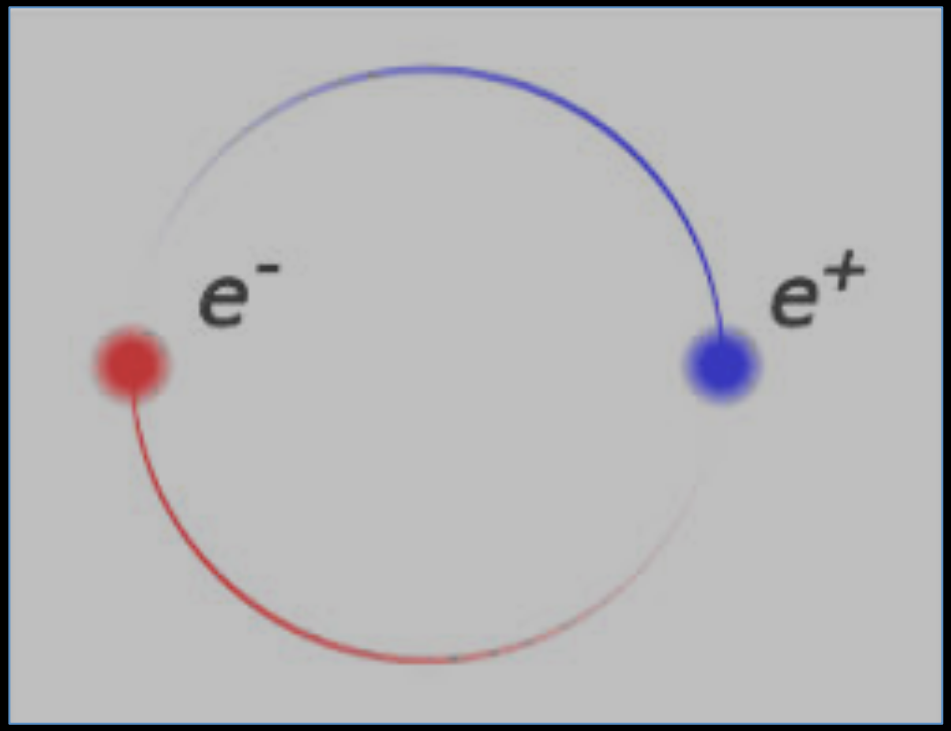
Higher Order Corrections to the Positronium Hyperfine Splitting

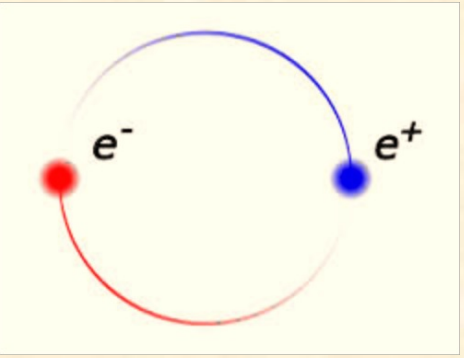


Greg Adkins
Franklin & Marshall College

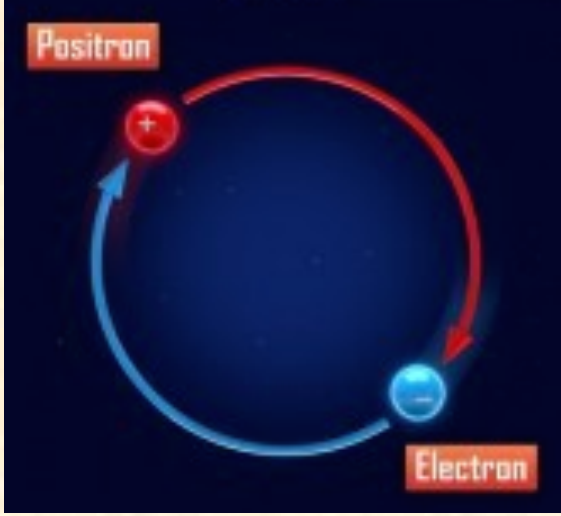
Higher Order Corrections to the Positronium Hyperfine Splitting

Motivation
Experimental Situation
History
Present Status

Acknowledgments
NSF PHY-1404268
Franklin & Marshall College



Positronium

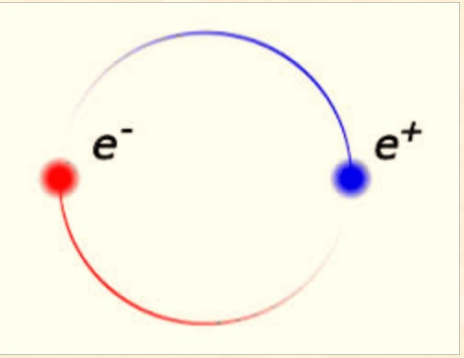


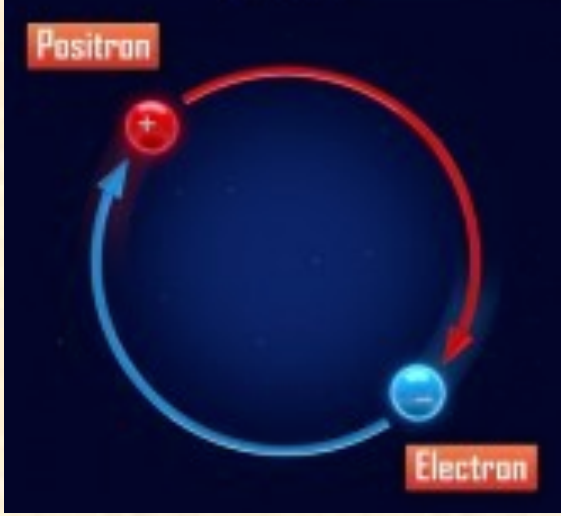
Positronium is the simplest bound system.

The constituents are structureless pointlike particles.

The binding is almost completely through the usual Coulombic attraction between particles of opposite charge. The structure of positronium can be described (in lowest approximation) by a Schrödinger equation (as with hydrogen but with reduced mass $m_r=m_e/2$).

While there are relativistic and other electromagnetic corrections to the properties of positronium, strong and weak effects are very small. (Coupling to the strong force is indirect, e.g. by hadronic vacuum polarization and is suppressed by the mass scale. Weak effects are intrinsically small.)



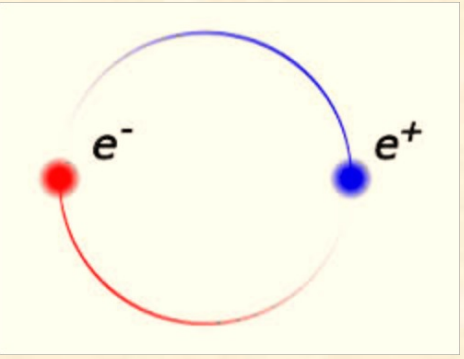


Why Study Positronium?

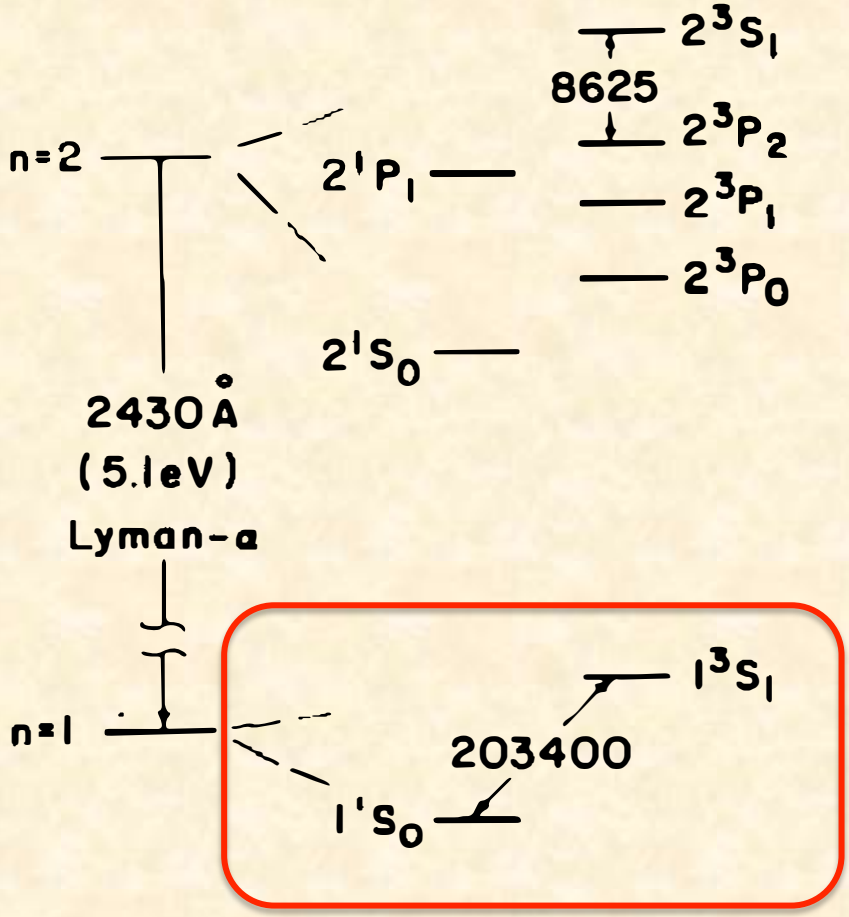
Positronium is intrinsically interesting. Many fundamental aspects of quantum field theory enter into its description. It differs from other exotic atoms in having large recoil effects, little sensitivity to hadronic physics, and is subject to real and virtual annihilation.

Positronium is accessible both to high precision experiments and to detailed calculations, so its study allows for a stringent test of the theory of bound state in QED (quantum electrodynamics) and quantum field theory generally.

Positronium is ideal for tests of fundamental symmetries and is useful in searches for "new physics".

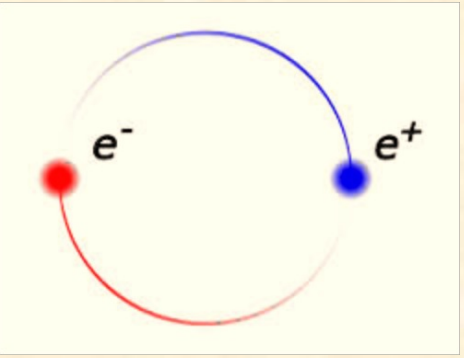


Positronium Spectrum: n=1 and n=2 and hyperfine splitting



Spectroscopic notation:

$$n^{2s+1}\ell_j$$

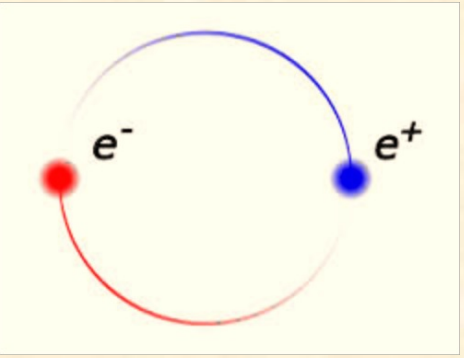


Hyperfine Structure

This table shows the two main periods of experimental work on the positronium hyperfine interval: the 1950s and the 1970s. The present era promises to be a third such exciting period of progress.

