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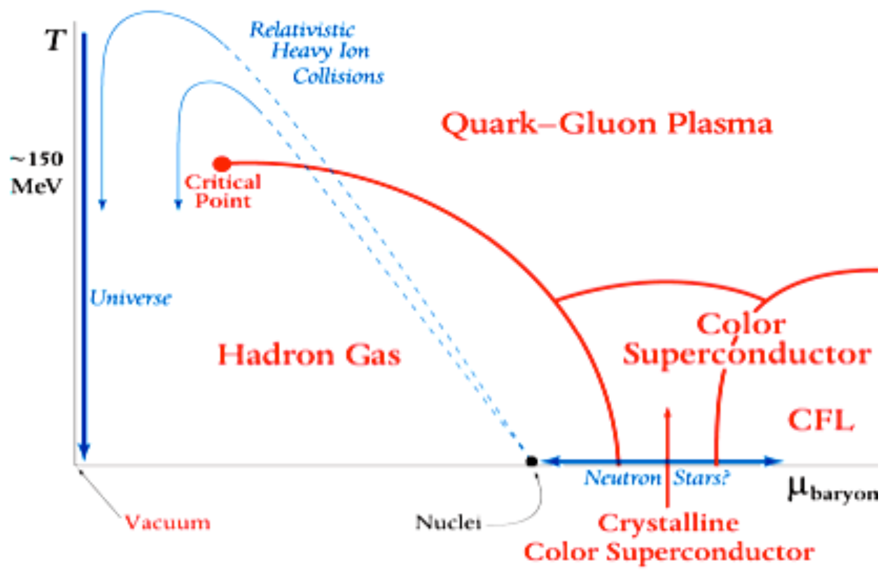
Determination of freeze-out conditions from fluctuations in the Hadron Resonance Gas model

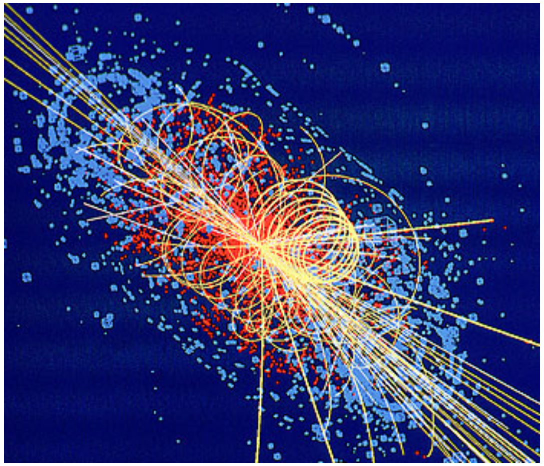
Fairness 2014, Vietri sul Mare, Italia – 22 September 2014

Based on: P.A., W.Alberico, R.Bellwied, M.Bluhm, V.Mantovani Sarti, M.Nahrgang, C.Ratti; arXiv:1403.4903[hep-ph]
M.Nahrgang, M.Bluhm, P.A., R.Bellwied, C.Ratti; arXiv: 1402.1238[hep-ph]

Introduction

- A lot of theoretical effort is devoted to study the QCD phase diagram.
- A transition from the confined phase to the deconfined phase (Quark-Gluon Plasma (QGP)) is predicted for large temperatures and/or densities.
- Experimentally, via Heavy Ion Collisions (HIC), we are able to create the QGP in the laboratory.





- Such condition of matter is hard to detect due to the nature of strong interactions.
- We have indirect clues of QGP formation.

Theoretical approaches

Using various approaches we get evidences of the formation of a phase similar to the QGP.

- Phenomenology
- Simulations of QCD on the lattice
- Hydrodynamics simulations

In my talk I will concentrate on the study of the higher order fluctuations of conserved charges that can be detected in the experiments.

Fluctuations ⟺ Moments

Starting from a given partition function we define the fluctuations of a set of conserved charges as:

$$\frac{p}{T^4} = \frac{\ln \mathcal{Z}}{VT^3} \qquad \chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n}\left(p/T^4\right)}{\partial\left(\mu_B/T\right)^l \partial\left(\mu_S/T\right)^m \partial\left(\mu_Q/T\right)^n}$$

The fluctuations of conserved charges are related to the moments of the multiplicity distributions of the same charge measured in HIC.

$$\delta N = N - \langle N \rangle$$

mean: $M = \langle N \rangle = VT^3\chi_1,$

variance: $\sigma^2 = \langle (\delta N)^2 \rangle = VT^3\chi_2,$

skewness: $S = \frac{\langle (\delta N)^3 \rangle}{\sigma^3} = \frac{VT^3\chi_3}{(VT^3\chi_2)^{3/2}},$

kurtosis: $k = \frac{\langle (\delta N)^4 \rangle}{\sigma^4} - 3 = \frac{VT^3\chi_4}{(VT^3\chi_2)^2};$

Fluctuations ⟺ Moments

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Taking ratios of these fluctuations we obtain simple quantities related to the moments of the distributions, avoiding any volume dependence

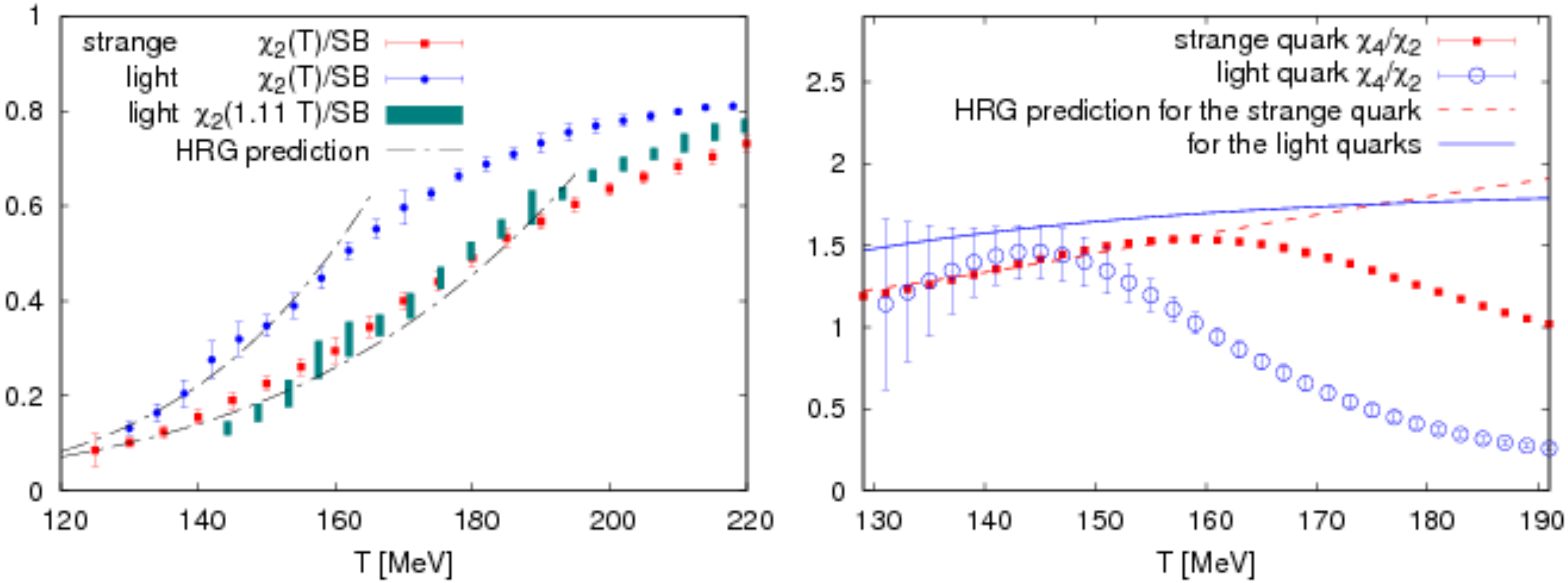
$$\sigma^2/M = \chi_2/\chi_1 \qquad k\sigma^2 = \chi_4/\chi_2$$

$$S\sigma = \chi_3/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

F.Karsch, Central Eur.J.Phys. (2012)

Lattice

These quantities can be calculated by means of simulations on the lattice. These results are in full equilibrium with respect to the BQS, and are obtained starting from first principles.



The HRG model predictions show a remarkable agreement with the lattice results (in the same conditions as the lattice simulations are performed).

R.Bellwied et al., PhysRevLett. (2013)

Hadron-Resonance Gas model

This model is based on the idea that a system of interacting hadrons in the ground state can be described by a gas of non-interacting resonances and hadrons.

$$p(T,\{\mu_k\}) = \sum_k (-1)^{B_k+1} \frac{d_k T}{(2\pi)^3} \int d^3\vec{p}\ \ln\left[1+(-1)^{B_k+1} e^{-(\sqrt{\vec{p}^{\,2}+m_k{}^2}-\mu_k)/T}\right]$$

$$n_k(T,\mu_k) = \left(\frac{\partial p}{\partial \mu_k}\right)_T = \frac{d_k}{(2\pi)^3} \int d^3\vec{p} \frac{1}{(-1)^{B_k+1} + e^{(\sqrt{\vec{p}^{\,2}+m_k{}^2}-\mu_k)/T}}$$

Where B_k, d_k and m_k denote respectively the baryon number, the degeneracy and the mass of the particle k, while its chemical potential in full chemical equilibrium is given by:

$$\mu_k = B_k \mu_B + Q_k \mu_Q + S_k \mu_S$$

HRG model flexibility

With the HRG we can perform changes which allow us to get closer to the experimental situation, this cannot be done with lattice simulations:

- Cuts in the kinematics's particle distribution ;
- Feed down correction (strong feed vs weak feed); differences among primary, secondary and tertiary vertex from experimental analysis;
- Explore easily the phase diagram in the hadron sector for arbitrarily high chemical potential ;
- Looking not just for the conserved charge (e.g. Protons) ;
- …

