Lattice QCD survey of spectroscopy and hadron interactions

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(RIKEN, Nishina Center)

HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

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Faisal Etminan (Univ. Birjand)
Takashi Inoue (Nihon Univ.)
Noriyoshi Ishii, Keiko Murano (RCNP, Osaka Univ.)
Hidekatsu Nemura, Kenji Sasaki,
Masanori Yamada (Univ. Tsukuba)

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Outline

✓ Introduction to lattice QCD

-- How to explore single hadron spectroscopy --

✓ Hadron scatterings and multi-hadron systems

✓ Potentials from LQCD simulations

-- HAL QCD method to extract LQCD potentials --

✓ Results on H-dibaryon & charmed tetraquarks (Tcc)

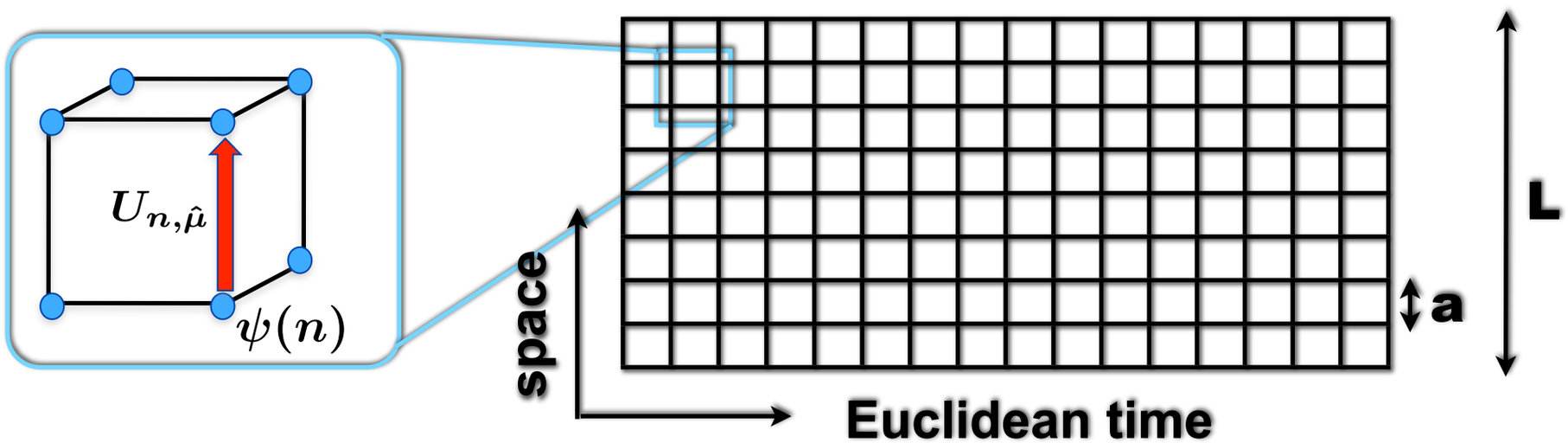
✓ Summary

Lattice QCD -- a brief introduction --

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{2} tr F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi$$

➡ **Monte Calro simulations in Euclidean space-time**

- **Quarks : ψ(n)**
- **Gluons : $U_{n,\mu}$**



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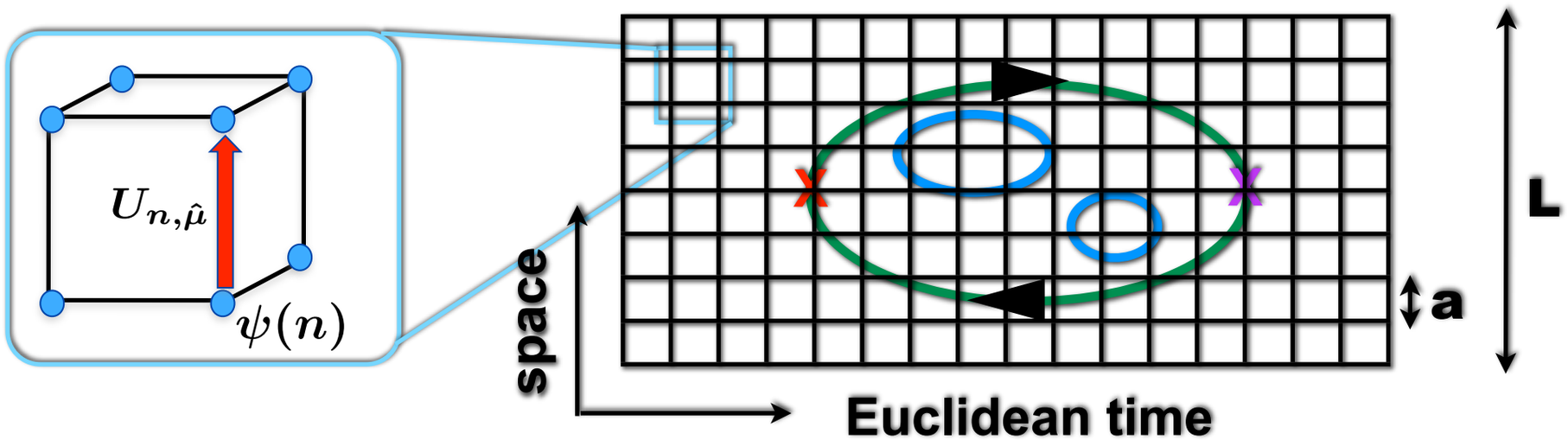

e.g.) meson masses $\langle C(\tau) \rangle = \sum_{\vec{x}} \langle 0 | \phi(x) \phi(0)^{\dagger} | 0 \rangle$ $\phi(x) = \bar{q}(x) \Gamma q(x)$

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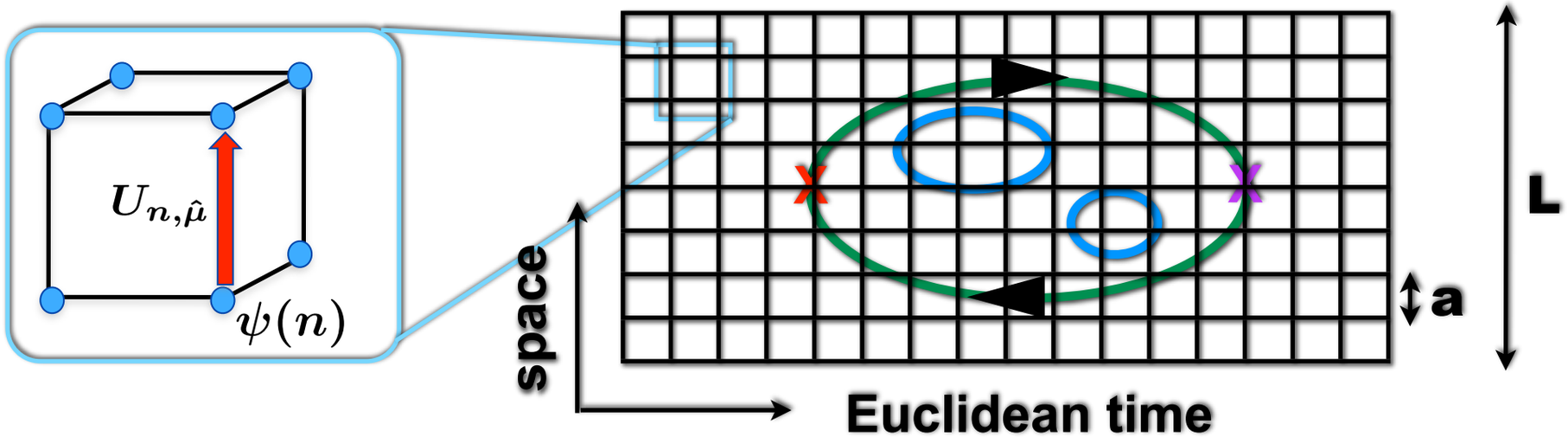

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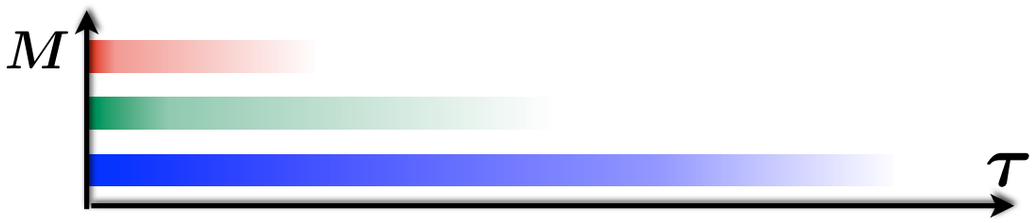
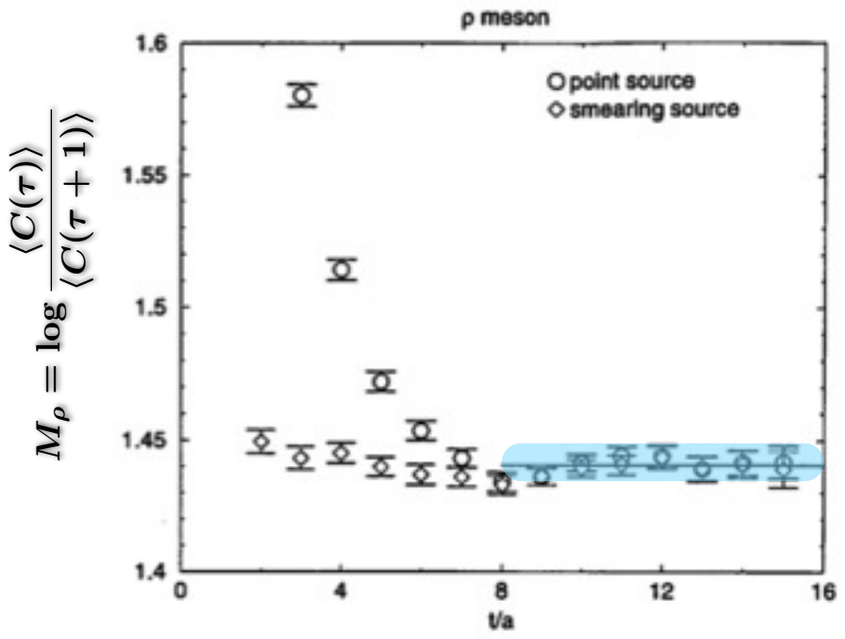
$$\langle C(\tau) \rangle = A_0 e^{-M_0 \tau} + A_1 e^{-M_1 \tau} + A_2 e^{-M_2 \tau} + \cdots$$



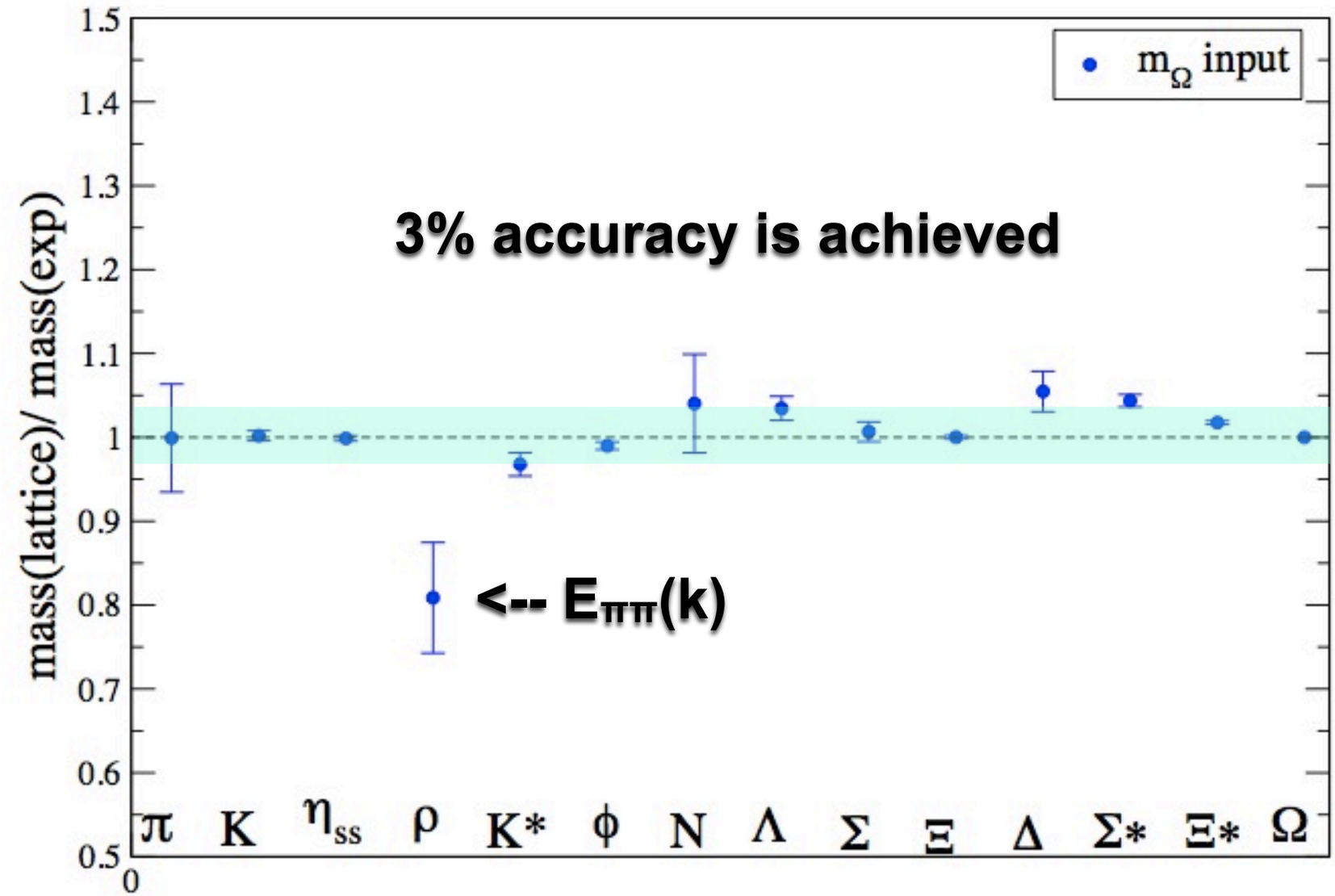
At large τ region, ground states dominate correlation functions (Ground state saturation)



Hadron spectroscopy with light quarks

- **Lattice QCD simulations for low-lying hadrons on physical point**

Aoki et al. (PACS-CS), PRD81, 074503, (2010).

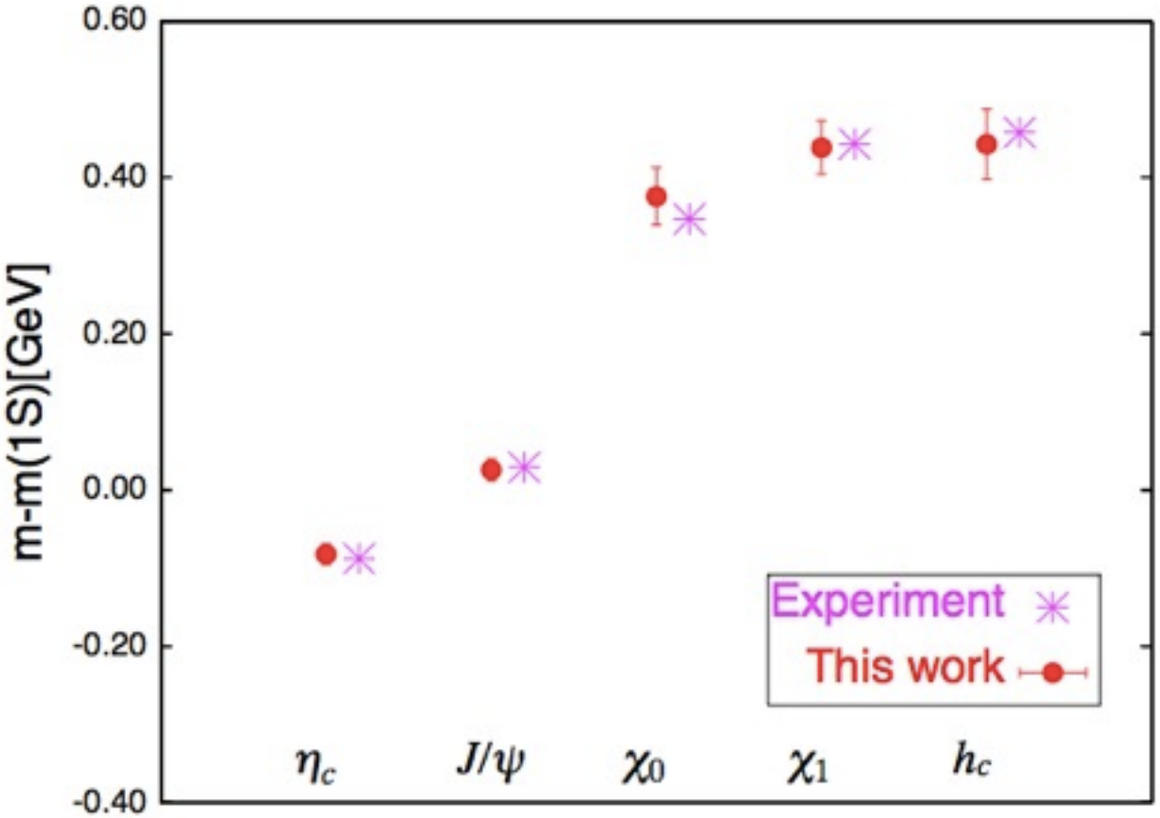


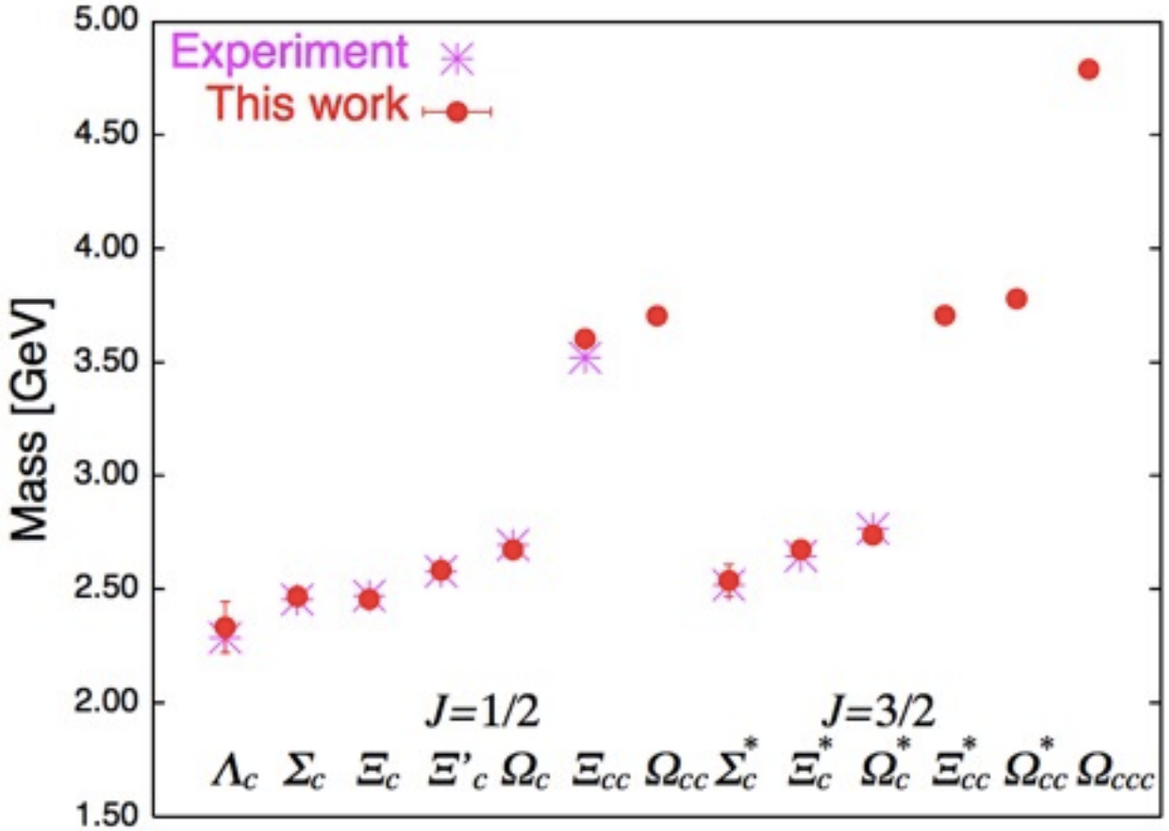
➡ **Gauge-invariant, non-perturbative & model-independent Numerical Experiments from LQCD can be possible**

Hadron spectroscopy in charm sector

✤ **Charm quark from Relativistic Heavy Quark action@physical point**

Namekawa et al. (PACS-CS), PRD84, 074505 (2011).
Namekawa et al. (PACS-CS), PRD87, 094512 (2013).





➡ **LQCD can predict undiscovered hadron properties (Ξcc*, Ωcc, ...)**

✓ **How can we study properties of bound/resonant multi-hadrons?**

Structure of bound/resonance state

T-matrix in formal scattering theory (N/D method)

$$T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s} - W_i} + \frac{1}{2\pi} \int_{s_+}^{\infty} ds' \frac{\rho(s')}{s' - s}$$

Interaction part is not determined within scattering theory

--> interactions faithful to phase shift from Lattice QCD

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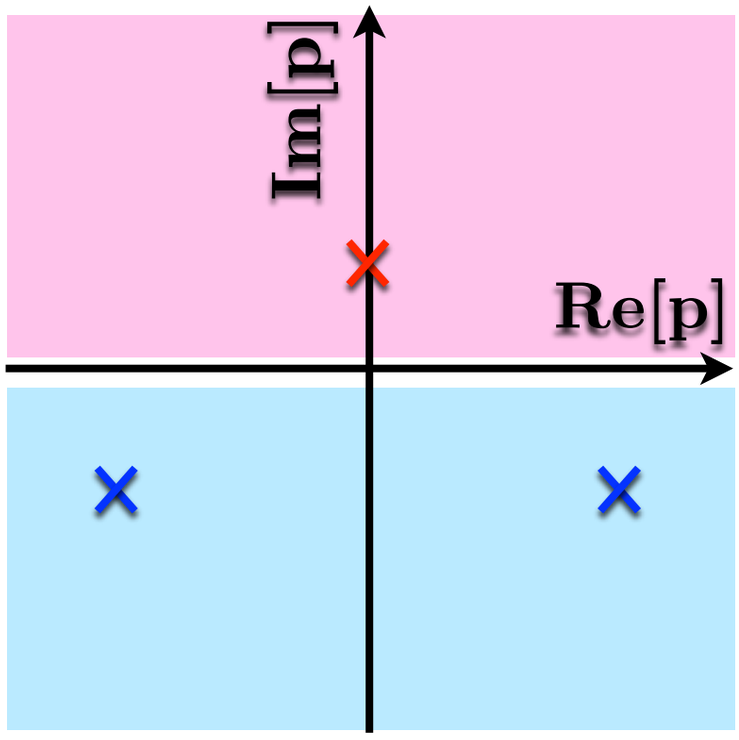


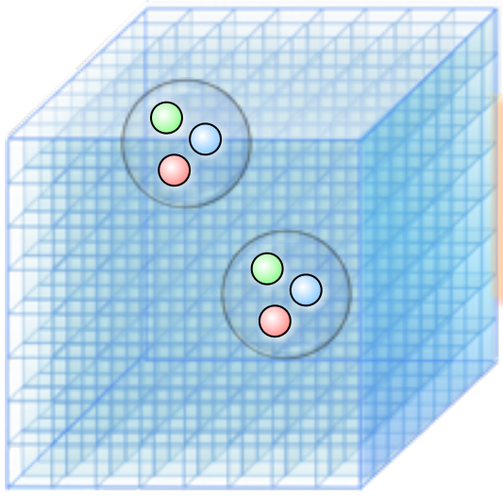
Bound states (physical sheet)

- binding energy --> T-matrix pole position
- coupling --> residue of pole
- Size --> mean-square radii

Resonance/Virtual states (unphysical sheet)

- Analytic continuation of T-matrix
- resonance energy --> T-matrix pole position
- coupling --> (complex) residue of pole?
- Size --> compositeness??

Hadron scattering on the lattice



- ***Lüscher's finite size formula***

  interaction energy --> phase shift

Lüscher, NPB354, 531 (1991).