TABLE I: Experimental Results for the Positronium Hyperfine Interval

Year	Frequency Shift	Precision	Experimenters
1951	227(34) GHz	15%	Deutsch and Dulit
1952	203.2(3) GHz	1477 ppm	Deutsch and Brown
1954	203350(50) MHz	246 ppm	Weinstein, Deutsch, and Brown
1955	203300(40) MHz	197 ppm	Weinstein, Deutsch, and Brown
1957	203330(40) MHz	197 ppm	Hughes, Marder, and Wu
1970	203403(12) MHz	59 ppm	Theriot, Beers, Hughes, and Zioch
1972	203396(5) MHz	25 ppm	Carlson, Hughes, Lewis, and Lindgren
1975	203387.0(1.6) MHz	8 ppm	Mills and Bearman
1977	203384(4) MHz	20 ppm	Carlson, Hughes, and Lindgren
1977	203384.9(1.2) MHz	6 ppm	Egan, Hughes, and Yam
1983	203387.5(1.6) MHz	8 ppm	Mills (“Line-shape effects”)
1984	203389.10(74) MHz	3.6 ppm	Ritter, Egan, Hughes, and Woodle
2014	203394.2 $(1.6)_{stat}(1.3)_{sys}$ MHz	10 ppm	Ishida, Namba, Asai, Kobayashi, Saito, Yoshida, Tanaka, and Yamamoto

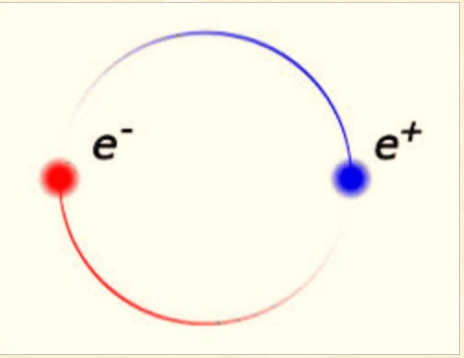


Hyperfine Structure

This table shows the two main periods of experimental work on the positronium hyperfine interval: the 1950s and the 1970s. The present era promises to be a third such exciting period of progress.

TABLE I: Experimental Results for the Positronium Hyperfine Interval

Year	Frequency Shift	Precision	Experimenters
1951	227(34) GHz	15%	Deutsch and Dulit
1952	203.2(3) GHz	1477 ppm	Deutsch and Brown
1954	203350(50) MHz	246 ppm	Weinstein, Deutsch, and Brown
1955	203300(40) MHz	197 ppm	Weinstein, Deutsch, and Brown
1957	203330(40) MHz	197 ppm	Hughes, Marder, and Wu
1970	203403(12) MHz	59 ppm	Theriot, Beers, Hughes, and Zioch
1972	203396(5) MHz	25 ppm	Carlson, Hughes, Lewis, and Lindgren
1975	203387.0(1.6) MHz	8 ppm	Mills and Bearman
1977	203384(4) MHz	20 ppm	Carlson, Hughes, and Lindgren
1977	203384.9(1.2) MHz	6 ppm	Egan, Hughes, and Yam
1983	203387.5(1.6) MHz	8 ppm	Mills ("Line-shape effects")
1984	203389.10(74) MHz	3.6 ppm	Ritter, Egan, Hughes, and Woodle
2014	203394.2 $(1.6)_{stat}(1.3)_{sys}$ MHz	10 ppm	Ishida, Namba, Asai, Kobayashi, Saito, Yoshida, Tanaka, and Yamamoto

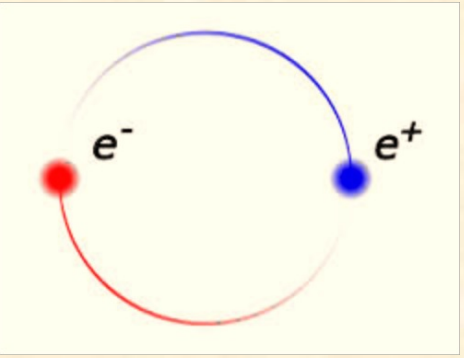


Hyperfine Structure

This table shows the two main periods of experimental work on the positronium hyperfine interval: the 1950s and the 1970s. The present era promises to be a third such exciting period of progress.

TABLE I: Experimental Results for the Positronium Hyperfine Interval

Year	Frequency Shift	Precision	Experimenters
1951	227(34) GHz	15%	Deutsch and Dulit
1952	203.2(3) GHz	1477 ppm	Deutsch and Brown
1954	203350(50) MHz	246 ppm	Weinstein, Deutsch, and Brown
1955	203300(40) MHz	197 ppm	Weinstein, Deutsch, and Brown
1957	203330(40) MHz	197 ppm	Hughes, Marder, and Wu
1970	203403(12) MHz	59 ppm	Theriot, Beers, Hughes, and Zioch
1972	203396(5) MHz	25 ppm	Carlson, Hughes, Lewis, and Lindgren
1975	203387.0(1.6) MHz	8 ppm	Mills and Bearman
1977	203384(4) MHz	20 ppm	Carlson, Hughes, and Lindgren
1977	203384.9(1.2) MHz	6 ppm	Egan, Hughes, and Yam
1983	203387.5(1.6) MHz	8 ppm	Mills ("Line-shape effects")
1984	203389.10(74) MHz	3.6 ppm	Ritter, Egan, Hughes, and Woodle
2014	203394.2 $(1.6)_{stat}(1.3)_{sys}$ MHz	10 ppm	Ishida, Namba, Asai, Kobayashi, Saito, Yoshida, Tanaka, and Yamamoto

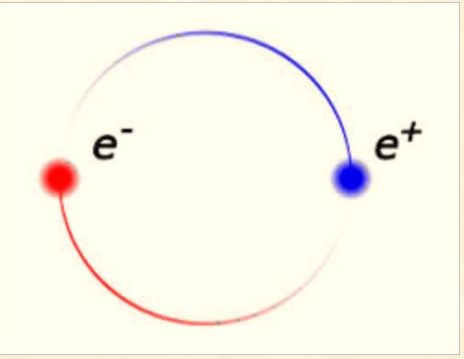


Hyperfine Structure

This table shows the two main periods of experimental work on the positronium hyperfine interval: the 1950s and the 1970s. The present era promises to be a third such exciting period of progress.

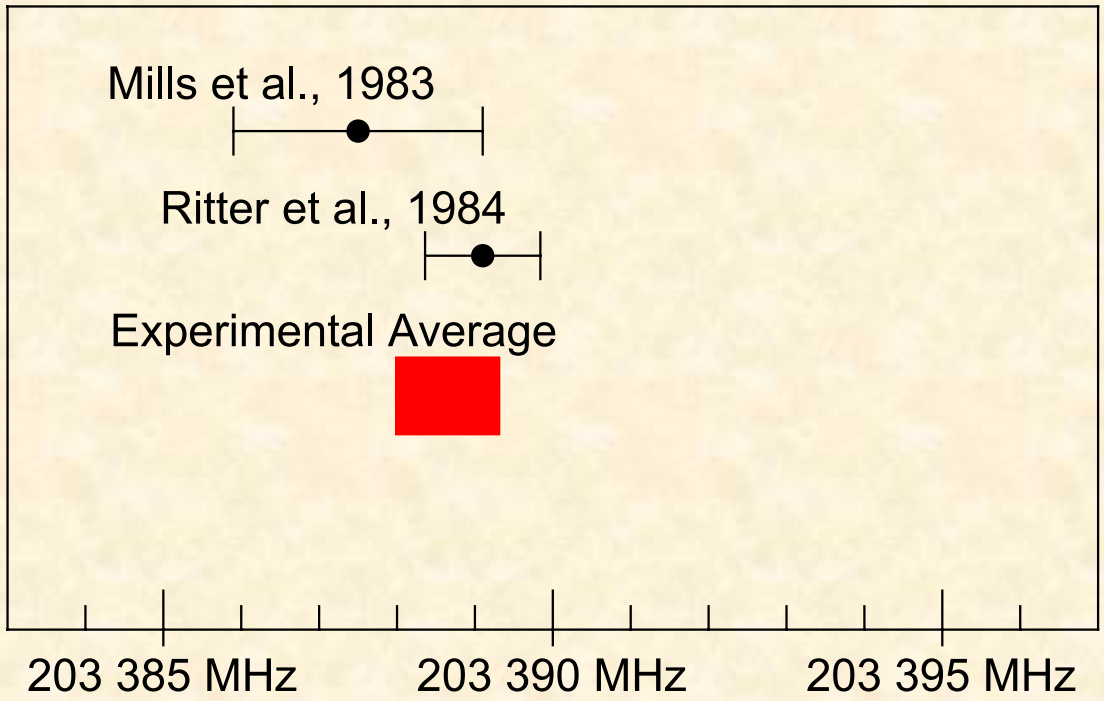
TABLE I: Experimental Results for the Positronium Hyperfine Interval

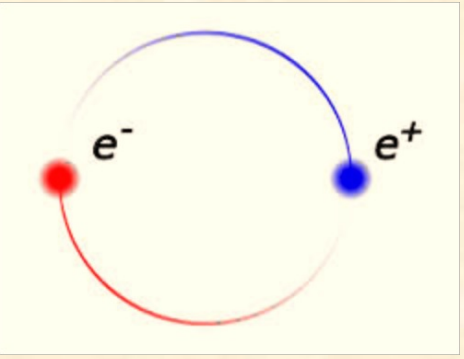
Year	Frequency Shift	Precision	Experimenters
1951	227(34) GHz	15%	Deutsch and Dulit
1952	203.2(3) GHz	1477 ppm	Deutsch and Brown
1954	203350(50) MHz	246 ppm	Weinstein, Deutsch, and Brown
1955	203300(40) MHz	197 ppm	Weinstein, Deutsch, and Brown
1957	203330(40) MHz	197 ppm	Hughes, Marder, and Wu
1970	203403(12) MHz	59 ppm	Theriot, Beers, Hughes, and Zioch
1972	203396(5) MHz	25 ppm	Carlson, Hughes, Lewis, and Lindgren
1975	203387.0(1.6) MHz	8 ppm	Mills and Bearman
1977	203384(4) MHz	20 ppm	Carlson, Hughes, and Lindgren
1977	203384.9(1.2) MHz	6 ppm	Egan, Hughes, and Yam
1983	203387.5(1.6) MHz	8 ppm	Mills ("Line-shape effects")
1984	203389.10(74) MHz	3.6 ppm	Ritter, Egan, Hughes, and Woodle
2014	203394.2 $(1.6)_{stat}(1.3)_{sys}$ MHz	10 ppm	Ishida, Namba, Asai, Kobayashi, Saito, Yoshida, Tanaka, and Yamamoto