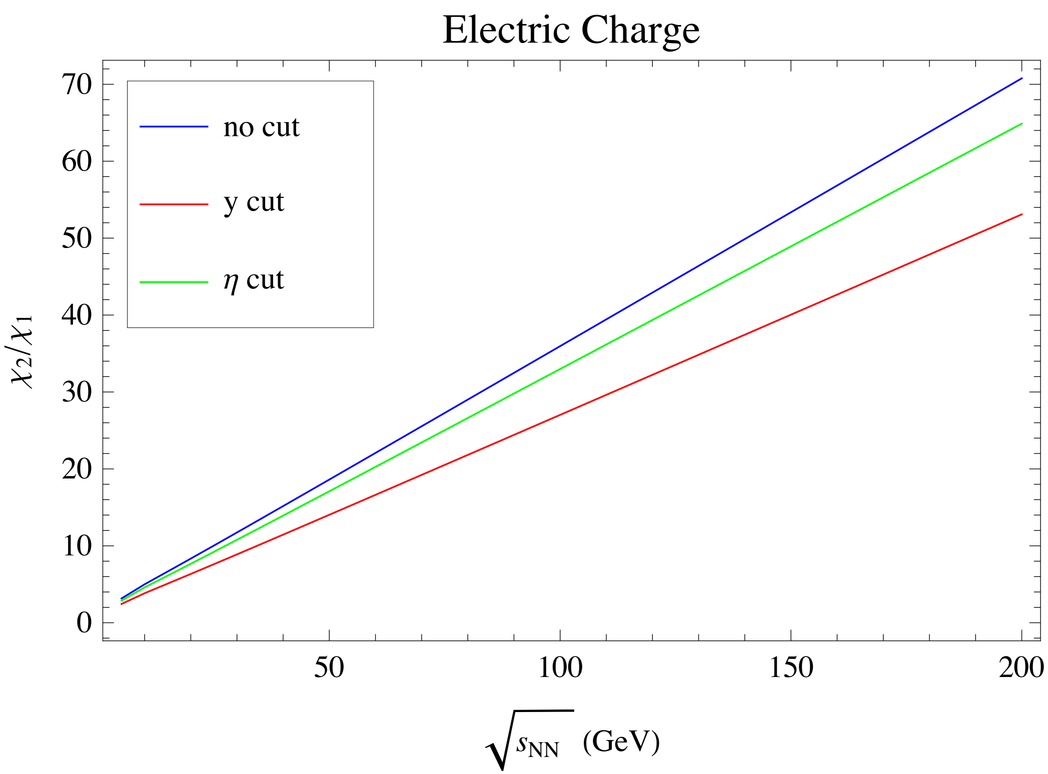
HRG model flexibility

With the HRG we can perform changes which allow us to get closer to the experimental situation, this cannot be done with lattice simulations:

- Solve external global conditions, in order to reduce the number of unknowns (μ_S and μ_Q are functions of T and μ_B) ;
- Study of higher order susceptibilities at finite μ_B (on the lattice one would need to calculate too many terms in the Taylor expansion) ;
- The possibility to apply microscopic corrections due to the thermal evolution of the system.

Kinematic cuts

In the experiment we cannot get the entire phase space of particle distribution, but just a window



$$n_k(T,\mu_k) = \frac{d_k}{4\pi^2}\int_{-y_{\mathrm{MAX}}}^{y_{\mathrm{MAX}}} \mathrm{d}y \int_{p_T^{\mathrm{MIN}}}^{p_T^{\mathrm{MAX}}} \mathrm{d}p_T \times$$

$$\times \frac{p_T\sqrt{p_T^2+m_k^2}\,\mathrm{Cosh}[y]}{(-1)^{B_k+1}+\exp((\mathrm{Cosh}[y]\sqrt{p_T^2+m_k^2}-\mu_k)/T)}$$

$$n_k(T,\mu_k) = \frac{d_k}{4\pi^2}\int_{-\eta_{\mathrm{MAX}}}^{\eta_{\mathrm{MAX}}} \mathrm{d}\eta \int_{p_T^{\mathrm{MIN}}}^{p_T^{\mathrm{MAX}}} \mathrm{d}p_T \times$$

$$\times \frac{p_T^2\,\mathrm{Cosh}[\eta]}{(-1)^{B_k+1}+\exp((\sqrt{p_T^2\,\mathrm{Cosh}[\eta]^2+m_k^2}-\mu_k)/T)}$$

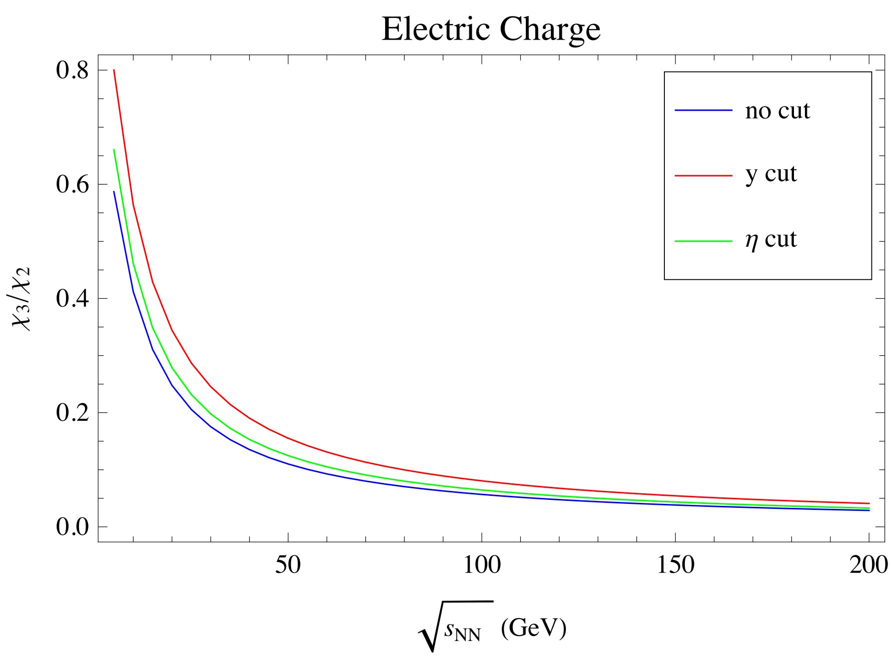
Experimental cuts

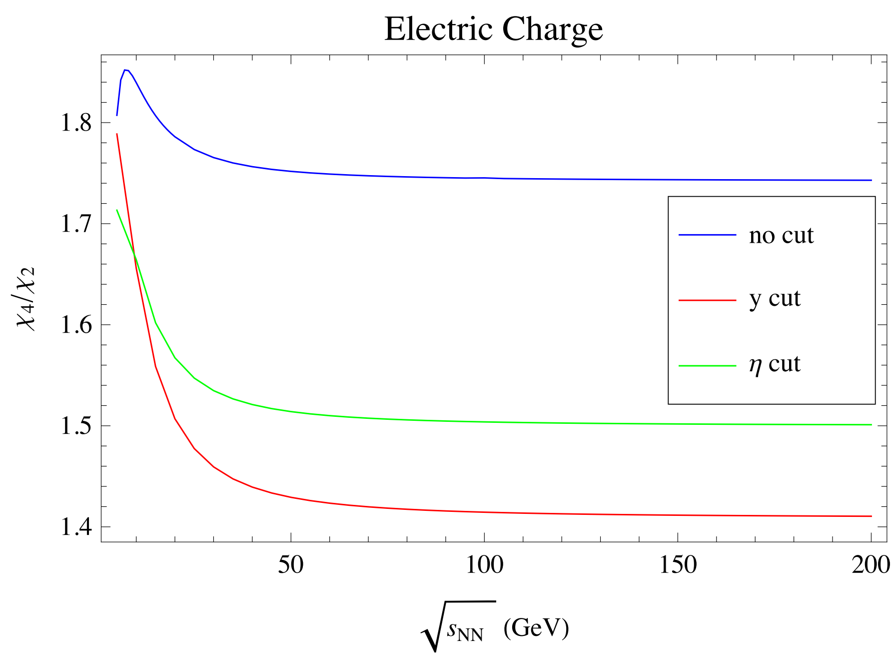
$$\text{"}y\text{"} \quad \rightarrow \quad 0.4 \leq p_T \leq 0.8\ (GeV)\,, \quad |y| \leq 0.5$$

$$\text{"}\eta\text{"} \quad \rightarrow \quad 0.2 \leq p_T \leq 2\ (GeV)\,, \quad |\eta| \leq 0.5$$

Kinematic cuts

In the experiment we cannot get the entire phase space of particle distribution, but just a window





Partial chemical equilibrum

Experimental situation:

D.Teaney, Phys.Rev. C (2001)
P.Huovinen, Nucl.Phys. A (2010)
M.Bluhm, P.A. et al., arXiv:1306.6188 (2013)

- The time scales for inelastic scatterings are much larger than the lifetime of the hadronic stage.
- So it is more reasonable that the hadronic phase in HIC is not in full but in partial chemical equilibrium (PCE).
- At $T<T_{ch}$ (chemical freeze-out temperature), only elastic and quasi-elastic scatterings (mediated by the formation and subsequent strong decay of resonances) are allowed.

$$\pi\pi \to \rho \to \pi\pi \qquad p\pi \to \Delta \to p\pi \qquad K\pi \to K^* \to K\pi$$

- Only a few particles are considered stable:

$$\text{Mesons: } \pi^0, \pi^+, \pi^-, K^+, K^-, K^0, \overline{K}^0, \eta$$

$$\text{Baryons: } p, n, \Lambda^0, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-, \Omega^-$$

Strong vs Weak

$$\text{Mesons: } \pi^0, \pi^+, \pi^-, K^+, K^-, K^0, \overline{K}^0, \eta$$

$$\text{Baryons: } p, n, \Lambda^0, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-, \Omega^-$$

- With this set of stable particles we are excluding weak decays; this can be done with a particular experimental analysis of secondary vertex.
- In this way we can get a more detailed description of the primordial situation immediately after hadronization.

Full vs Partial chemical equilibrum

The main consequence of the PCE is the conservation of the effective number of stable particles N_i.

Resonances contribute to the effective number of stable species through their branching ratios $d_{r\to i}$.

$$\bar{N}_i = N_i + \sum_r d_{r\to i}\, N_r$$

As a consequence, each stable hadron acquires an effective chemical potential.

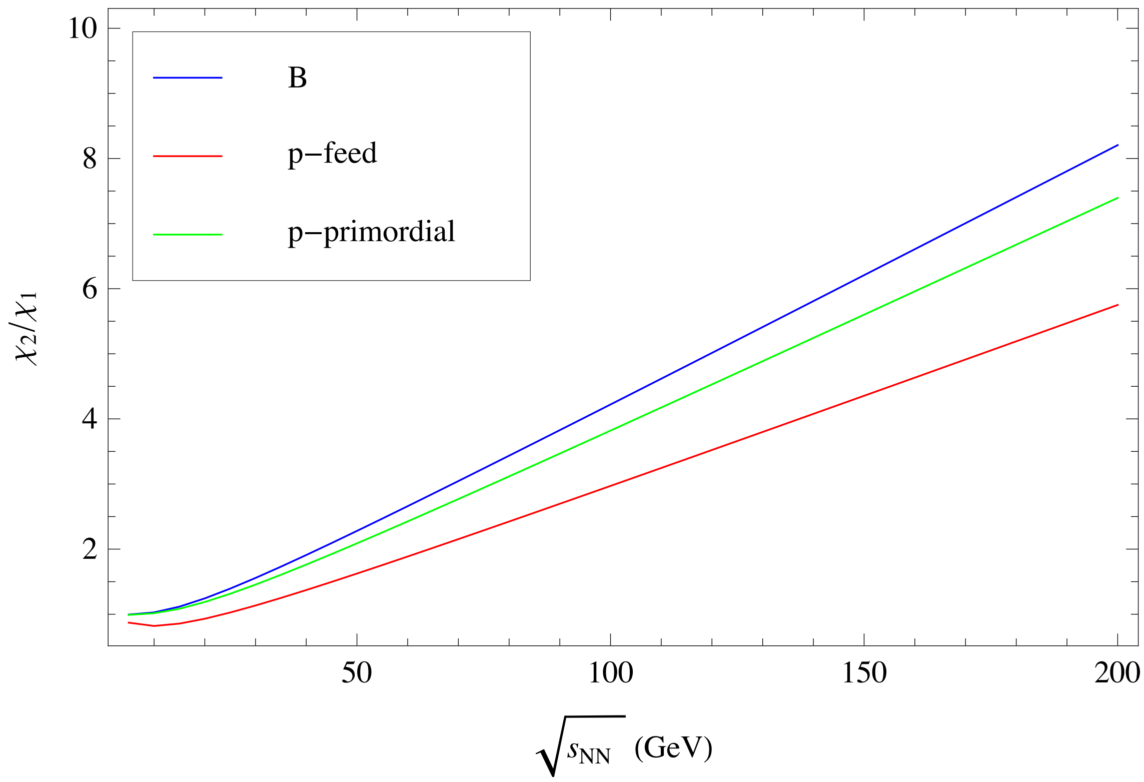
The chemical potentials of the resonances can be written in terms of the stable states in which they decay.

$$\mu_r = \sum_i d_{r\to i}\, \mu_i$$

This turns out to be crucial in order to calculate the feed-down contribution to the higher order cumulants.