- ***Scattering parameters***

$$k cot\delta(k) = \frac{1}{a} - \frac{1}{2} r_e k^2 + \cdots$$

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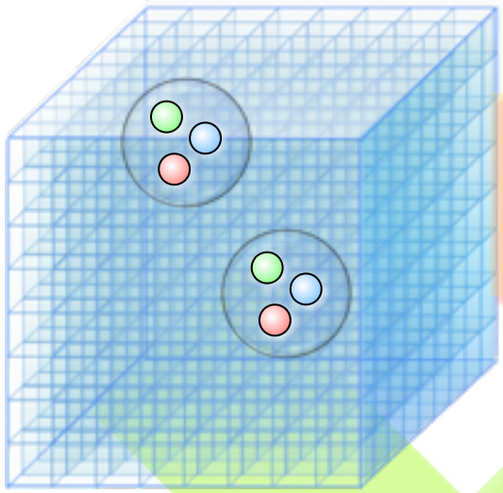

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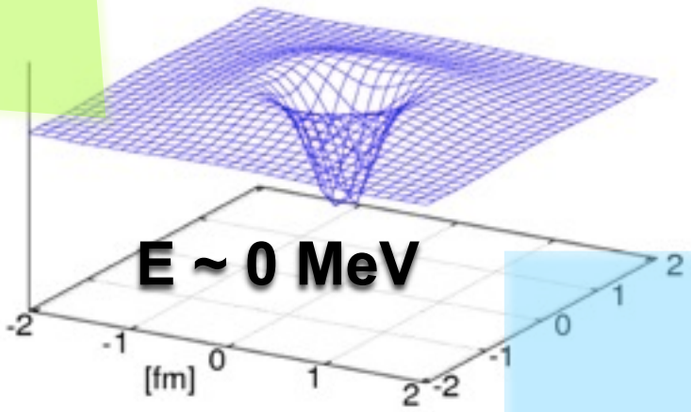
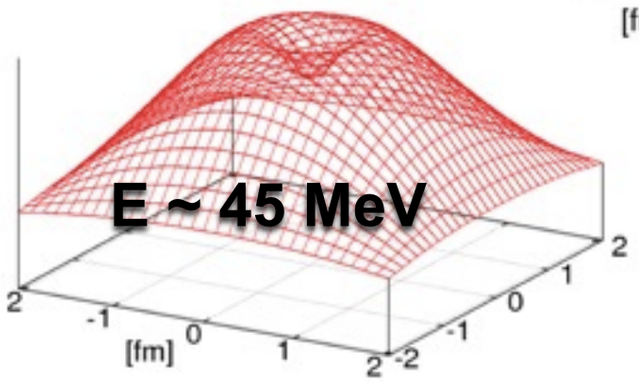
Kurth et al., JHEP 1312 (2013) 015.

- ***NBS wave function***





Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
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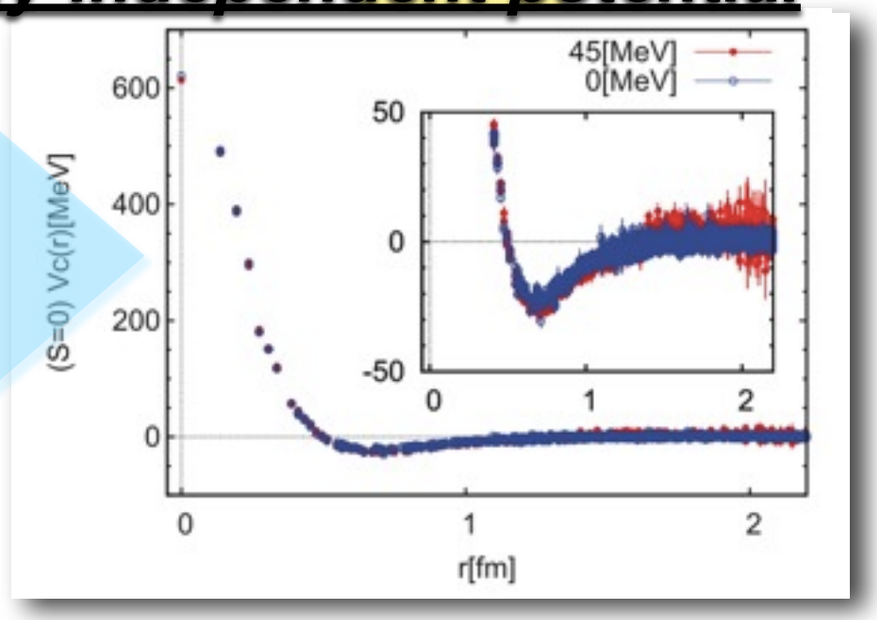
- ***Energy-independent potential***



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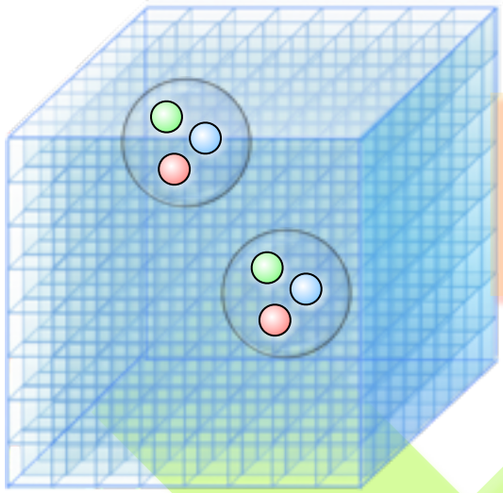

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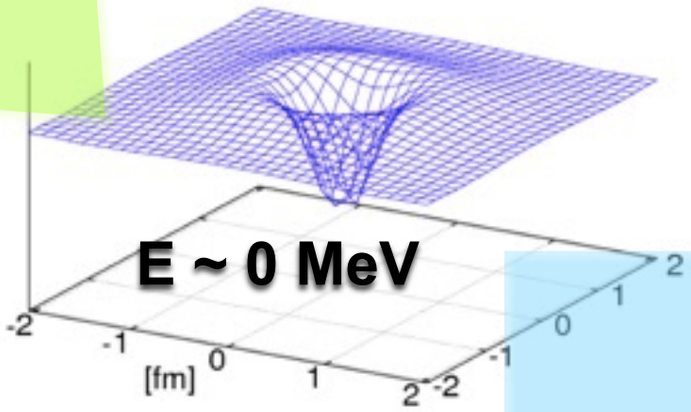

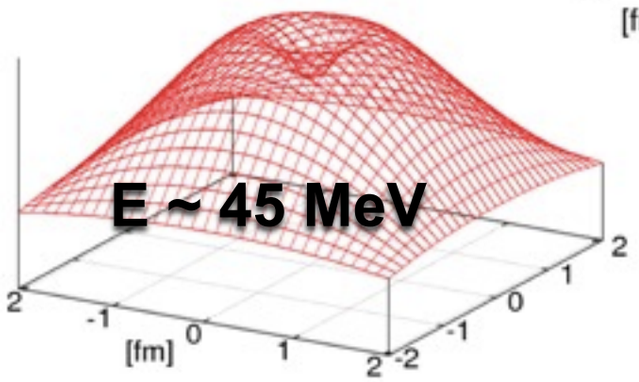

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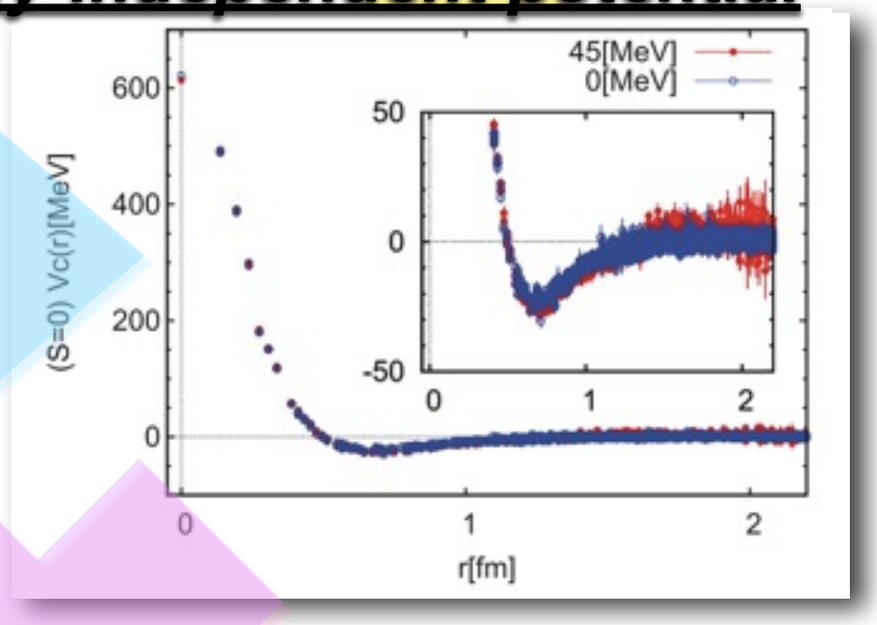


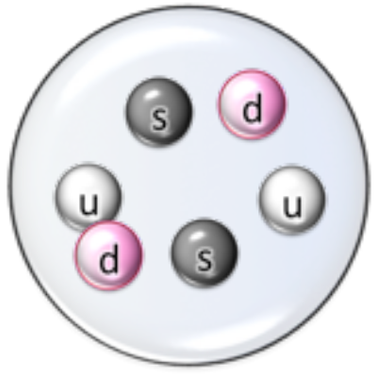
✓ **LQCD potentials can be applied to...**

Properties of hadrons & Nuclei, construction of EOS, etc.

Topics covered : Exotic candidates

Possible candidates of multi-quark hadrons in quark models

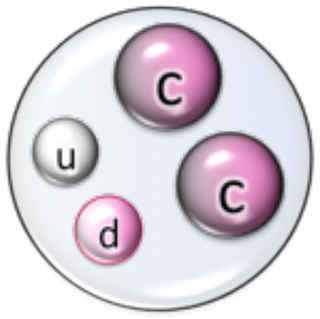
1) H-dibaryon : $I(J^P)=0(0^+)$



✓ No Pauli blocking in flavor singlet baryon-baryon channels
✓ Attractive color magnetic forces

R. L. Jaffe, PRL38 (1977).

2) Doubly charmed tetraquark (Tcc) : $I(J^P) = 0(1^+)$



✓ Attractive color magnetic forces
✓ "Good diquark" picture?

H. J. Lipkin, PLB172 (1986).

S. Zouzou et al., Z. Phys. C30 (1986).

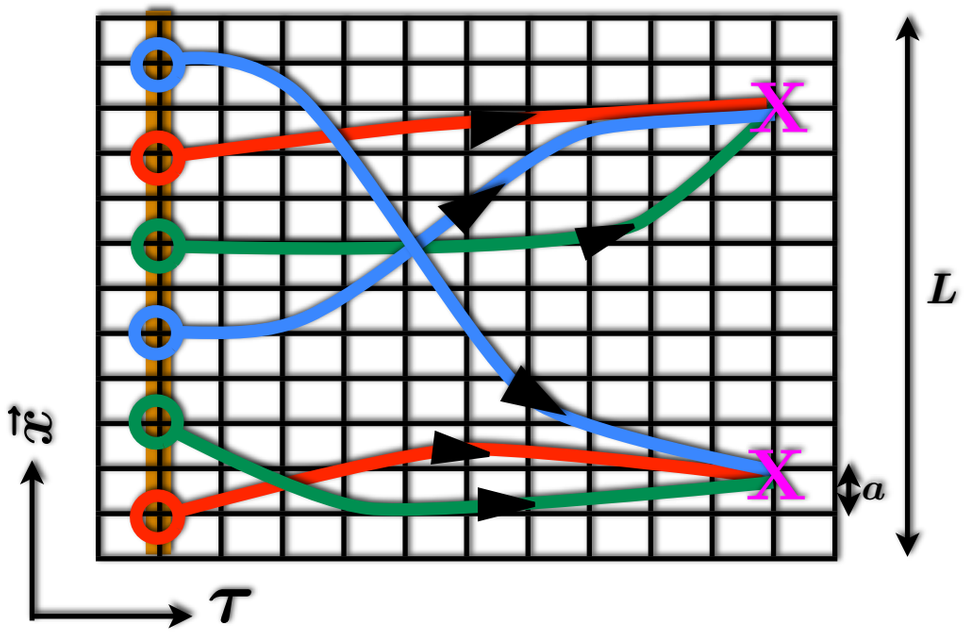
Predicted B.E. and structures highly depends on model parameters
--> Lattice QCD study of hadron interactions is performed

Scattering on the lattice

Key quantity : Equal-time Nambu-Bethe-Salpeter (NBS) wavefunction

$$\psi(\vec{r},\tau) = \sum_{\vec{x}} \langle 0|\phi_1(\vec{x}+\vec{r},\tau)\phi_2(\vec{x},\tau)\mathcal{J}^\dagger(\tau=0)|0\rangle$$

$$= \sum_{W(\vec{k})} A_{W(\vec{k})} \exp\left[-W(\vec{k})\tau\right] \psi_{W(\vec{k})}(\vec{r}) \qquad \psi_{W(\vec{k})}(\vec{r}) \equiv \sum_{\vec{x}} \langle 0|\phi_1(\vec{x}+\vec{r})\phi_2(\vec{x})|W(\vec{k}),B,J^P\rangle$$



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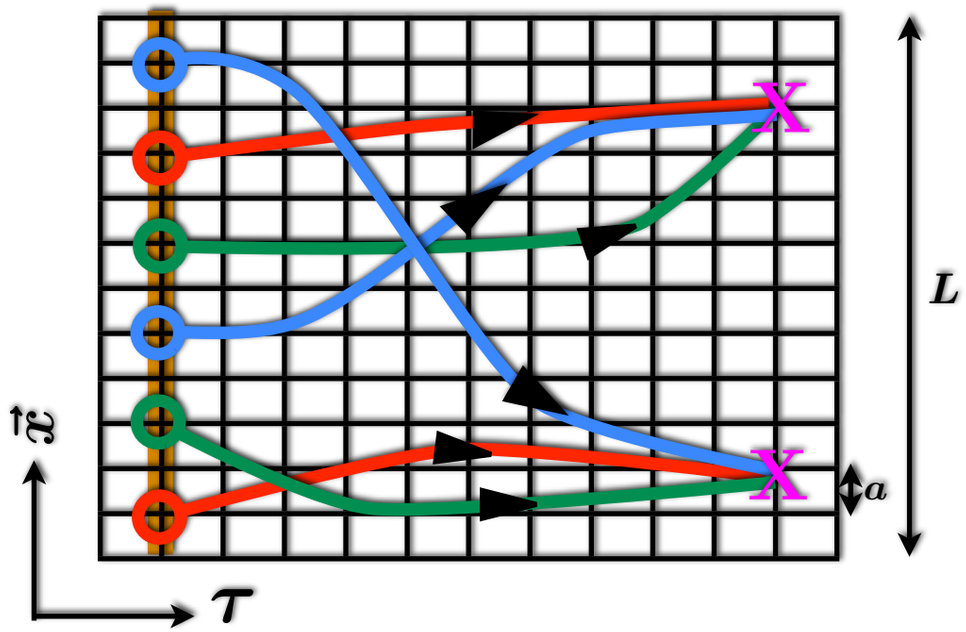

- **Helmholtz eq. of NBS wave func.**

$$(\nabla^2 + \vec{k}^2)\psi_{W(\vec{k})}(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

$$\psi^{(l)}_{W(\vec{k})}(r) \sim \frac{e^{i\delta_l(k)}}{kr} \sin\left(kr + \delta_l(k) - l\pi/2\right)$$

- **NBS wave func. in QFT ~ wave func. in Q.M.**

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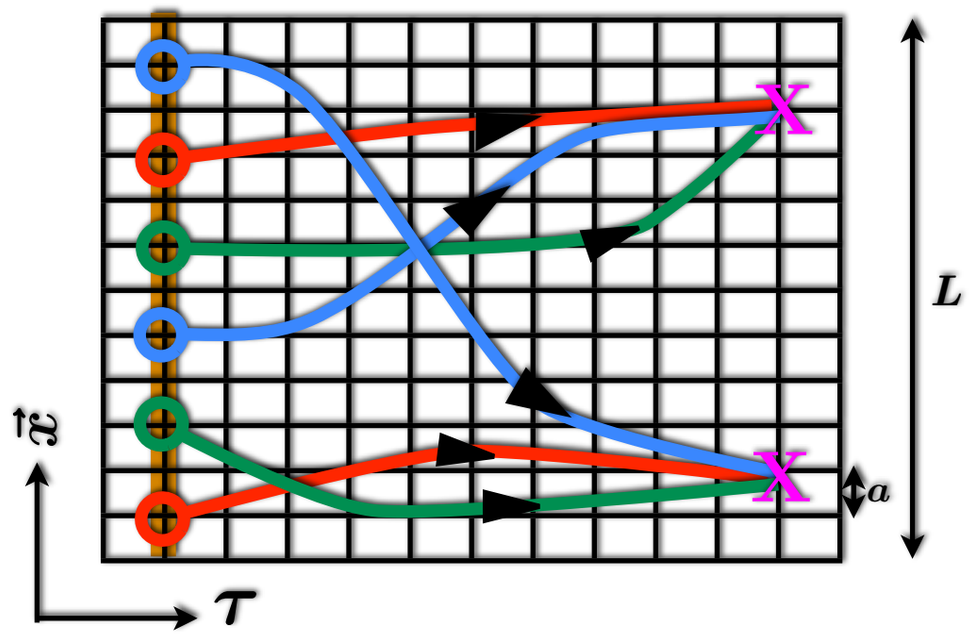

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- **NBS wave func. in QFT ~ wave func. in Q.M.**

- **Temporal correlation, W(k) : phase shift (Lüscher's formula)** M. Lüscher, NPB354, 531 (1991).
- **Spacial correlation, ψ(r) : potential --> observable** CP-PACS Coll., PRD71, 094504(2005). Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).

Lattice QCD potential -- HAL QCD --

Full details, see, Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

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Half off-shell T-matrix in interacting region:

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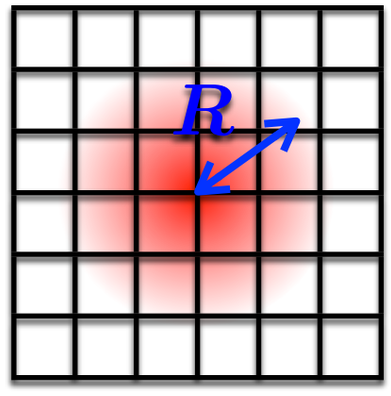

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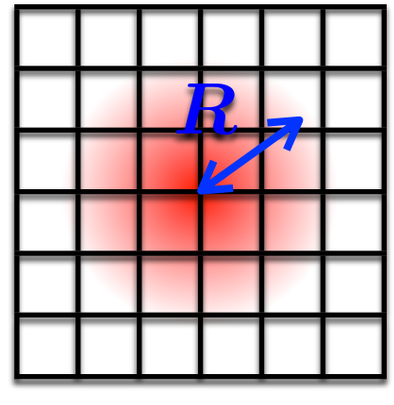
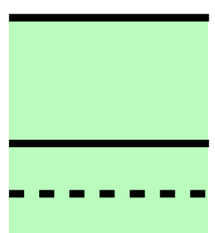


Energy-independent potential faithful to phase shift

$$U(\vec{r}, \vec{r}') = \int^{W_{\rm th}} \frac{dW}{2\pi}\, \mathcal{K}_W(\vec{r})\psi_W^*(\vec{r}')$$



inelastic: $W_{th}=MB_1B_2$

elastic: B_1+B_2

above W_{th}: coupled-channel analysis

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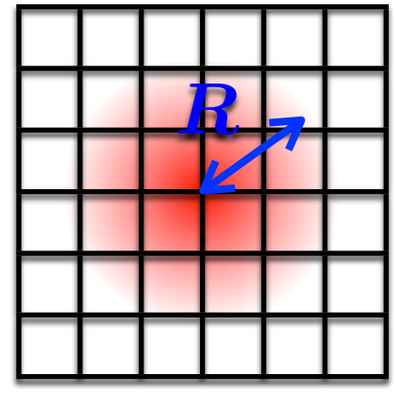

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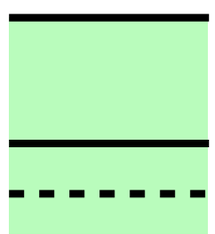

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Such potentials satisfy Schrödinger-type equations :

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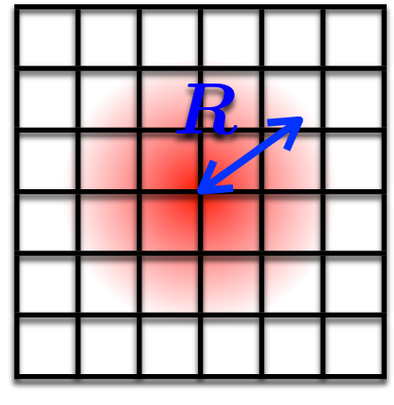

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Energy-independent potential faithful to phase shift

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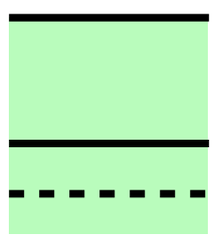


inelastic: $W_{th}=MB_1B_2$

elastic: B_1+B_2

above W_{th}: coupled-channel analysis

Such potentials satisfy Schrödinger-type equations :

$$(\nabla^2 + \vec{k}^2)\psi_{W(\vec{k})}(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}')\psi_{W(\vec{k})}(\vec{r}')$$

Technical thing to be solved for multi-hadron systems :

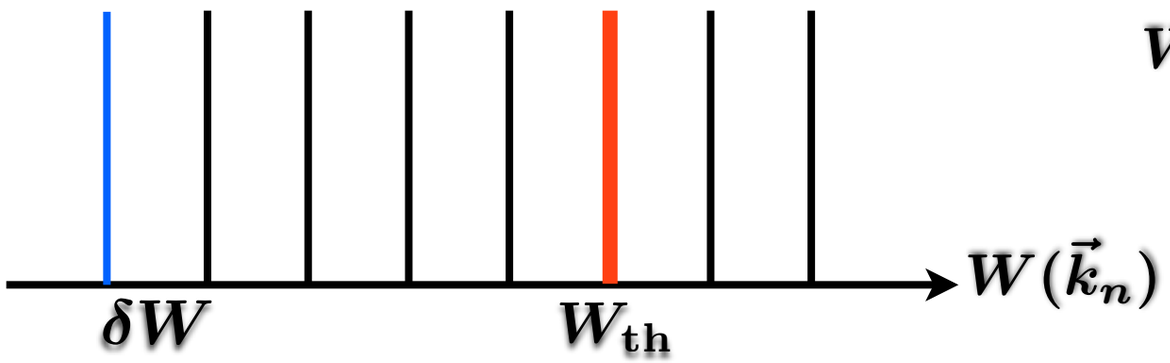
identifying single-energy states in simulations is very tough

Multi-hadron systems : challenge

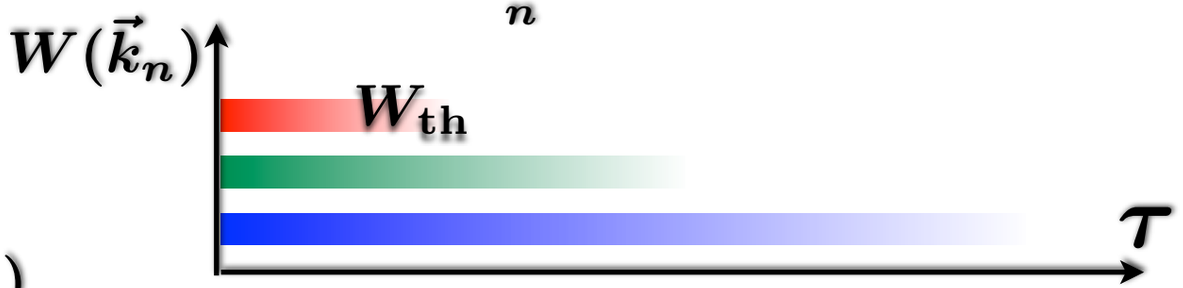
Why is it so hard to extract single-state in multi-hadron simulations?