Hyperfine Structure

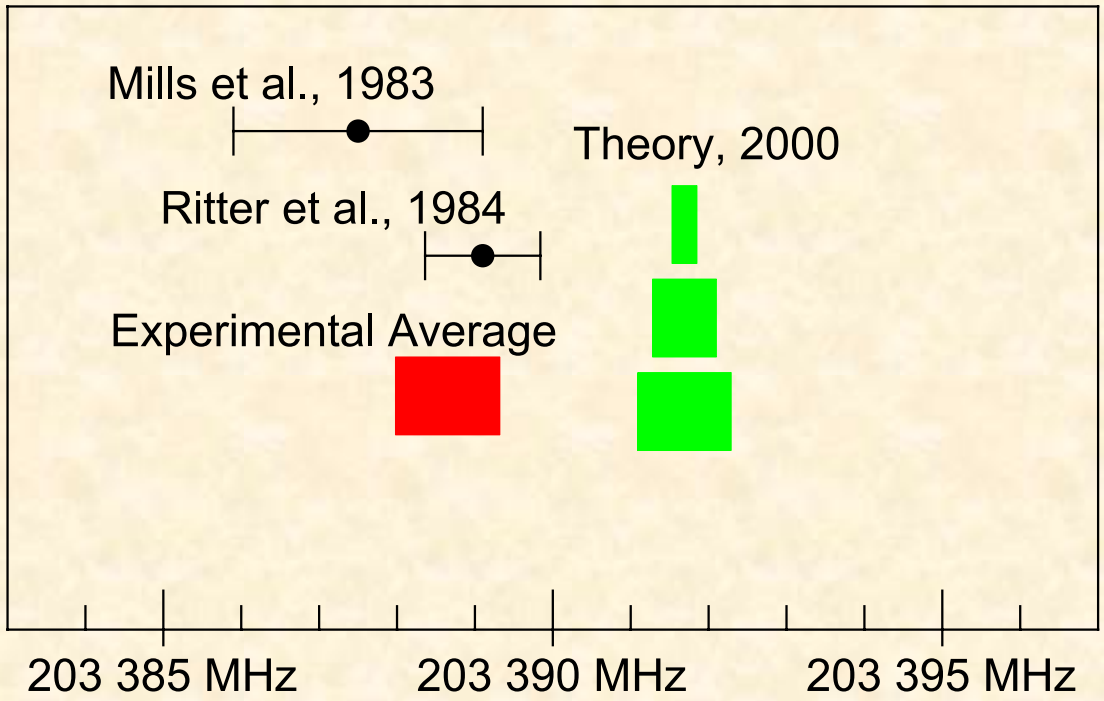
The experimental situation for the hyperfine structure is somewhat problematic. The experimental results shown are the result of many years of work of increasing precision, completed in 1984.

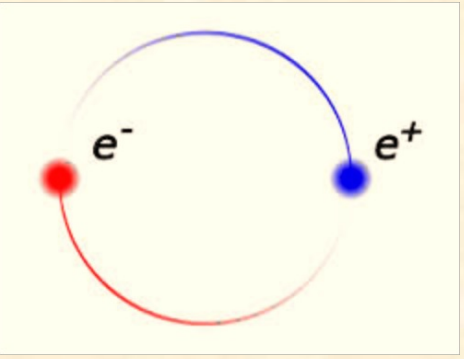




Hyperfine Structure

The experimental situation for the hyperfine structure is somewhat problematic. The experimental results shown are the result of many years of work of increasing precision, completed in 1984.



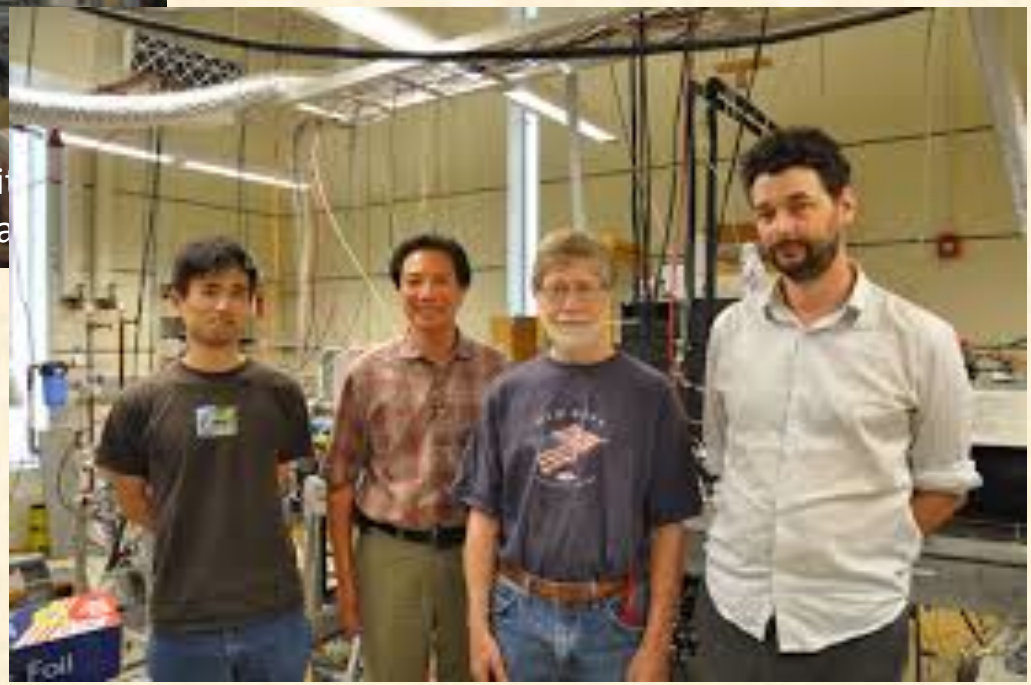


Hyperfine Structure: Experiment

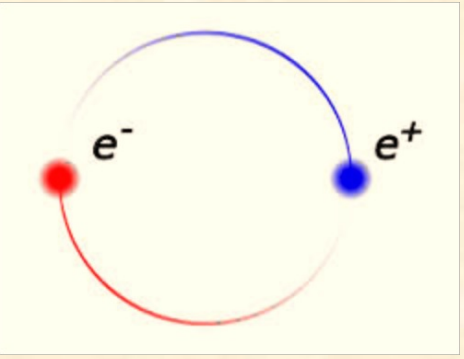
Two groups are currently pursuing measurements of the positronium hyperfine splitting: at U. Tokyo and at U.C. Riverside

Akira Miyazaki
The University of Tokyo

collaborated with
T. Yamazaki, T. Suehara,
T. Namba, S. Asai, T. Kobayashi, H. Sai
Y. Tatematsu, I. Ogawa, T. Idehara, and S. Sa

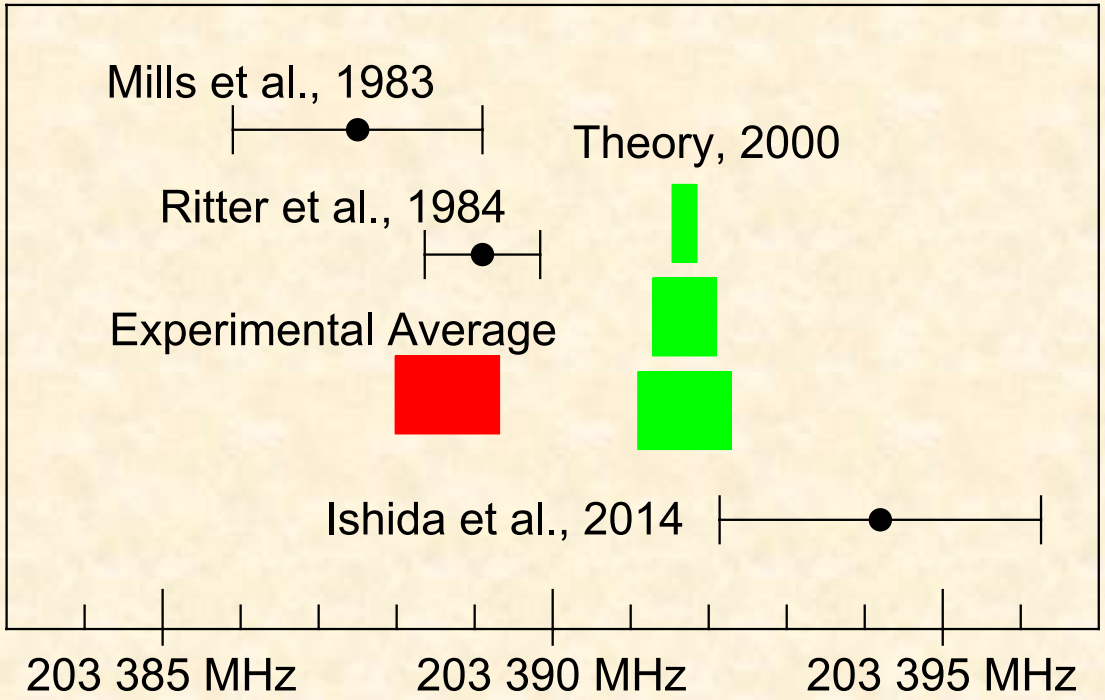


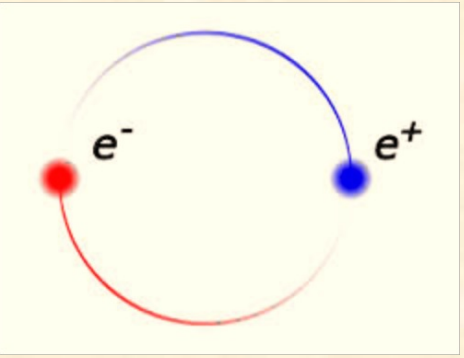
Allen Mills and collaborators
U.C. Riverside



Hyperfine Structure
Experiment vs. Theory

A recent result by the Tokyo group throws perhaps a new light on the situation. Additional experimental work with the promise of new ppm measurements is ongoing.





Hyperfine Structure

The fine and hyperfine structure comes from small corrections to the energy levels due to relativity, magnetic effects, recoil and virtual annihilation.

e.g. Hyperfine Structure

From the electron's perspective, there is a magnetic field produced by the positron due to the positron magnetic moment:

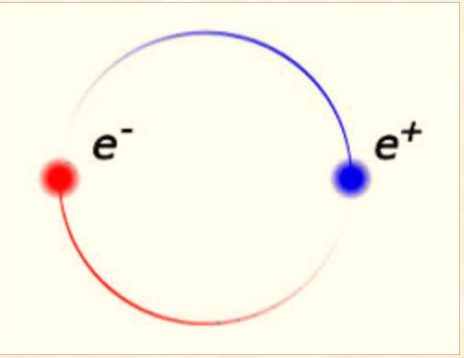
$$\vec{B} \propto \vec{\mu}_p \propto \vec{S}_p$$

The interaction energy is

$$H_{\mathrm{HFS}} = -\vec{\mu}_e \cdot \vec{B} \propto \vec{S}_e \cdot \vec{S}_p$$

This interaction separates o-Ps from p-Ps. This "Fermi splitting" is

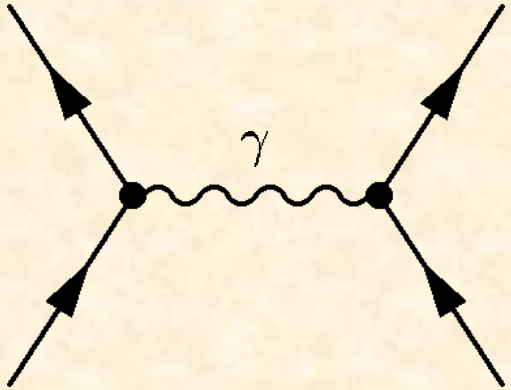
$$\Delta E_F = \frac{1}{3} m\alpha^4$$



Hyperfine Structure

Another contribution to the hyperfine structure is the process of virtual annihilation. Due to charge parity invariance, this only affects orthopositronium (which has C=-1).

