

τ -Decay and Hadronic Spectral Functions

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- $eL\sigma M$ where Scalars, Pseudoscalars, Vectors and Axialvectors are genuine degrees of freedom
- reduce complexity of QCD interaction by **effective hadron hadron interaction in a model with hadronic dofs and symmetries known from the QCD Lagrangian.**
- however several works suggest that the a_1 is dynamically created from $\rho\pi$ e.g.
 - Wagner, Leupold, Phys.Rev.D78:053001,2008
 - L. Roca, E. Oset and J. Singh, Phys. Rev. D72:014002,2005
- is a model where the a_1 is chiral partner of ρ and an explicit degree of freedom applicable to describe more complicated systems such as the τ spectral function?
- more elaborate methods for precise determination of masses and decay widths are available.
- We want to construct a phenomenological model that describes the low lying resonances in the vacuum with reasonable precision, therefore we first have to identify the states within the multiplets.

$$\mathcal{L} = \mathcal{L}_{\text{meson}} + \mathcal{L}_{\text{baryon}} + \mathcal{L}_{\text{dilaton}} + \mathcal{L}_{\text{weak}}$$

$$\begin{aligned} \mathcal{L}_{\text{meson}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\ & + c_1 (\det \Phi - \det \Phi^\dagger)^2 + \text{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \frac{g_2^2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\ & + \text{chirally invariant vector and axialvector four-point interaction vertices} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{baryon}} = & \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\ & - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi^\dagger \Psi_{2L}) \\ & - M (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} - \bar{\Psi}_{2R} \Psi_{1L}) \end{aligned}$$

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2} (\partial^\mu G)^2 - \frac{1}{4} \frac{m_G}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right)$$

$$\begin{aligned} \mathcal{L}_{\text{weak}} = & \delta_w \frac{g \cos \theta_C}{2} \text{Tr}[W_{\mu\nu} L^{\mu\nu}] + \delta_{\text{em}} \frac{e}{2} \text{Tr}[B_{\mu\nu} R^{\mu\nu}] + \frac{1}{4} \text{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2] \\ & + \frac{g}{2\sqrt{2}} (W_\mu^- \bar{u}_{\nu\tau} \gamma_\mu (1 - \gamma_5) u_\tau + \text{h.c.}) \end{aligned}$$

$N_F = 2$ and $N_F = 3$ meson multiplets:

(Pseudo-)Scalars $\Phi_{ij} \simeq \langle q_L \bar{q}_R \rangle_{ij} \simeq \frac{1}{\sqrt{2}} (q_i \bar{q}_j - q_i \gamma_5 \bar{q}_j)$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0)}{\sqrt{2}} + \frac{i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{*+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0)}{\sqrt{2}} + \frac{i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{*0} + iK^0 \\ K_0^{*-} + iK^- & \bar{K}_0^{*0} + i\bar{K}_0^0 & \sigma_S + i\eta_S \end{pmatrix}$$

Lefthanded $L_{ij}^\mu \simeq \langle q_L \bar{q}_L \rangle_{ij} \simeq \frac{1}{\sqrt{2}} (q_i \gamma^\mu \bar{q}_j + q_i \gamma_5 \gamma^\mu \bar{q}_j)$

$$L^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{*+} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} + K_1^0 \\ K^{*-} + K_1^- & \bar{K}^{*0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu$$

Righthanded $R_{ij}^\mu \simeq \langle q_R \bar{q}_R \rangle_{ij} \simeq \frac{1}{\sqrt{2}} (q_i \gamma^\mu \bar{q}_j - q_i \gamma_5 \gamma^\mu \bar{q}_j)$

$$R^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{*+} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{*0} - K_1^0 \\ K^{*-} - K_1^- & \bar{K}^{*0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu$$

Mesonic Lagrangian with Global Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, arXiv:1208.0585 [hep-ph]

Global Chiral Symmetry:

$$\begin{aligned} \mathcal{L}_{\text{meson}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\ & + c_1 (\det \Phi - \det \Phi^\dagger)^2 + \text{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \frac{g_2^2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \text{ch. inv. 4-point interactions among (pseudo-)scalars and (axial-)vectors} \end{aligned}$$

