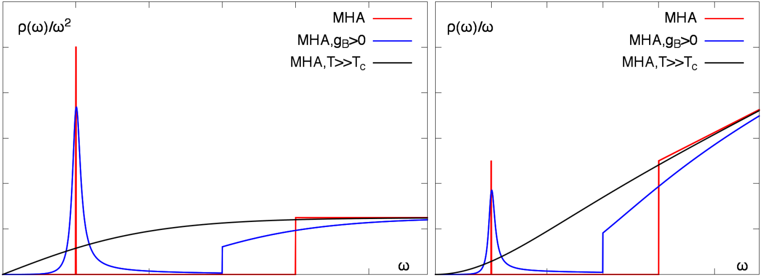
$$\frac{\rho_{MHA}(\omega)}{\omega^2} \sim A\delta(\omega - m) + B\Theta(\omega - s_0)$$



Expectations: Transport phenomena and bound-state dissociation

- More realistically: (Thermal) modification of bound states and continuum threshold

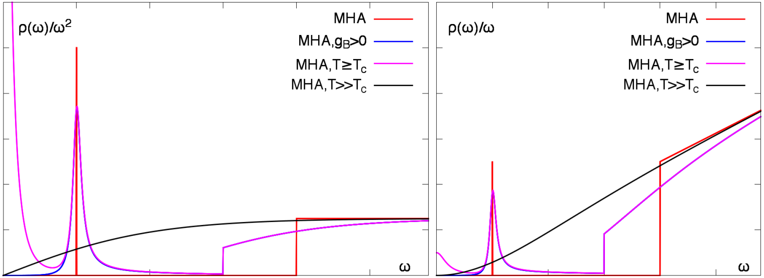
$$\frac{\rho^{g_B>0}_{MHA}(\omega)}{\omega^2} \sim A' \frac{g \tanh(\omega\beta)^3}{(\omega-m)^2+g_B^2} + B'\Theta(\omega - s_0')\tanh(\omega\beta)$$



Expectations: Transport phenomena and bound-state dissociation

- Also, emergence of transport peaks around $\omega = 0$

$$\frac{\rho_{MHA}^{T \gtrsim T_c}(\omega)}{\omega^2} \sim A' \frac{g \tanh(\omega\beta)^3}{(\omega - m)^2 + g_B^2} + B'\Theta(\omega - s_0') \tanh(\omega\beta) + \frac{C'}{\omega^2} \frac{\tanh(\omega\beta)}{(\omega/g')^2 + 1}$$



Basics of SPF reconstruction

Basics of SPF reconstruction

The process of reconstruction is an ill-posed problem

- Only a finite number of points is available to approximate a continuous function.
- Recall, in a typical spline interpolation:
  - F= True function;
  - D = Data;
  - I = Approximation to F;
  - B(k) = Basis function;
  - c(k) = weights;

Approximate F with

$$I_k = \sum_k c(k)B(k)$$

by searching for $[D - I_k] = \mathrm{min}.$

Basics of SPF reconstruction

In the reconstruction only the transformed data $D = \mathcal{L}[F]$ is known:

- Minimize $[D - I_k] = \min$ with

$$I_k = \mathcal{L}[\sum_k c(k)B(k)]$$

- Approximate $F = \mathcal{L}^{-1}[F']$ with the un-transformed $B(k)$ and $c(k)$.

Two approaches:

1. Fix the basis functions and minimize: $I_k = \sum_k c(k)\mathcal{L}[B(k)]$.
2. Define the basis functions by specific Ansätze: $I_{k'}^{Ansatz} = \mathcal{L}[A(c(k'))]$.