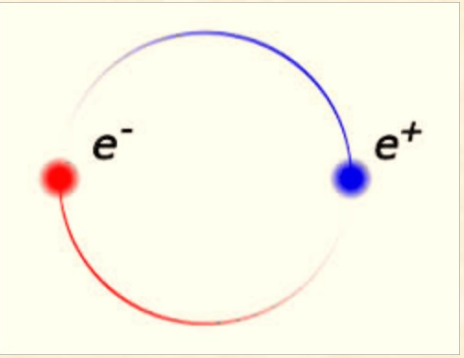
The contribution of this process to the o-Ps energy, and hence to the hyperfine splitting, is (for the n=1 triplet o-Ps state)



$$\Delta E_A = \frac{1}{4} m\alpha^4$$

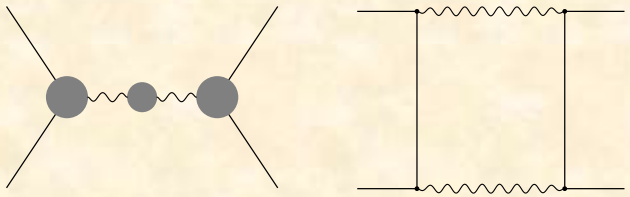
The total (lowest order) splitting is

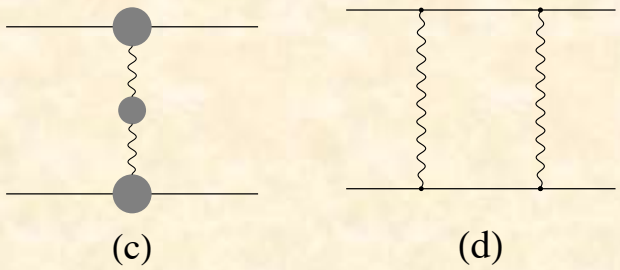
$$\Delta E_{\mathrm{LO}} = \frac{7}{12} m\alpha^4 = 204386 MHz$$



Hyperfine Structure History

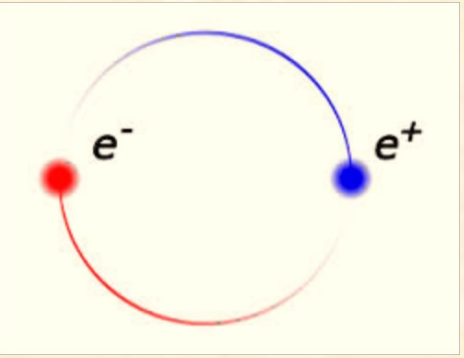
The calculation of corrections to the ps hfs commenced in 1952 with the work of Karplus and Klein. The one-loop graphs are shown below. The complete result at this order is $\left(-\frac{1}{2}\ln 2-\frac{8}{9}\right)\frac{m\alpha^5}{\pi}=-1.235\frac{m\alpha^5}{\pi}=-1005.497\,MHz$





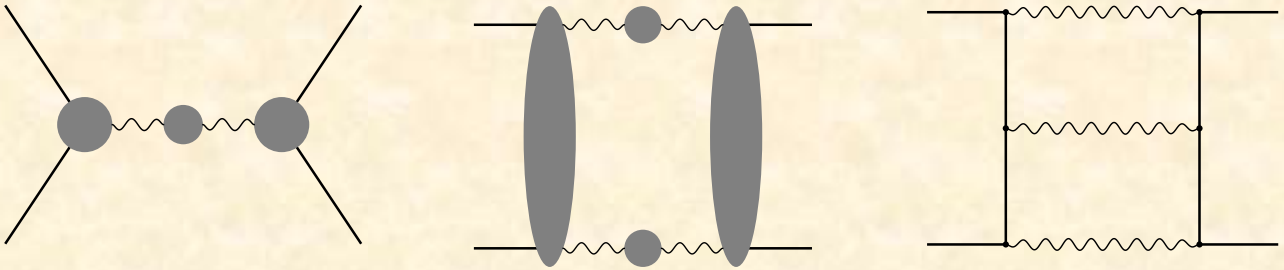
(a)	$1\gamma A$	$\left(-\frac{11}{9}\right)\frac{m\alpha^5}{\pi}$	$-1.222\frac{m\alpha^5}{\pi}$
(b)	$2\gamma A$	$\left(\frac{1}{2}-\frac{1}{2}\ln 2\right)\frac{m\alpha^5}{\pi}$	$0.153\frac{m\alpha^5}{\pi}$
(c)	$1\gamma E$	$\left(\frac{1}{3}\right)\frac{m\alpha^5}{\pi}$	$0.333\frac{m\alpha^5}{\pi}$
(d)	$2\gamma E$	$\left(-\frac{1}{2}\right)\frac{m\alpha^5}{\pi}$	$-0.500\frac{m\alpha^5}{\pi}$

[This calculation appears in the texts of Schwinger (1970) and Itzykson and Zuber (1980).]

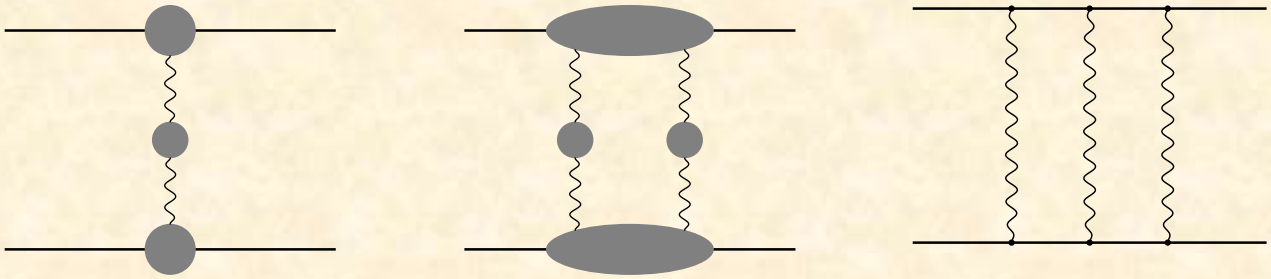


Hyperfine Structure History

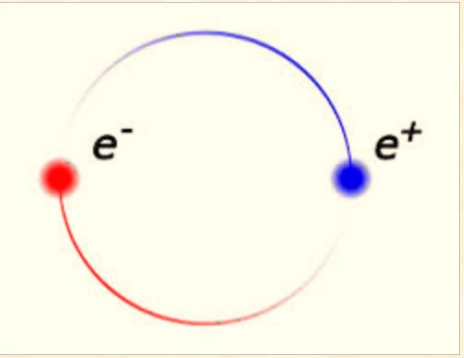
The calculation of two-loop $(O(\alpha^2))$ corrections to the ps hfs began in 1970 and was completed by 1998 and involved many workers.



(a) (b) (c)



(d) (e) (f)



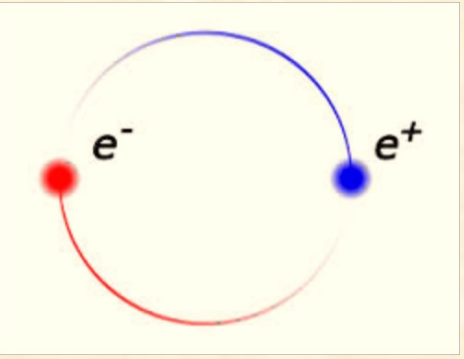
Hyperfine Structure History

The first contributions to be successfully completed were the two-loop logs, done by 1979.

$$1\gamma A \quad \frac{1}{24}\ln\left(\frac{1}{\alpha}\right)m\alpha^6 \quad 3.825\,MHz$$

$$3\gamma E \quad \frac{1}{6}\ln\left(\frac{1}{\alpha}\right)m\alpha^6 \quad 15.300\,MHz$$

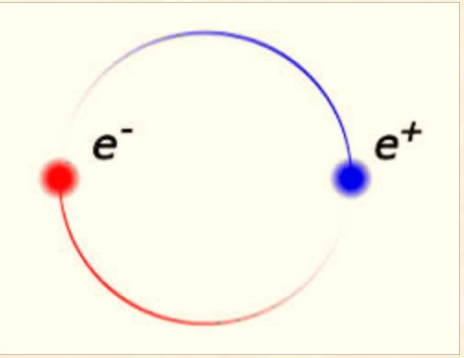
$$\text{total} \quad \frac{5}{24}\ln\left(\frac{1}{\alpha}\right)m\alpha^6 \quad 19.125\,MHz$$



Hyperfine Structure History

The two-loop constants were completed by 1998:

$1\gamma A$	$-1.2401 m\alpha^6/\pi^2$	$-2.344\,MHz$	1997
$2\gamma A$	$-0.3206 m\alpha^6/\pi^2$	$-0.606\,MHz$	1993
$3\gamma A$	$-0.5126 m\alpha^6/\pi^2$	$-0.969\,MHz$	1988
$1\gamma E$	$-0.1356 m\alpha^6/\pi^2$	$-0.256\,MHz$	–
$2\gamma E$	$-5.3824 m\alpha^6/\pi^2$	$-10.175\,MHz$	1983, 1998
$3\gamma E$	$3.7141 m\alpha^6/\pi^2$	$7.021\,MHz$	1997
total	$-3.8771 m\alpha^6/\pi^2$	$-7.329\,MHz$	1998

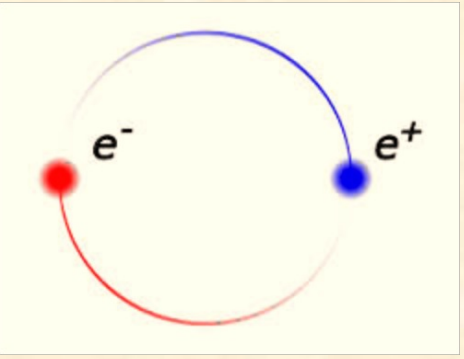


Hyperfine Structure History

The analytic value for the two-loop correction was obtained in 1999 by Czarnecki, Melnikov, and Yelkhovsky.