$U(N_F)_L \times U(N_F)_R$ Transformation:

$$\Phi \rightarrow U_L \Phi U_R^\dagger, \quad L^\mu \rightarrow U_L L^\mu U_L^\dagger, \quad R^\mu \rightarrow U_R R^\mu U_R^\dagger$$

Covariant Derivative:

$$D^\mu \Phi = \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu)$$

Field Strength Tensors:

$$L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu, \quad R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$$

Explicit Breaking of Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]

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$U(1)_A$ -Anomaly

$$c_1 (\det \Phi - \det \Phi^\dagger)^2$$

non-vanishing quark masses, NO isospin breaking

$$\text{Tr}[H(\Phi + \Phi^\dagger)], \quad H = h_a t^a \quad (N_f = 3, \Delta)$$

remaining symmetry is $U(2)_V$

Spontaneous Breaking of Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, arXiv:1208.0585 [hep-ph]

Global Chiral Symmetry:

$$\begin{aligned} \mathcal{L}_{\text{meson}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\ & + c_1 (\det \Phi - \det \Phi^\dagger)^2 + \text{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \text{Tr} \left[\left(\frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \frac{g_2^2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \text{ch. inv. 4-point interactions among (pseudo-)scalars and (axial-)vectors} \end{aligned}$$

Spontaneous breaking of global chiral symmetry by non-zero scalar condensate

$$\sigma \rightarrow \sigma + \phi, \quad \phi = Z_\pi f_\pi$$

- i) $m_\rho^2 = m_1^2 + \frac{\phi^2}{2} (h_1 + h_2 + h_3)$, $m_{a_1}^2 = m_1^2 + (g_1 \phi)^2 + \frac{\phi^2}{2} (h_1 + h_2 - h_3)$
- ii) 3 point interaction vertices and mixing terms in $(D^\mu \Phi)^\dagger D_\mu \Phi$ that are proportional to the VEV ϕ .
- iii) unphysical mixing between axial-vector and pseudoscalar fields, diagonalization by shift of axial vectors; $P \rightarrow Z_\pi P$

$U(2)_L \times U(2)_R$ Symmetry in the Baryonic Sector

S. Gallas, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 014004 arXiv:0907.5084 [hep-ph]

S. Gallas, F. Giacosa and G. Pagliara, Nucl. Phys. A 872 (2011) 13 arXiv:1105.5003 [hep-ph]

Baryons in the mirror assignment:

$$\begin{aligned}
\mathcal{L}_{\text{baryon}} = & \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\
& - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi^\dagger \Psi_{2L}) \\
& - M (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} - \bar{\Psi}_{2R} \Psi_{1L})
\end{aligned}$$

$U(2)_L \times U(2)_R$ Transformation

$$\Psi_{1R} \rightarrow U_R \Psi_{1R}, \quad \Psi_{1L} \rightarrow U_L \Psi_{1L}$$

$$\Psi_{2R} \rightarrow U_L \Psi_{2R}, \quad \Psi_{2L} \rightarrow U_R \Psi_{2L}$$

Covariant Derivative

$$D_{1R}^\mu = \partial^\mu - ic_1 R^\mu, \quad D_{1L}^\mu = \partial^\mu - ic_1 L^\mu$$

$$D_{2R}^\mu = \partial^\mu - ic_2 R^\mu, \quad D_{2L}^\mu = \partial^\mu - ic_2 L^\mu$$

- allows for **chirally invariant mass term** generated by the gluon and/or tetraquark condensate
- Nucleons N, N^* are real chiral partners $N(1650)$ is favoured as chiral partner of $N(939)$
- yields correct nuclear matter saturation

Scale Invariance and the Glueball

S. Janowski, D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 84 (2011) 054007 arXiv:1103.3238 [hep-ph]

Scale invariance of the QCD Lagrangian is broken on the quantum level

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2} (\partial^\mu G)^2 - \frac{1}{4} \frac{m_G}{\Lambda^2} \left(G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right)$$