➡ **momentum excitations of systems**

$$\vec{k}_n = \frac{2\pi\vec{n}}{L} \qquad \delta W = \frac{4\pi^2}{\mu L^2} + \cdots$$



$$\langle O(\tau)\rangle = \sum_n A_n \exp(-W(\vec{k}_n)\tau)$$



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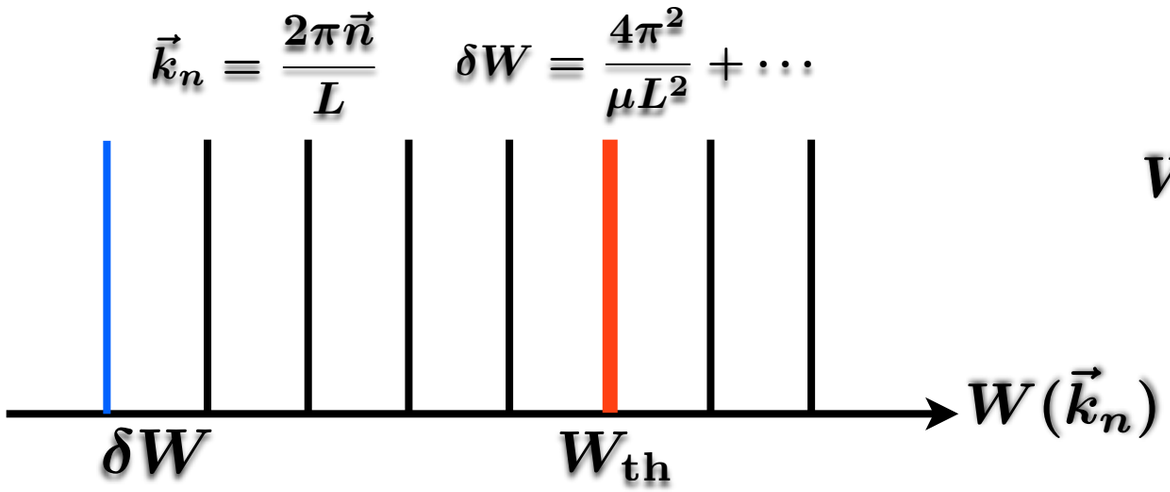

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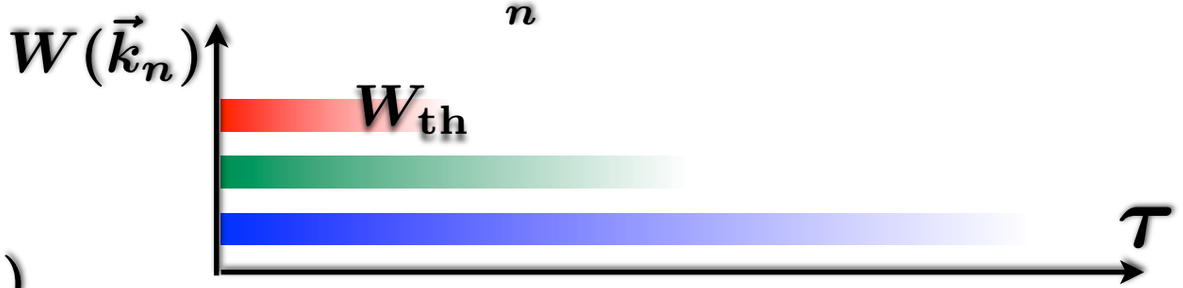
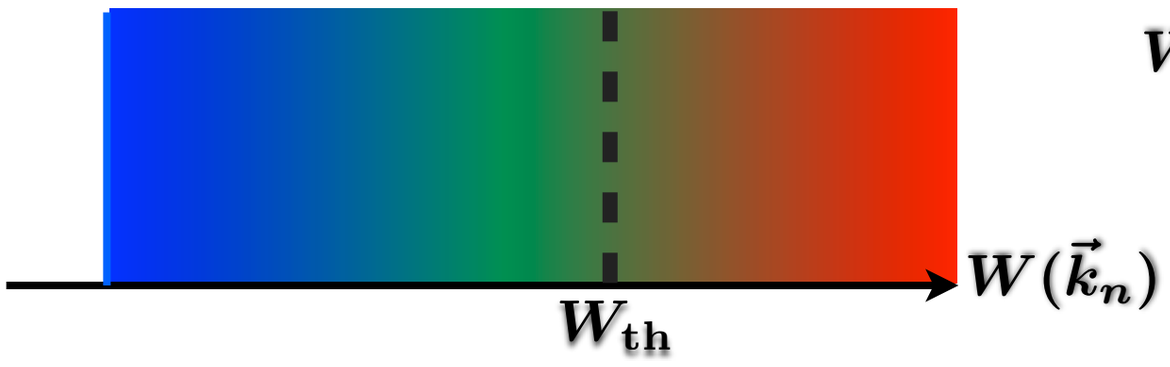
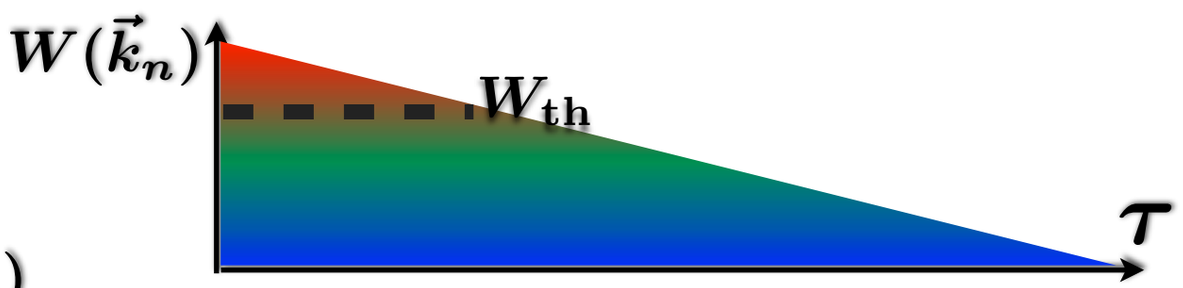


- **Large L limit** : **dense spectrum** --> **momentum excited state contaminations**





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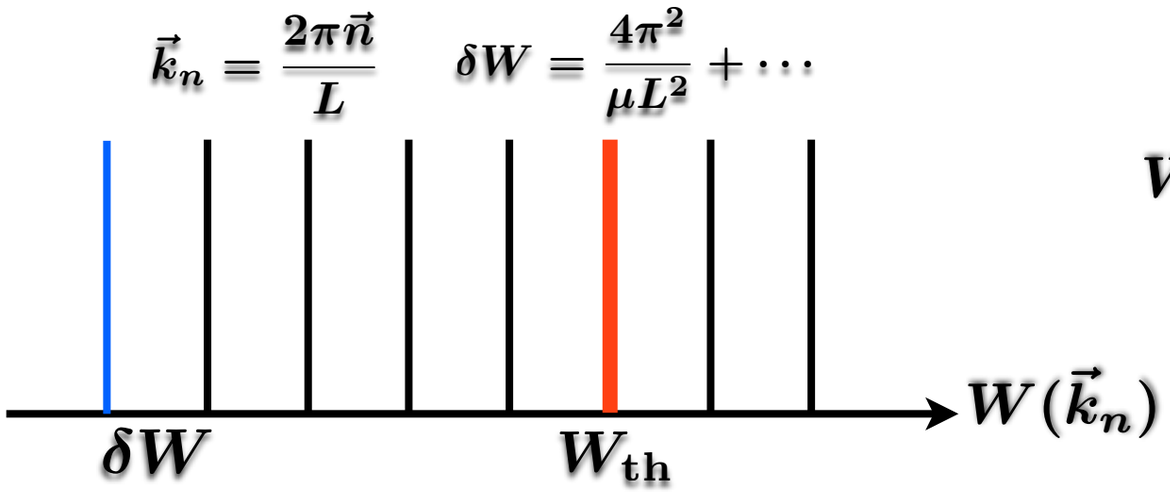

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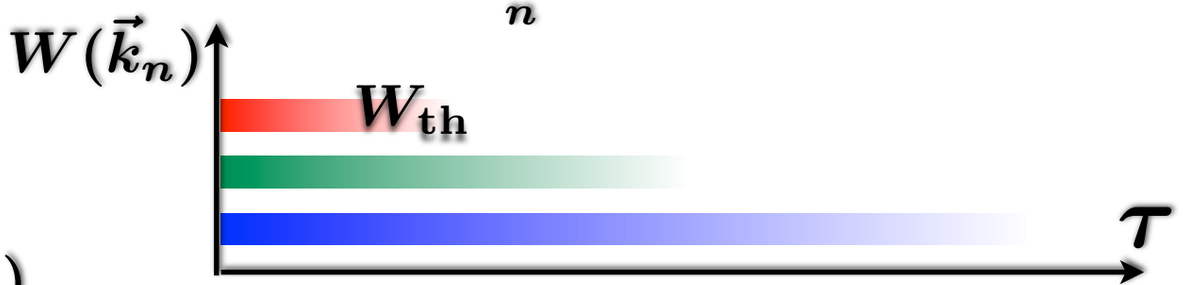

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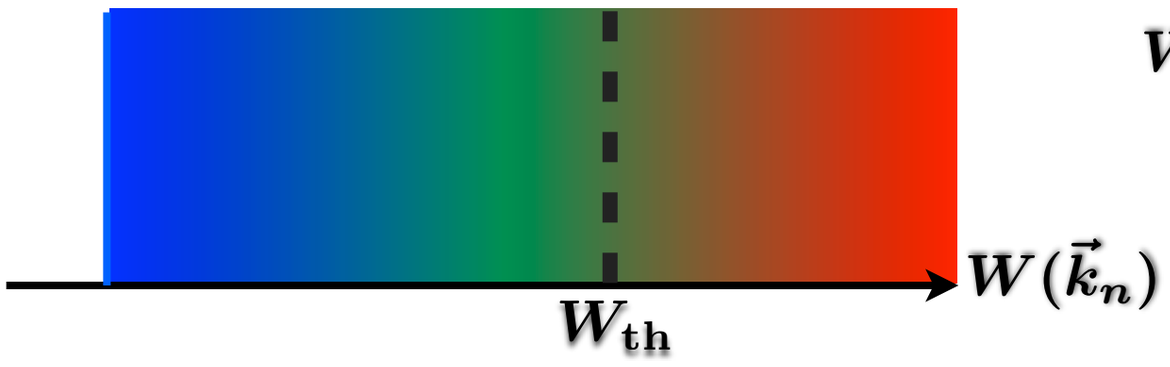
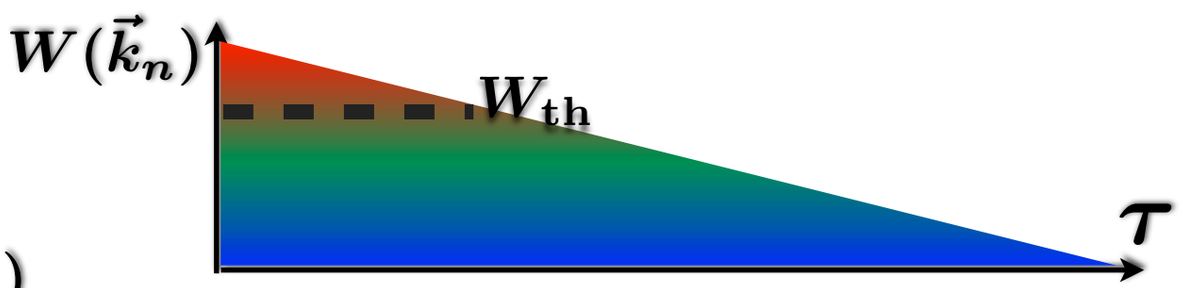
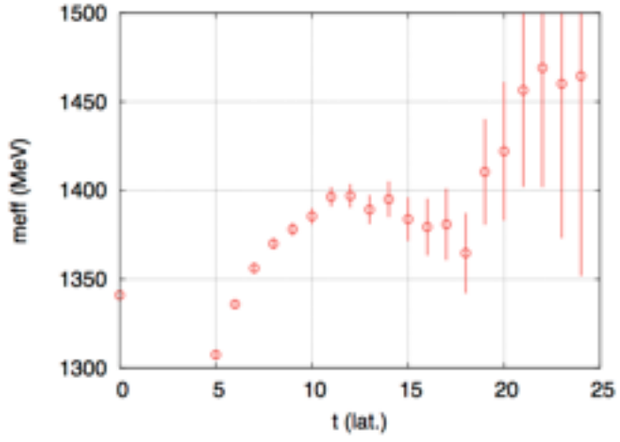




- **Signal becomes rapidly bad for multi-hadrons :**
  - ✓ **pion :** $S/N \sim \text{const.}$
  - ✓ **nucleon :** $S/N \sim \exp[-A(m_N - 3/2m_\pi)\tau]$



Solution = energy-independent kernel

✓ **Definition of energy-independent potential (below inelastic threshold):**

$$\psi_{W(\vec{k})}(\vec{r}) = \langle 0|\phi_1(\vec{r}+\vec{x})\phi_2(\vec{x})|W(\vec{k})\rangle$$

$$(\vec{k}^2 + \nabla^2)\psi_{W(\vec{k})}(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r},\vec{r}')\psi_{W(\vec{k})}(\vec{r}')$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

$$W(\vec{k}) = \sqrt{m_1^2+\vec{k}^2} + \sqrt{m_2^2+\vec{k}^2}$$

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✓ **Extract energy-independent potential from time-dependent Schrödinger-type eq.**

Ishii et al.(HAL QCD), PLB712, 437(2012).

$$R(\vec{r},\tau) \equiv \psi(\vec{r},\tau)e^{(m_1+m_2)\tau} \quad (\tau > \tau_{\rm th})$$

$$\left[-\partial_\tau + \nabla^2/2\mu + \partial_\tau^2/8\mu + \mathcal{O}(\delta^2)\right]R(\vec{r},\tau) = \int d\vec{r}' U(\vec{r},\vec{r}')R(\vec{r}',\tau)$$

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✓ **Velocity expansion:**

$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

$$\Rightarrow V(\vec{r},\nabla) = \underbrace{V_C(\vec{r}) + \sigma_1\cdot\sigma_2 V_\sigma(\vec{r}) + S_{12}V_T(\vec{r})}_{\text{(LO)}} + \underbrace{\vec{L}\cdot\vec{S}V_{LS}(\vec{r})}_{\text{(NLO)}} + \mathcal{O}(\nabla^2)$$

✓ **Calculate observable: phase shift, binding energy, ...**

Advantage: We can obtain potentials w/o identifying single-energy state

NN potential from HAL QCD method

Central force of singlet NN system

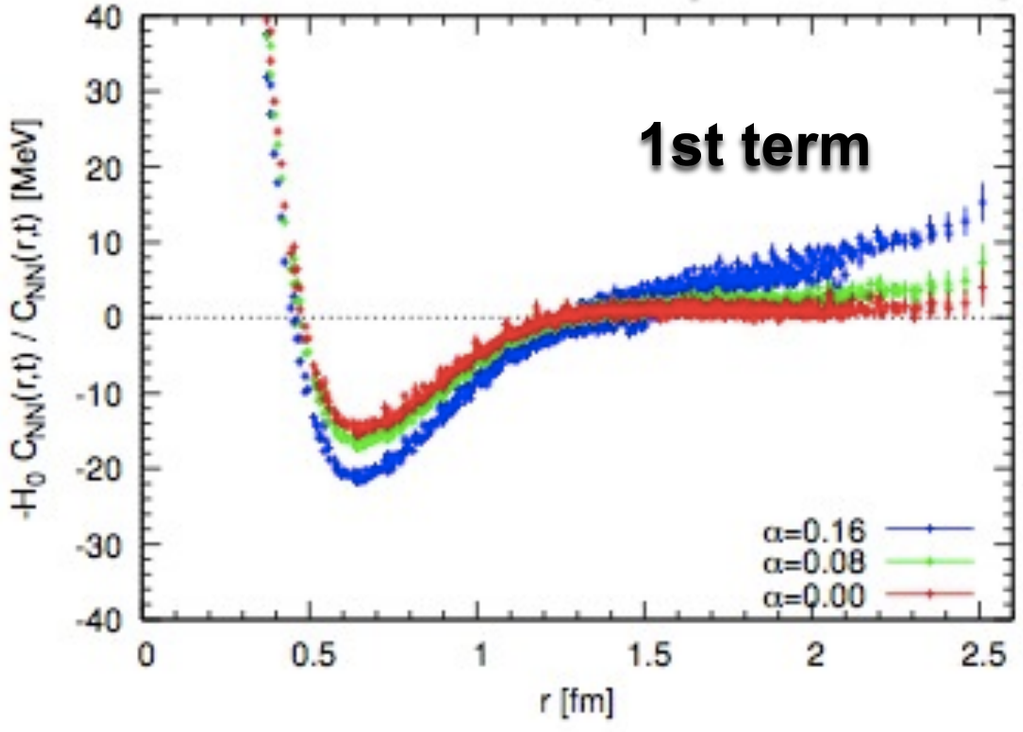
Ishii et al.(HAL QCD), PLB712, 437(2012).

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t) + \frac{1}{4M_N}\frac{(\partial/\partial t)^2 R(\vec{r},t)}{R(\vec{r},t)}$$

Examine source operator dependence: possible contaminations are different

$$f(x,y,z) = 1 + \alpha\big(\cos(2\pi x/L) + \cos(2\pi y/L) + \cos(2\pi z/L)\big)$$

❖ **N_f=2+1 full QCD : m_π~0.7GeV, L=3 fm**



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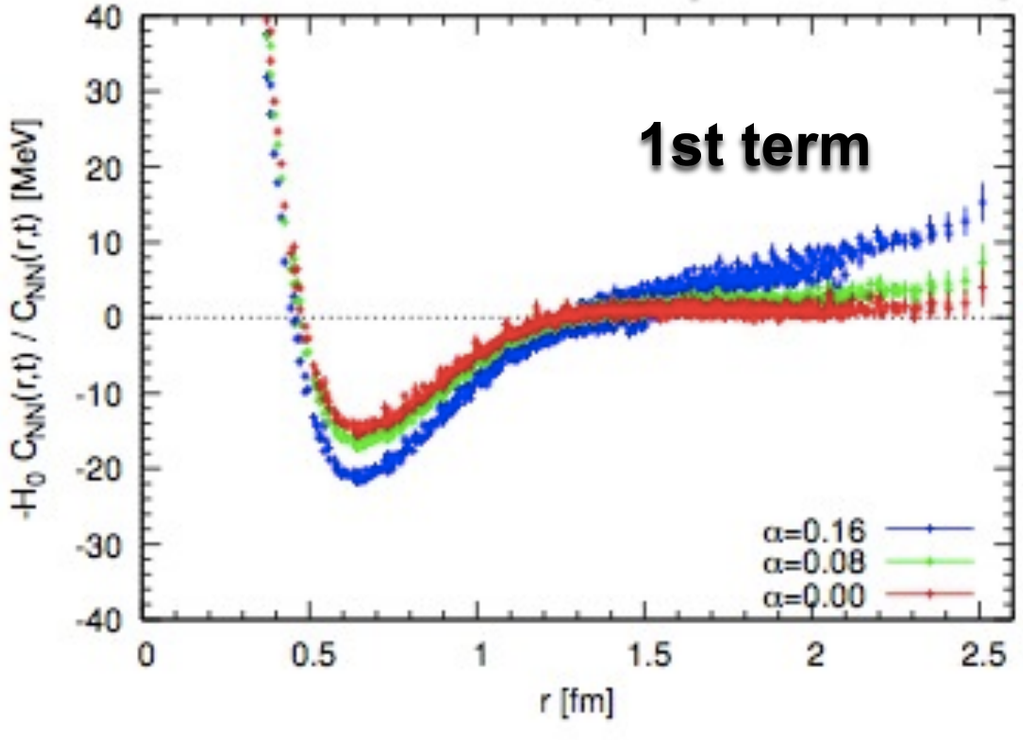
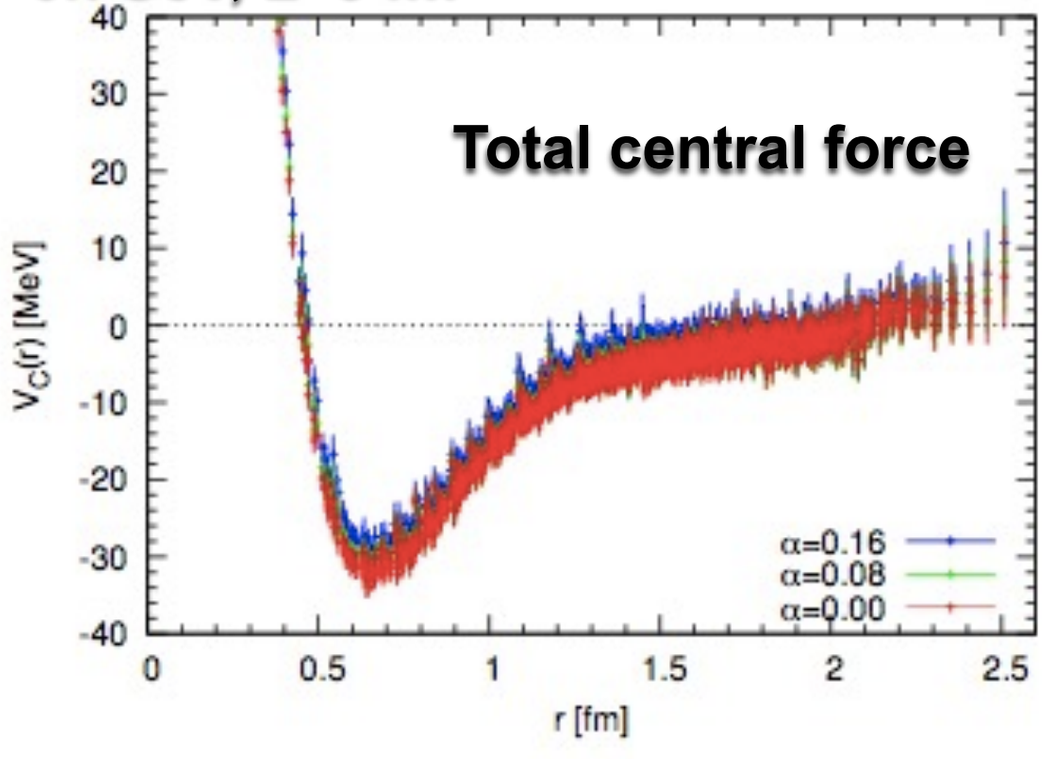
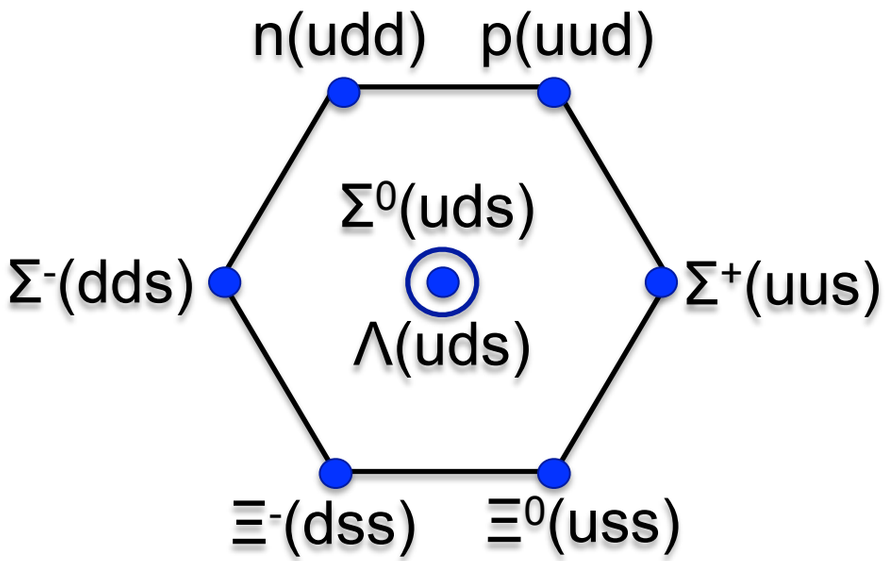




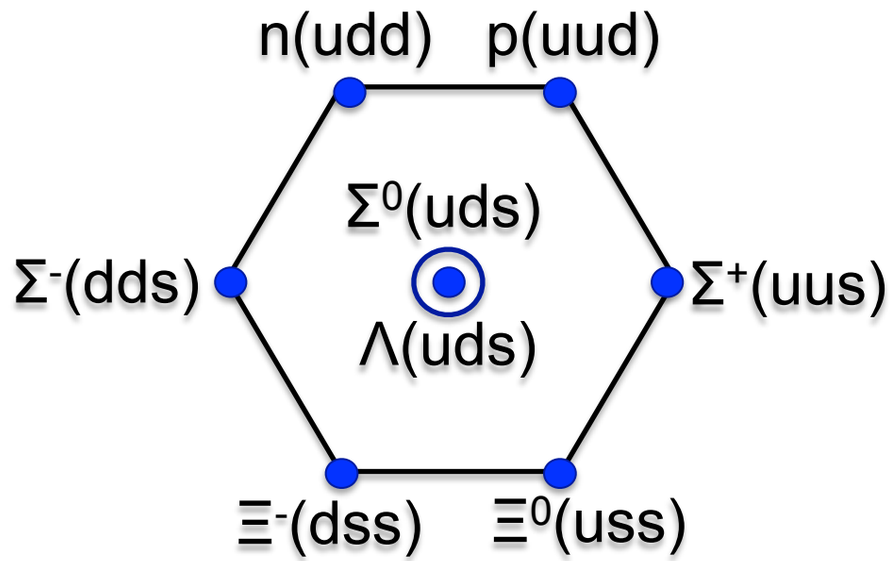
Source operator dependence disappears

Single-energy-state saturation is not a question!!