Feed down correction

The detector can usually detect just charged particles, like pions, kaons and protons; this means that experimentally we have just samples of conserved charges.



B = Baryon number ;

p-feed = all the contributions from particles and strong resonances which decay into a proton ;

p-primordial = the protons which do not come from the feed

No cuts are applied to the plot.

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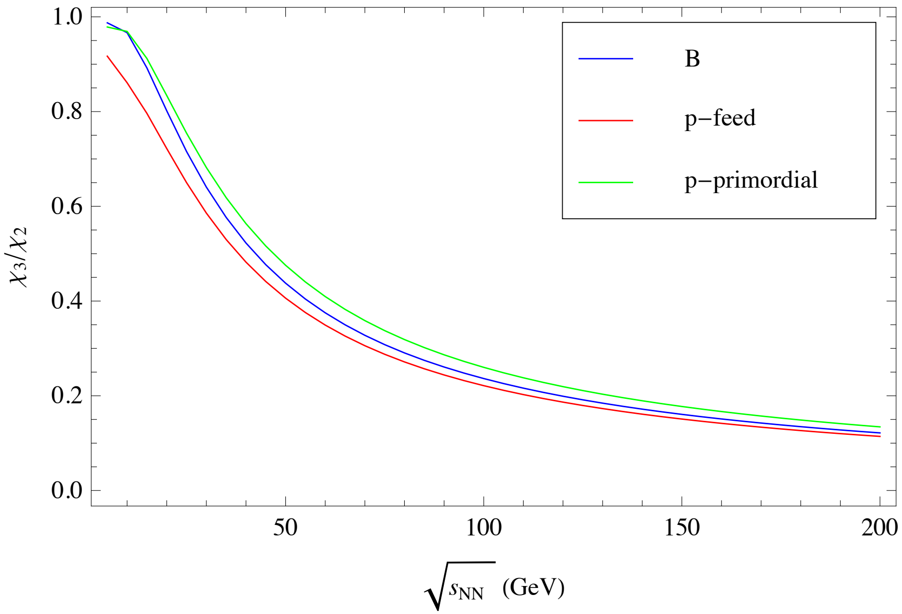
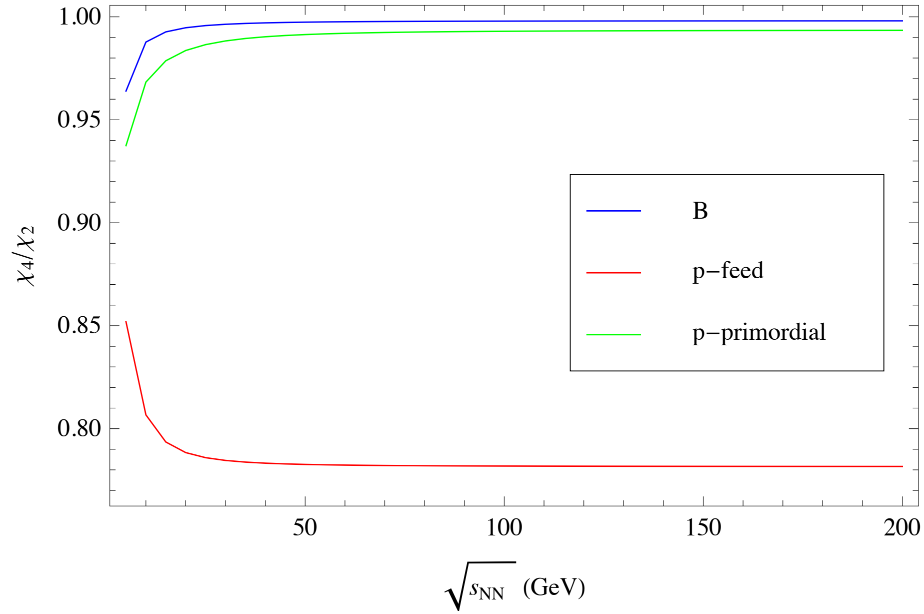
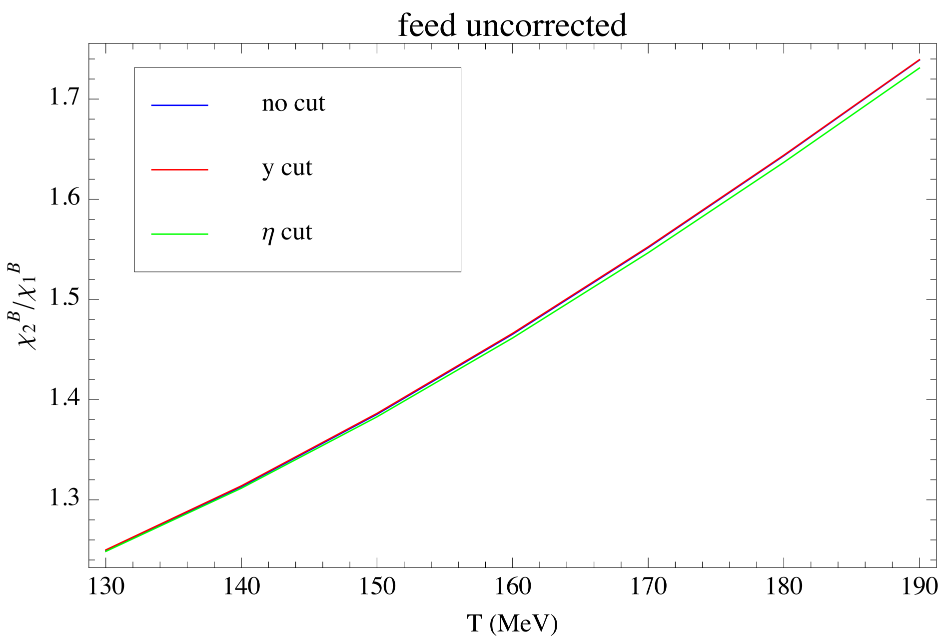


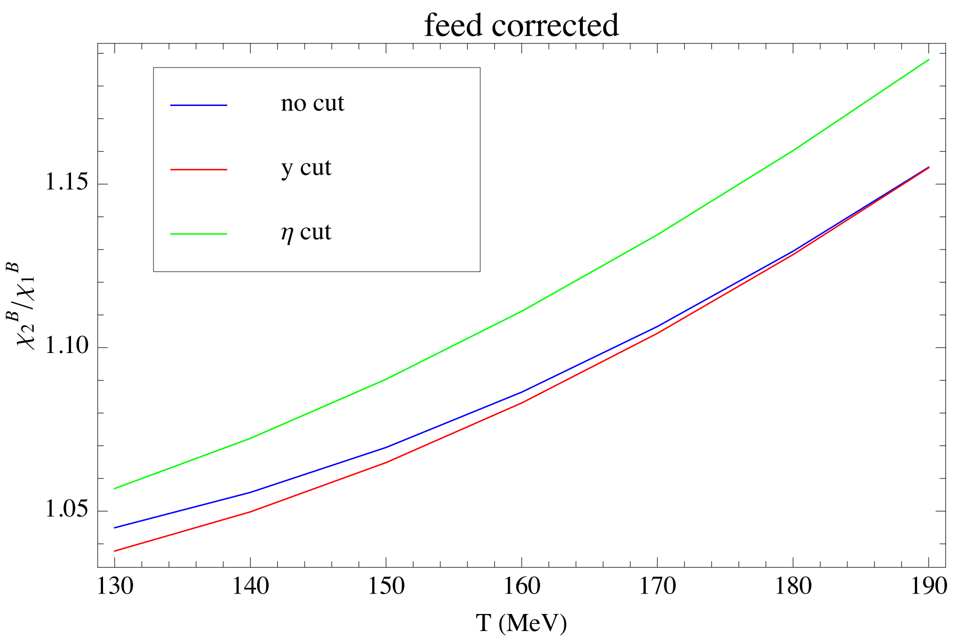


No cuts are applied to the plots.

Feed-down + Cuts

Applying together feed-down corrections and kinematic cuts can lead to unexpected results.





Applied cuts

$$0.4 \le p_T \le 0.8\,(GeV)\ ,\ \ |y| \le 0.5\ ,\ \ |\eta| \le 0.5$$

Freeze-out

Once hadronization happens the system is driven to the PCE; the stable particle abundance at this point stays the same until the detection (T_{kin} = temperature of kinetic freeze-out).

$$T_{hadr} \geq T_{ch} \geq T_{kin}$$

The concept of chemical freeze-out is enormously important:
we can relate the hadron yields (and their higher moments) detected in the experiment to the microscopic situation immediately after the hadronization.

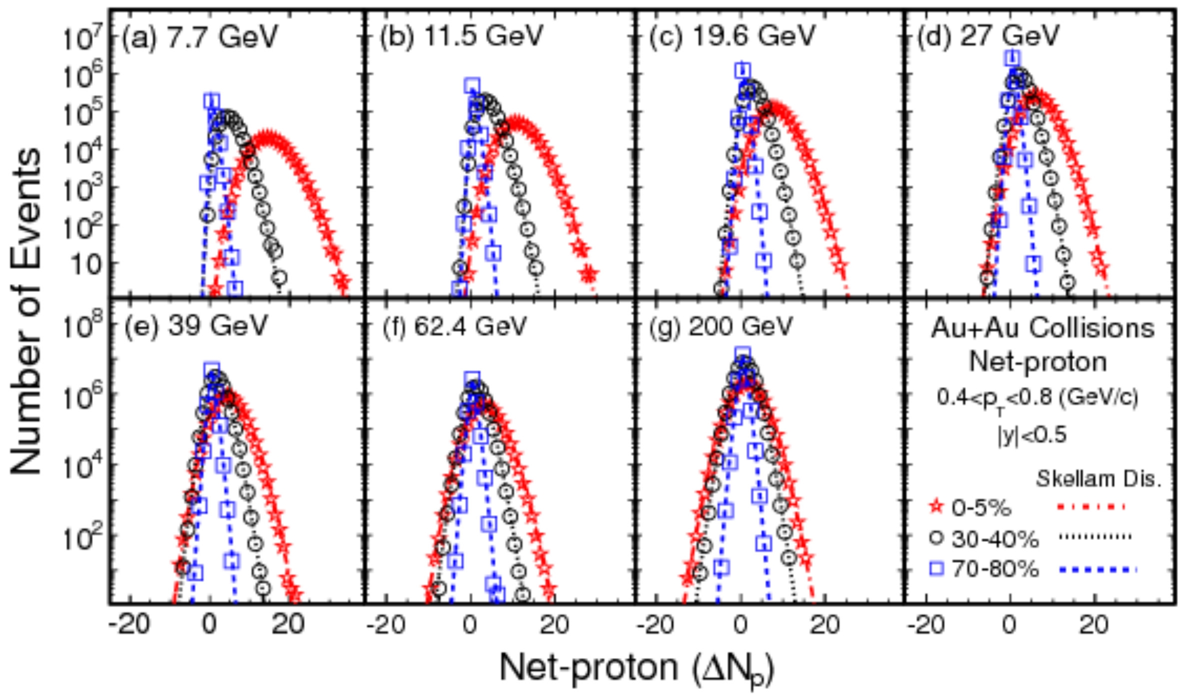
So we use the HRG model to calculate the freeze-out parameters in order to fit the experimental results.