\mathcal{L}_σ is in principle scale invariant, only mass terms and $U(1)_A$ -anomaly break scale invariance

$$x^\mu \rightarrow \lambda^{-1} x^\mu, \quad \varphi(x) \rightarrow \lambda \varphi(\lambda^{-1} x), \quad \Psi(x) \rightarrow \lambda^{\frac{3}{2}} \Psi(\lambda^{-1} x)$$

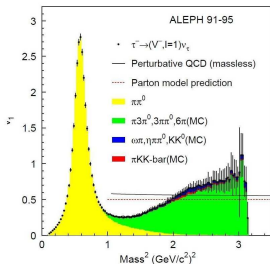
Scalar glueball is associated with fluctuations of the dilaton potential

Ground state of dilaton G_0 is related to the gluon condensate $G_0 = \Lambda = \frac{\sqrt{11}}{2m_G} C^2$

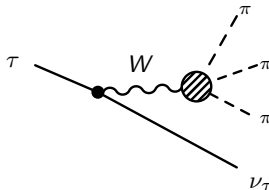
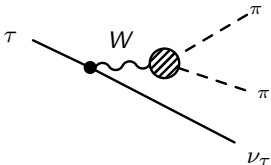
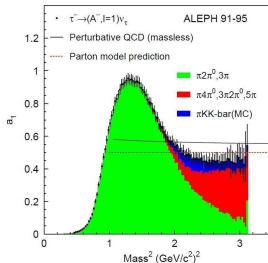
- favours $q\bar{q}$ interpretation of $f_0(1370)$ as chiral partner of the pion ($f_0(500)$ is disfavoured) and $f_0(1500)$ is 75% glueball
- scale invariance extended to $\mathcal{L}_{\text{meson}}, \mathcal{L}_{\text{baryon}}$ by parametrization of meson and baryon masses by G

τ -Decay

Vector $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$



Axial Vector $\tau^- \rightarrow 2\pi^0 / -\pi^- / +\nu_\tau$



- semileptonic τ -decay involves strong and weak interactions
- effective electroweak interactions of hadrons in the vacuum

What do we know about weak interaction?

- $SU(2)_L \times U(1)_Y$ gauge symmetry with gauge fields W^μ and B^μ
- Weinberg mixing between $SU(2)_L \times U(1)_Y$ gauge fields B^μ, W_3^μ to physical interaction fields A^μ, Z_0^μ .
- Cabibbo mixing, flavour eigenstates are not weak eigenstates.
- Charged interaction violates symmetry under Charge and Parity transformations but preserves combined CP;
 W_\pm^μ act on left-handed particles, right-handed antiparticles only

$$P_L = \frac{1-\gamma_5}{2} \quad P_R = \frac{1+\gamma_5}{2} .$$

- Charged bosons induce flavour-changing processes.



Linear Sigma Model with Weak Interaction

i) transformation of composite quarks

$$\Phi_{kl} \simeq \langle \bar{q}_R q_L \rangle_{kl}, \quad q_k \xrightarrow{U(1)_Y} e^{iy_k \Theta_Y(X)} q_k, \quad y_k = 2(Q_k - I_{3k})$$

ii) $U(1)_Y$ Transformation:

Scalar and Pseudoscalar Fields:

$$\Phi \xrightarrow{U(1)_Y} \Phi U_Y^\dagger \simeq \Phi + i\Theta_Y \Phi t_3$$

Gauge Field:

$$B^\mu \xrightarrow{U(1)_Y} U_Y B^\mu U_Y^\dagger + \frac{i}{g'} U_Y \partial^\mu U_Y^\dagger$$

Righthanded Fields:

$$R^\mu \xrightarrow{U(1)_Y} U_Y R^\mu U_Y^\dagger$$

iii) Covariant Derivative:

$$D_Y^\mu \Phi = \partial^\mu \Phi - ig_1 [L^\mu \Phi - \Phi (R^\mu + \frac{g'}{g_1} B_\mu t_3)]$$

iv) Field Strength Tensor:

$$R^{\mu\nu} = (\partial^\mu R^\nu - ig' [B^\mu, R^\nu]) - (\partial^\nu R^\mu - ig' [B^\nu, R^\mu])$$