Basics of SPF reconstruction

- The maximum entropy method (MEM) uses the first epproach.
  - Based on Bayes theorem.
  - The minimization is extended to incorporate a Shannon-Jaynes-Entropy term.
  - The basis functions are fixed to

$$\frac{\rho(\omega)}{\omega} = m(\omega) \exp \sum_k c(k) B(k, \omega)$$

    where the prior information (or default model) $m(\omega)$ is introduced.
- Caveats and problems in the past:
  - Past: Divergent kernel.
  - Past: small lattice sizes, often staggered (further reduction of points).
  - Impact of default model?
  - Accuracy of data enough to converge close to true solution?
  - Interpretation of MEM artefacts in SPF result?

Basics of SPF reconstruction

- Recent analyses started using also the second approach.
  - The minimization is done as highlighted above.
  - The spectral function is determined by a fixed Ansatz depending on a number of parameters $c(k')$

$$\rho(\omega) = \rho_{Ansatz}(\omega, c(k'))$$

- Caveats:
  - Choice of Ansatz?
  - Dependence on Ansatz?
  - Insensitivity of correlator (deviation from δ-functions is very small)?

An incomplete review of recent lattice studies

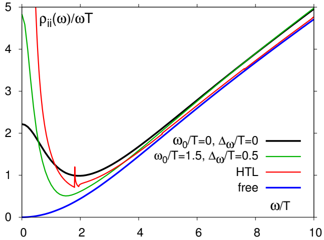
Lattice calculations have studied the SPF using EFT methods (NRQCD, HQET) and direct computations.

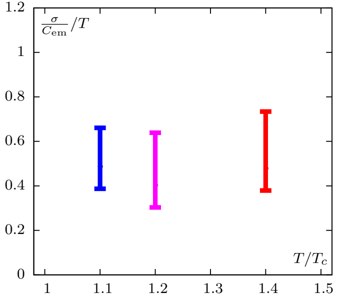
- Access to different phenomena in the SPF's:
  - Bound-states only (color: orange).
  - Transport only (color: brown).
  - Transport and bound-states (color: red).
- In the following recent lattice studies in all three categories are highlighted[2].

[2] Naturally incomplete, my sincere apologies to everybody not mentioned.

Electrical conductivity in the quenched continuum [1012.4963], [1301-2.7436], Lat'14

Jumping to conclusions





Details:

- Local current correlators are calculated on quenched ensembles
- The continuum limit is taken in $T/T_c \in [1.1 : 1.45]$.
- Reconstruction using Ansatz

Results:

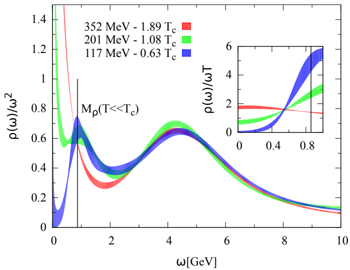
- Ansatz for SPF:

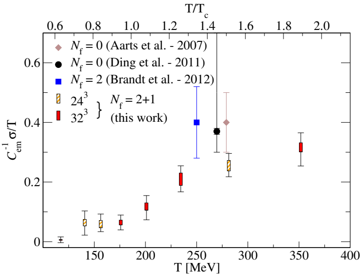
$$\rho(\omega) = \rho_{peak}(\omega) + k\rho_{free}(\omega).$$

- First reliable determination of σ_{el}/T.
- No info on ρ-dissociation.

Electrical conductivity accross the phase transition [1307.6763], [1310.7466]

Jumping to conclusions





Details:

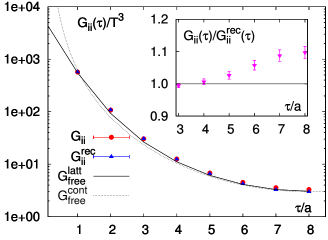
- Conserved current correlators on anisotropic $N_f = 2 + 1$ Wilson-Clover ensembles
- Sea-quarks with $m_\pi \sim 400$MeV in $T/T_c \in [0.63 : 1.90]$
- Reconstruction using MEM

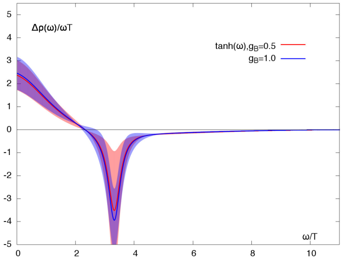
Results:

- Analysis can track σ_{el}/T accross the phase transition.
- No detailed info on ρ-dissociation.