Experimental multiplicity distribution

Measuring event by event the quantities of interest we get a distribution; from this we can calculate the moments.

Due to exact charge conservation we should have no fluctuations.

They come from the finite window of kinematic acceptance.



STAR coll., PhysRevLett (2014)

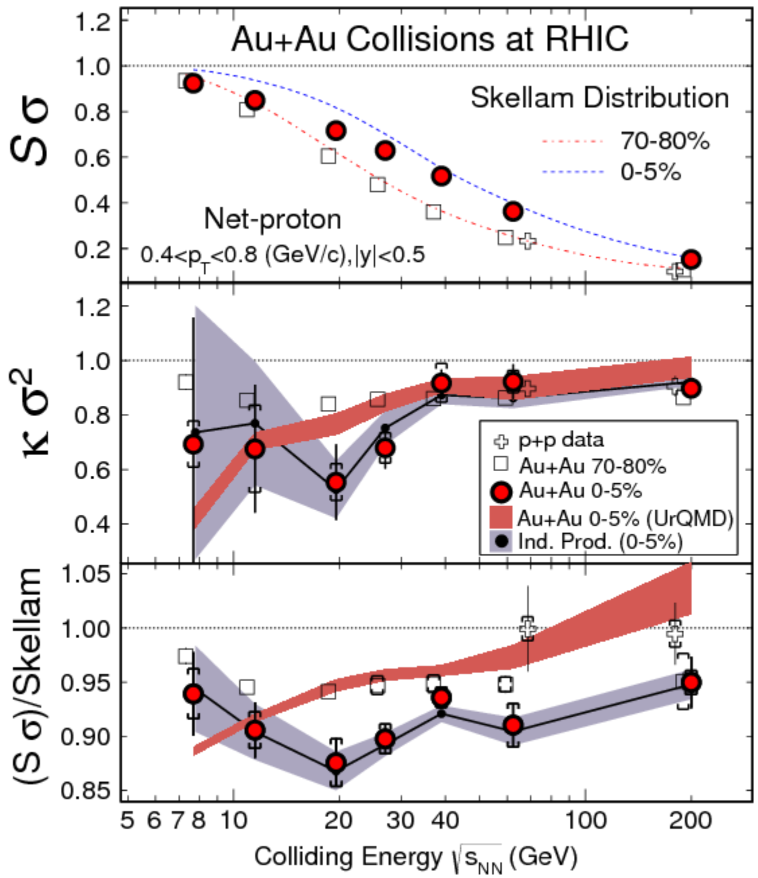
Freeze-out: experiment

The higher the order of the moments involved, the higher the expected sensitivity to the f.o. parameters of the quantities measured.

In the last years the interest in these higher order cumulants increased, leading to new results.

Unfortunately the higher the order, the bigger the error bars!!!

In order to be really sensitive in our fit, we were forced to use the lower moments products (with small error bar), and used the higher order one just as a crosscheck.



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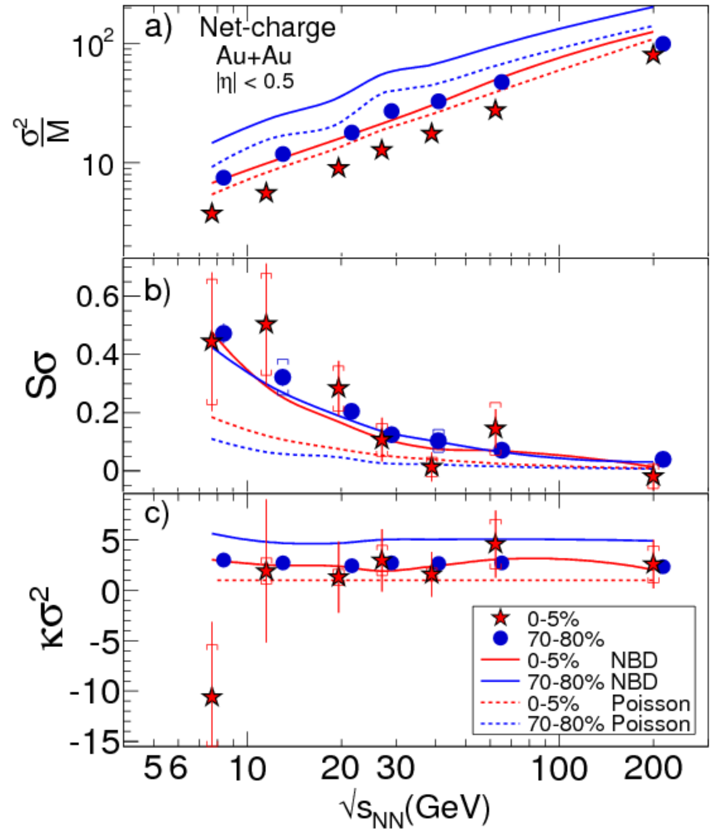


STAR coll., arXiv 1402.1558

Ex^2 : Extra Experimental issues

- Due to the configuration of the experiment the particle yields get a contamination for particular kinematics (proton spallation);
- Neutrally charged particles cannot be detected, but their contribution could be still sizeable in the yields of their isospin partner, related ot other conserved charges (B or S) (isospin randomization).

The first can be fixed just performing more time consuming calculations, taking care of the correct kinematics window for each particle.

For the other one we need to apply some corrections, due to the regeneration of the strong resonances during the evolution of the hadronic phase towards the detectors.

Isospin randomization

For example in the net-proton moments we have to take into account the following isospin randomization precesses

$$p(n) + \pi^0(\pi^+) \rightarrow \Delta^+ \rightarrow n(p) + \pi^+(\pi^0)\,,$$
$$p(n) + \pi^-(\pi^0) \rightarrow \Delta^0 \rightarrow n(p) + \pi^0(\pi^-)$$

For globally conserved charges we do not need to take such processes into account , due to the global conservation laws (which still apply in strong decays).

Isospin randomization

It has been studied that the lifetime of such resonances is short enough to justify a complete isospin randomization during the hadron phase evolution, for most of the collision energies reached in the experiments.

In the end, at the detection we cannot distinguish between a primordial proton and a proton coming from a neutron plus a pion.

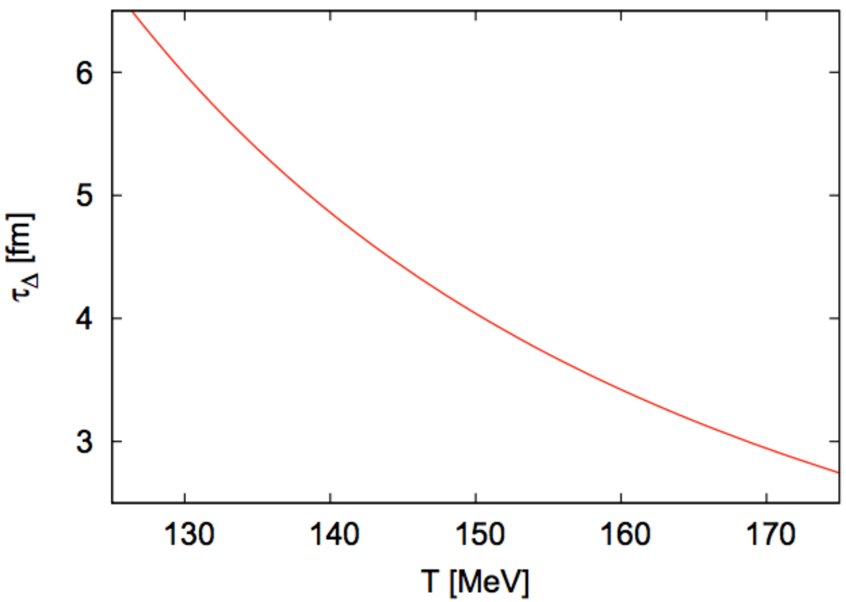


FIG. 1: Mean time τ_Δ of a rest nucleon to form Δ^+ or Δ^0 in the hadronic medium as a function of temperature T.

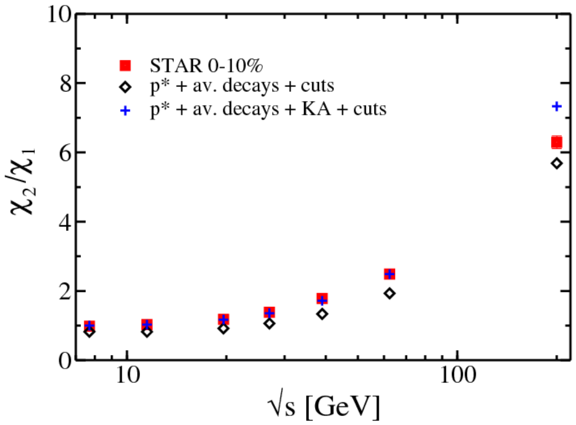
M.Kitazawa and M.Asakawa, Phys. Rev. C (2012)

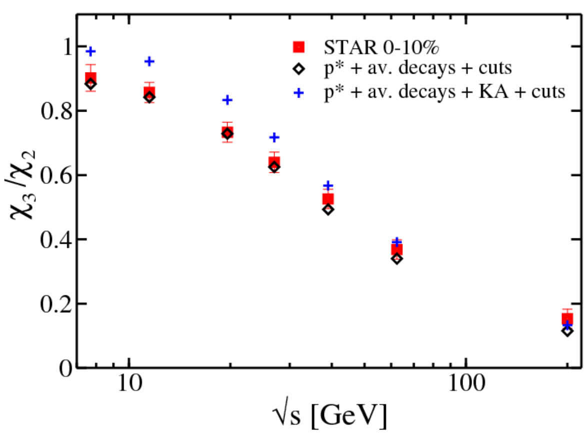
Isospin randomization

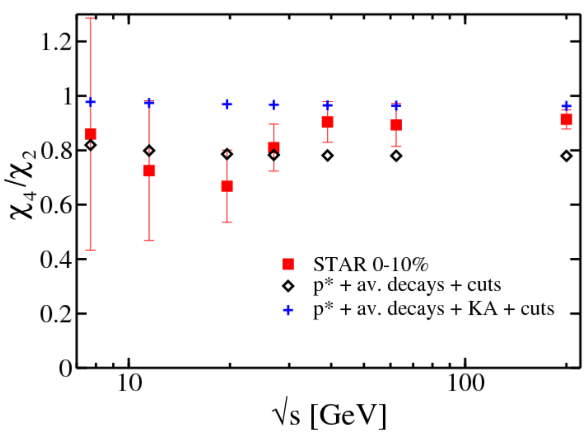
Below I show the effect of the isospin randomization correction to the net-proton distribution.