Local $SU(2)_L \times U(1)_Y$ Symmetry

$SU(2)_L \times U(1)_Y$ Transformations

$$\Phi \xrightarrow{SU(2)_L \times U(1)_Y} U_L \Phi U_Y^\dagger$$

$$L^\mu \xrightarrow{SU(2)_L} U_L L^\mu U_L^\dagger \quad W^\mu \xrightarrow{SU(2)_L} U_L W^\mu U_L^\dagger + \frac{i}{g} U_L \partial^\mu U_L^\dagger$$

$$R^\mu \xrightarrow{U(1)_Y} U_Y R^\mu U_Y^\dagger \quad B^\mu \xrightarrow{U(1)_Y} U_Y B^\mu U_Y^\dagger + \frac{i}{g'} U_Y \partial^\mu U_Y^\dagger$$

$SU(2)_L \times U(1)_Y$ Covariant Derivative:

$$D^\mu \Phi = \partial^\mu \phi - ig_1 \left[(L^\mu + \frac{g}{g_1} W^\mu) \Phi - \Phi (R^\mu + \frac{g'}{g_1} B_\mu t_3) \right]$$

Field Strength Tensors:

$$R^{\mu\nu} = (\partial^\mu R^\nu - ig' [B^\mu, R^\nu]) - (\partial^\nu R^\mu - ig' [B^\nu, R^\mu])$$

$$L^{\mu\nu} = (\partial^\mu L^\nu - ig [W^\mu, L^\nu]) - (\partial^\nu L^\mu - ig [W^\nu, L^\mu])$$

Weinberg Mixing and Cabibbo Mixing

Weinberg Mixing:

neutral bare $SU(2)_L \times U(1)_Y$ gauge fields B^μ , W_3^μ are related to the physical fields A^μ , Z^μ by

and

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

$$e = g' \cos\theta_W = g \sin\theta_W$$

Cabibbo mixing:

strong isospin eigenstates d, s, b are related to the weak eigenstates by the CKM matrix

$$N_f = 2$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Linear Sigma Model with Weak Interaction

$$\begin{aligned} \mathcal{L}_{\text{weak}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] + \delta_w \frac{g \cos \theta_C}{2} \text{Tr}[W_{\mu\nu} L^{\mu\nu}] + \delta_{\text{em}} \frac{e}{2} \text{Tr}[B_{\mu\nu} R^{\mu\nu}] \\ & + \frac{1}{4} \text{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2] + \frac{g}{2\sqrt{2}} (W_\mu^- \bar{u}_{\nu\tau} \gamma_\mu (1 - \gamma_5) u_\tau + \text{h.c.}) \end{aligned}$$

The Decay $\tau \rightarrow W \nu_\tau$ is well known from SM.

Gauge invariant mixing term $\simeq \delta_w s W_\mu \rho^\mu$.

Covariant Derivative with Physical Interaction Fields:

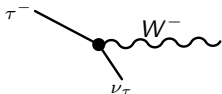
$$\begin{aligned} D^\mu \Phi \equiv & \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ie[A^\mu t_3, \Phi] - ig \cos \theta_C (W_1^\mu t_1 + W_2^\mu t_2) \Phi \\ & - ig \cos \theta_W (Z^\mu \Phi + \tan^2 \theta_W \Phi Z^\mu) \end{aligned}$$