Generalized BB force : fate of H-dibaryon



X

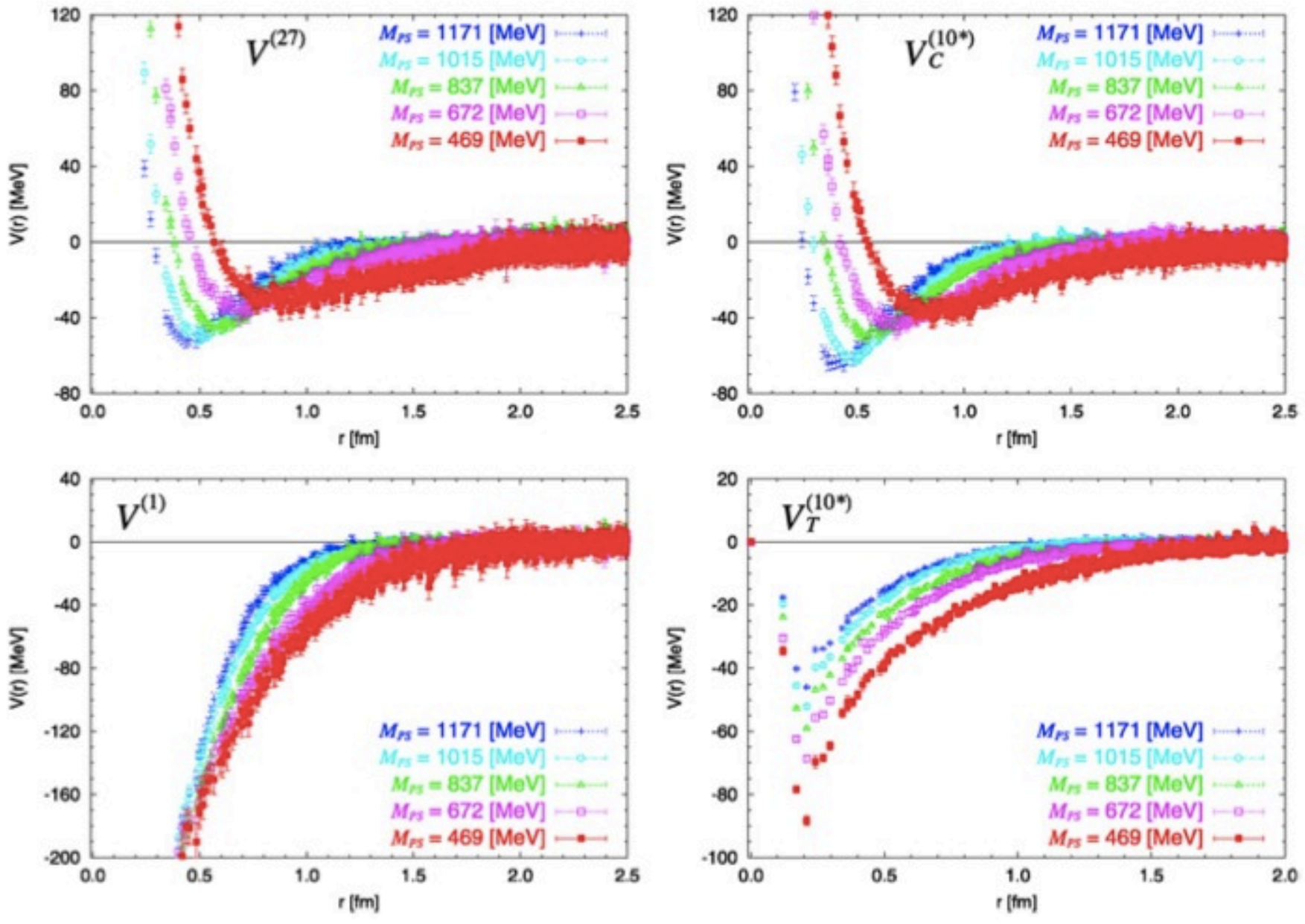


$$= 27 + 8 + 1 + \overline{10} + 10 + \overline{8}$$

Generalized BB potentials in $SU(3)_F$ limit

- **Full QCD [$SU(3)_F$] : m_π~0.47-1.17GeV, L=3.9 fm**

Inoue et al. (HAL QCD), PRL106 (2011), NPA881 (2012).



27-plet: NN ($^1S_0$), 10*-plet: NN ($^3S_0$), 1-plet: H-dibaryon channel

No Pauli-blocking, attractive color-magnetic force

Structure of bound H-dibaryon

✓ **Definition of LQCD potentials**

$$\psi_{W(\vec{k})}(\vec{r}) = \langle 0|\phi_1(\vec{r}+\vec{x})\phi_2(\vec{x})|W(\vec{k})\rangle$$

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✓ **Solve Schrödinger equation**

- **binding energy**
- **mean-square radius**

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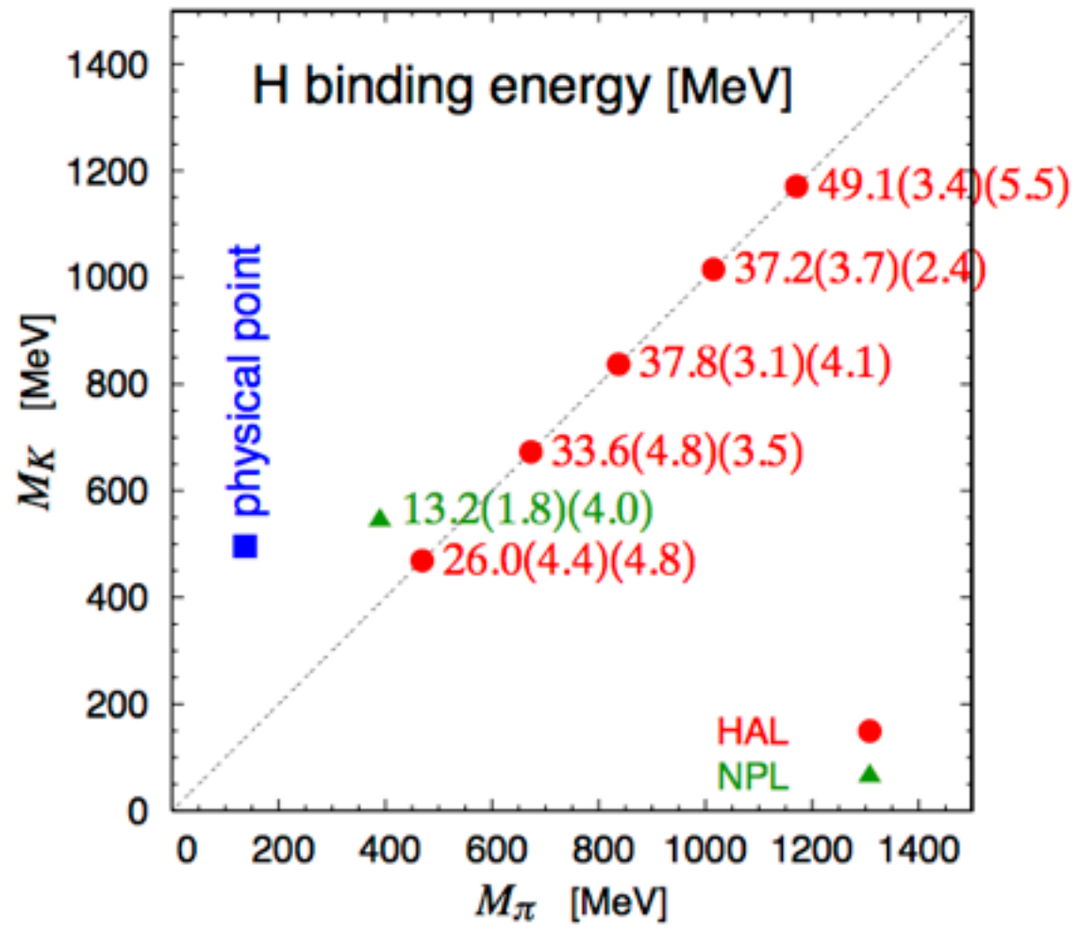
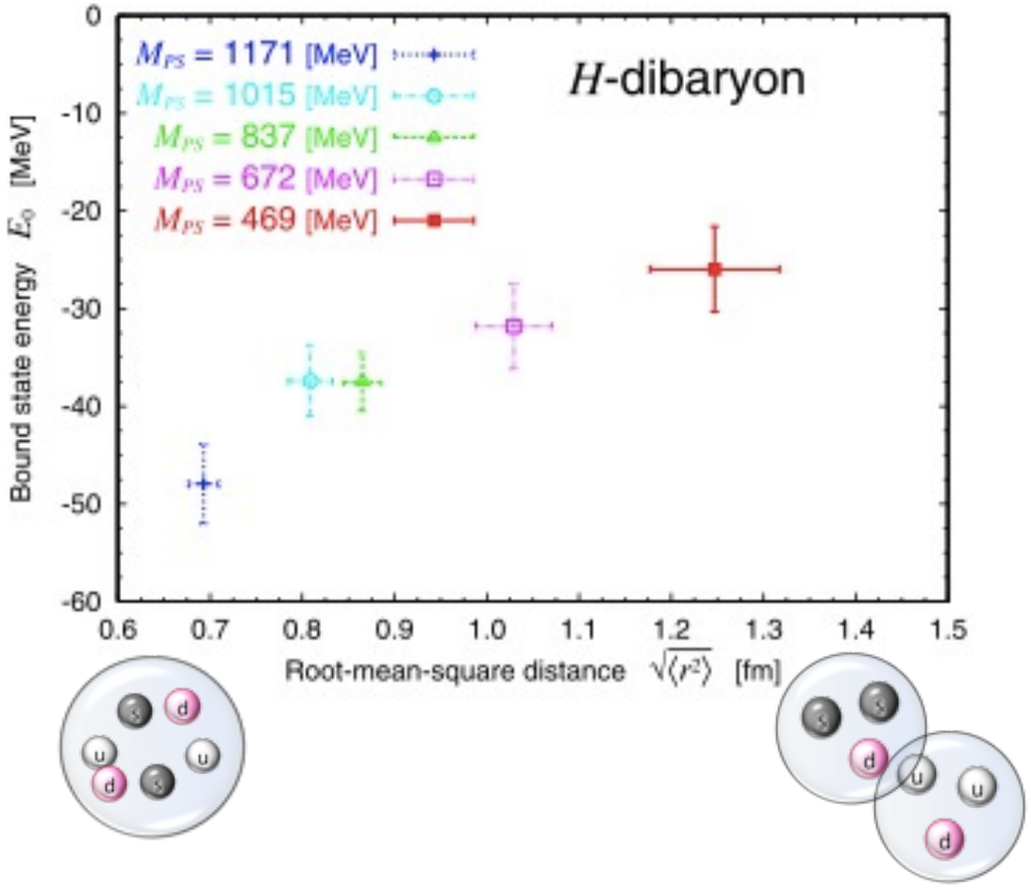


see also, Beane et al. (NPLQCD), PRL106 (2011).



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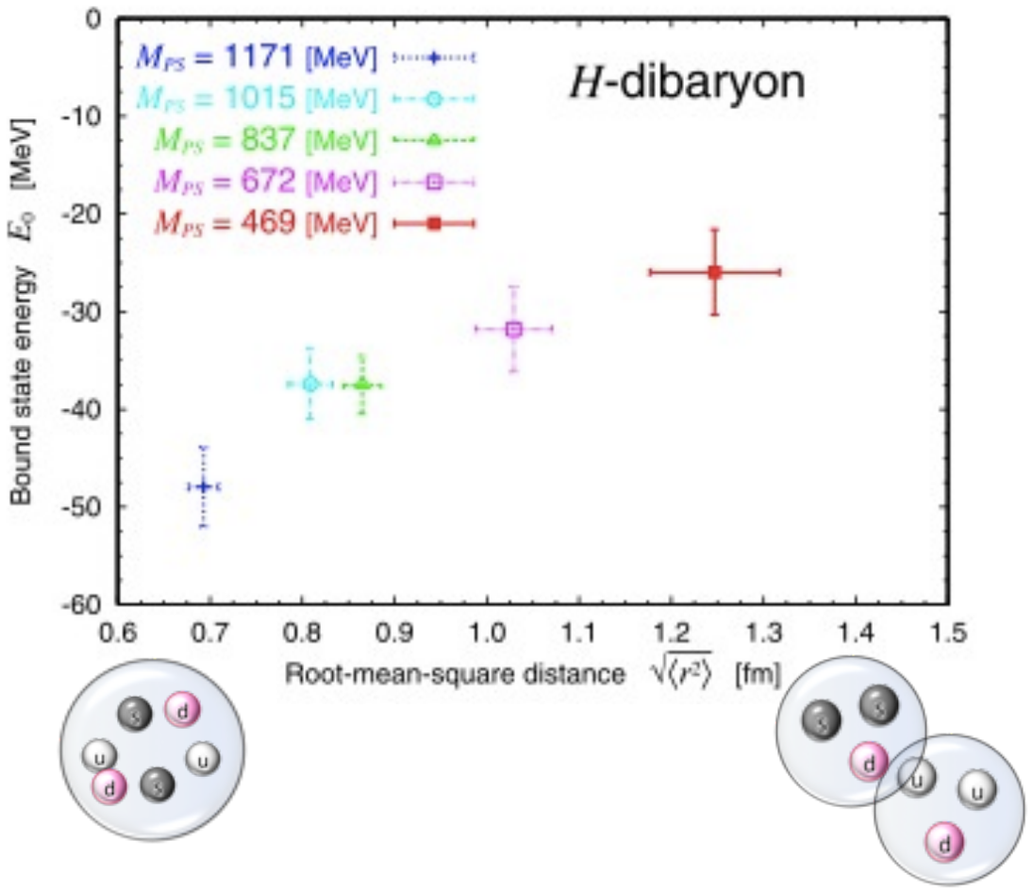

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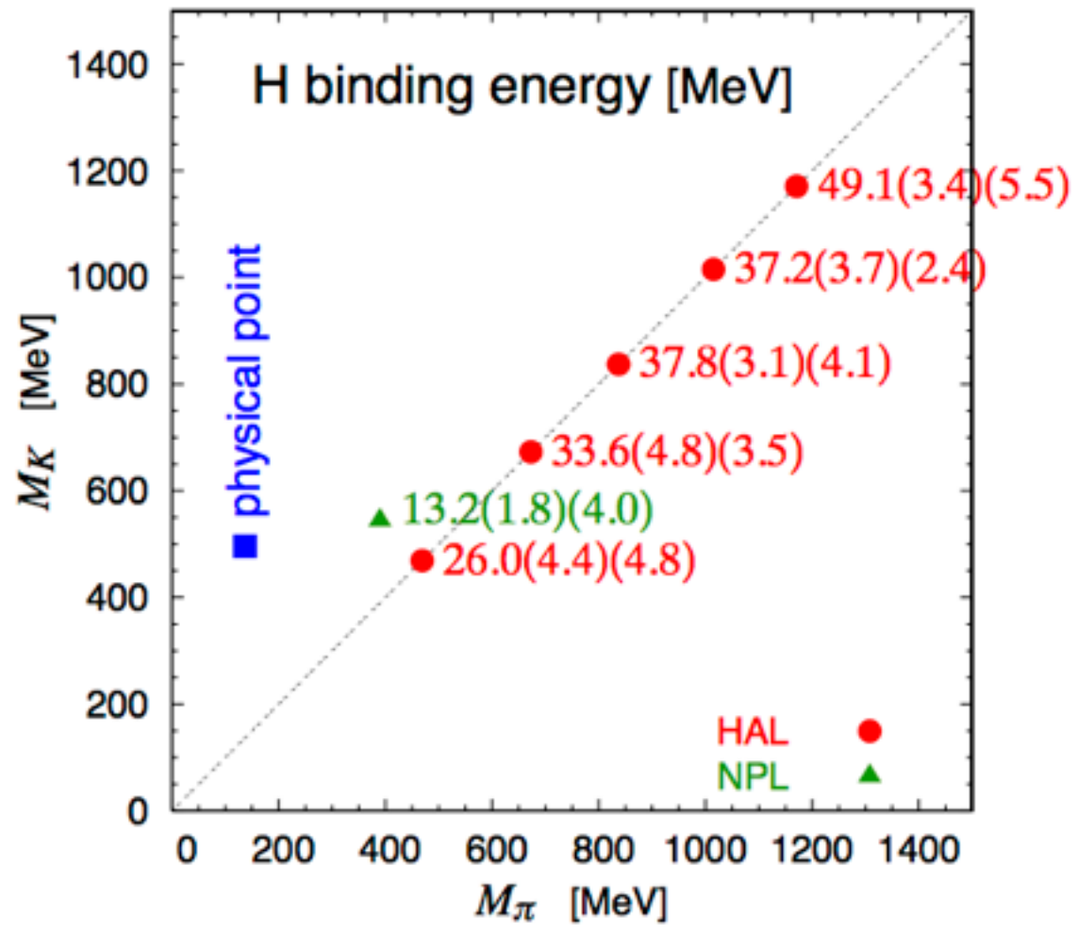


see also, Beane et al. (NPLQCD), PRL106 (2011).

✓ **Fate of H-dibaryon?**

- **Coupled-channel HAL QCD approach**

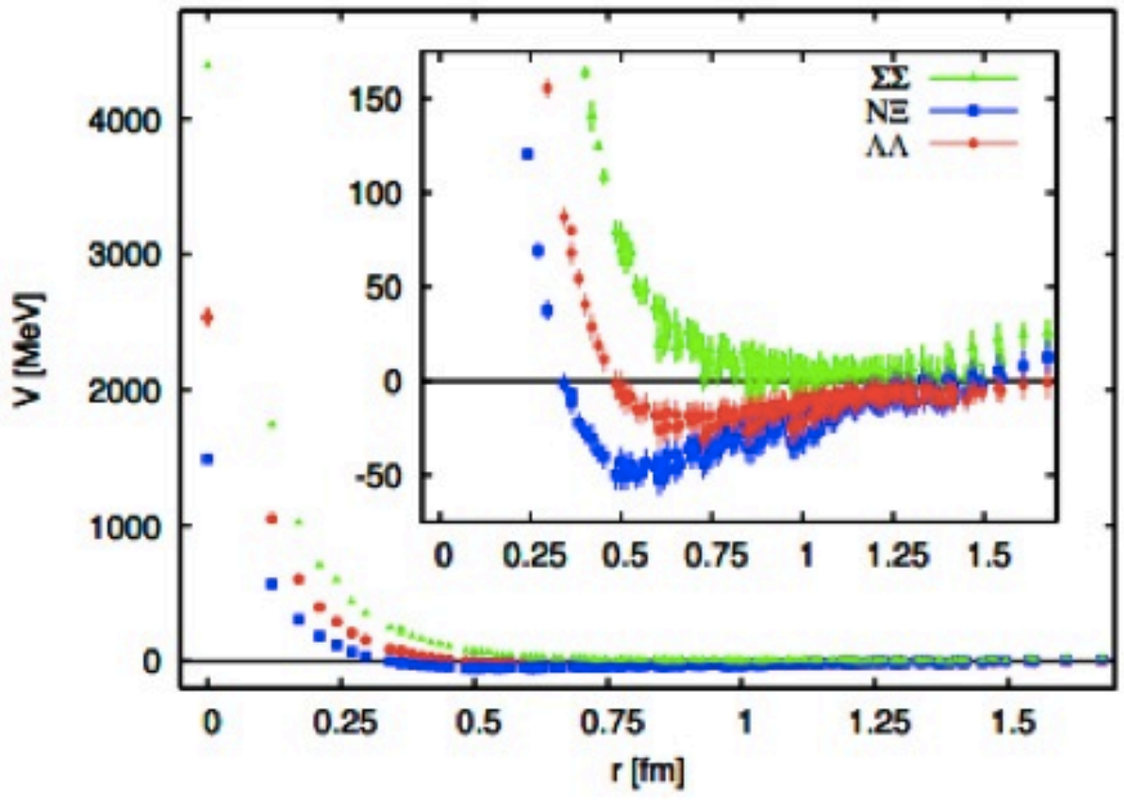
Aoki et al. (HAL QCD), Proc. Jpn. Acad., Ser. B, 87 (2011).
PTEP 2012, 01A105 (2012).

Coupled-channel hyperon potentials

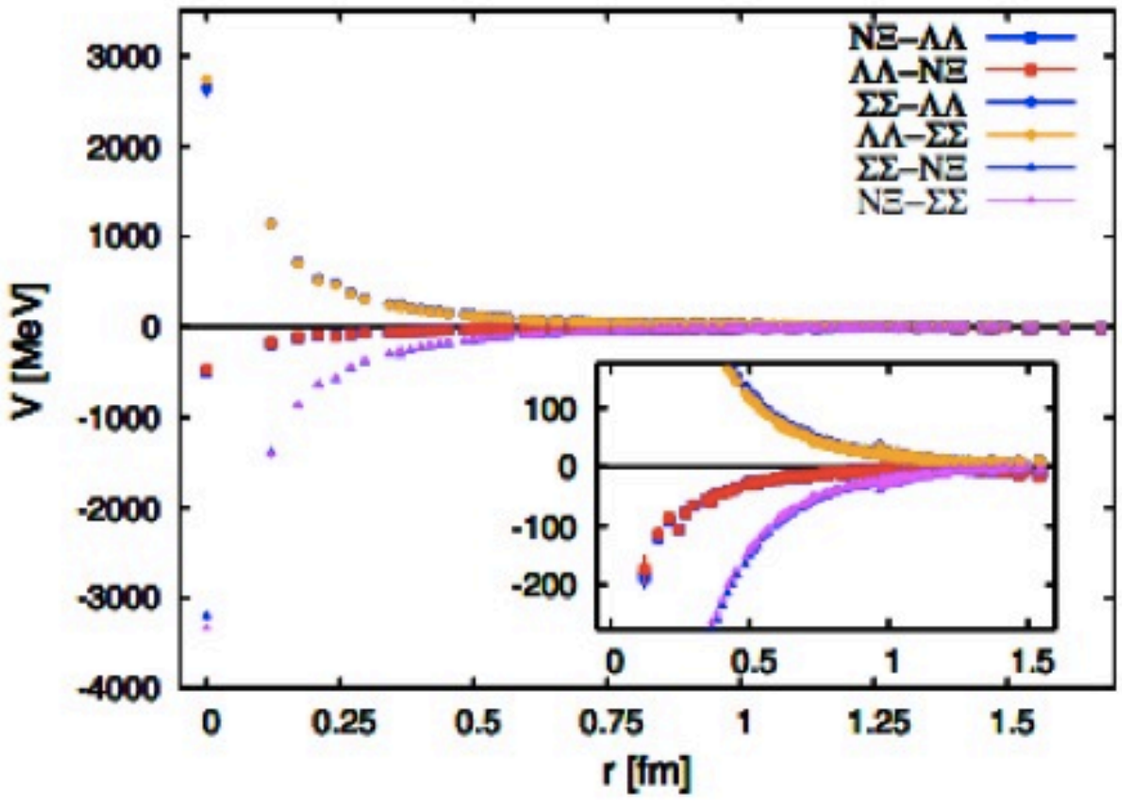
- **N_f=2+1 full QCD @m_π~0.87GeV, L=2 fm** Sasaki, (HAL QCD), PTEP 2012, 01A105 (2012).

$$\begin{pmatrix} \langle \Lambda\Lambda| \\ \langle \Sigma\Sigma| \\ \langle N\Xi| \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{27}{40}} & -\sqrt{\frac{8}{40}} & -\sqrt{\frac{5}{40}} \\ -\sqrt{\frac{1}{40}} & -\sqrt{\frac{24}{40}} & \sqrt{\frac{15}{40}} \\ \sqrt{\frac{12}{40}} & \sqrt{\frac{8}{40}} & \sqrt{\frac{20}{40}} \end{pmatrix} \begin{pmatrix} \langle \mathbf{27}| \\ \langle \mathbf{8}_s| \\ \langle \mathbf{1}| \end{pmatrix}$$

diagonal part



off-diagonal part

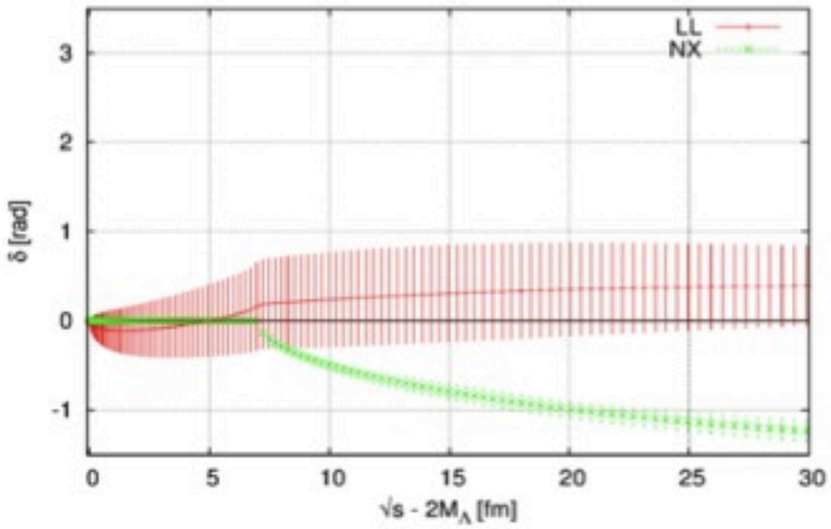


- **Coupled-channel calculation on the lattice is quite tough, but now is possible**
- **NΞ channel is the most attractive**
- **Large channel coupling effect**