This correction changes significantly the moments behavior; especially it shows a different value for χ_4/χ_2.

This correction has been crucial in our study, especially in the study of the high energy collisions (as I will show).

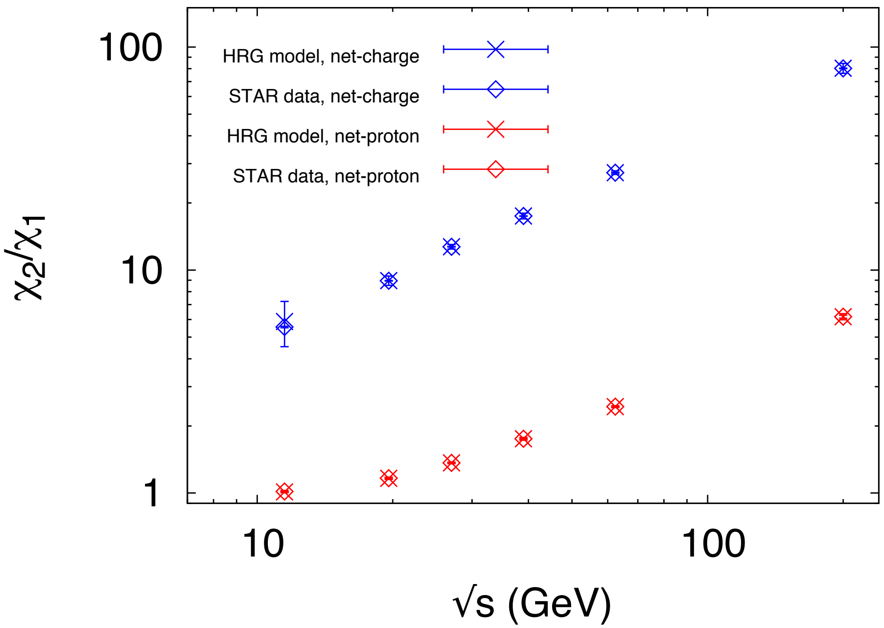


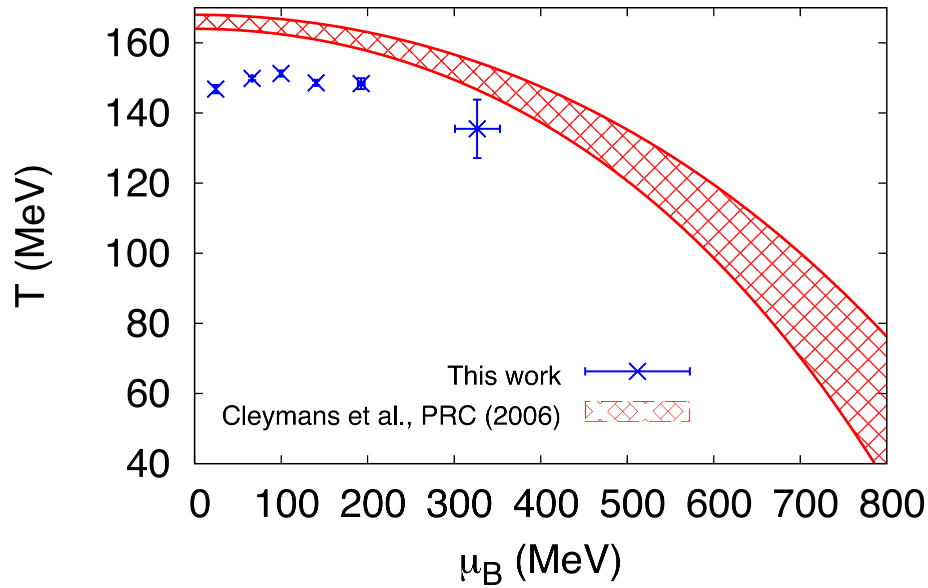




Results

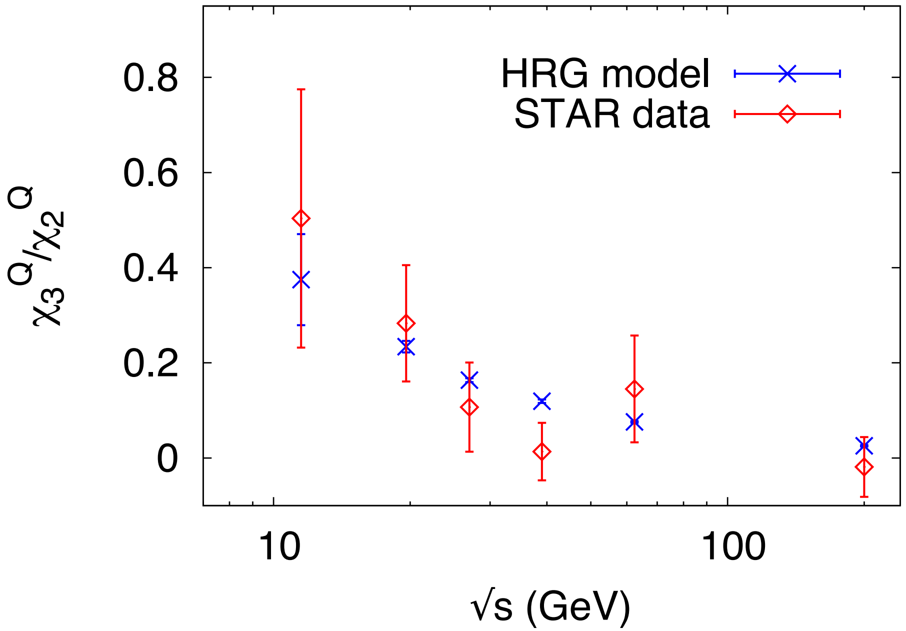
Below I show the freeze-out parameters that I obtain fitting the lower moment product for electric charge and net-proton distribution, for 6 different collision energies.

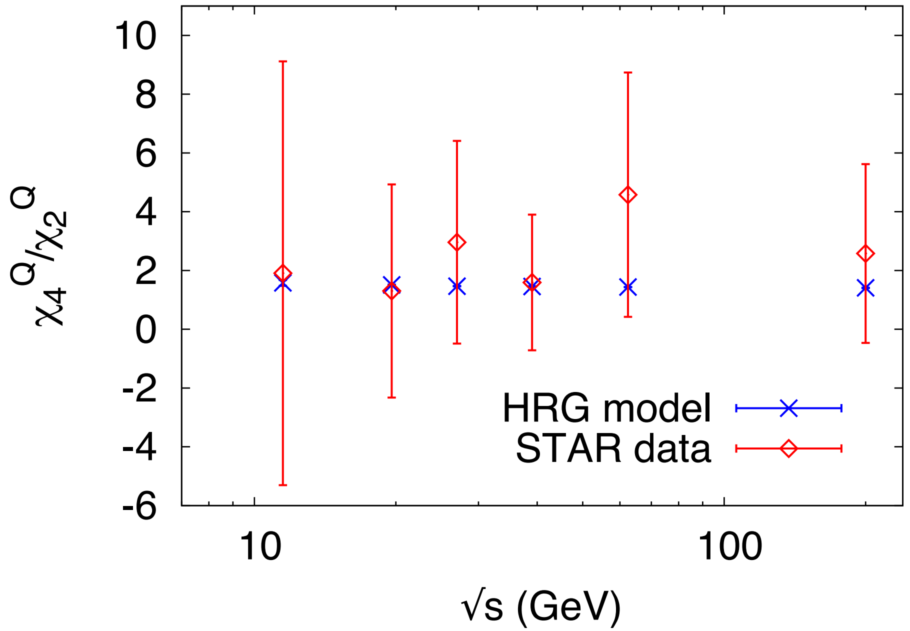




Results – Electric charge

Using the new freeze-out parameters the higher order cumulants are in good agreement with the experimental data