Field Strength Tensors:

$$\begin{aligned} L^{\mu\nu} \equiv & \{ \partial^\mu L^\nu - ie[A^\mu t_3, L^\nu] - ig \cos \theta_C [W_1^\mu t_1 + W_2^\mu t_2, L^\nu] - ig \cos \theta_W [Z^\mu, L^\nu] \} \\ & - \{ \partial^\nu L^\mu - ie[A^\nu t_3, L^\mu] - ig \cos \theta_C [W_1^\nu t_1 + W_2^\nu t_2, L^\mu] - ig \cos \theta_W [Z^\nu, L^\mu] \} \\ R^{\mu\nu} \equiv & \{ \partial^\mu R^\nu - ie[A^\mu t_3, R^\nu] - ig \sin \theta_W [Z^\mu, R^\nu] \} \\ & - \{ \partial^\nu R^\mu - ie[A^\nu t_3, R^\mu] - ig \sin \theta_W [Z^\nu, R^\mu] \} \end{aligned}$$

Vector Channel

Common to all channels is the process:



$$\Gamma_{\tau^- \rightarrow W^- 2\nu_\tau}(s) \sim \frac{|p(m_\tau^2, s, m_\nu^2)|}{m_\tau^2} \left| \begin{array}{c} \tau^- \\ \text{---} \\ \bullet \\ \text{---} \\ W^- \\ \text{---} \\ \nu_\tau \end{array} \right|^2$$

Vector Channel

$$\Gamma_{W^- \rightarrow \pi^- 2\pi^0}(s) \sim \frac{1}{s} \left| \begin{array}{c} W^- \\ \text{---} \\ \circ \\ \text{---} \\ \pi^0 \\ \text{---} \\ \pi^- \end{array} \right|^2 |p(s, m_\pi)|$$

$$\left| \begin{array}{c} W^- \\ \text{---} \\ \circ \\ \text{---} \\ \pi^0 \\ \text{---} \\ \pi^- \end{array} \right|^2 = \left| \begin{array}{c} W^- \\ \text{---} \\ \bullet \\ \text{---} \\ \pi^0(k_1) \\ \text{---} \\ \pi^-(k_2) \end{array} \right|^2 + \left| \begin{array}{c} W^- \\ \text{---} \\ \bullet \\ \text{---} \\ \rho^- \\ \text{---} \\ \bullet \\ \text{---} \\ \pi^0(k_1) \\ \text{---} \\ \pi^-(k_2) \end{array} \right|^2$$

Axial-Vector Channel

Axial-Vector Channel

$$\Gamma_{W^- \rightarrow \pi^- 2\pi^0}(s) \sim \frac{1}{s} \frac{2}{2 \cdot 3} \int \left| W^- \begin{array}{c} \nearrow \pi^0 \\ \circlearrowleft \\ \searrow \pi^- \\ \searrow \pi^0 \end{array} \right|^2 dm_{12}^2 dm_{23}^2$$

$$\begin{aligned} & \left| W^- \begin{array}{c} \nearrow \pi^0 \\ \circlearrowleft \\ \searrow \pi^- \\ \searrow \pi^0 \end{array} \right|^2 \\ &= \frac{1}{2} \left| 2 \begin{array}{c} W^- \begin{array}{c} \nearrow \pi^0 \\ \bullet \\ \searrow \pi^- \\ \searrow \pi^0 \end{array} + \begin{array}{c} W^- \begin{array}{c} \nearrow \pi^0(k_1) \\ \bullet \\ \searrow \pi^-(k_2) \\ \bullet \\ \searrow \pi^0(k_3) \end{array} + \begin{array}{c} W^- \begin{array}{c} \nearrow \pi^0(k_3) \\ \bullet \\ \searrow \pi^-(k_2) \\ \bullet \\ \searrow \pi^0(k_1) \end{array} \right. \\ &+ 2 \begin{array}{c} W^- \begin{array}{c} \nearrow \pi^0 \\ \bullet \\ \searrow \pi^- \\ \bullet \\ \searrow \pi^0 \end{array} + \begin{array}{c} W^- \begin{array}{c} \nearrow \pi^0(k_1) \\ \bullet \\ \searrow \pi^-(k_2) \\ \bullet \\ \searrow \pi^0(k_3) \end{array} + \begin{array}{c} W^- \begin{array}{c} \nearrow \pi^0(k_3) \\ \bullet \\ \searrow \pi^-(k_2) \\ \bullet \\ \searrow \pi^0(k_1) \end{array} \left. \right|^2 \end{aligned}$$