EM current and sum rules [1212-2.4200]

Jumping to conclusions





Details:

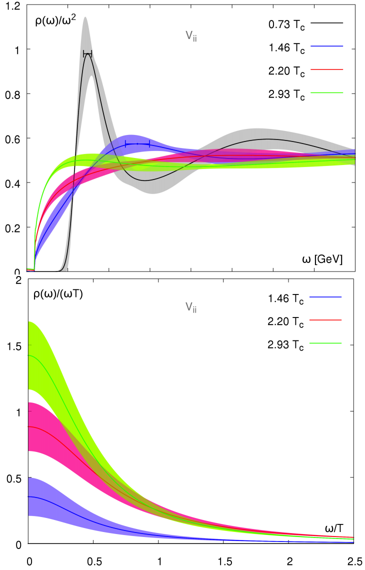
- Local current correlators on $N_f = 2$ Wilson-Clover ensembles
- A sum rule $\int(\rho_T(\omega) - \rho_V(\omega))/\omega = 0$ and $T = 0$ lattice data used to constrain the SPF.
- Sea-quarks with $m_\pi = 270$MeV and $T \sim 1.2T_c$
- Reconstruction using Ansatz

Results:

- Reliable determination of σ_{el}/T.
- Significant spectral weight in the ρ-region, model-dependent! $\Rightarrow$ closer inspection to be published soon!

Direct computation of charmonia [1204.4945]

Jumping to conclusions



Details:

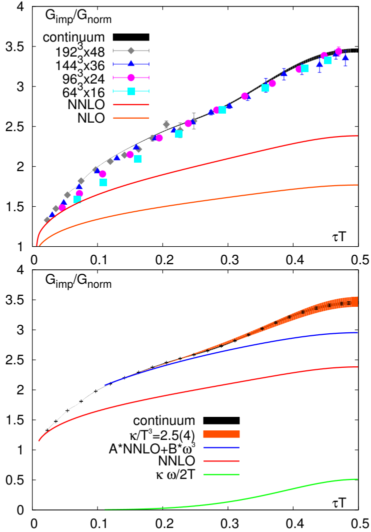
- Full QCD on large, quenched ensembles.
- Includes both transport and bound states
- Quenched, no sea-quarks.
- Reconstruction using MEM

Results:

- Dissociation of J/Ψ around $1.46\,T_c^{quench}$.
- Emergence of transport peak with $\kappa/T^3 \sim 4-7$.

HQ diffusion from EFT [1311.3759], QM'14

Jumping to conclusions



Details:

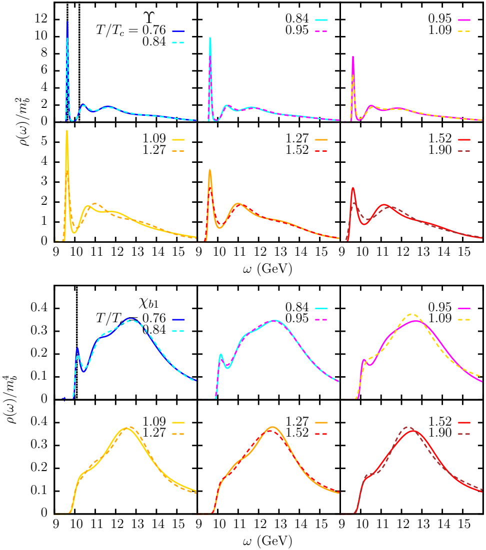
- E-field correlator defines clean probe for heavy-quark diffusion.
- Calculation is extrapolated to the continuum limit.
- Quenched, no sea-quarks.
- Reconstruction using Ansatz

Results:

- Ansatz for diffusion coefficient:
  $$\rho(\omega) = \max\left[\mathrm{A}\rho_{\mathrm{NNLO}}(\omega) + \mathrm{Bw}^3, \frac{\omega\kappa}{2\mathrm{T}}\right]$$
- Ansatz based on NNLO and constant contribution for κ.
- $\kappa/T^3 = 2.5(4)$.
- Open question: How to reconcile with full QCD computation?