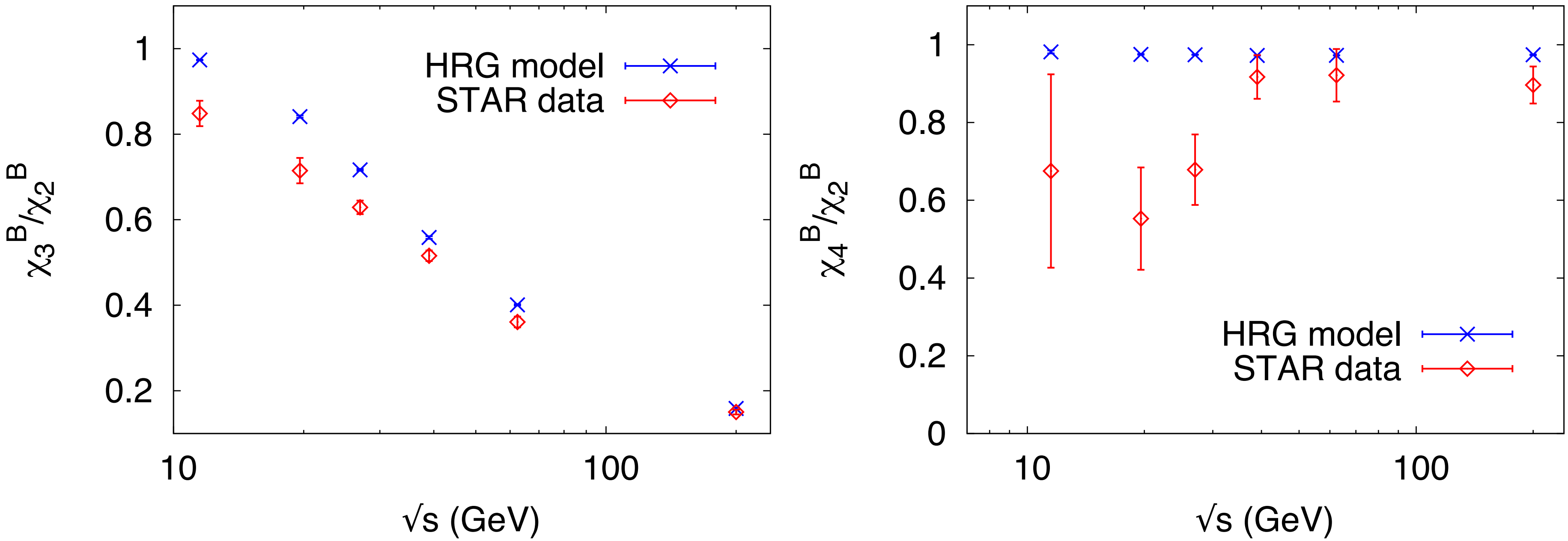




Results – Protons

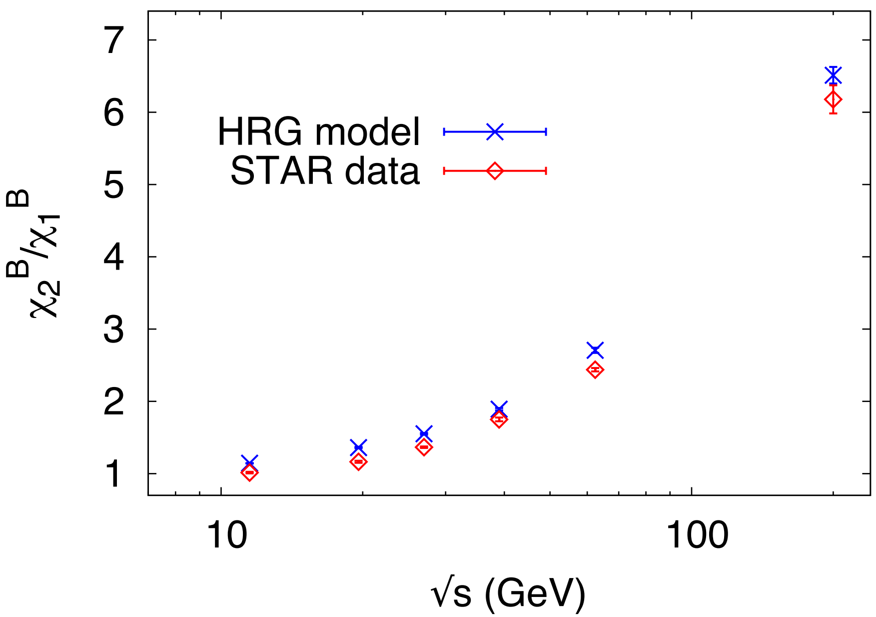
Using the new freeze-out parameters the higher order cumulants are in good agreement with the experimental data

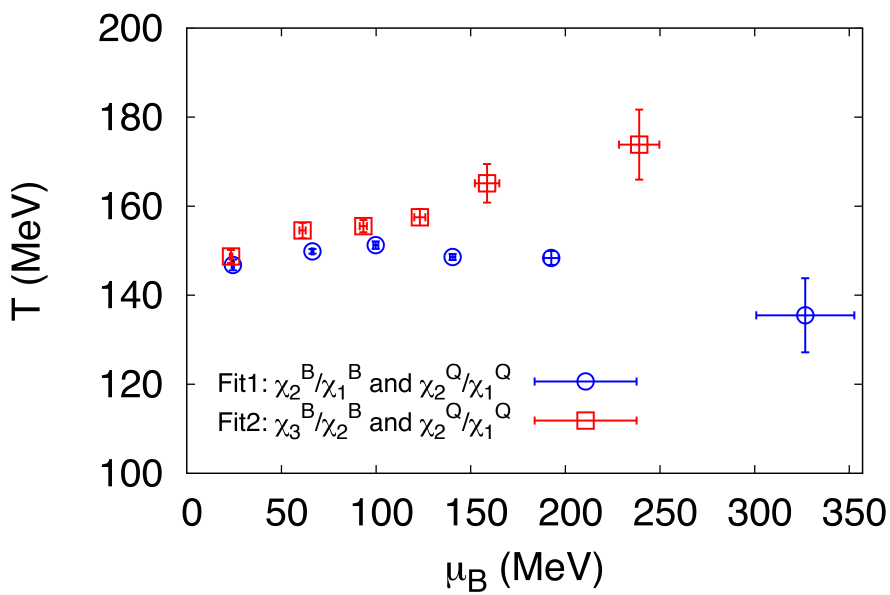
Looking at χ_3/χ_2 we understand as expected that the isospin randomization are overestimated decreasing the collision energy.



Results

Just as a crosscheck I show what happens fitting the χ_3/χ_2 for the proton (which has still small error bars) together with the χ_2/χ_1 for the charge. The result obtained is non-physical.



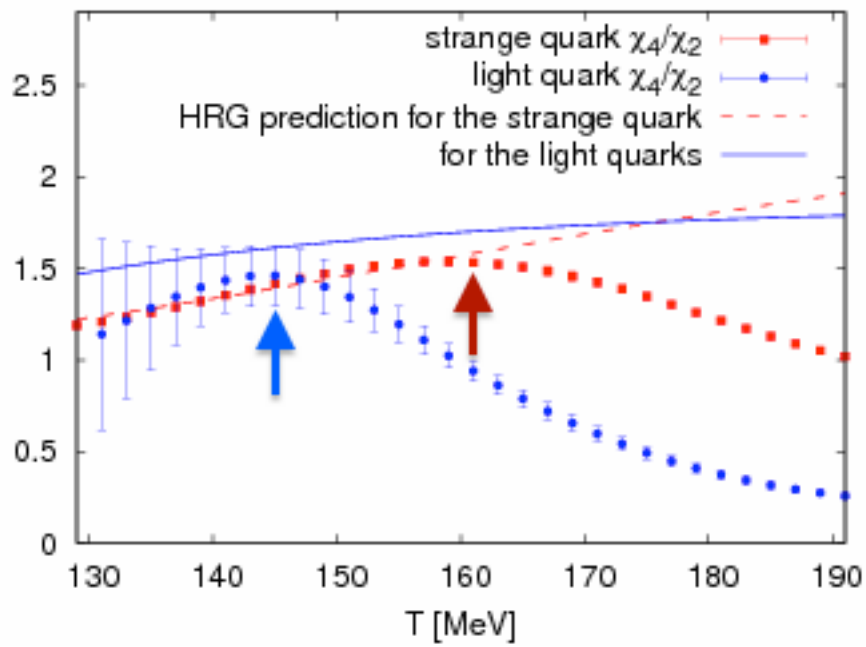


Conclusions & Outlook

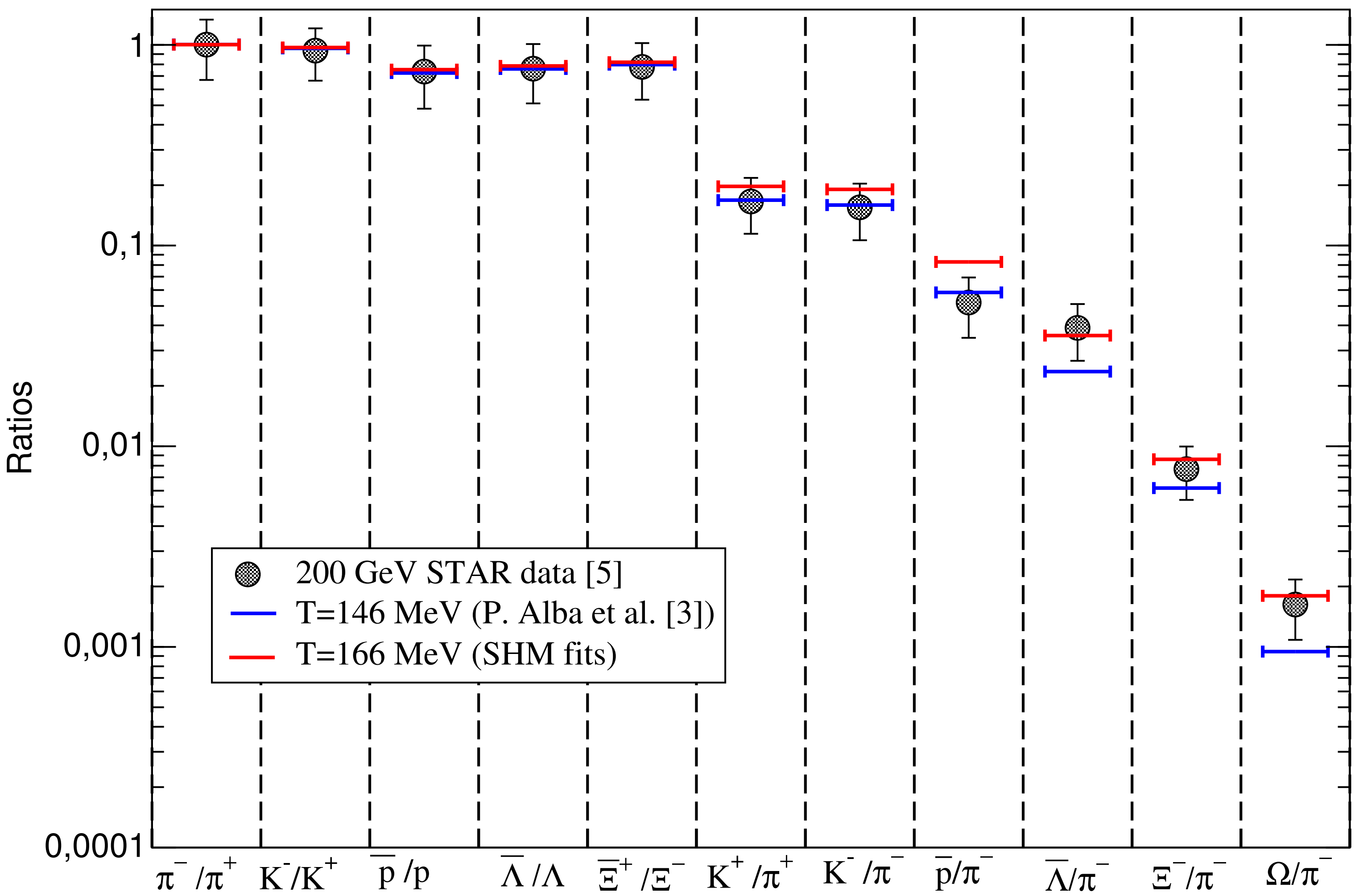
- Studying the lower order cumulants we were able to find a common freeze-out surface to describe the experimental data for Q and B.
- The temperature we find is about 20 MeV lower than the one found in previous studies; respect to them we are analyzing quantities strictly dominated by light flavor.
- We are looking for new corrections to solve the higher order moments inconsistency for the net-proton distribution.
- We are looking at the strange sector (V. Mantovani on Friday), waiting for the data coming from RHIC and LHC.