- $\Gamma(W^- \rightarrow \pi^- 2\pi^0) \simeq 1\%$ and $\Gamma(a_1^- \rightarrow \pi^- 2\pi^0) \simeq 1\%$
- in principle also contributions of σ resonance $\Gamma_{W^- \rightarrow \sigma 2\pi^0 \pi^-} \simeq 0$

Spectral Functions

Spectral Density is taken as the **Imaginary Part** of the Propagator

$$d(s) = -\frac{1}{\pi} \text{Im}[\Delta(s)], \quad \Delta(s) = \frac{1}{s - m_0^2 + g^2 \text{Re}[\Sigma(s)] + g^2 i \text{Im}[\Sigma(s)]}$$

Optical Theorem:

$$g^2 \text{Im}[\Sigma(s)] = \sqrt{s} \Gamma(s)$$

Resonance Mass:

$$m_{\text{res.}}^2 = m_0^2 - g^2 \text{Re}[\Sigma(m_r)]$$

Sum Rules:

$$\int_0^\infty d_{\rho \rightarrow 2\pi}(s, m_\rho, \Gamma_\rho) ds = 1, \quad \int_0^\infty d_{a_1 \xrightarrow{\rho\pi} 3\pi}(s, m_{a_1}, m_\rho) ds = 1$$

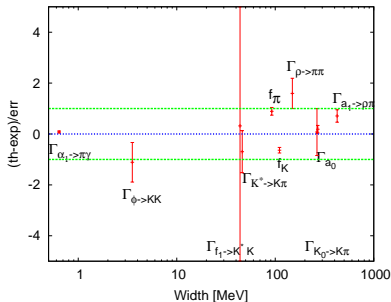
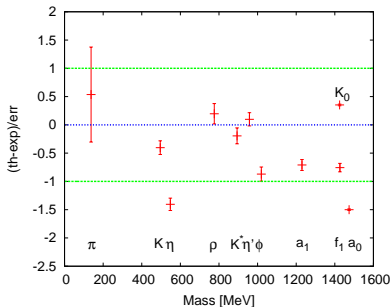
Spectral Functions

$$d_\rho(s) = \frac{1}{N_\rho} \frac{1}{\pi} \frac{\sqrt{s} \Gamma_{\rho\pi\pi}(s)}{(s - m_\rho^2)^2 + (m_\rho^2 \Gamma_\rho)^2}, \quad d_{a_1}(s) = \frac{1}{N_{a_1}} \frac{1}{\pi} \frac{\sqrt{s} \Gamma_{a_1\rho\pi}(s)}{(s - m_{a_1}^2)^2 + (m_{a_1}^2 \Gamma_{a_1})^2}$$

$m_\rho, \Gamma_\rho, m_{a_1}, \Gamma_{a_1}$ are now fit parameters

Quest for the Parameters

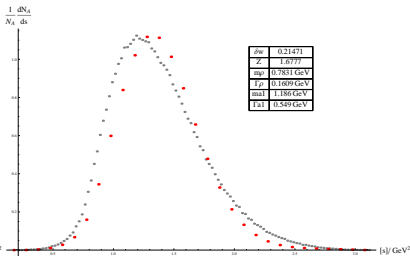
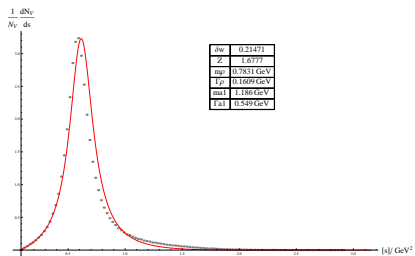
D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke
Scalar mesons in a linear sigma model with (axial-)vector mesons



[arXiv:hep-ph/1208.0585]

- global fit of 13 parameters; test model
- 21 decay widths and masses

Results with $m_\rho, \Gamma_\rho, m_{a_1}, \Gamma_{a_1}$ and pion renormalisation constant Z as obtained from $N_f = 3$