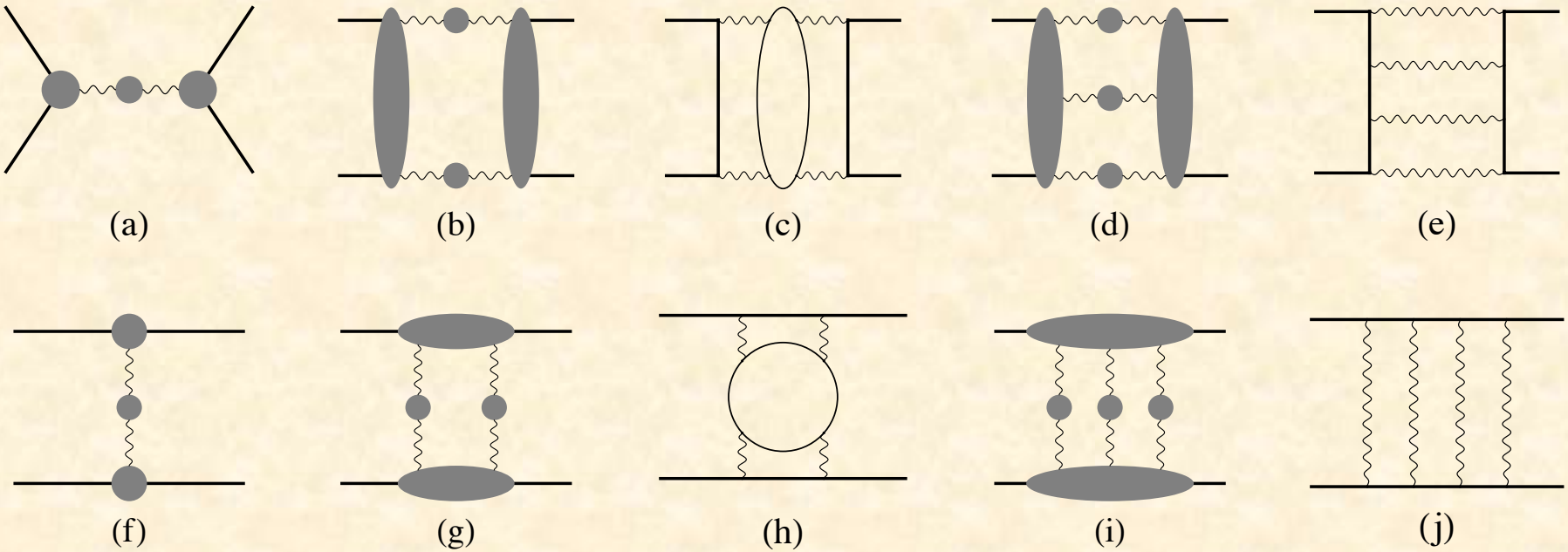
$$\Delta E_{20} = \left\{ -\frac{53}{32}\zeta(3) + \frac{221}{24}\zeta(2)\ln 2 - \frac{5197}{576}\zeta(2) + \frac{1}{2}\ln 2 + \frac{1367}{648} \right\} \frac{m\alpha^6}{\pi^2}$$

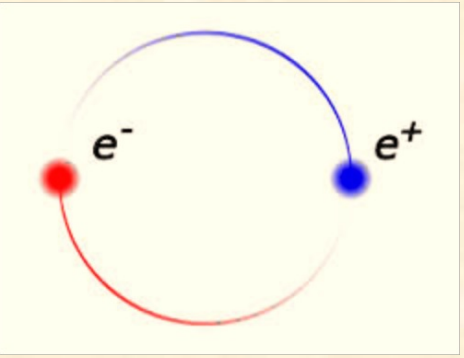
$$= -3.8771 \frac{m\alpha^6}{\pi^2} = -7.329 MHz$$



Hyperfine Structure
Three-Loop Contributions

The present challenge for the positronium hyperfine structure is to complete the calculation of all three-loop corrections. A schematic representation of this set of graphs is shown below.





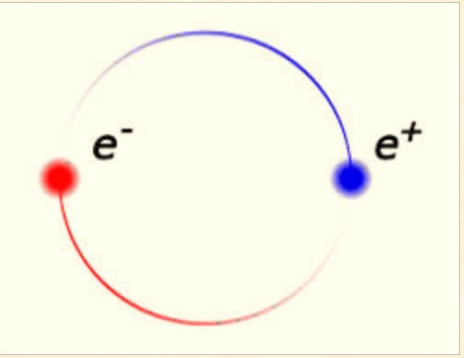
Hyperfine Structure
Three-Loop Contributions

The three loop logs have already been done.

$$-\frac{7}{8}\ln^2\left(\frac{1}{\alpha}\right)\frac{m\alpha^7}{\pi} \qquad -0.918\,MHz \qquad \text{Karshenboim (1993)}$$

$$\left(-\frac{17}{3}\ln 2+\frac{217}{90}\right)\ln\left(\frac{1}{\alpha}\right)\frac{m\alpha^7}{\pi} \qquad -0.323\,MHz$$

Kniehl & Penin (2000)
Melnikov &Yelkhovsky (2001)
Hill (2001)



Hyperfine Structure
Status as of 2000

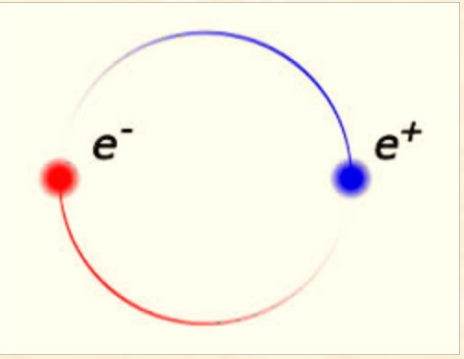
The theoretical formula for the hfs can be written as

$$\Delta E = m\alpha^4\left\{C_0 + C_1\frac{\alpha}{\pi} + C_{21}\alpha^2\ln\left(\frac{1}{\alpha}\right) + C_{20}\left(\frac{\alpha}{\pi}\right)^2 + C_{32}\frac{\alpha^3}{\pi}\ln^2\left(\frac{1}{\alpha}\right) + C_{31}\frac{\alpha^3}{\pi}\ln\left(\frac{1}{\alpha}\right) + C_{30}\left(\frac{\alpha}{\pi}\right)^3 + \cdots\right\}$$

where all terms through the three loop logs were known by 2000.
The numerical value was found to be

$$\Delta E = 203389.69 MHz$$

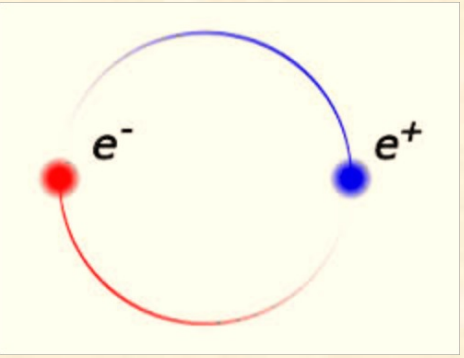
with an uncertainty variously estimated to be 0.16MHz to 0.6MHz.
This uncertainty is comparable to the experimental uncertainty, and should be reduced by computing the three-loop non-log contributions.



Hyperfine Structure
Status as of 2000

The various known contributions to the energy splitting are as shown:

Coefficient	Term	Contribution
$C_0 = 7/12 = 0.583$	$m\alpha^4$	$204386.631 MHz$
$C_1 = -1.235$	$m\alpha^5/\pi$	$-1005.497 MHz$
$C_{21} = 0.208$	$m\alpha^6 \ln(1/\alpha)$	$19.125 MHz$
$C_{20} = -3.877$	$m\alpha^6/\pi^2$	$-7.329 MHz$
$C_{32} = -0.875$	$m(\alpha^7/\pi)\ln^2(1/\alpha)$	$-0.918 MHz$
$C_{31} = -1.517$	$m(\alpha^7/\pi)\ln(1/\alpha)$	$-0.323 MHz$
total		$203391.69 MHz$

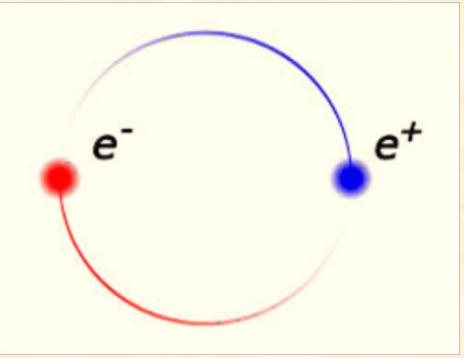


Hyperfine Structure
Status as of 2000

The natural order of ("hard") three-loop corrections is

$$\frac{m\alpha^7}{\pi^3} \approx 0.00439 MHz$$

None of the C's is particularly big—the largest being C_{20}=-3.877. In order to describe a correction that amounts to 0.6MHz, C_{30} would have to be quite large—about $14\pi^2 \approx 138$. Is such a large C possible? Apparently it is.



Hyperfine Structure
Three-Loop Contributions

The first work on three-loop constants was by Simona Marcu (Master of Science thesis, University of Alberta, 2011), working under the direction of Alexander Penin, using the techniques of NRQED/pNRQED.

Typical energies and momenta of Coulombic bound states satisfy

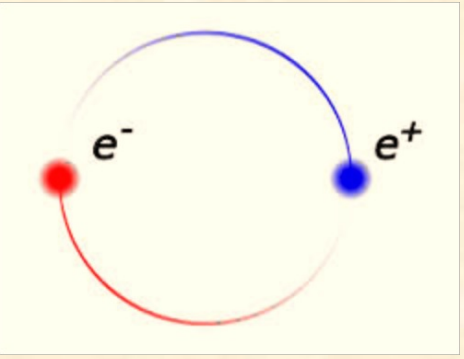
$$E_{\rm bound} \sim m_e\alpha^2 \; ; \; p_{\rm bound} \sim m_e\alpha \;\; ({\rm so\ r} \sim 1/({\rm m_e}\alpha))$$

Exchange photon types:

Hard photon: $$E, k \sim m_e \quad \Rightarrow \quad V(t,r) \sim \delta(t)\delta(r)$$

Soft photon: $$E, k \sim \alpha m_e \quad \Rightarrow \quad V(t,r) \sim \delta(t)V(r)$$

Ultrasoft photon: $$E, k \sim \alpha^2 m_e \quad \Rightarrow \quad V(t,r) \sim V(t,r) \quad ({\rm but} \;\; kr << 1)$$

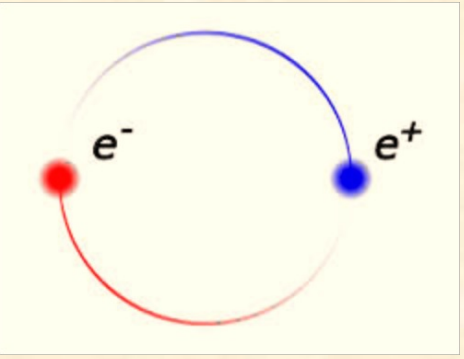


Hyperfine Structure
Three-Loop Contributions

Marcu evaluated the contribution of ultrasoft photon exchange using dimensional regularization in the $\overline{\rm MS}$ scheme (which implies that the associated hard and soft contributions must be computed in the same scheme). Her result was a contribution to the hfs of

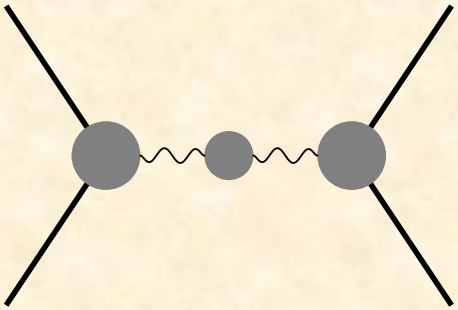
$$\Delta E = \frac{7}{12} m\alpha^4 \times \frac{\alpha^3}{\pi} \{18.8646(17)\} = 0.477 MHz$$

(3/7 of this can be attributed to the annihilation channel and 4/7 to the exchange channel.)