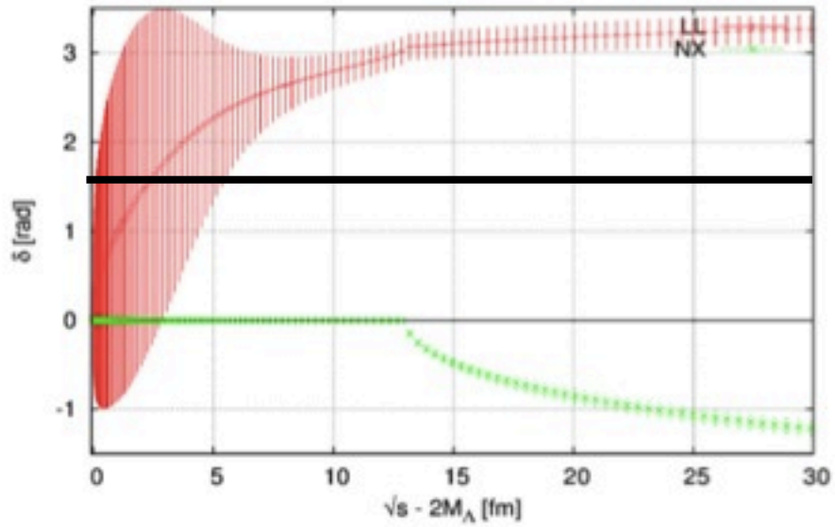
ΛΛ-NΞ phase shifts & H-dibaryon

❖ **Nf=2+1 full QCD @m_π~0.41-0.70GeV, L=2.9 fm**

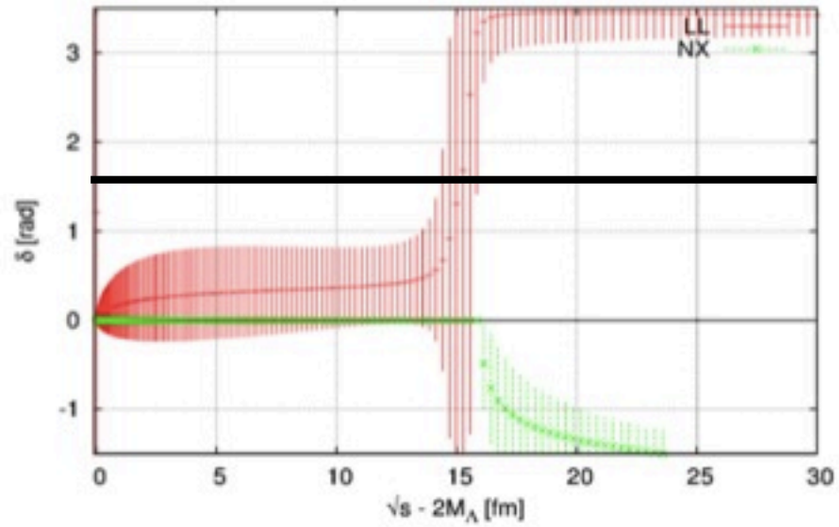
- **m_π = 700 MeV**



- **m_π = 570 MeV**



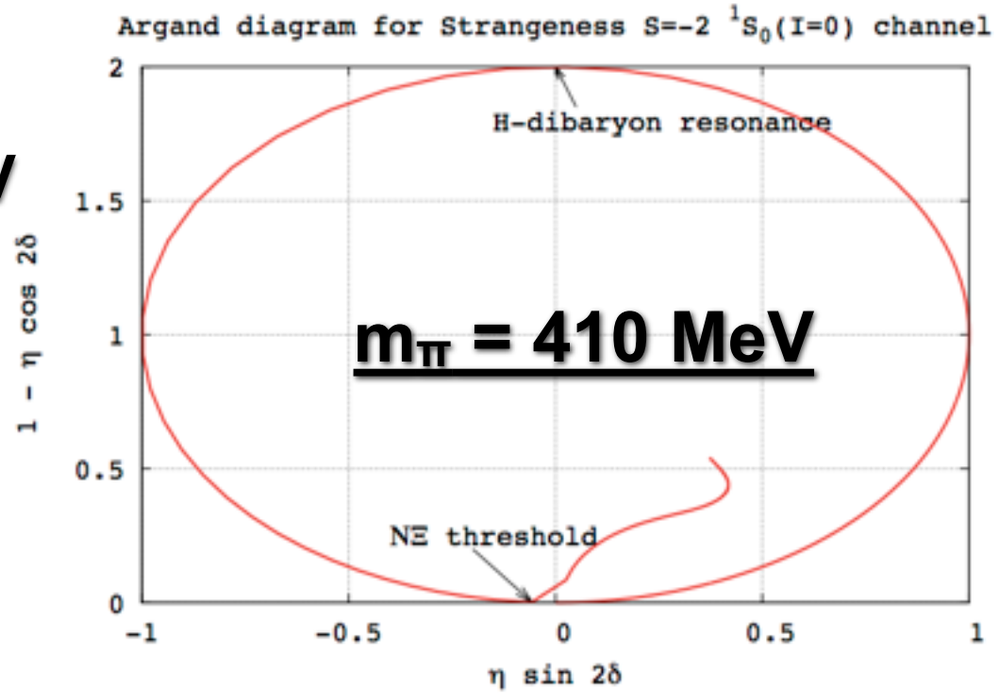
- **m_π = 410 MeV**



- **H-dibaryon is bound @m_π=700MeV**
- **H-dibaryon becomes resonance @m_π=410, 570MeV**

see also, extrapolation to physical point

Shanahan, Thomas, Young, arXiv:1308.1748 [nucl-th].



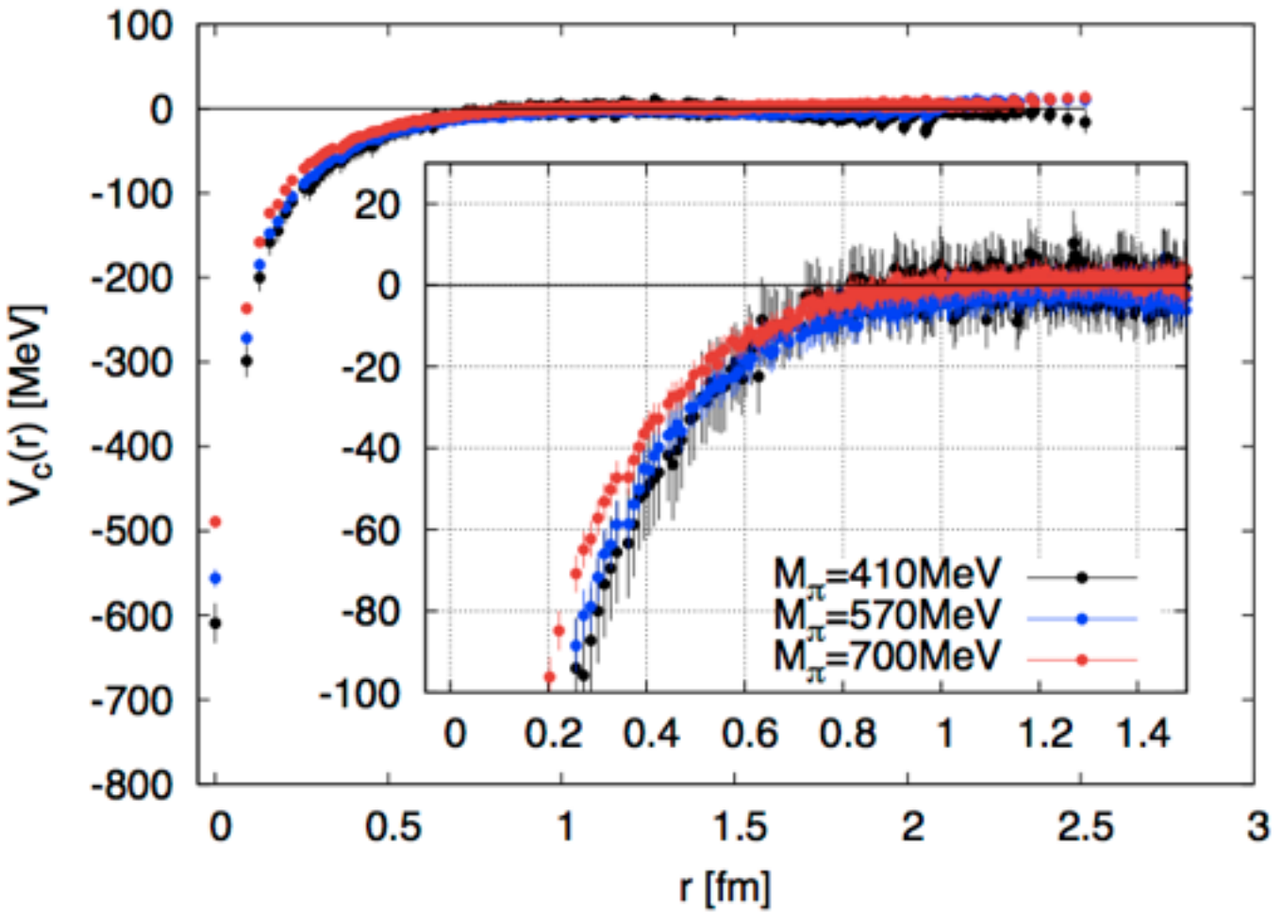
H-dibaryon is unlikely bound state...

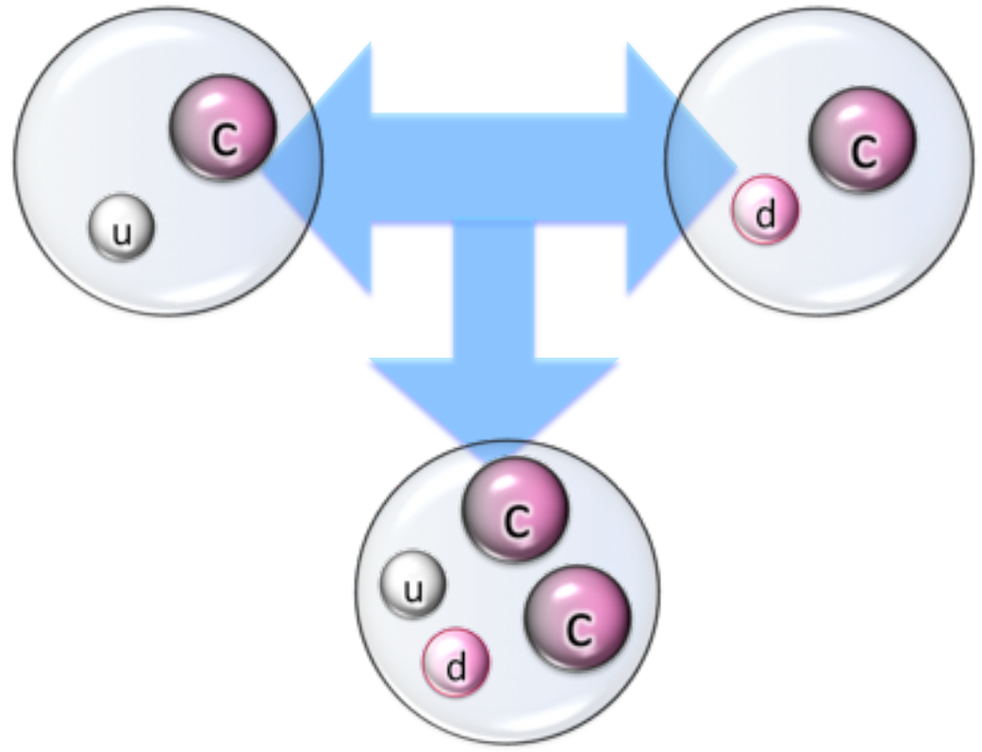
Tetraquark : Tcc in $I(J^P)=0(1^+)$

S-wave DD* potential : Tcc in I=0

- **Nf=2+1 full QCD@m_π~0.41-0.70GeV, L=2.9 fm**
  - **Relativistic heavy quark (RHQ) action is employed for charm quarks**

Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).



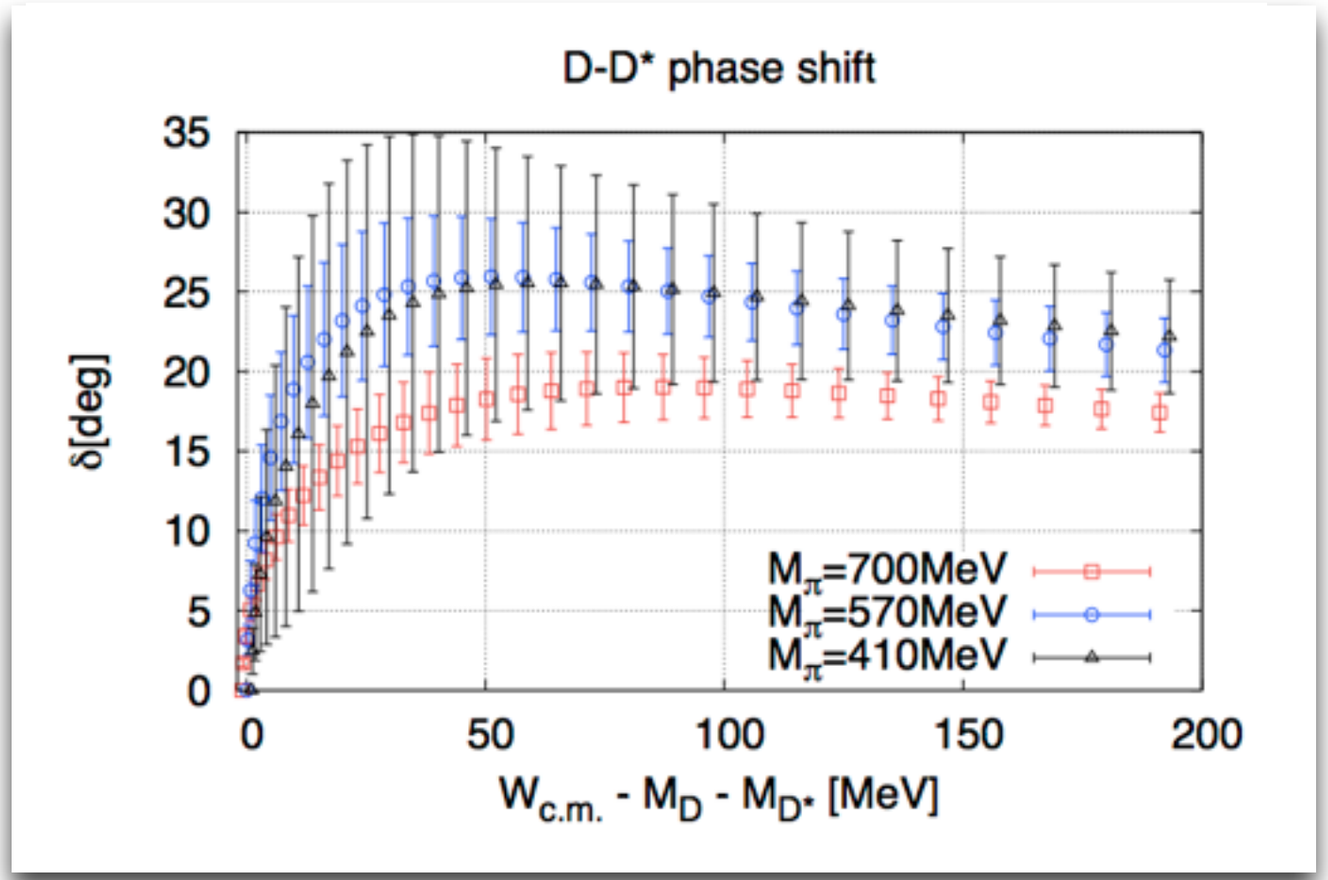


- **Entirely attractive S-wave potentials**
- **Small quark mass dependence**
- **Check whether bound Tcc exist or not --> phase shift analysis**

S-wave DD* phase shift : Tcc in I=0

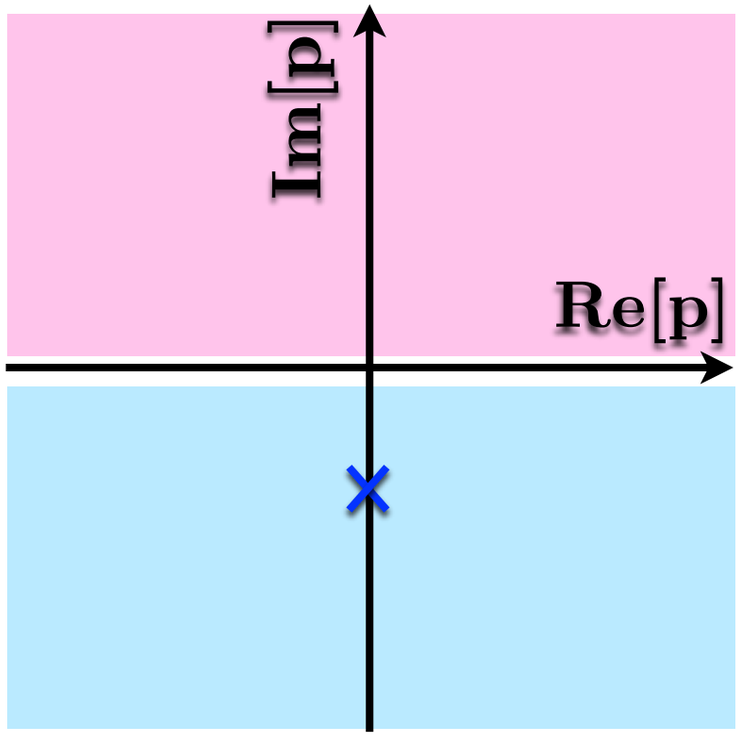
- **solve Schrödinger equation --> phase shift**

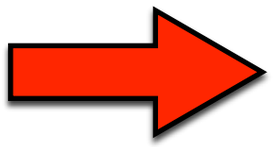
Y. Ikeda et al. (HAL QCD), PLB729, 85 (2014).

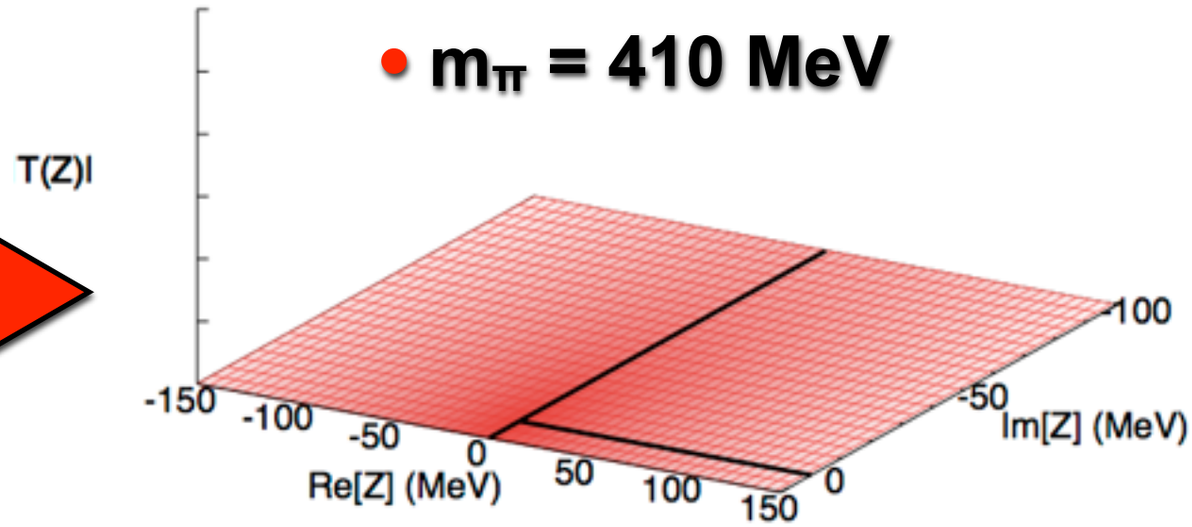


- **Attraction is not strong enough to generate bound state**
- **Rapid increase at threshold of DD* phase shift --> effect of virtual state?**
  - ➡ **perform analytic continuation of Lippmann-Schwinger equation**
  - ➡ **examine pole position**

I=0 DD* T-matrix on complex energy plane

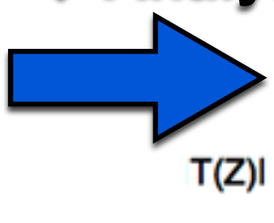


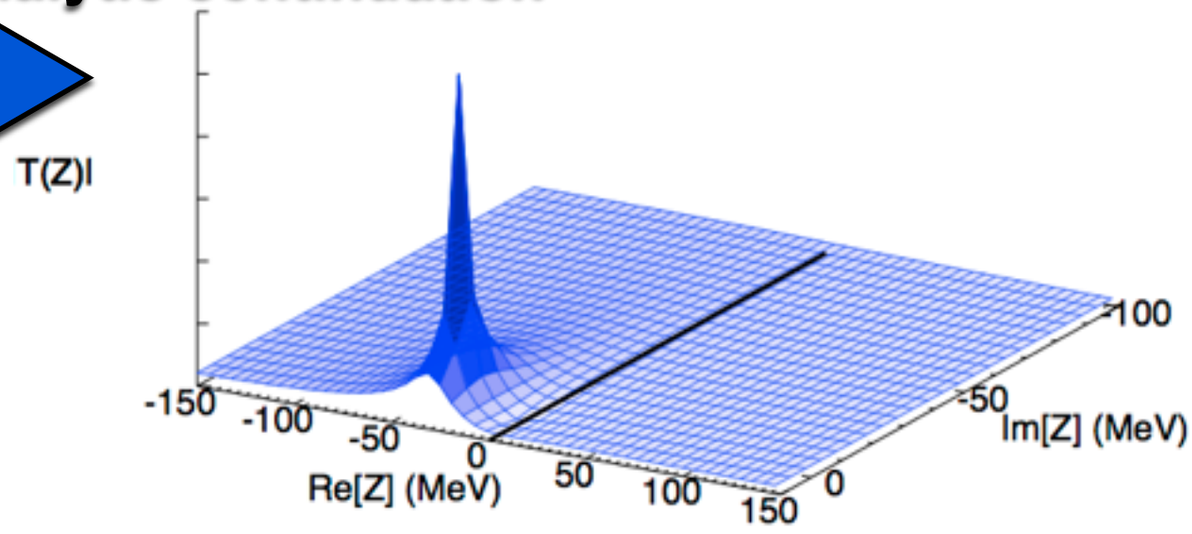




- m_π = 410 MeV

❖ Analytic continuation





- **Virtual** pole on the DD* unphysical energy plane
- Origin of rapid increase of scattering phase shift

Summary

- **Search for exotic hadrons on the lattice from HAL QCD method**
  - ➡ **Full QCD simulation (light quark)**
  - ➡ **Charm quarks: Relativistic Heavy Quark action**

✓ **Energy-independent potentials are derived from time-dependent Schrödinger equation of Nambu-Bethe-Salpeter wave functions**

✓ **Property of bound H-dibaryon @m_π=0.47-1.17GeV in SU(3)$_F$ limit**

✓ **H-dibaryon resonance @m_π=0.41-0.70GeV w/o SU(3)$_F$ symmetry**

✓ **Tcc (I=0) : S-wave DD* potential is attractive,**
but not strong enough to form bound state @m_π=0.41-0.70GeV

- **Future direction:**

➡ **Physical point simulations of hadron potentials**

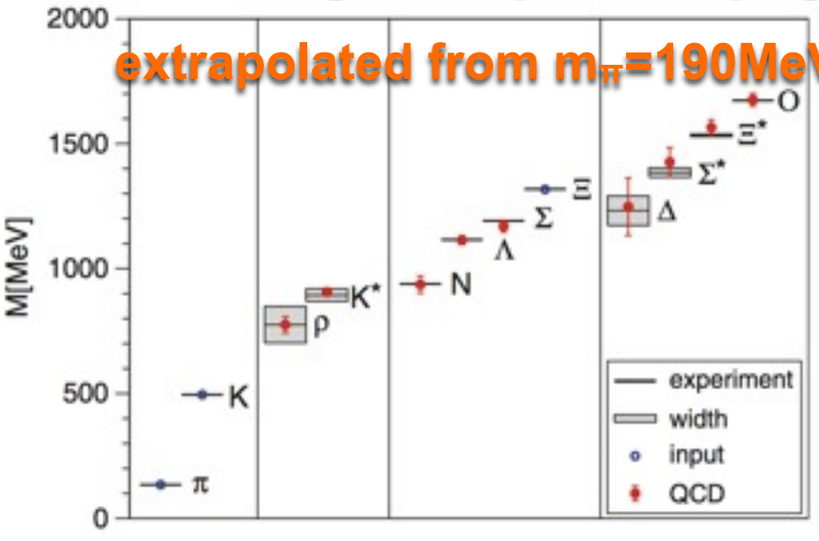
Backup

LQCD@physical point

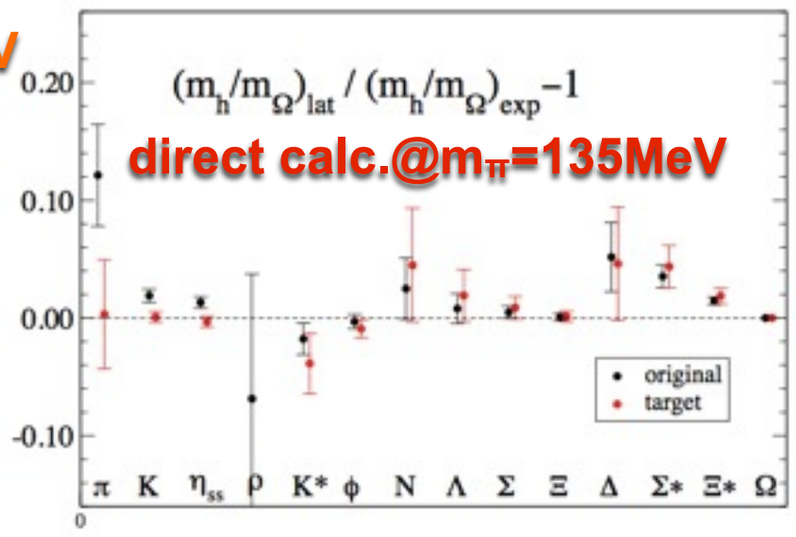
- Quark mass (m_Q --> $m_{phys.}$)
- Lattice spacing (a --> 0)
- Lattice volume (1/L --> 0)

Lattice QCD simulation with physical quark masses, finer lattice, large volume is desirable

- **Physical point = physical quark mass**

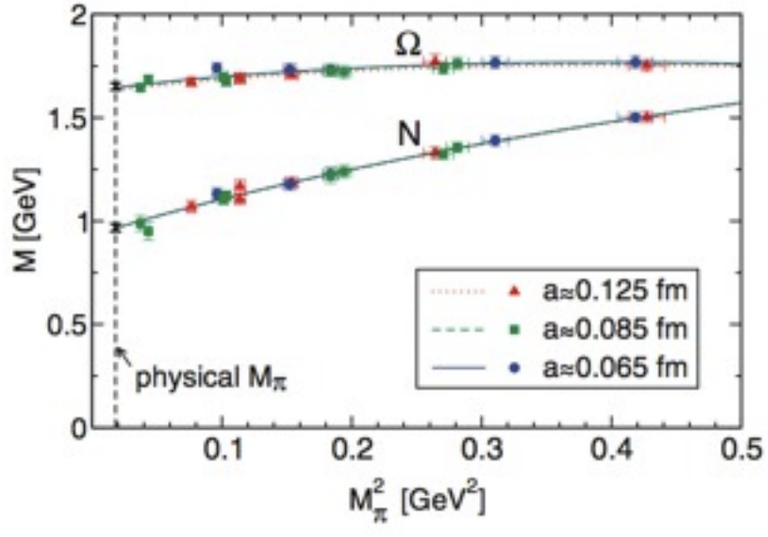


BMW Coll., Science (2008).