Thanks for the attention

Backup Slides



strange quark χ_4/χ_2
light quark χ_4/χ_2
HRG prediction for the strange quark
for the light quarks
2.5
2
1.5
1
0.5
0
130
140
150
160
170
180
190
T [MeV]



Ratios
1
0,1
0,01
0,001
0,0001
200 GeV STAR data [5]
T=146 MeV (P. Alba et al. [3])
T=166 MeV (SHM fits)
π^-/π^+
K^-/K^+
$\bar{p}/p$
$\bar{\Lambda}/\Lambda$
$\bar{\Xi}^+/\Xi^-$
K^+/π^+
K^-/π^-
$\bar{p}/\pi^-$
$\bar{\Lambda}/\pi^-$
Ξ^-/π^-
Ω/π^-

Strangeness cumulants

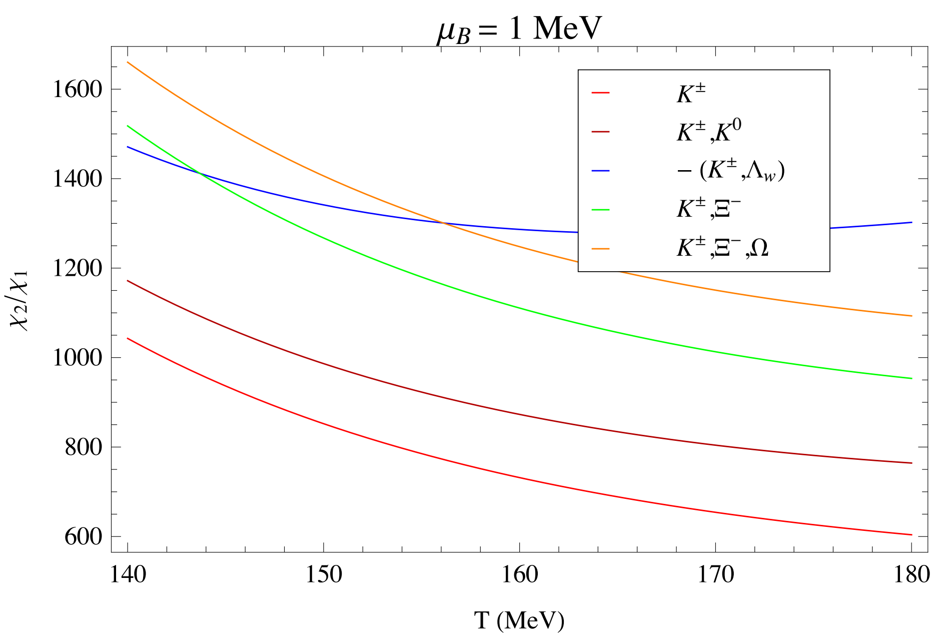
We want to check whether the strange quarks hadronize at higher temperatures than the light ones; the chemical potentials stay the same because the system is the same; only the temperature might change due to the different masses.

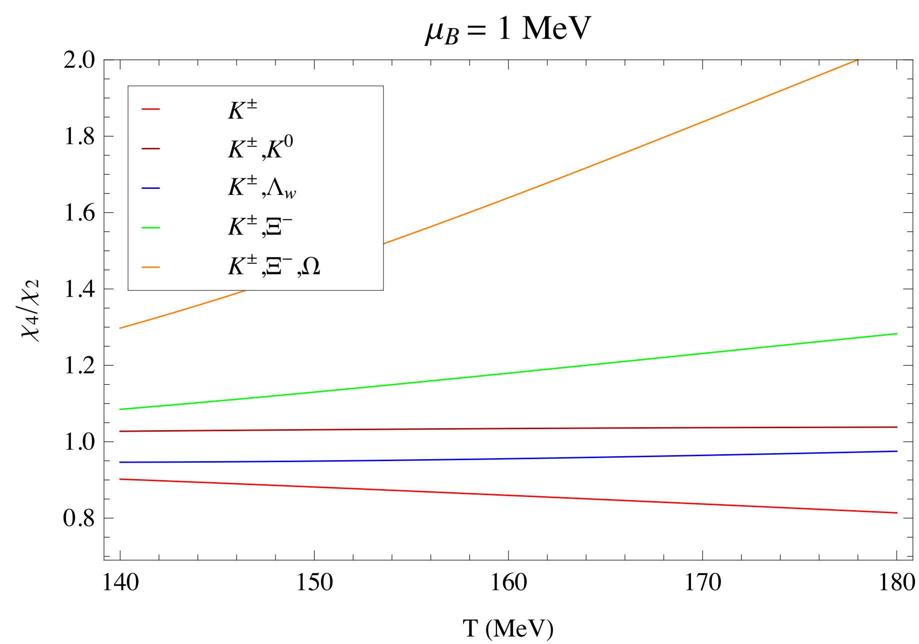
We call this a flavor hierarchy.

Strange particle sets

The strange particles are harder to measure; this is the reason why no data about strangeness has been published until now.

We can do some exploratory studies, about the set of strange particles which gives the clearest signal.





work in progress

Some references

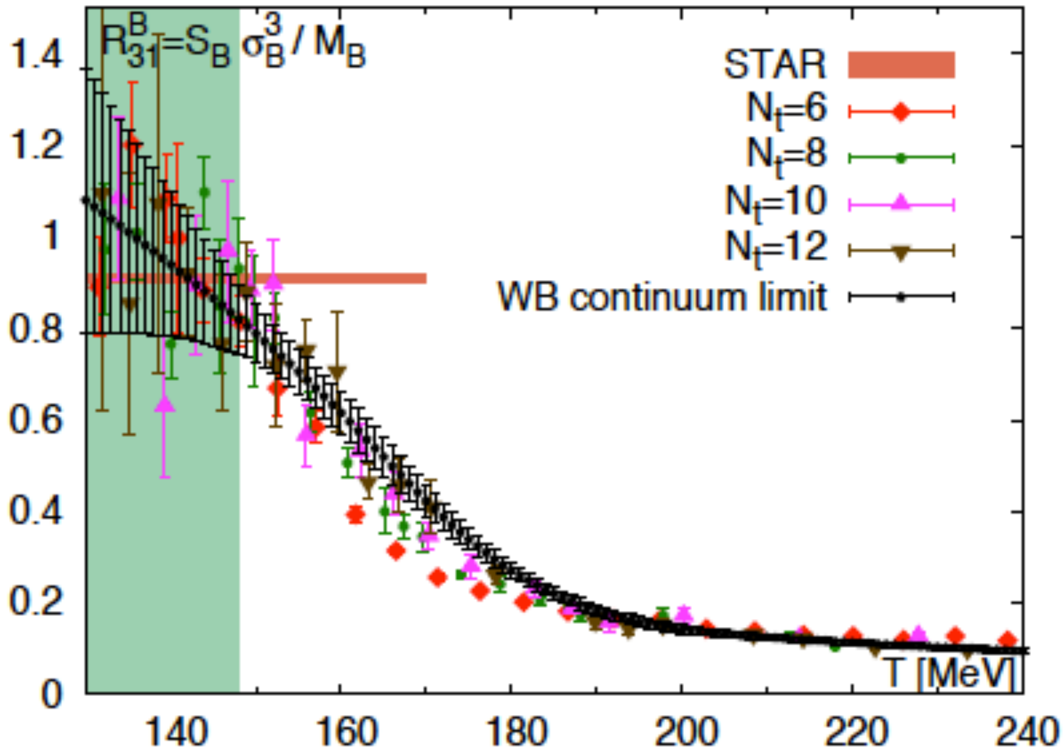
Study about the use of the gran-canonical enesemble to reproduce the experimental results with kinematic cuts: M. Nahrgang, T. Schuster, M. Mitrovski, R. Stock and M. Bleicher, Eur. Phys. J. C 72 (2012) 2143 [arXiv: 0903.2911 [hep-ph]]

Estimastion of the influence of the cuts on resonances and not on the feed: S. Jeon and V. Koch, Phys. Rev. Lett. 83 (1999) 5435 [nucl-th/ 9906074]

Hadronic phase duration: C. Nonaka and S. A. Bass, Phys. Rev. C 75, 014902 (2007) [nucl-th/0607018]; D. Teaney, arXiv:nucl-th/0204023

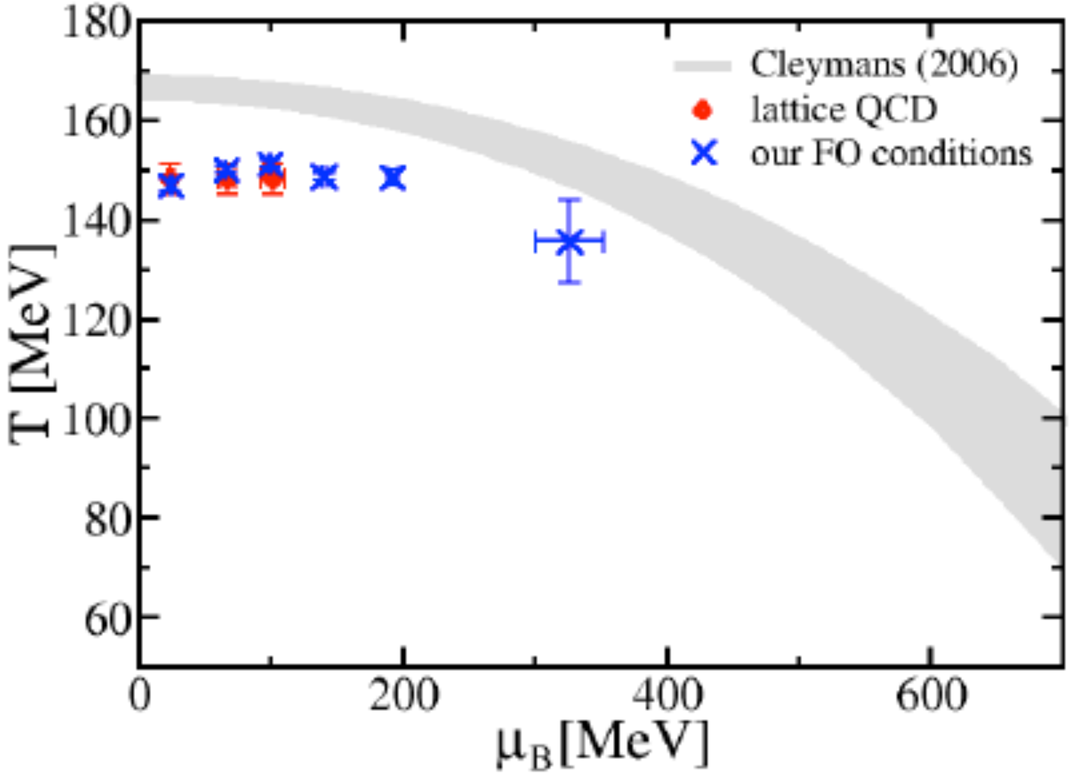
Some notes

We note here that while in our model approach all momentum integrals are evaluated at the chemical freeze-out, the final kinematics which is subject to the acceptance cuts is determined at lower kinetic freeze-out temperature. In principle a study of the evolution of the thermal distributions of the particles, until the kinetic freeze-out taking elastic scatterings in the thermally equilibrated hadronic phase into account, would be needed in order to implement the kinematic cuts more realistically.