Bottomonium dissociation from NRQCD [1402.1601]

Jumping to conclusions



Details:

- NRQCD: Separation of scales excludes transport phenomena.
- Clean probe for bound-state dissociation of very heavy mesons.
- Reconstruction via MEM.

Results:

- Υ exhibits ground state peak throughout.
- χ_{b_1} seems to dissociate $\Rightarrow$ but cut-off close and large.

Summary

Summary

Status

- A number of lattice groups are attacking the problem of spf reconstruction (at finite temperature).
- Electrical conductivities show good agreement over a number of lattice setups and reconstruction methods.
- HQ diffusion needs to be further explored, but new studies are underway.
- First results on bottomonium dissociation from lattice (NR)QCD.

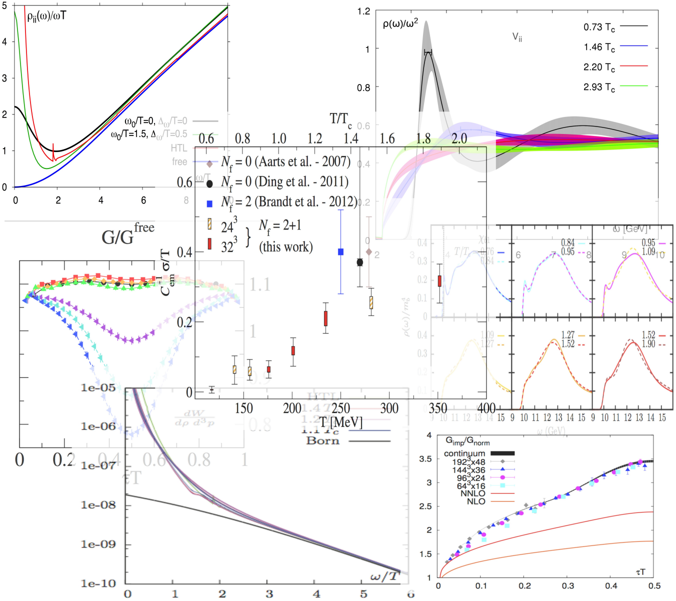
Caveats

- SPF reconstruction is the central difficulty.
- Need high accuracy data and a large lattice sizes.
- Large scale (, expensive) lattice calculations required.

Perspectives

- New calculations on larger lattices are under way:
  - anisotropic $N_f = 2 + 1$, charmonia, shown at Lat'14.
  - quenched, isotropic bottomonium using full QCD, shown at Lat'14.
  - $N_f = 2$, $N_t = 24, 20, 12$ shown at QM'14.
- New reconstruction methods are being developed, shown at QM'14 and Lat'14.
- Improved codes to boost accuracy at reasonable computational cost.
- Theoretical understanding of finite momentum SPF is extending, shown at MITP 07 2014 ⇒ Extend also lattice calculations.
- Also: Updates in vacuum vector spectroscopy from lattice QCD.

Lots to do!



ρii(ω)/ωT
ω0/T=0, Δω/T=0
ω0/T=1.5, Δω/T=0.5
HTL
free
ρ(ω)/ω²
Vii
0.73 Tc
1.46 Tc
2.20 Tc
2.93 Tc
T/Tc
Nf = 0 (Aarts et al. - 2007)
Nf = 0 (Ding et al. - 2011)
Nf = 2 (Brandt et al. - 2012)
24³
32³
Nf = 2+1
(this work)
C_em^-1 σ/T
T [MeV]
G/G^free
Born
ω/T
ω [GeV]
G_imp/G_norm
continuum
192³x48
144³x36
96³x24
64³x16
NNLO
NLO
τT