PACS-CS Coll., PRD81 (2010).

- **Continuum limit (a --> 0)**



BMW Coll., Science (2008).

- **Thermodynamic limit (1/L --> 0): on-going@K-computer**

10fm^3, phys. point, full QCD gauge configuration by PACS-CS Coll.

- 4th strongest in the world

Nambu-Bethe-Salpeter wave function

Full details, see, Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Equal-time Nambu-Bethe-Salpeter(NBS) amplitudes (e.g., spin-less, equal-mass)

$$\Psi(\vec{x}_1,t;\vec{x}_2,t) \equiv \langle 0|\phi(x_1)\phi(x_2)|W(\vec{k});in\rangle = \psi_{W(\vec{k})}(\vec{r})e^{-iW(\vec{k})t} \quad \text{(C.M. frame)}$$

$$W(\vec{k}) = 2\sqrt{m^2+\vec{k}^2}$$

Klein-Gordon equations at large r (r>R):

$$(\partial_t^2 - \nabla_i^2 + m^2)\phi(\vec{x}_i,t) = 0 \quad (i=1,2)$$

$$\Rightarrow \quad (\nabla_r^2 + \vec{k}^2)\psi_{W(\vec{k})}(\vec{r}) = 0$$

- **Spatial correlation ψ(r) is NBS wave function and satisfies Helmholtz equation**
- **Asymptotic form of NBS wave function:**

$$\psi_W^{(l)}(\vec{r}) \sim \frac{e^{i\delta_l(k)}}{kr}\sin\left(kr+\delta_l(k)-l\pi/2\right)$$

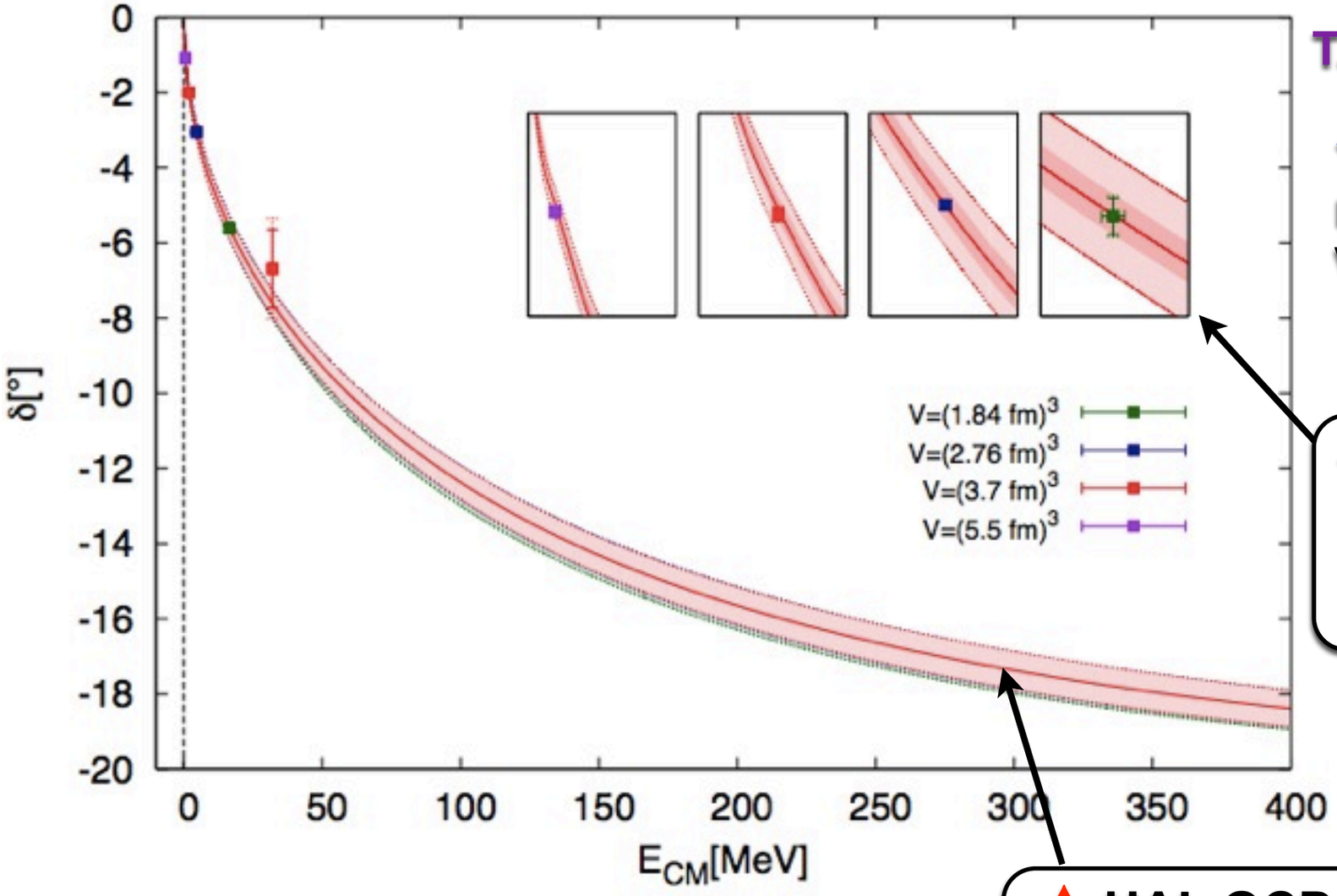
faithful to scattering phase shift

NBS wave functions in quantum field theory

~ wave functions in quantum mechanics

HAL QCD & Lüscher's methods

I=2 ππ S-wave phase shift from HAL QCD & Lüscher's methods



T. Kurth et al., JHEP 1312 (2013) 015.

❖ **Quench QCD**

$m_\pi \sim 940$ MeV, $a=0.115$ fm

$V \times T = (16^3, 24^3, 32^3, 48^3) \times 128$

✦ **Lüscher's method (points)**

$$k\cot\delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in Z} \frac{1}{|\vec{n}|^2 - \tilde{k}^2}$$

✦ **HAL QCD (red band)**

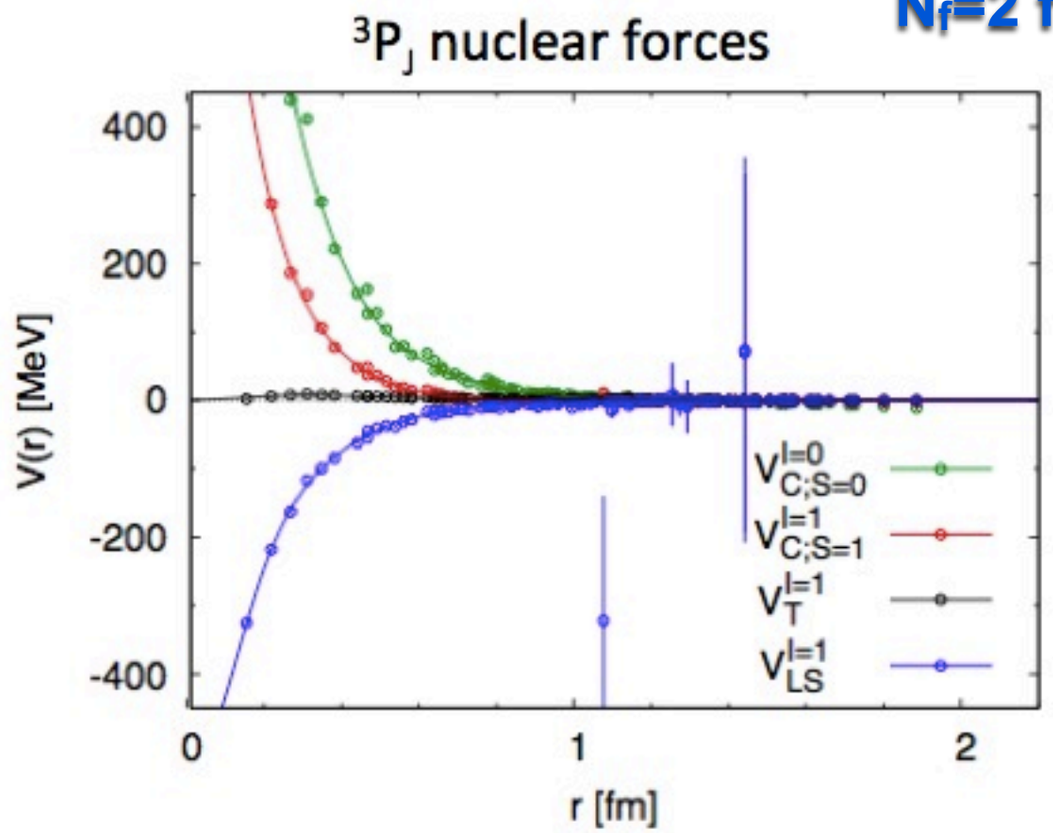
$$\left[-\partial_\tau + \nabla^2/2\mu + \partial_\tau^2/8\mu \right] R(\vec{r}, \tau) = V(\vec{r}) R(\vec{r}, \tau)$$

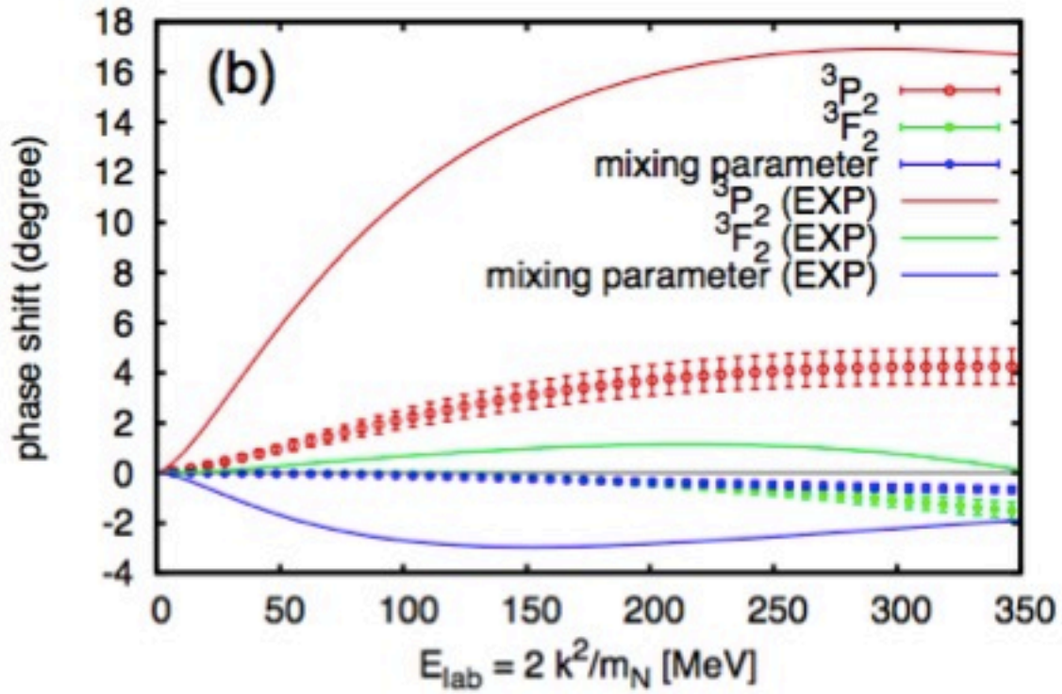
Both HAL QCD and Lüscher's finite size methods agree

NN potential : parity-odd sector

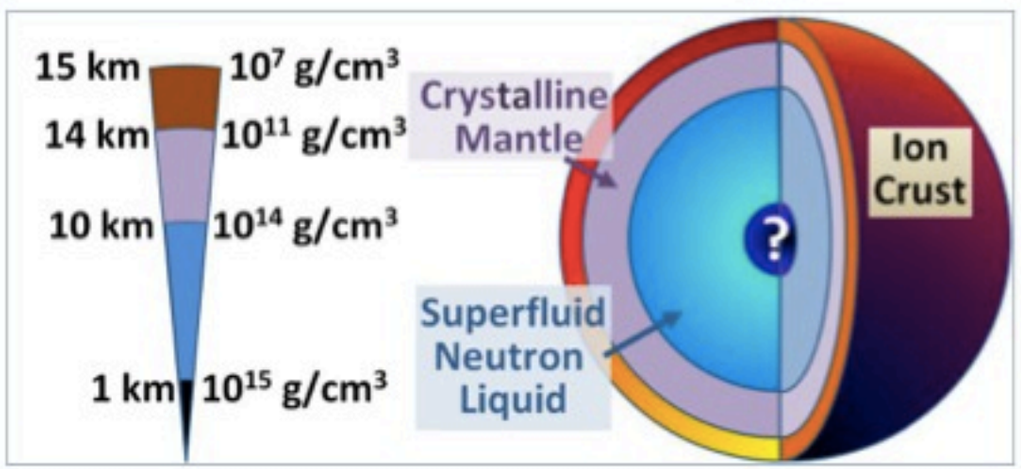
Murano (HAL QCD Coll.), arXiv:1305:2293 (2013).

N_f=2 full QCD, m_π~1.1 GeV





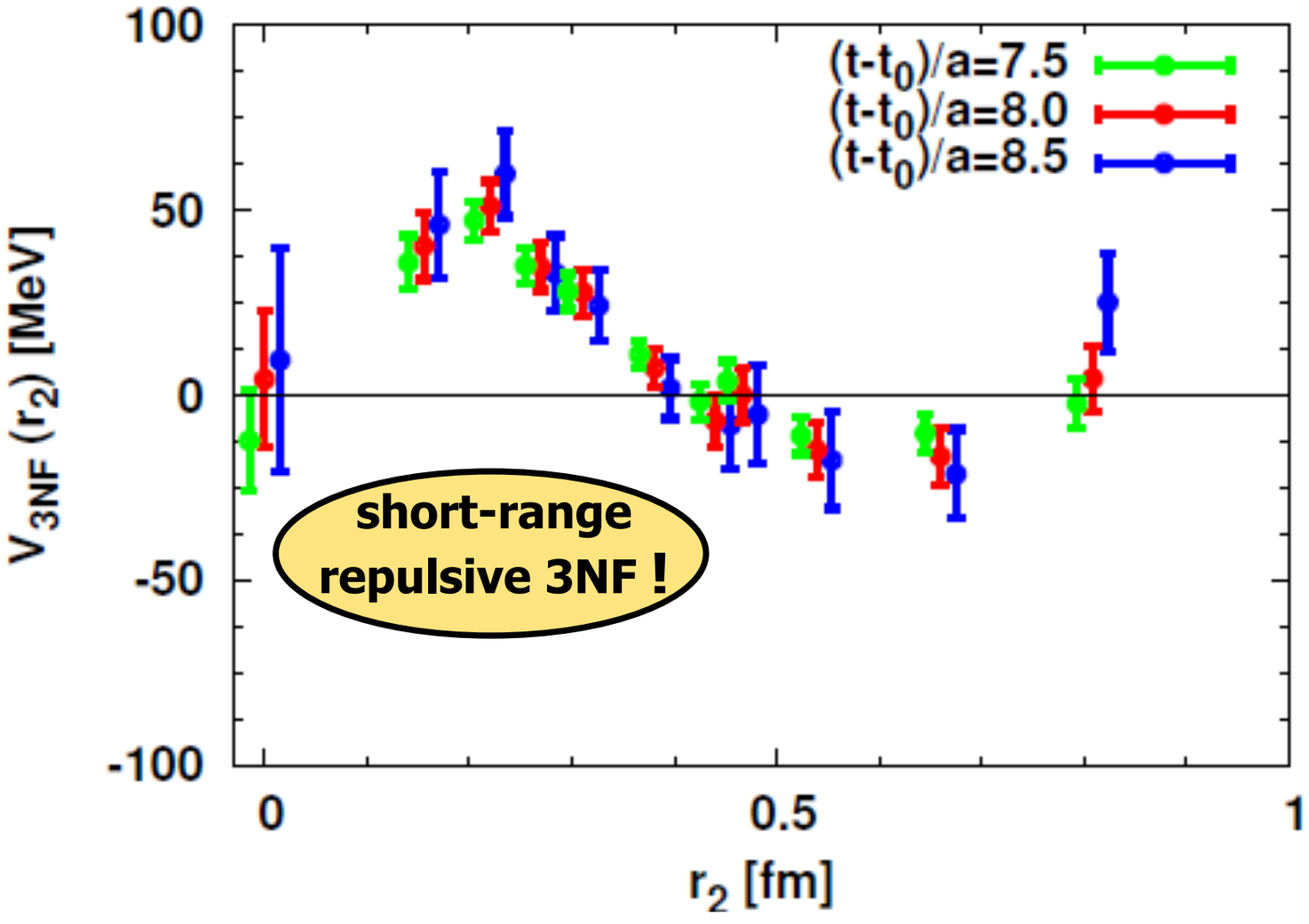
- LS potential has great influence on the shell model and the magic number of nuclei.
- Qualitative behaviors are reproduced. But the strength is not enough.
- Attraction in 3P2 channel
  - ➔ P-wave neutron superfluid in neutron star

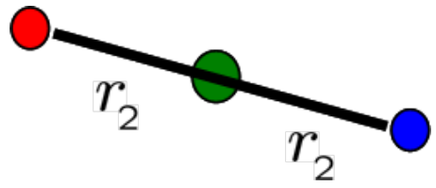


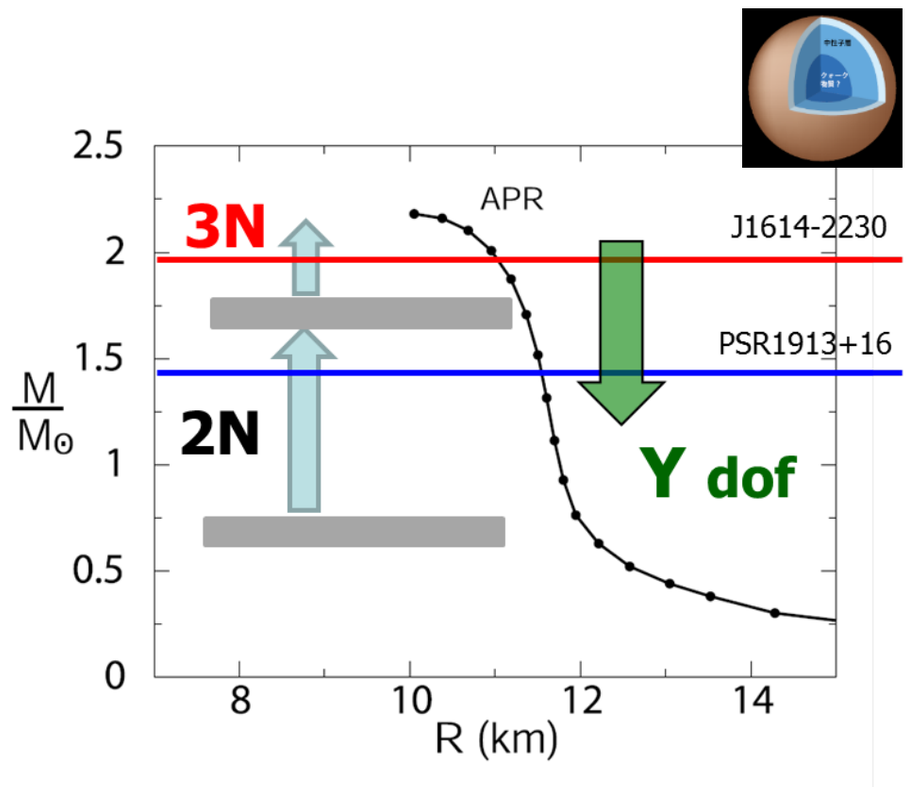
Three-body potentials (3NF)

Doi (HAL QCD Coll.), PTP127 (2012).

N_f=2 full QCD, m_π~1.1 GeV







- 3-body NBS wave function is calculated in the linear setup (not for potential)
- Short-range repulsive 3NF is observed

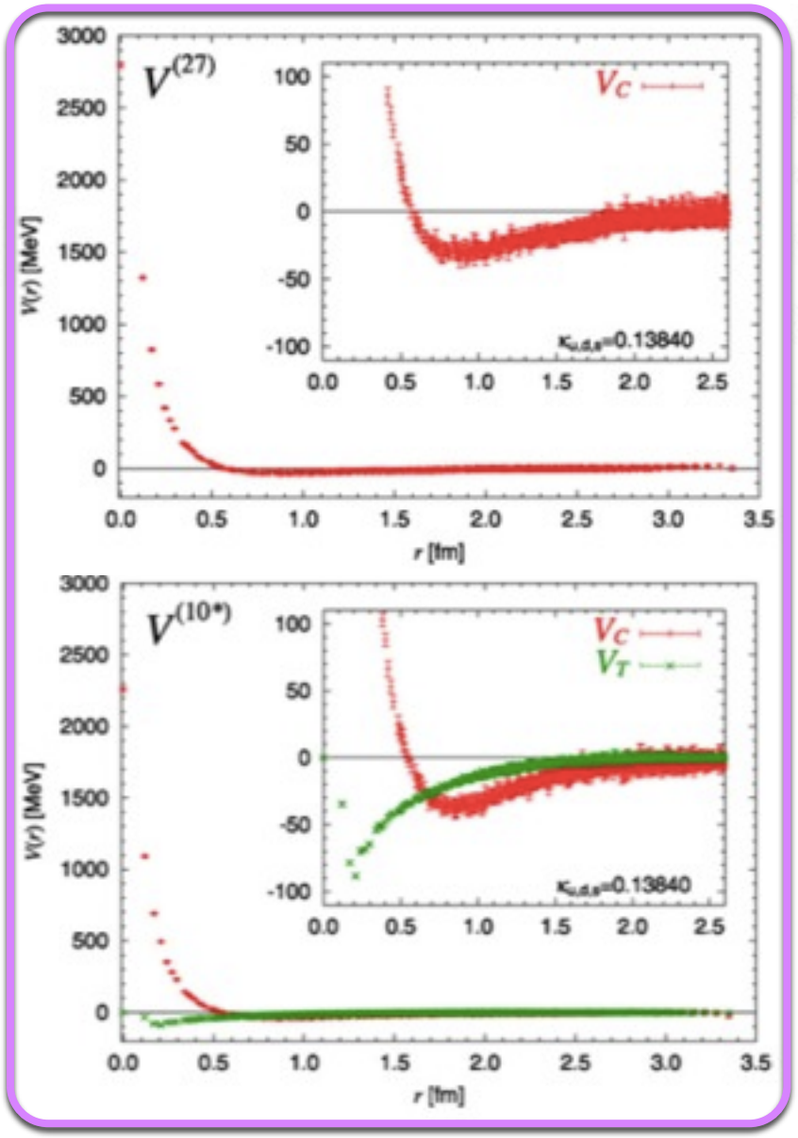
Generalized BB potentials in $SU(3)_F$ limit

❖ **Full QCD in $SU(3)_F$ limit : m_π~0.47GeV, L=3.9 fm**

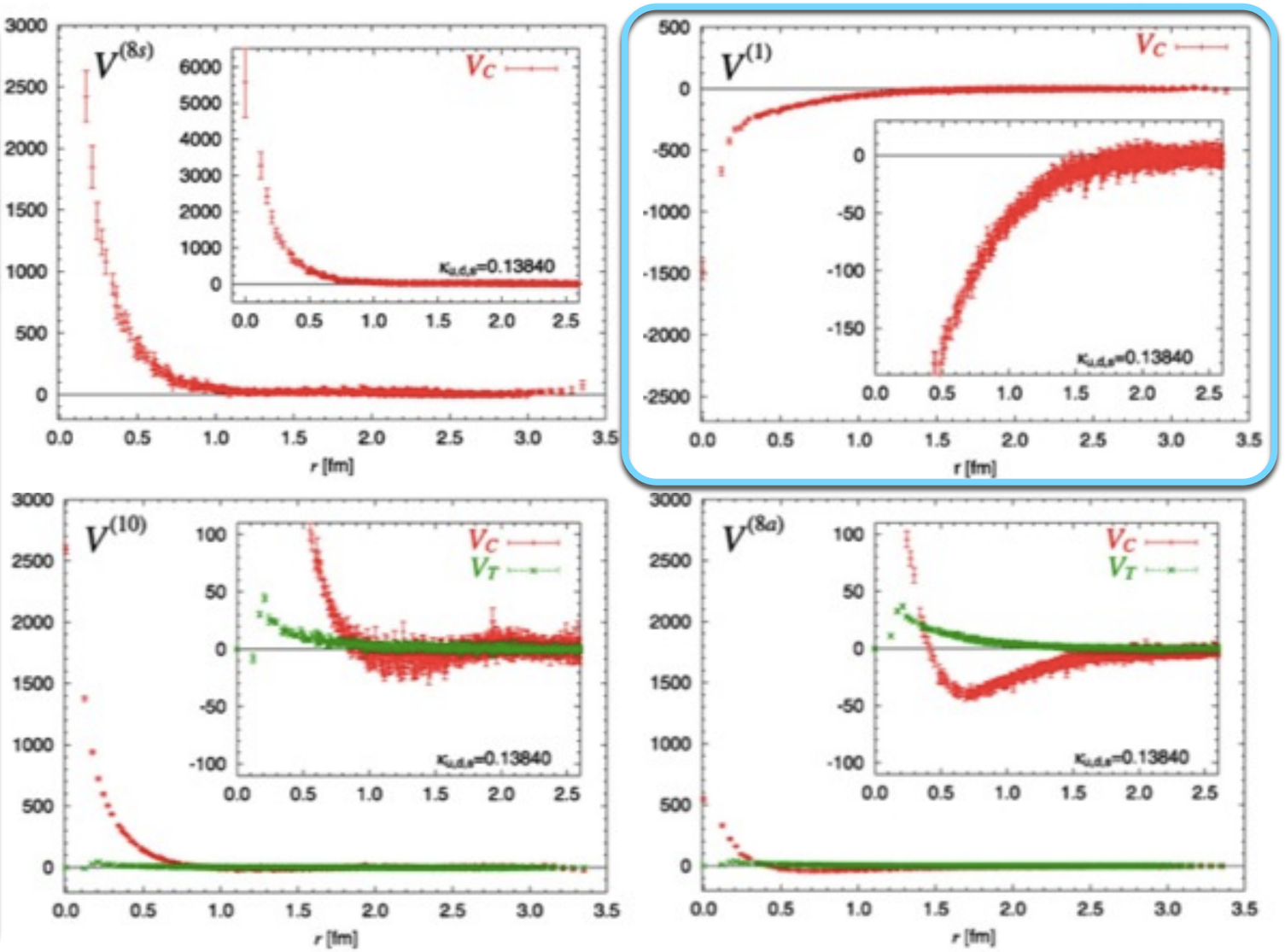
Inoue et al. (HAL QCD), PRL106 (2011), NPA881 (2012).