Hyperfine Structure
Three-Loop Contributions

The one-photon-annihilation contribution was obtained this year by Baker, Marquard, Penin, Piclum, and Steinhauser [PRL 112, 120401 (2014)]. Their result is dominated by the ultrasoft contribution, which is already present in the result of Marcu. They used results for the multi-loop diagrams that were recently obtained for QCD.



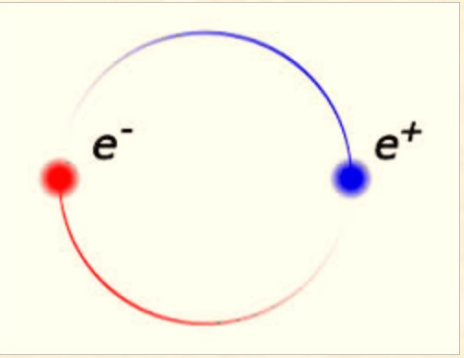
Their result is:

$$\Delta E = \frac{7}{12} m\alpha^4 \times \left(\frac{\alpha}{\pi}\right)^3 \{84.8(5)\} = 0.2174(13) MHz$$

of which the ultrasoft contribution is:

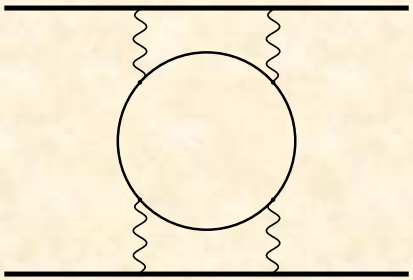
$$\Delta E_{\rm us} = \frac{7}{12} m\alpha^4 \times \left(\frac{\alpha}{\pi}\right)^3 \{79.79\} = 0.2044 MHz$$

in agreement with the result of Marcu.



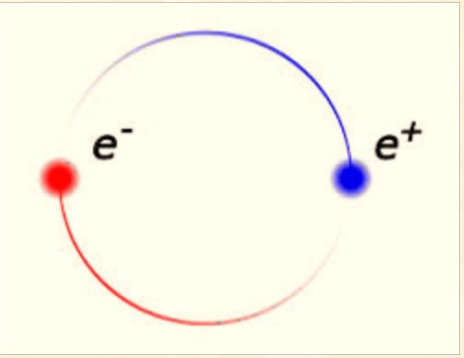
Hyperfine Structure
Three-Loop Contributions

We (Adkins and Fell) calculated one set of three-loop graphs—those involving the exchange of two photons that undergo a light-by-light scattering process [PRA 89, 052518 (2014)]. For this contribution various apparent IR and UV divergences cancel. All momenta can be taken to be hard, so this set of light-by-light graphs contributes at $O(m\alpha^7/\pi^3)$.



The contribution of this set of graphs is

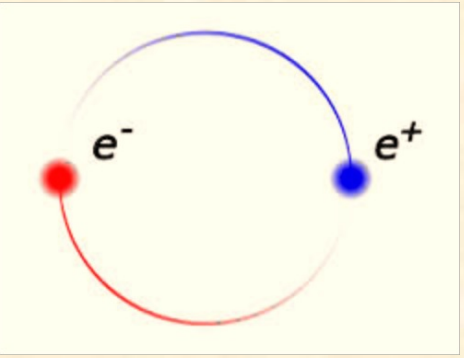
$$\Delta E = -0.2354\frac{m\alpha^7}{\pi^3} = -1.033kHz$$



Hyperfine Structure
Three-Loop Contributions

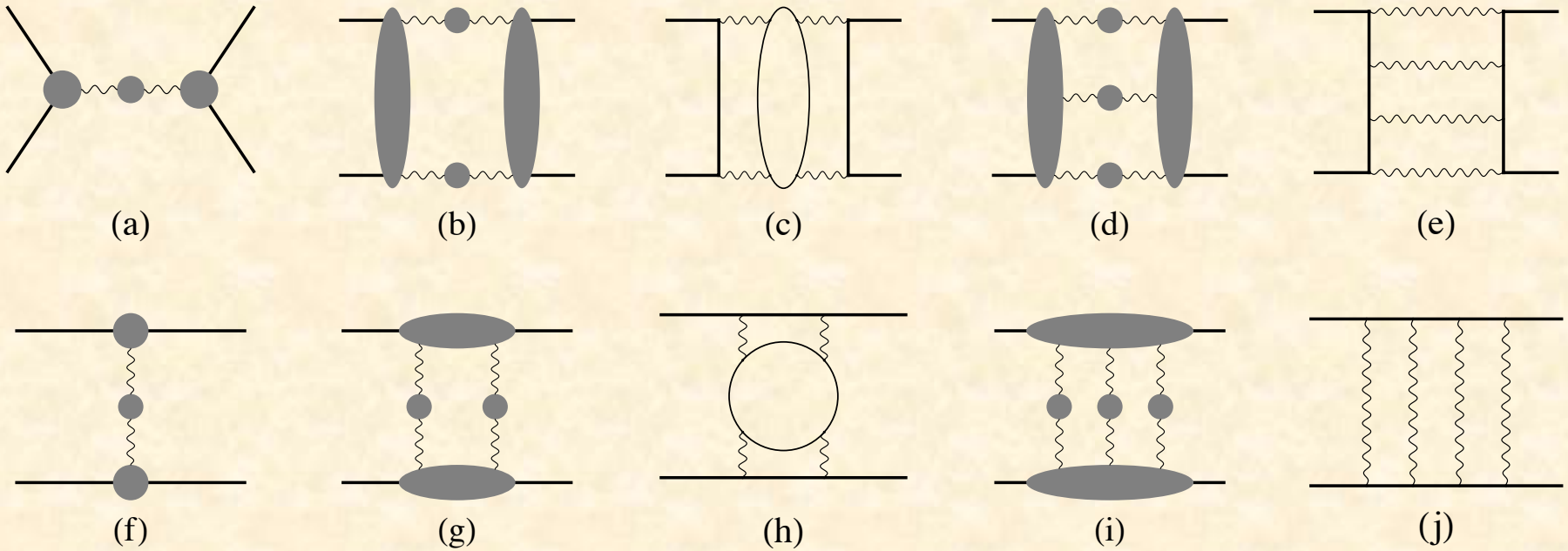
Eides and Shelyuto have obtained results for several gauge-invariant sets of graphs that contribute in the two-photon-exchange channel [Phys. Rev. D 89, 111301(R), 2014]. These contributions involve "hard" photons exclusively. They also evaluated the light-by-light scattering contribution and verified the previous result. The net contribution of all such graphs is

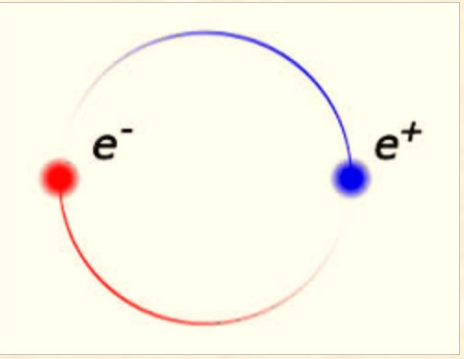
$$\Delta E = -1.2917(1)\frac{m\alpha^7}{\pi^3} = -5.672kHz$$



Hyperfine Structure
Three-Loop Contributions

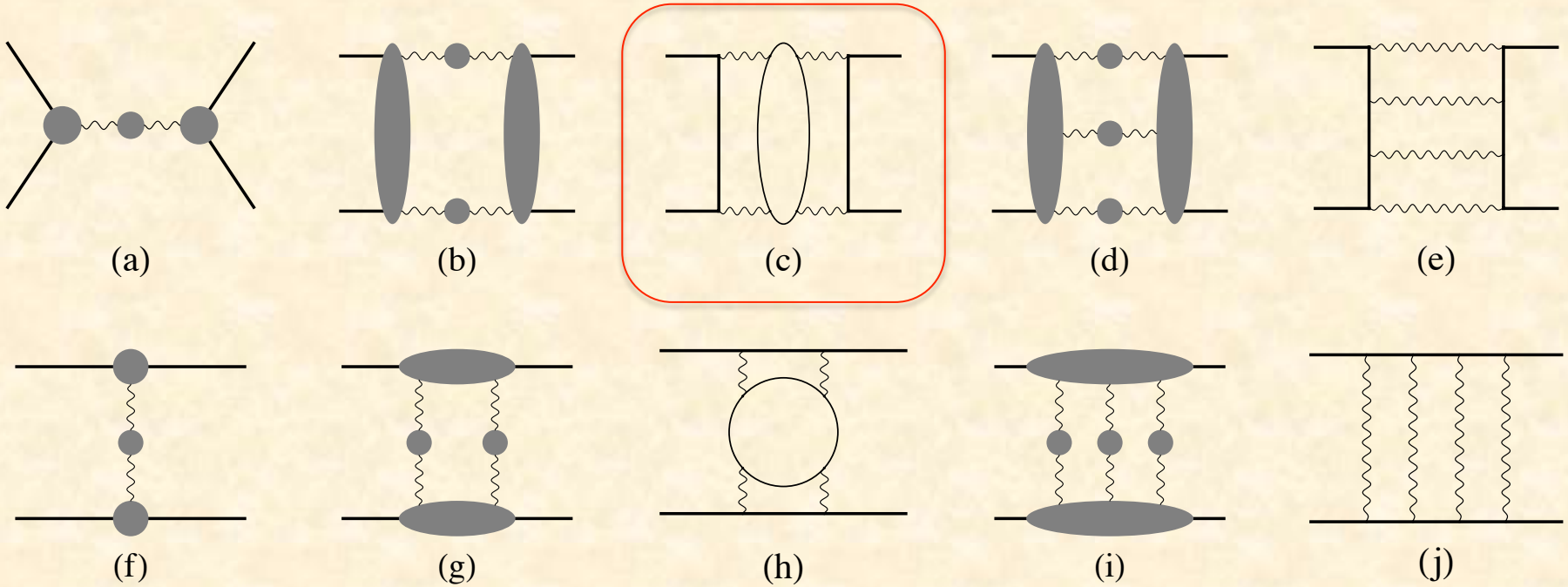
We have recently obtained a preliminary value for another set of three-loop graphs—those involving light-by-light scattering in the annihilation channel (graphs (c) below).