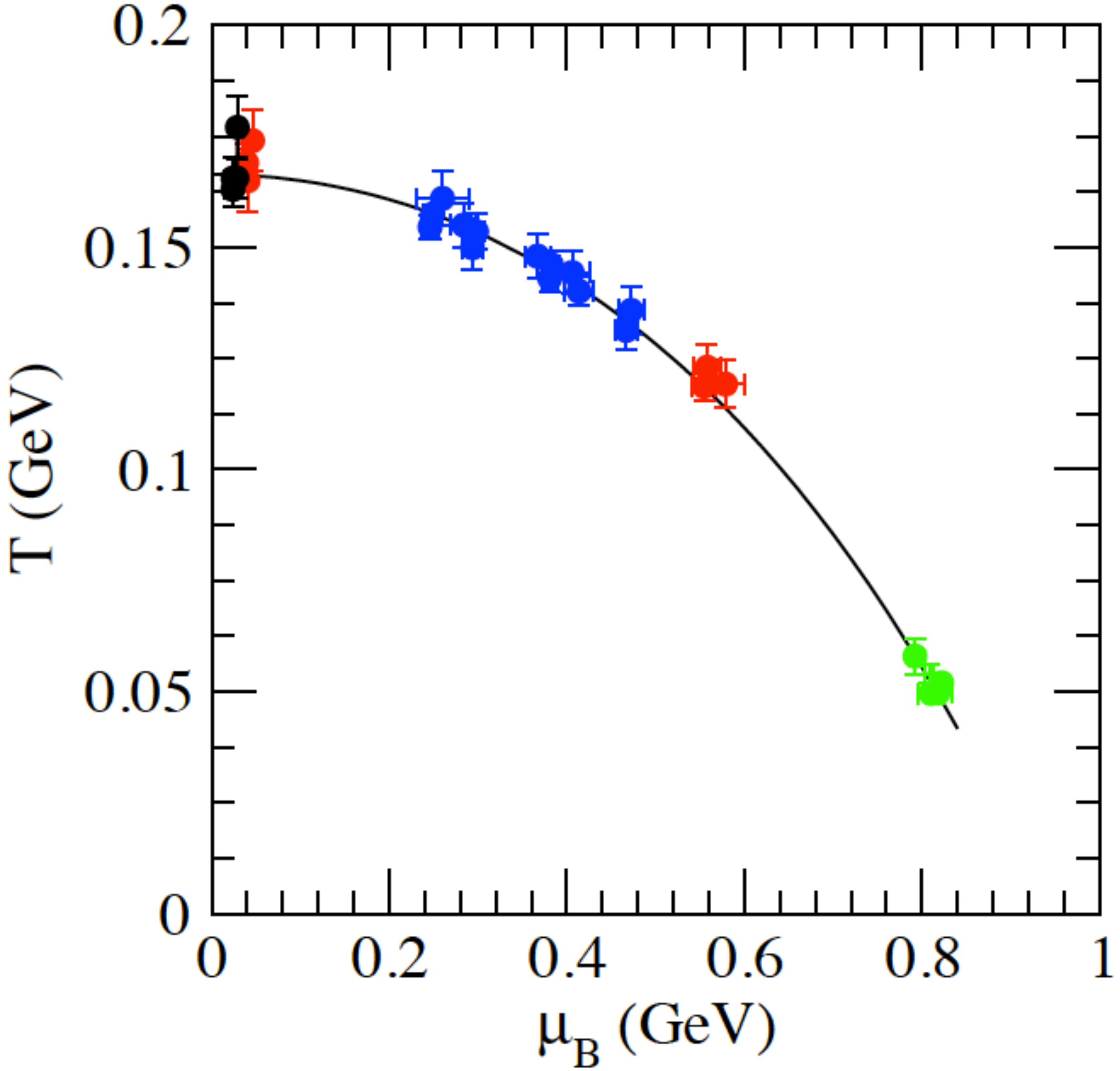
WB Collaboration: arXiv 1403.4576; STAR data from 1309.5681

$$T \leq 148 \pm 4 \text{ MeV}$$

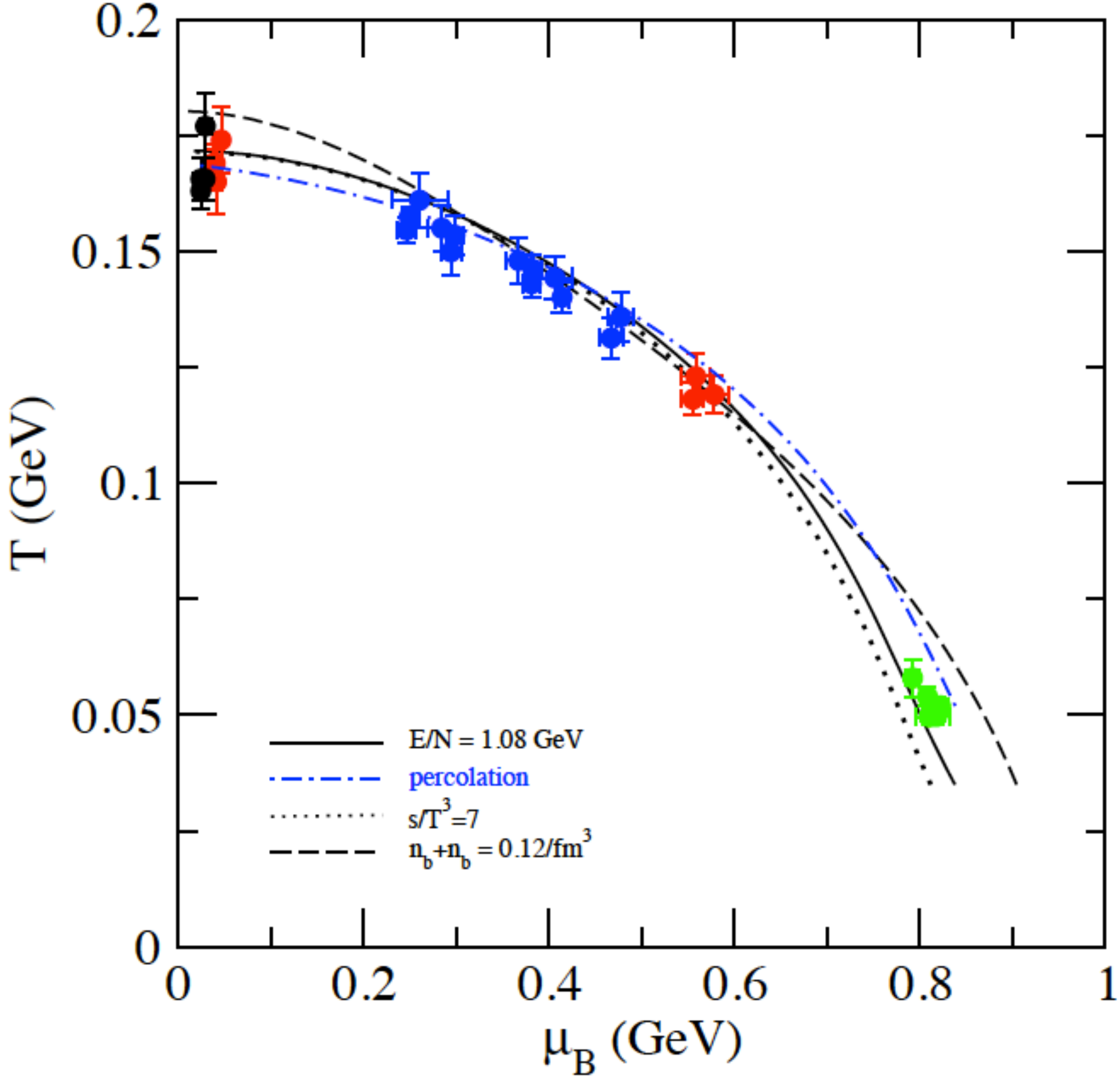


Cleymans (2006)
lattice QCD
our FO conditions
T [MeV]
μ_B[MeV]
180
160
140
120
100
80
60
0
200
400
600

$\sqrt{s}$ [GeV]	$\mu_{B,ch}$ [MeV]	T_{ch} [MeV]
11.5	326.7±25.9	135.5±8.3
19.6	192.5±3.9	148.4±1.6
27	140.4±1.4	148.5±0.7
39	99.9±1.4	151.2±0.8
62.4	66.4±0.6	149.9±0.5
200	24.3±0.6	146.8±1.2



0.2
0.15
0.1
0.05
0
T (GeV)
0
0.2
0.4
0.6
0.8
1
μB (GeV)



0.2
0.15
0.1
0.05
0
T (GeV)
0
0.2
0.4
0.6
0.8
1
μB (GeV)
E/N = 1.08 GeV
percolation
s/T3=7
nb+nb = 0.12/fm3

$$\chi_{n,s}^{B} \sim \begin{cases} -(2\kappa_q)^{n/2} h^{(2-\alpha-n/2)/\beta\delta} f_s^{(n/2)}(z) & , \text{ for } \mu_q/T = 0, \text{ and } n \text{ even} \\ -(2\kappa_q)^{n} \left(\frac{\mu_q}{T}\right)^{n} h^{(2-\alpha-n)/\beta\delta} f_s^{(n)}(z) & , \text{ for } \mu_q/T > 0 \end{cases} .$$

where we used $2-\alpha = \beta\delta(1+1/\delta)$. As $\alpha = -0.2131(34)$ is negative in the 3-dimensional, $O(4)$ universality class, the 4th order moments of the net baryon number fluctuations do not diverge yet in the chiral limit at the chiral transition temperature, $z = 0$. The first divergent moment is obtained for $n = 6$ if $\mu_q/T = 0$ and for $n = 3$ if $\mu_q/T > 0$.

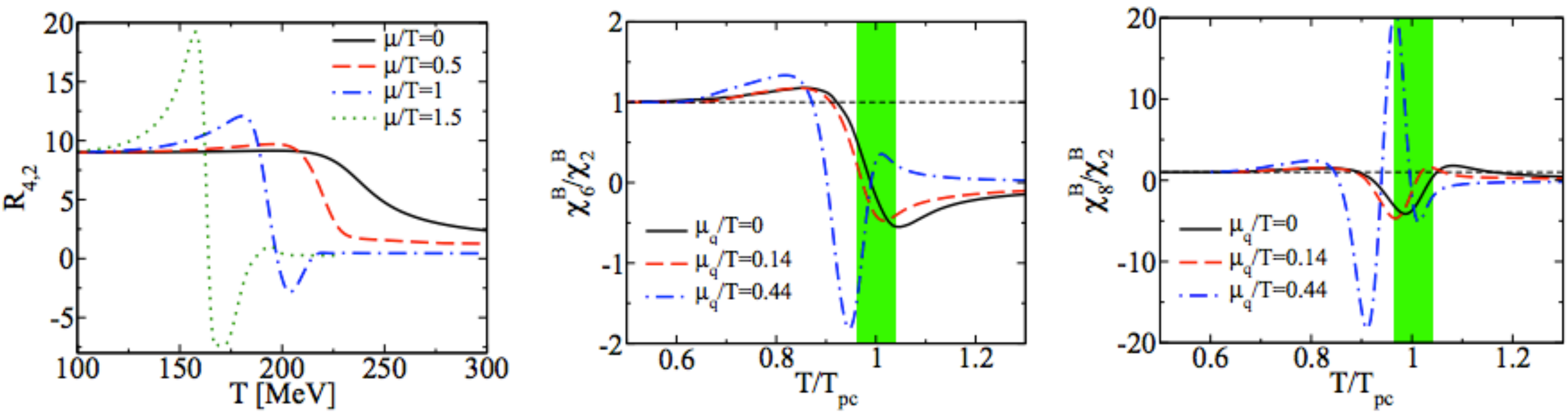


Figure 1. The temperature dependence of kurtosis $R_{4,2} := 9\chi_4^B/\chi_2^B$ and the higher order, χ_6^B/χ_2^B and χ_8^B/χ_2^B, ratios of cumulants for different μ_q/T calculated in the PQM model within the FRG approach [6, 8]. The T_{pc} is the pseudo-critical temperature obtained in the model at the physical pion mass.