$^1S_0$

$^3S_1 - ^3D_1$



NN channels



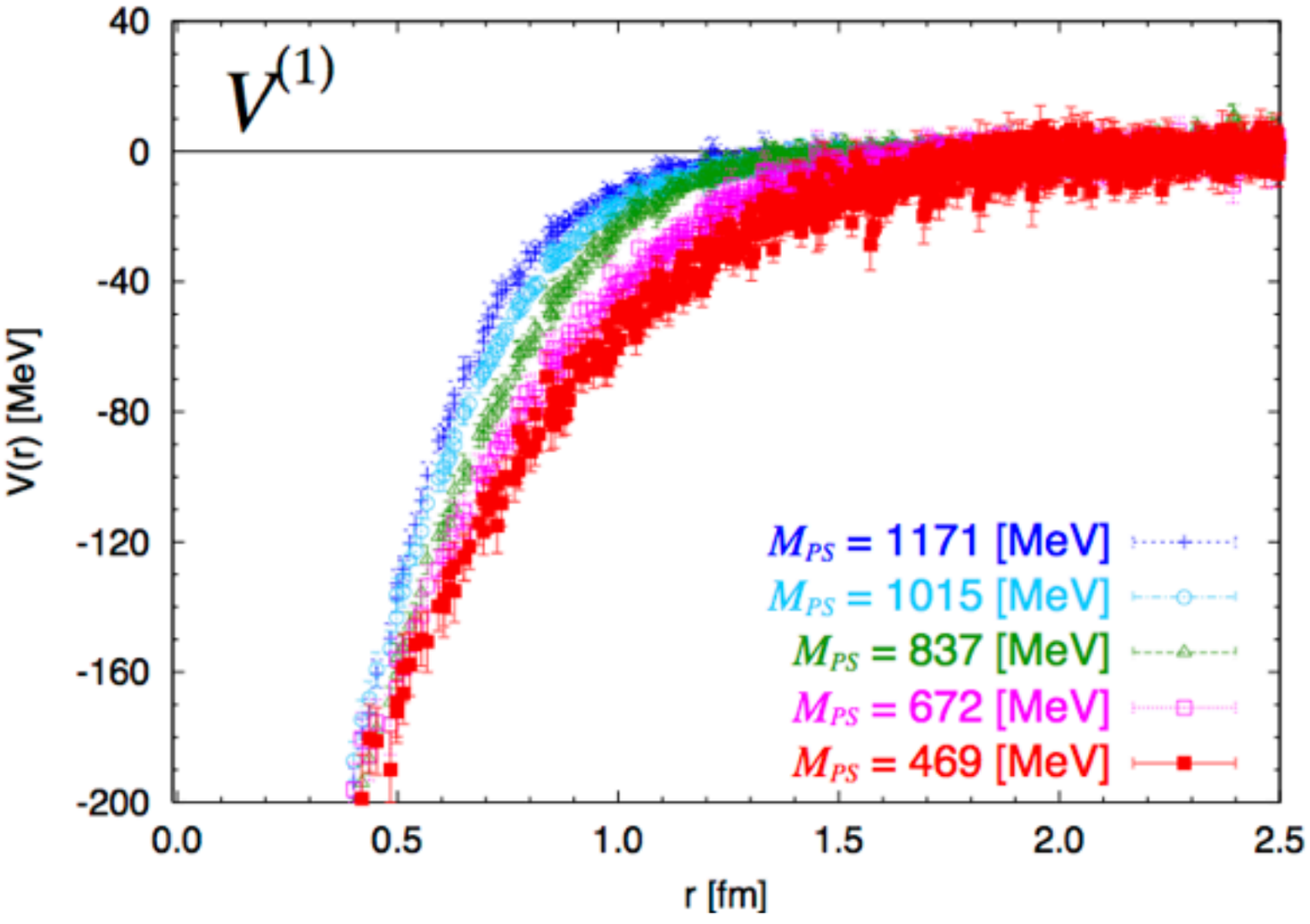
1-plet: H-dibaryon channel

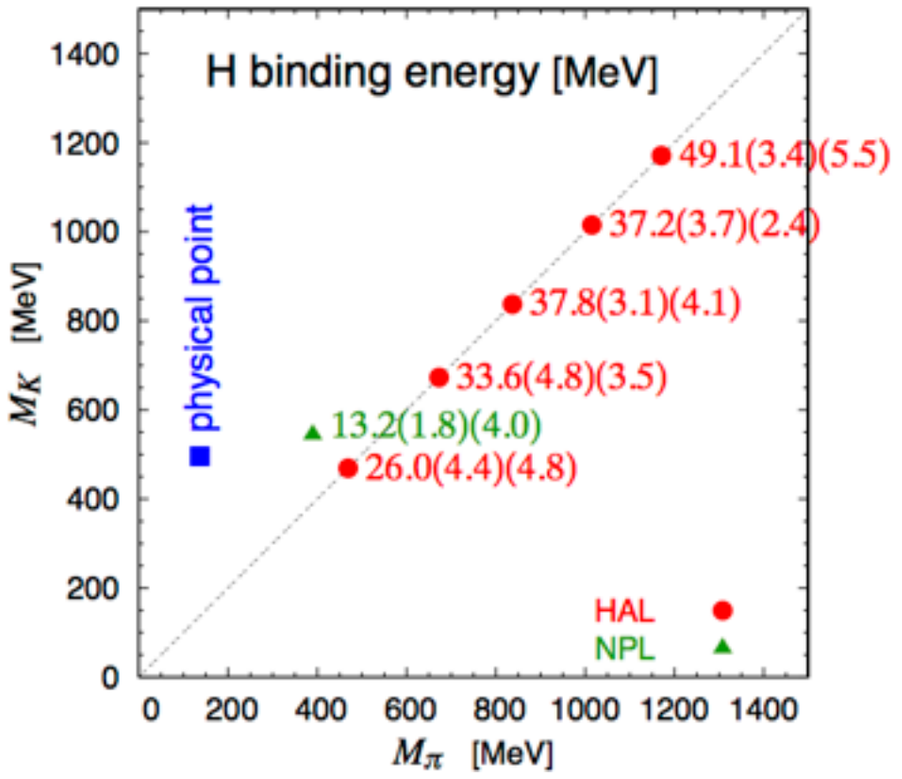
No Pauli-blocking, attractive color-magnetic force

YY potential & H-dibaryon

N_f=3 full QCD

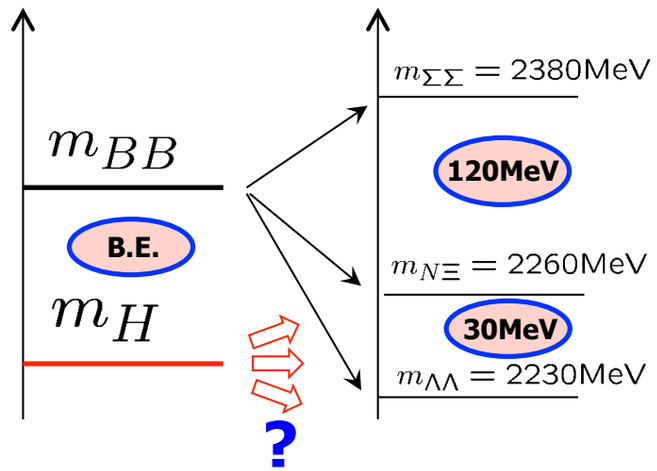
Inoue, [HAL QCD Coll.], PRL106 (2011).
Inoue, [HAL QCD Coll.], PTEP 2012, 01A105 (2012).





- **YY flavor-singlet potential @ SU(3)$_F$ limit**
- **Entirely attractive (No Pauli blocking)**
- **Bound H-dibaryon found**
- **Fate of H-dibaryon off SU(3)$_F$ limit**

SU(3) lat ➡ Physical point



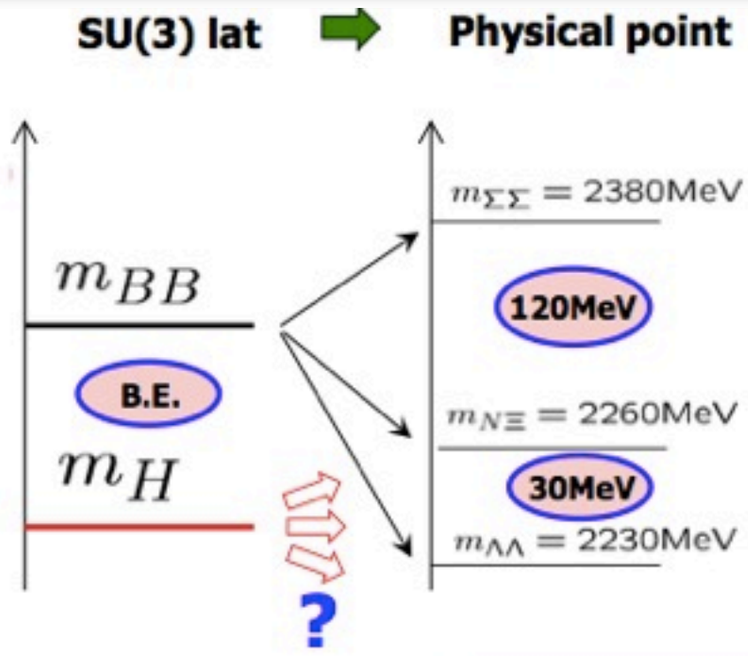
Coupled-channel formalism

Coupled channel extension for

$$\Lambda\Lambda - N\Xi - \Sigma\Sigma$$

is possible by employing the triple

$$\Psi_n(\vec{x}-\vec{y}) \equiv \begin{bmatrix} \langle 0|\Lambda(\vec{x})\Lambda(\vec{y})|n,in\rangle \\ \langle 0|N(\vec{x})\Xi(\vec{y})|n,in\rangle \\ \langle 0|\Sigma(\vec{x})\Sigma(\vec{y})|n,in\rangle \end{bmatrix}$$



$$E \equiv 2\sqrt{m_\Lambda^2 + \vec{p}_{\Lambda\Lambda}^{\,2}} = \sqrt{m_N^2 + \vec{p}_{N\Xi}^{\,2}} + \sqrt{m_\Sigma^2 + \vec{p}_{N\Xi}^{\,2}} = 2\sqrt{m_\Sigma^2 + \vec{p}_{\Sigma\Sigma}^{\,2}}$$

Argument parallel to the single-channel NN case leads a **coupled-channel Schrodinger eq.**

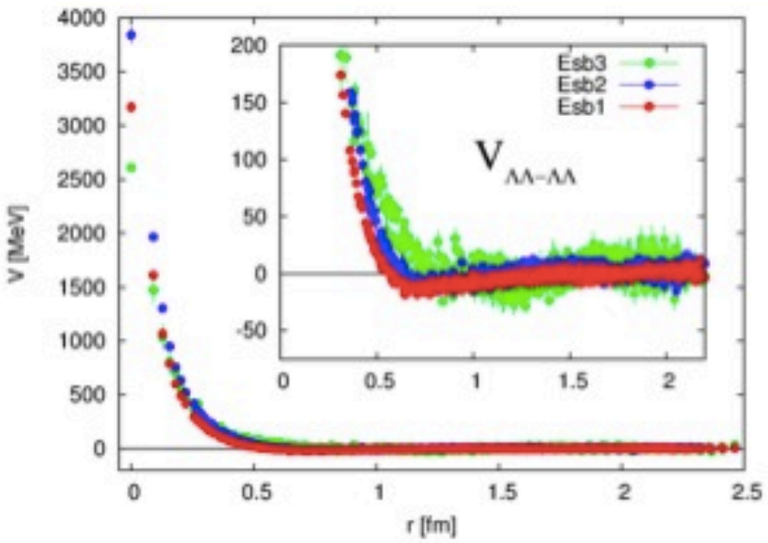
[S.Aoki et al., Proc.Japan Acad.B87(2011)509.]

$$\begin{bmatrix} \left(\frac{\vec{p}_{\Lambda\Lambda}^{\,2}}{2\mu_{\Lambda\Lambda}} + \frac{\Delta}{2\mu_{\Lambda\Lambda}}\right)\psi_{\Lambda\Lambda}(\vec{r};n) \\ \left(\frac{\vec{p}_{N\Xi}^{\,2}}{2\mu_{N\Xi}} + \frac{\Delta}{2\mu_{N\Xi}}\right)\psi_{N\Xi}(\vec{r};n) \\ \left(\frac{\vec{p}_{\Sigma\Sigma}^{\,2}}{2\mu_{\Sigma\Sigma}} + \frac{\Delta}{2\mu_{\Sigma\Sigma}}\right)\psi_{\Sigma\Sigma}(\vec{r};n) \end{bmatrix} = \int d^3r' \begin{bmatrix} U_{\Lambda\Lambda;\Lambda\Lambda}(\vec{r},\vec{r}') & U_{\Lambda\Lambda;N\Xi}(\vec{r},\vec{r}') & U_{\Lambda\Lambda;\Sigma\Sigma}(\vec{r},\vec{r}') \\ U_{N\Xi;\Lambda\Lambda}(\vec{r},\vec{r}') & U_{N\Xi;N\Xi}(\vec{r},\vec{r}') & U_{N\Xi;\Sigma\Sigma}(\vec{r},\vec{r}') \\ U_{\Sigma\Sigma;\Lambda\Lambda}(\vec{r},\vec{r}') & U_{\Sigma\Sigma;N\Xi}(\vec{r},\vec{r}') & U_{\Sigma\Sigma;\Sigma\Sigma}(\vec{r},\vec{r}') \end{bmatrix} \cdot \begin{bmatrix} \psi_{\Lambda\Lambda}(\vec{r}';n) \\ \psi_{N\Xi}(\vec{r}';n) \\ \psi_{\Sigma\Sigma}(\vec{r}';n) \end{bmatrix}$$

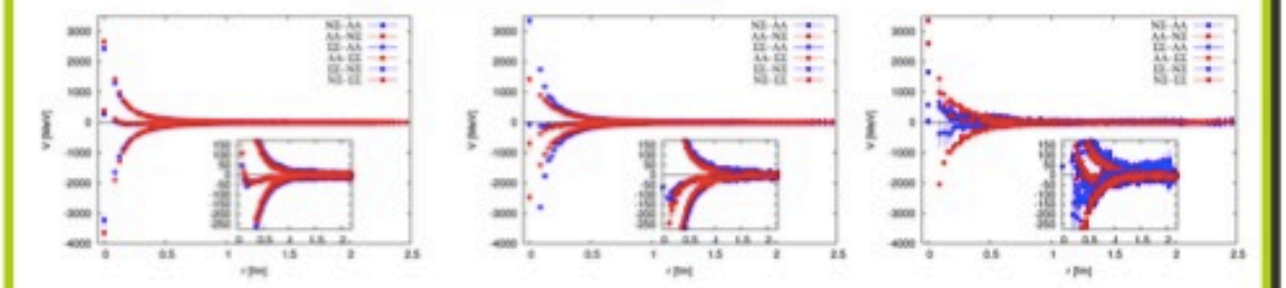
(Derivation is parallel, but notation is quite lengthy.)

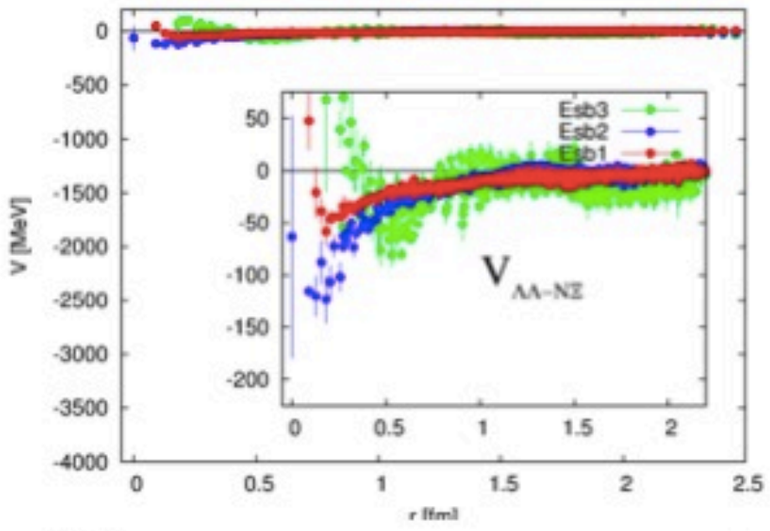
$^1S_0$ I=0 particle basis

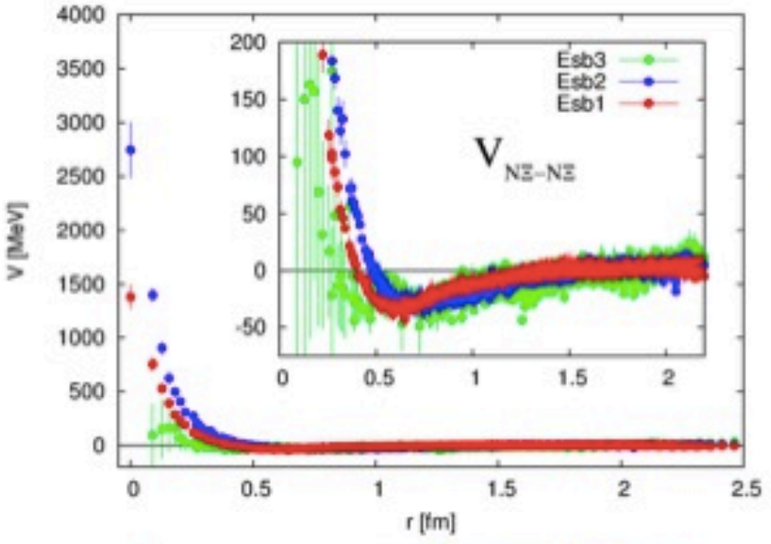
Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV



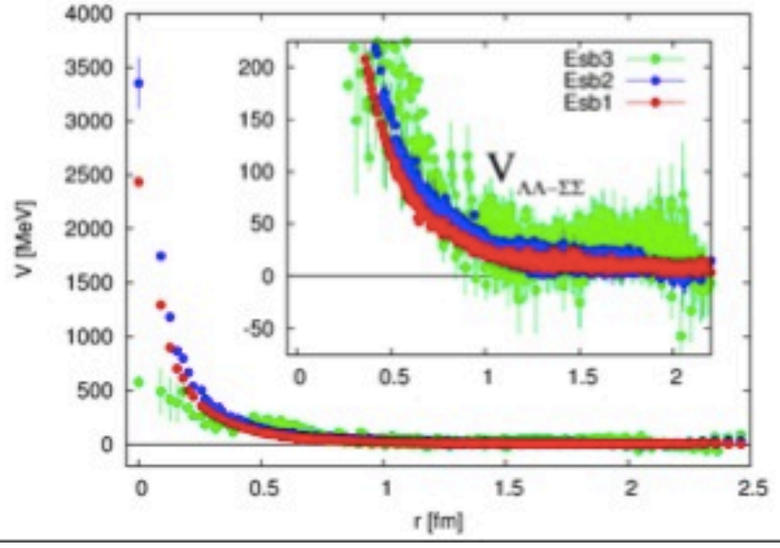
Hermiticity

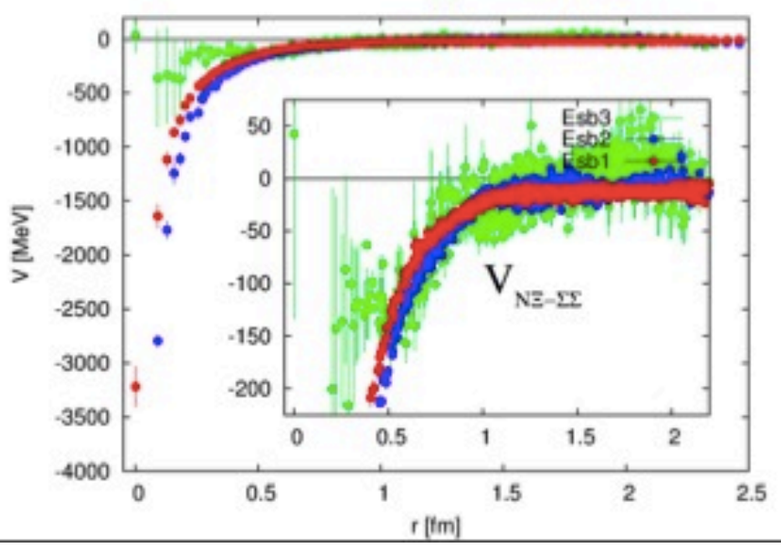


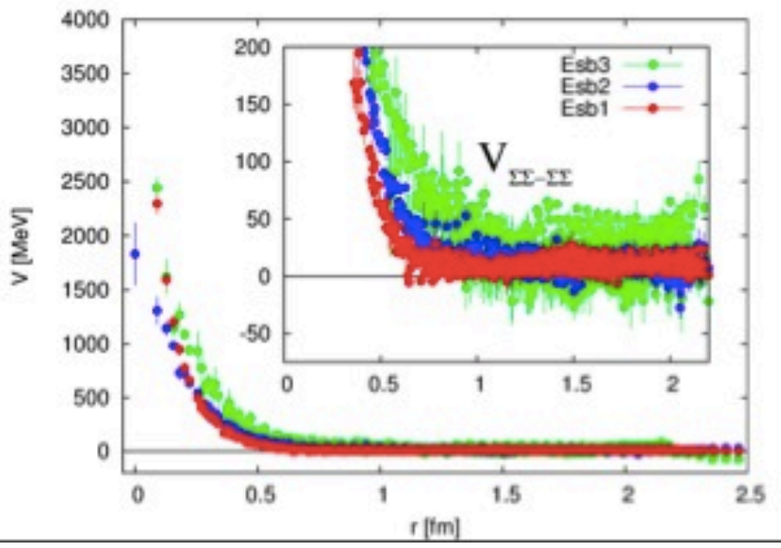




$$\begin{pmatrix} \langle \Lambda\Lambda| \\ \langle \Sigma\Sigma| \\ \langle N\Xi| \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{27}{40}} & -\sqrt{\frac{8}{40}} & -\sqrt{\frac{5}{40}} \\ -\sqrt{\frac{1}{40}} & -\sqrt{\frac{24}{40}} & \sqrt{\frac{15}{40}} \\ \sqrt{\frac{12}{40}} & \sqrt{\frac{8}{40}} & \sqrt{\frac{20}{40}} \end{pmatrix} \begin{pmatrix} \langle \mathbf{27}| \\ \langle \mathbf{8}_s| \\ \langle \mathbf{1}| \end{pmatrix}$$