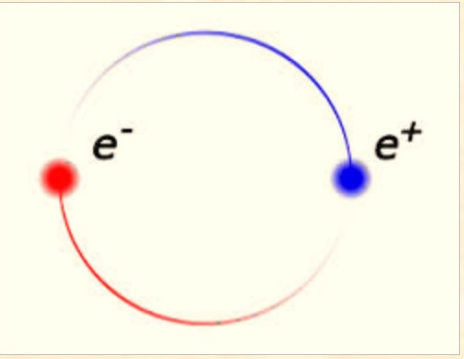




Hyperfine Structure
Three-Loop Contributions

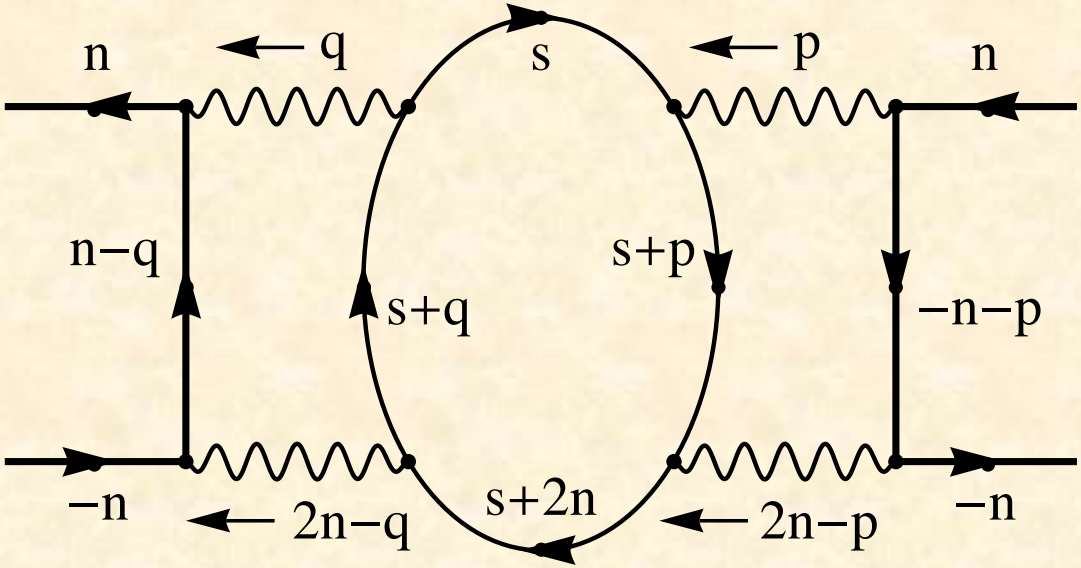
We have recently obtained a preliminary value for another set of three-loop graphs—those involving light-by-light scattering in the annihilation channel (graphs (c) below).

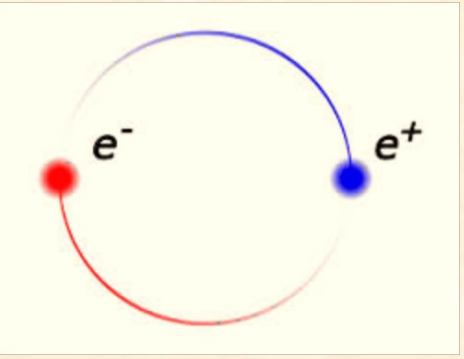




Hyperfine Structure
Three-Loop Contributions

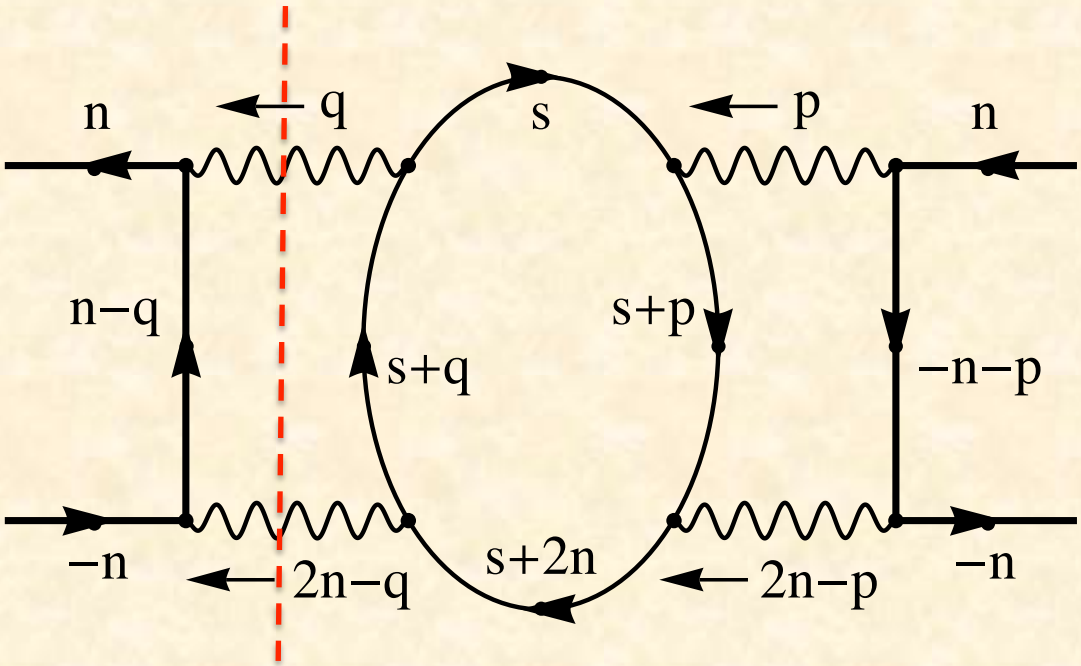
This graph (and its partners with crossed photons) is interesting because it contains an imaginary part—that was confirmed to reproduce the known decay-rate correction due to light-by-light scattering.

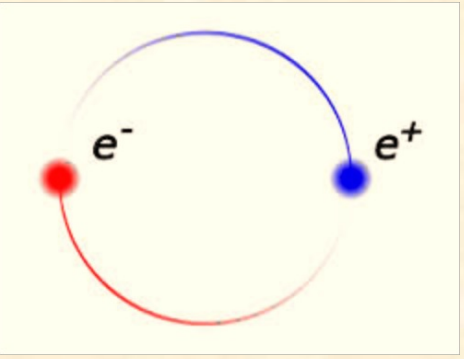




Hyperfine Structure
Three-Loop Contributions

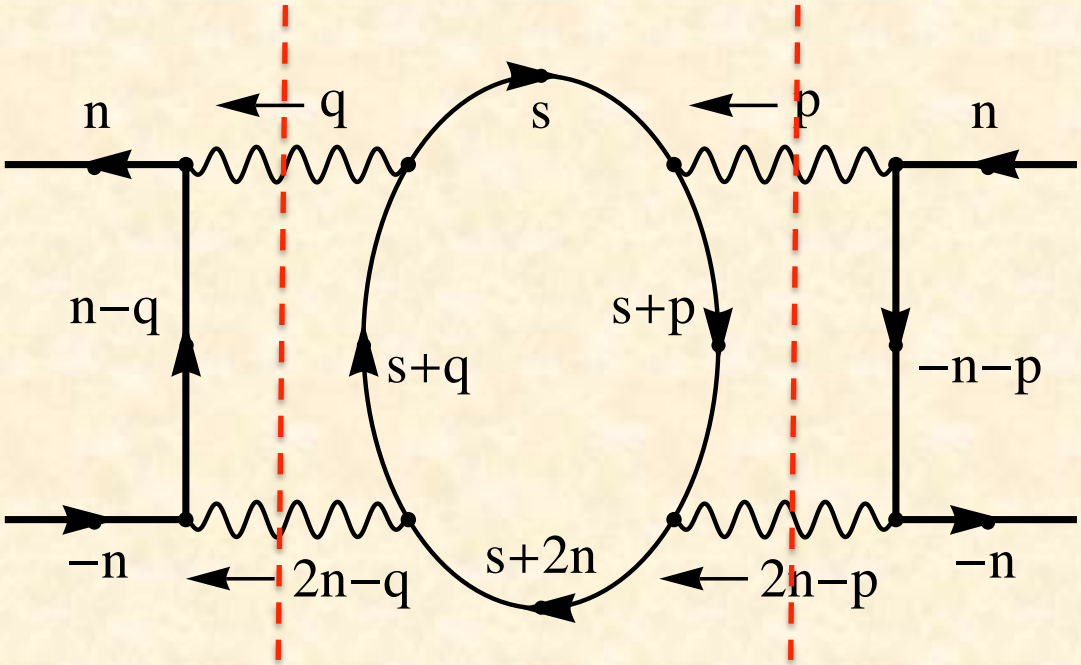
This graph (and its partners with crossed photons) is interesting because it contains an imaginary part—that was confirmed to reproduce the known decay-rate correction due to light-by-light scattering.

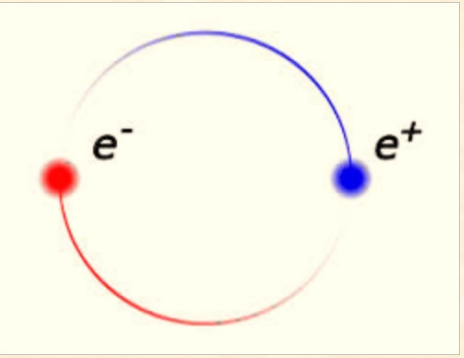




Hyperfine Structure
Three-Loop Contributions

This graph (and its partners with crossed photons) is interesting because it contains an imaginary part—that was confirmed to reproduce the known decay-rate correction due to light-by-light scattering.