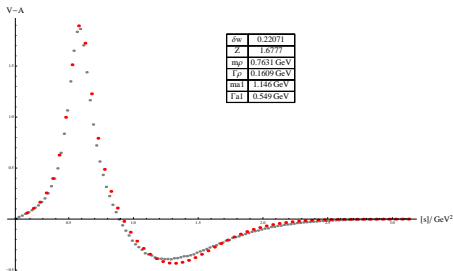


- Only one free parameter δ_w which describes the mixing between the charged weak bosons and the (axial-)vector mesons!
- ρ and a_1 mesons can be considered to be chiral partners
 $W\rho$ mixing $\sim \delta_w s$ Wa_1 mixing $\sim (\delta_w s + g_1 \phi^2)$
- the parameters have errors within range $\sim 5\%$ therefore we can still improve our results

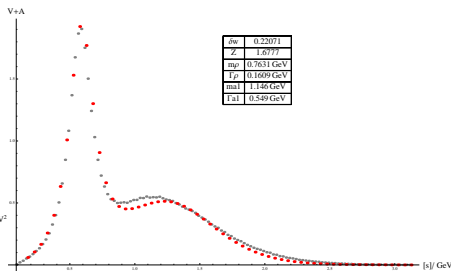
Inclusive Spectral Functions VMA and VPA

Vector Channel not independent on values of m_{a_1} and Γ_{a_1} .

→ Inclusive Spectral Functions $V - A$ and $V + A$



$V - A$

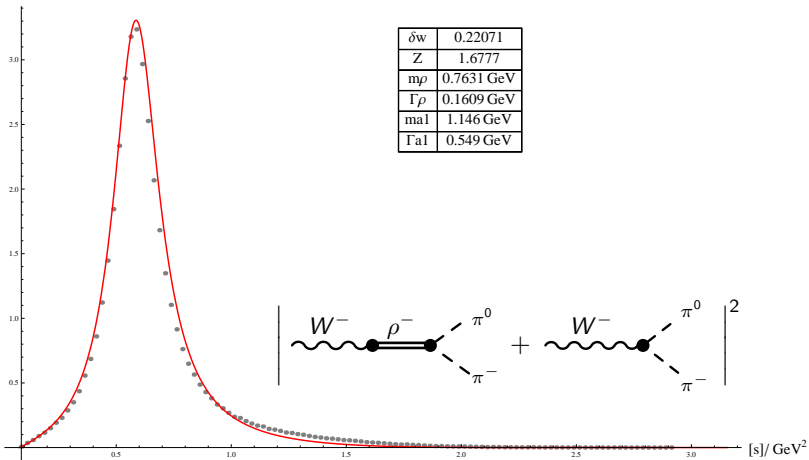


$V + A$

Vector Channel Spectral Function $\tau \rightarrow 2\pi\nu_\tau$

Coherent sum $|W \xrightarrow{\text{direct}} 2\pi + W \xrightarrow{\rho} 2\pi|^2$

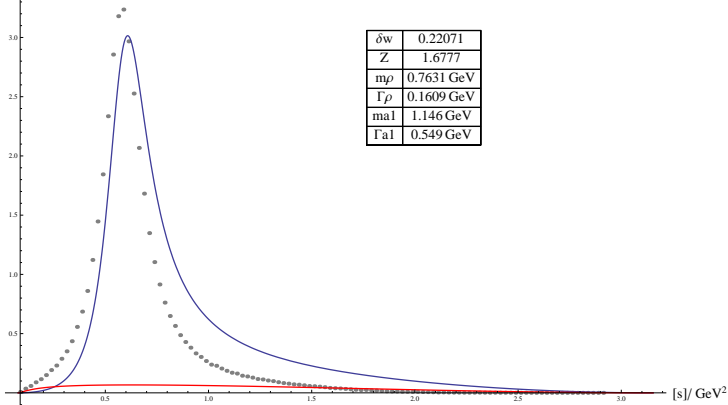
$$\frac{1}{N_V} \frac{dN_V}{ds}$$



Vector Channel Spectral Function $\tau \rightarrow \pi^- \pi^0 \nu_\tau$

$$W^- \rightarrow \pi^- \pi^0 \text{ and } W^- \rightarrow \rho^- \rightarrow \pi^- \pi^0$$

$$\frac{\Gamma(W^- \rightarrow \pi^- \pi^0)}{\Gamma(W^- \rightarrow \rho^- \rightarrow \pi^- \pi^0)} \simeq 0.02$$