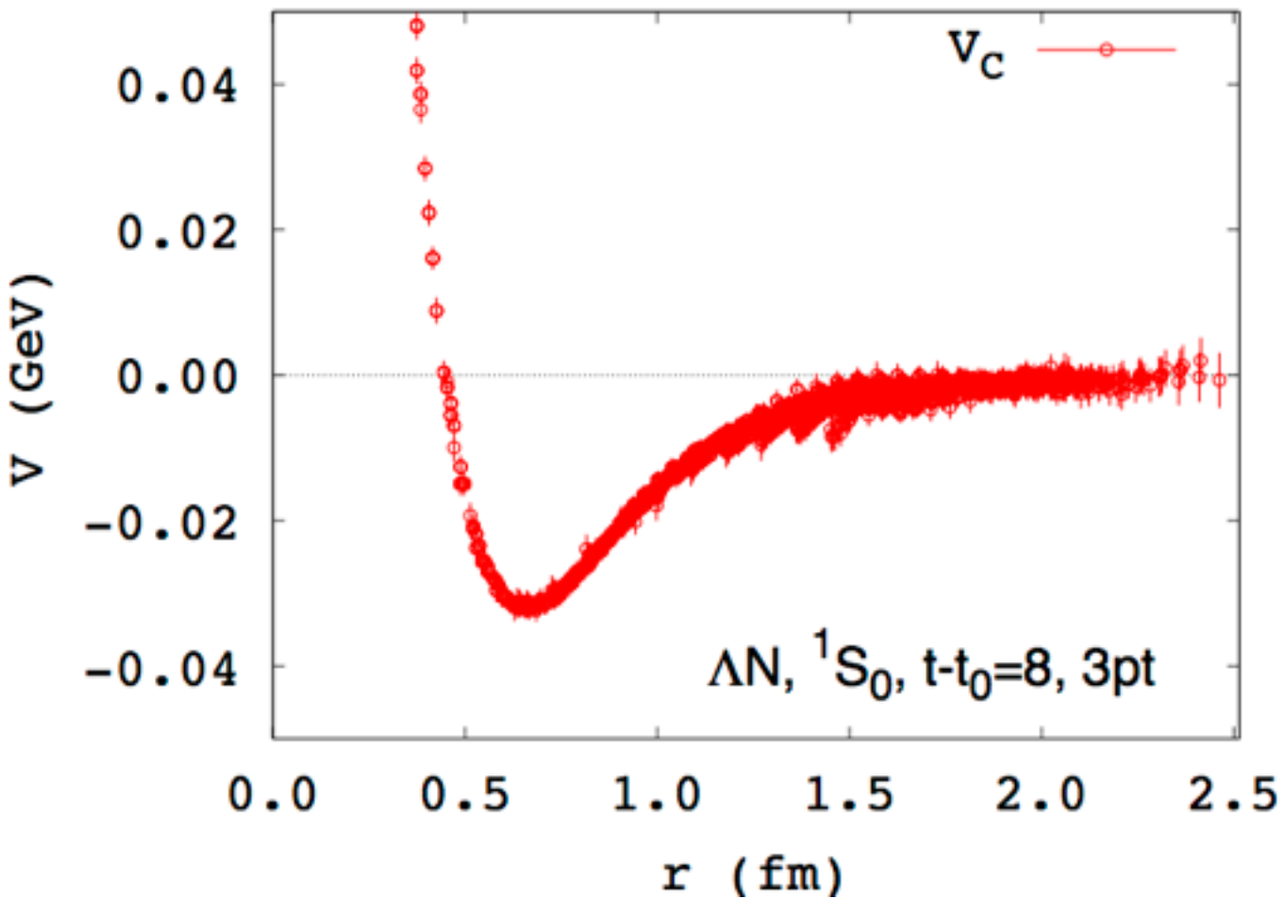


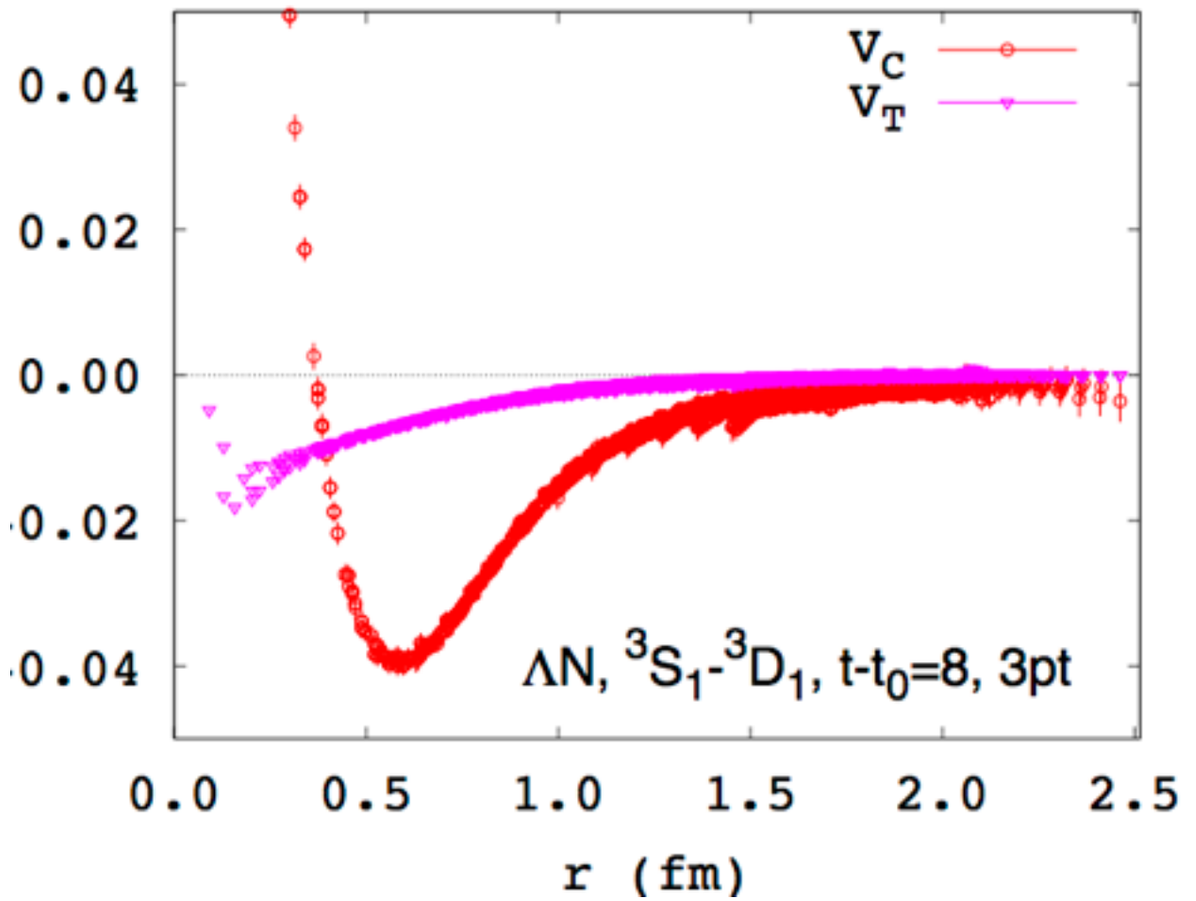


YN potentials

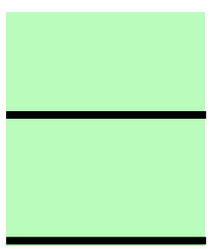
Nemura, [HAL QCD Coll.], PTEP 2012, 01A105 (2012).

N_f=2+1 full QCD, m_π~700 MeV





- **Effective ΛN potential (ΣN is renormalized)**
- **$^3S_1$ channel is more attractive than $^1S_0$ at simulated m_π**

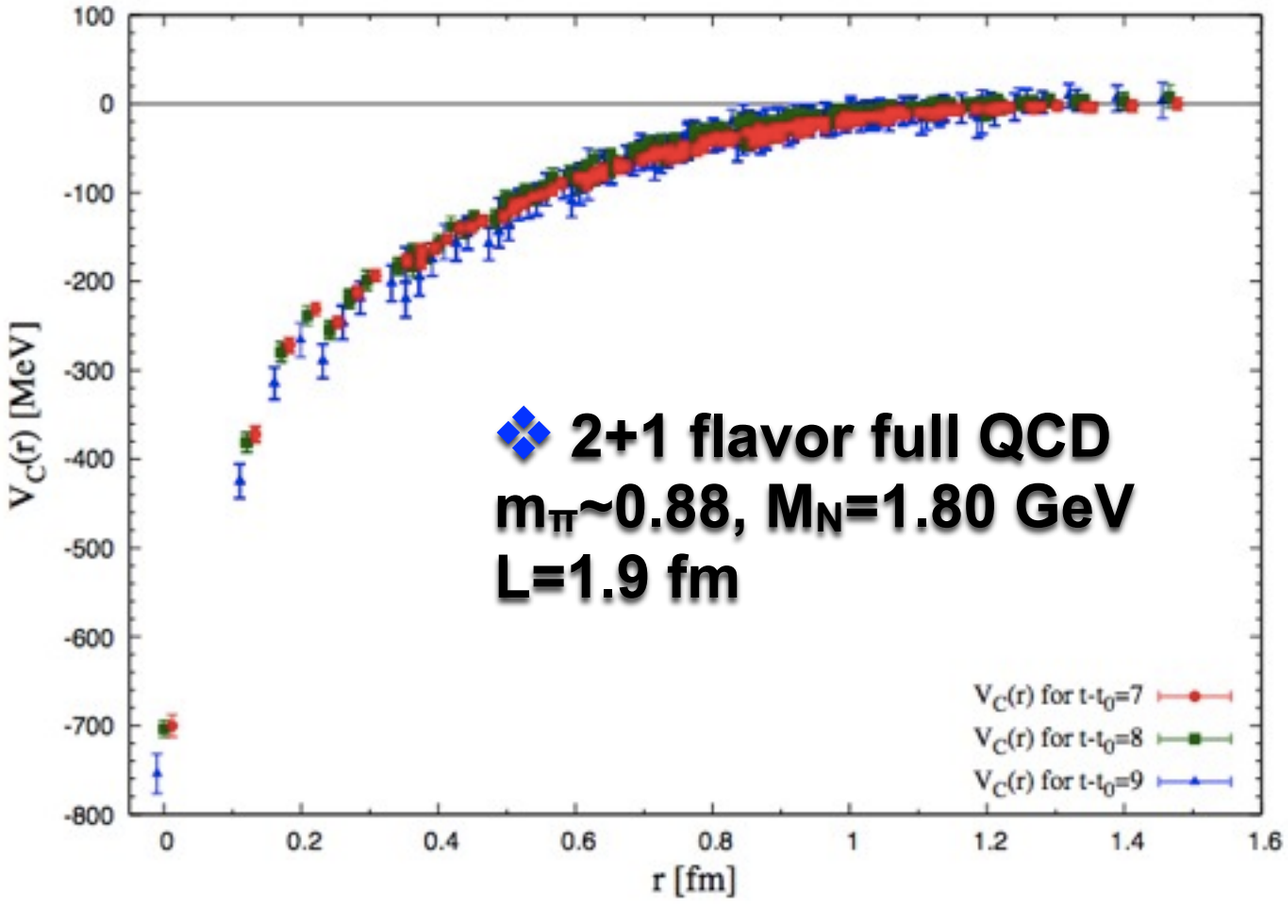


inelastic : ΣN

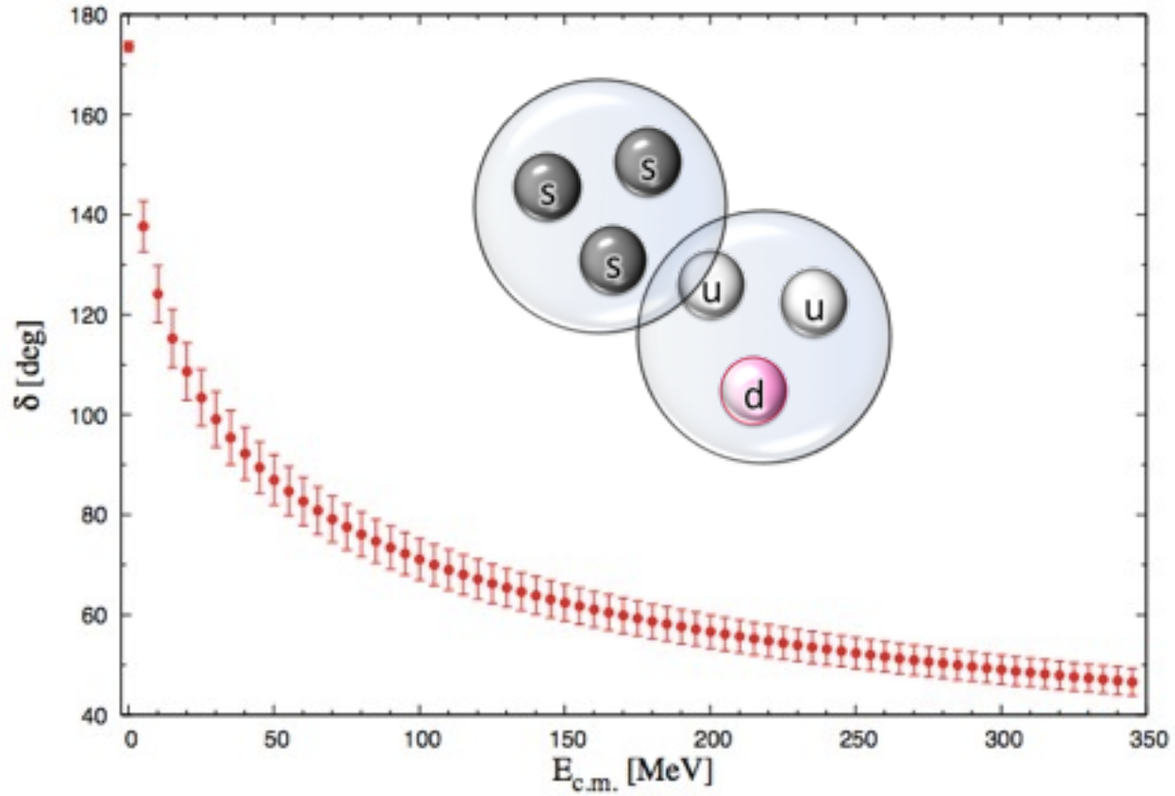
elastic : ΛN

NΩ potential & phase shift

F. Etminan et al. (HAL QCD), arXiv:1403.7284[hep-lat].



❖ 2+1 flavor full QCD
m_π~0.88, M_N=1.80 GeV
L=1.9 fm



- **Attractive** S-wave potential in $I(J^P)=1/2(2^+)$ channel
- $a_{N\Omega}$ = -1.28(0.13) fm, r_e = 0.50(0.03) fm
- Bound state is found with B.E.=18.9(5.0) MeV

Lattice QCD Setup : charm quarks

✤ Tsukuba-type Relativistic Heavy Quark (RHQ) action

S. Aoki et al., PTP109, 383 (2003)

Leading cutoff errors, $O((m_Q a)^n)$ and $O(a\Lambda_{QCD})$, are removed by adjusting RHQ parameters, $\{m_0, \nu, r_s, C_E, C_B\}$.

$$S^{\mathrm{RHQ}} = \sum_{x,y} \bar{q}(x) D_{x,y} q(y)$$

$$D_{x,y} = m_0 + \gamma_0 D_0 + \nu \gamma_i D_i - a r_t D_0^2 - a r_s D_i^2 - a C_E \sigma_{0i} F_{0i} - a C_B \sigma_{ij} F_{ij}$$

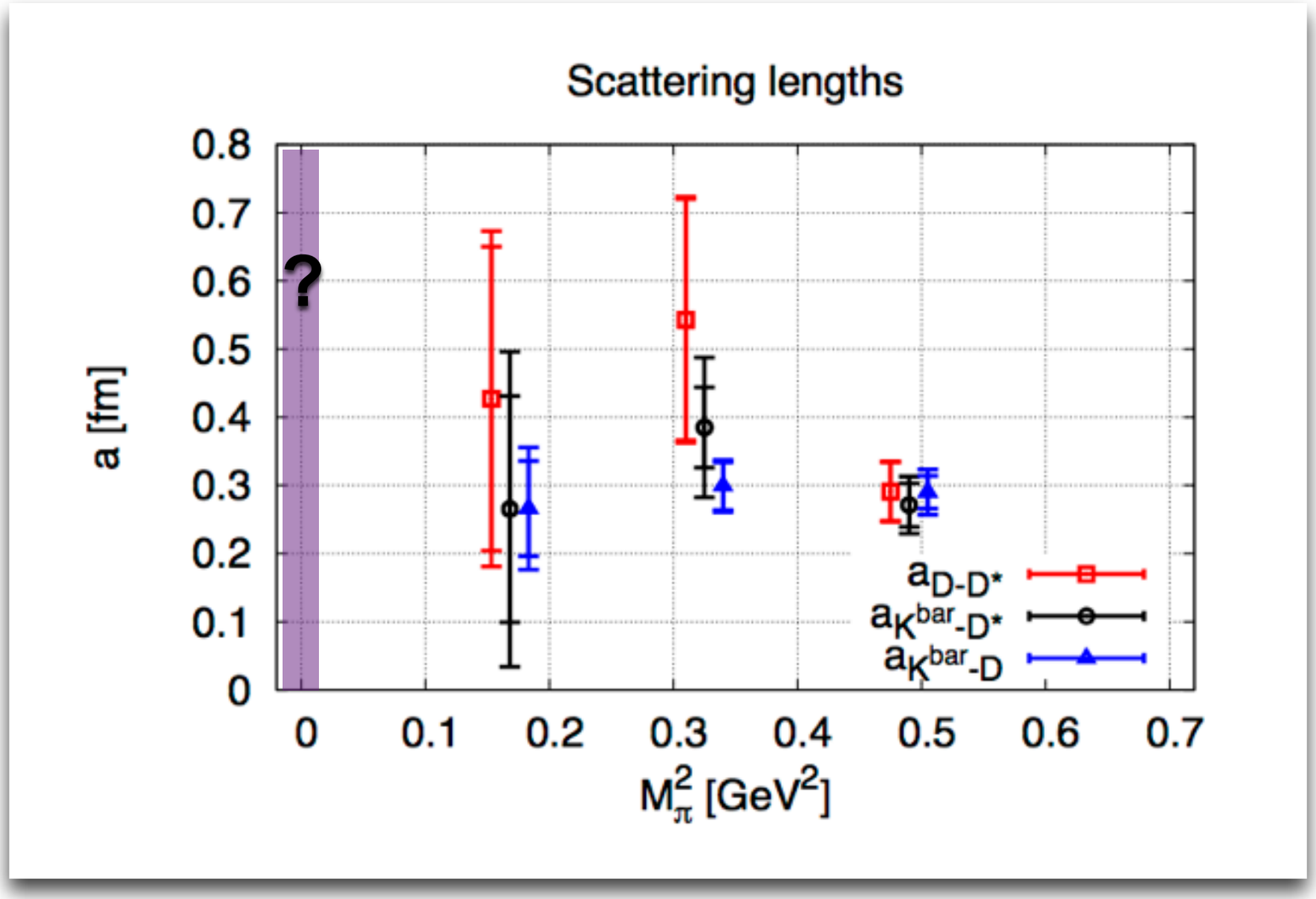
- We are allowed to choose r_t=1 (c.f. Wilson parameter)
- We are left with $O((a\Lambda_{QCD})^2)$ error (~ a few %)

We employ RHQ parameters tuned by Namekawa et al (PACS-CS coll).

Y. Namekawa et al., PRD84, 074505 (2011)

Charmed meson mass [conf.1, conf.2, conf.3] (MeV)
$M_{\eta c}$=3024(1), 3005(1), 2988(2) [PDG:2981]
$M_{J/\psi}$=3142(1), 3118(1), 3097(2) [PDG:3097]
M_D=1999(1), 1946(1), 1912(1) [PDG:1865 (D^0)]
M_{D^*}=2159(4), 2099(6), 2059(8) [PDG:2007 (D^{*0})]

Scattering lengths : I=0 channel



- stat. & syst. errors are included

➡ DD* scattering length becomes more attractive, as decreasing m_q

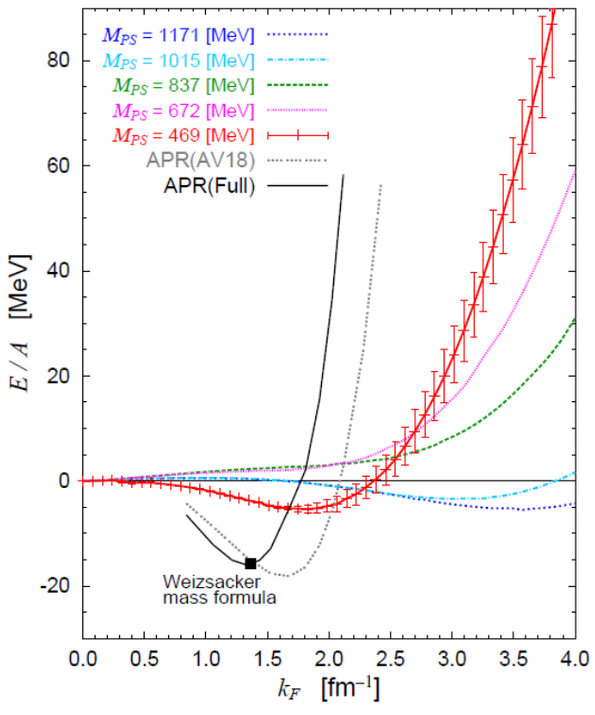
Nucleonic EOS from Lattice QCD

- **Full QCD in $SU(3)_F$ limit : m_π~0.47-1.17GeV, L=3.9 fm**

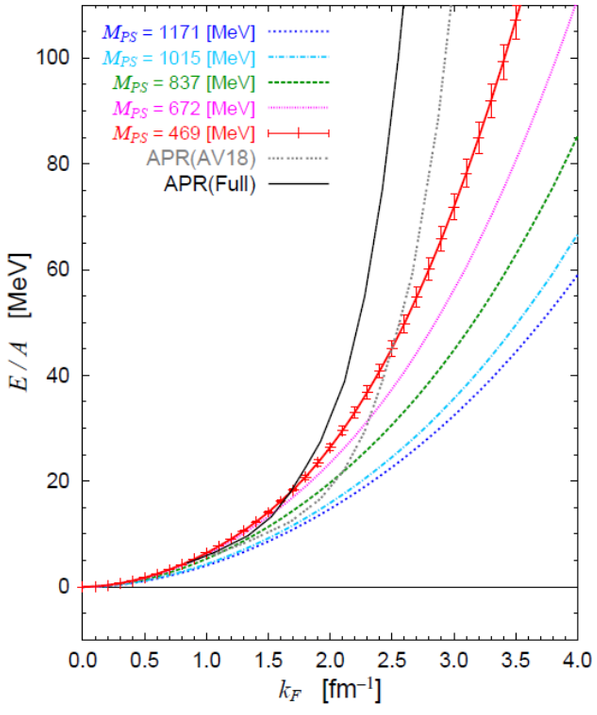
Inoue et al., [HAL QCD] PRL 111 (2013).

- **$^{2S+1}L_J = {}^1S_0, {}^3S_1\text{-}{}^3D_1$ NN potentials employed**
- **Brückner-Hartree-Fock many-body theory**

Nuclear matter

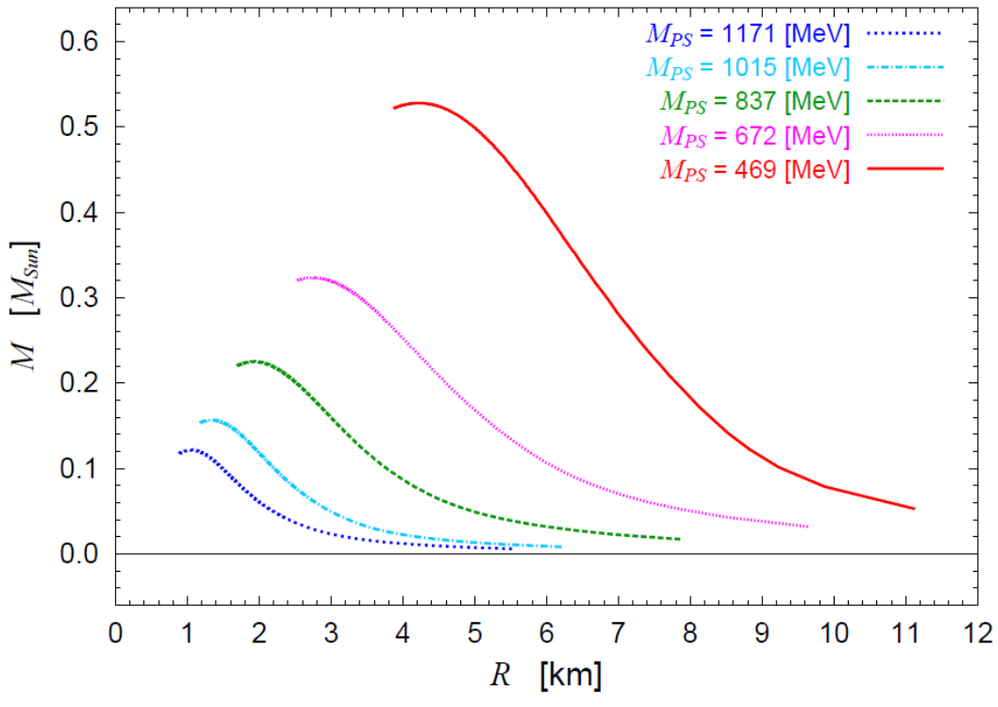


Neutron matter



- **EOS becomes stiff as decreasing m_q**
- **Saturation point is found**
- **Y-mixing is on-going**

- **Tolman-Oppenheimer-Volkoff equation**

$$\frac{dP(r)}{dr} = -\frac{(E(r)+P(r))(M(r)+4\pi r^3 P(r))}{r(r-2GM(r))}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 E(r)$$