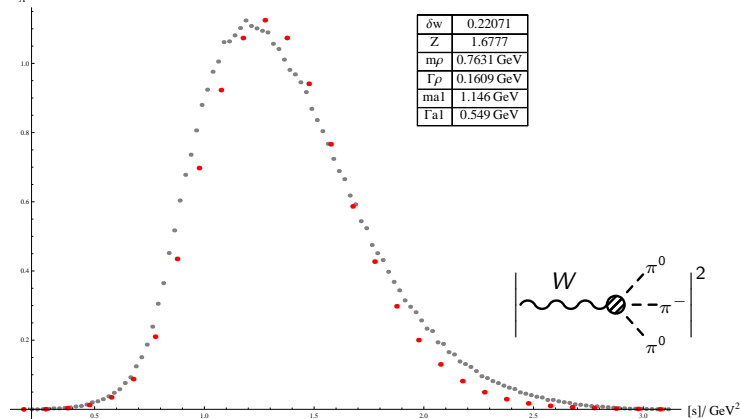
$$\frac{1}{N_V} \frac{dN_V}{ds}$$


Axial Vector Channel Spectral Function

$$\tau^- \rightarrow \pi^- 2\pi^0 + \pi^+ 2\pi^- \nu_\tau$$

Coherent Sum $|W^- \rightarrow \rho\pi \rightarrow 3\pi + W \xrightarrow{a_1} \rho\pi \rightarrow 3\pi|^2$

$$\frac{1}{N_A} \frac{dN_A}{ds}$$

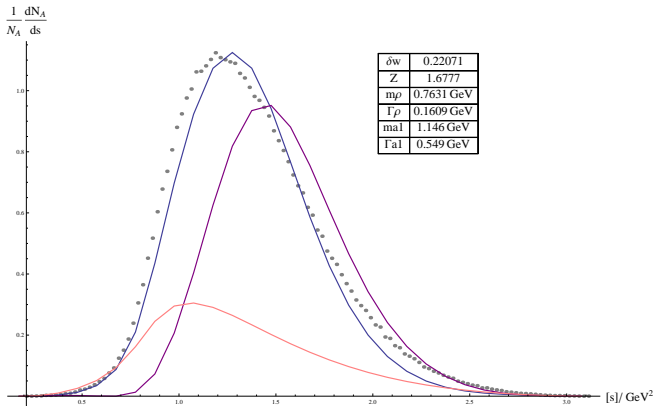


Axial Vector Channel Spectral Function

$$\tau^- \rightarrow \pi^- 2\pi^0 + \pi^+ 2\pi^- \nu_\tau$$

$$W^- \xrightarrow{\rho^- / \pi^0 / -} 2\pi^- / \pi^0 / + \quad \text{and} \quad W^- \rightarrow a_1^- \xrightarrow{\rho^- / \pi^0 / -} 2\pi^- / \pi^0 / +$$

$$\frac{\Gamma(W^- \xrightarrow{\text{direct}} \pi^- 2\pi^0)}{\Gamma(W^- \xrightarrow{\text{coh.}} \rho^- \pi^0 \rightarrow \pi^- 2\pi^0)} \simeq 0.02$$



Conclusion

- We described the decay of the τ lepton in an effective hadronic model
- Can we use our effective chiral model to describe the phenomenology of the low energy resonances in the vacuum? Yes!
- Can we consider a_1 to be a $\bar{q}q$ state and obtain a reasonable description of phenomenology? Yes!
- Can we consider ρ and a_1 to be chiral partners? Yes!
- We also have a very nice example of Vector Meson Dominance in the weak hadron sector.