Hyperfine Structure
Three-Loop Contributions

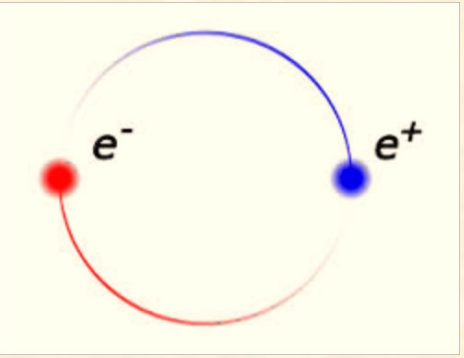
The net contribution of the light-by-light scattering graph in the annihilation channel is

$$\Delta E = \{1.58377(8) - 1.01626(2)\, i\}\, \frac{m\alpha^7}{\pi^3}$$

giving a contribution to the hfs of -6.95kHz and a decay rate correction

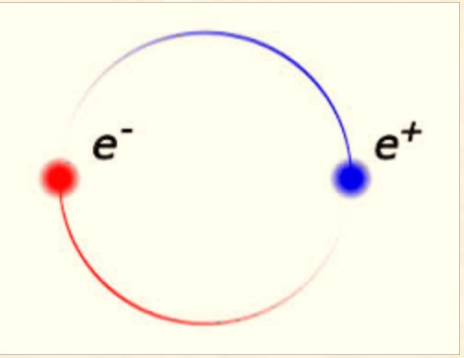
$$\Delta\Gamma = 1.294 \left(\frac{\alpha}{\pi}\right)^2 \Gamma_0(\text{p-Ps})$$

consistent with the known result.
(Adkins, Fell, Parsons, Salinger, and Wang)



Hyperfine Structure
Three-Loop Contributions

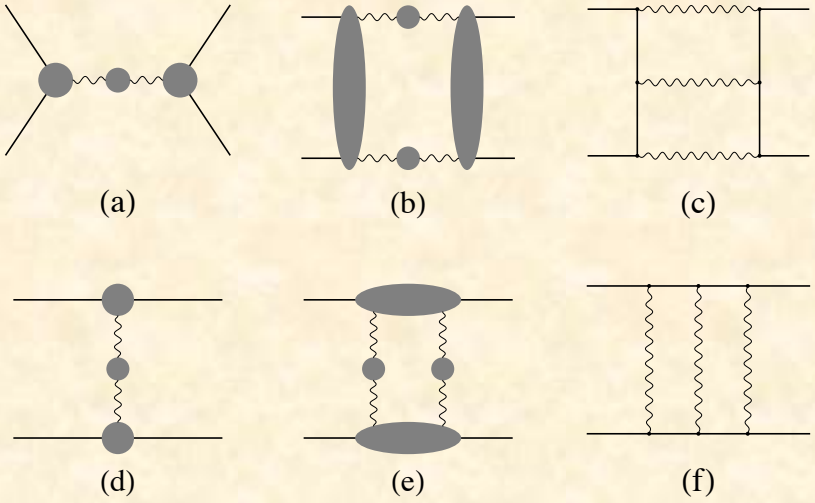
Why should we expect to be getting the correct answers?

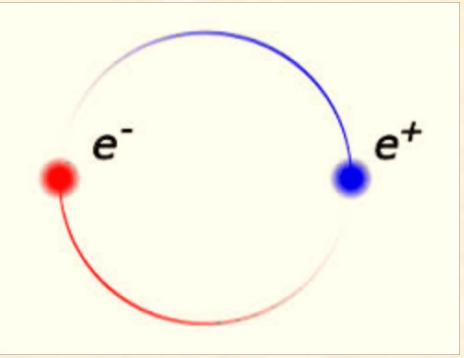


Hyperfine Structure
Three-Loop Contributions

Why should we expect to be getting the correct answers?

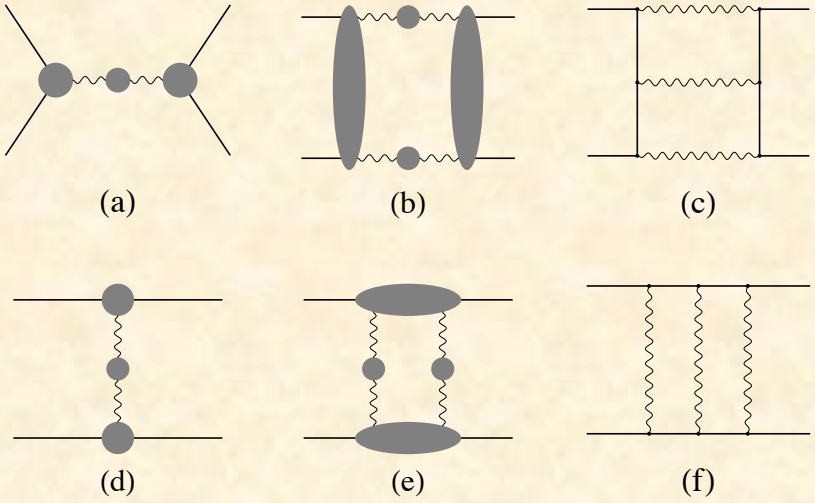
Remember what happened with the two-loop calculation…



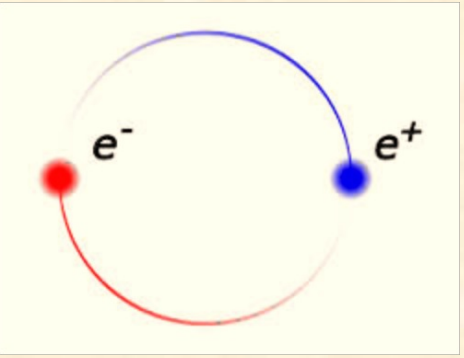


Hyperfine Structure
Three-Loop Contributions

Why should we expect to be getting the correct answers?

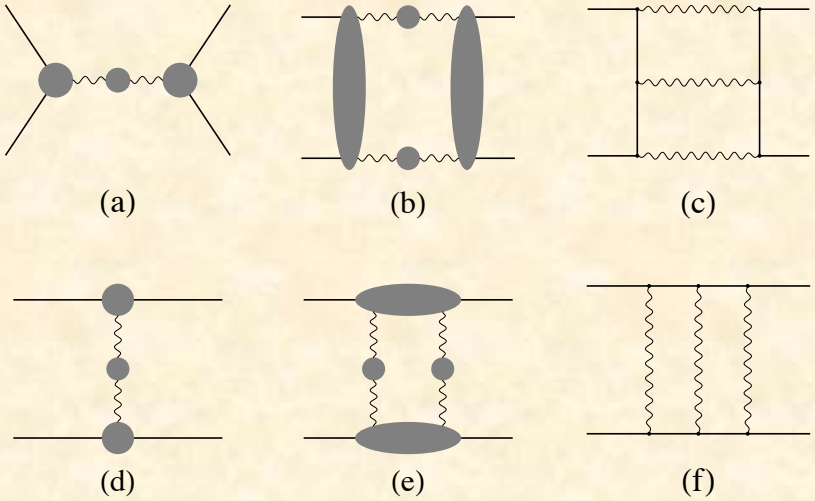


(c) Error in regularization of IR
-0.907MHz ➔ -0.969MHz



Hyperfine Structure
Three-Loop Contributions

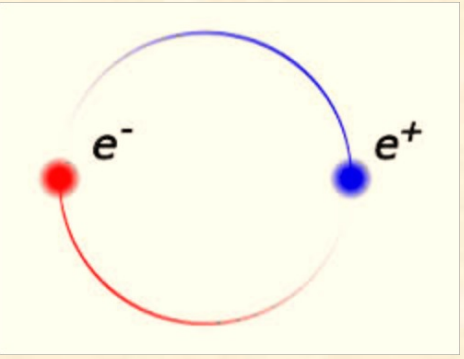
Why should we expect to be getting the correct answers?



(c) Error in regularization of IR

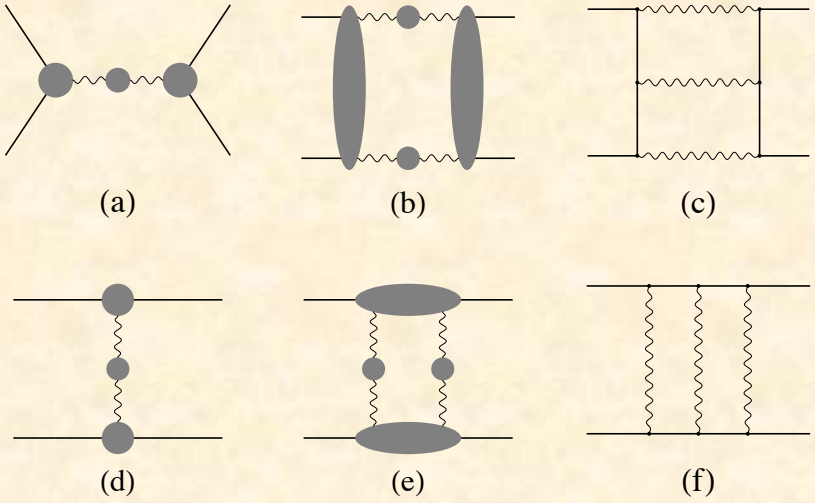
(b) Calculational mistakes

13.131MHz ➔ -0.606MHz



Hyperfine Structure
Three-Loop Contributions

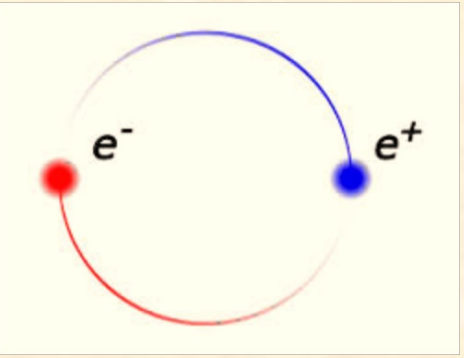
Why should we expect to be getting the correct answers?



(c) Error in regularization of IR

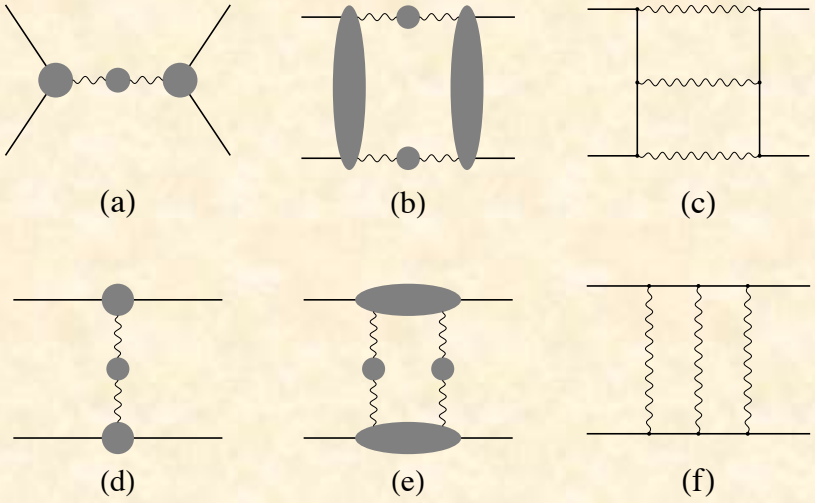
(b) Calculational mistakes

(a) Done by two groups
pre-publication interaction helpful
in sorting out differences



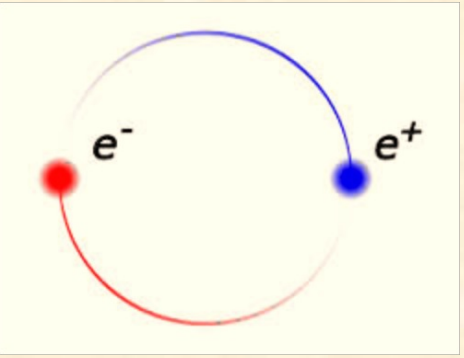
Hyperfine Structure
Three-Loop Contributions

Why should we expect to be getting the correct answers?



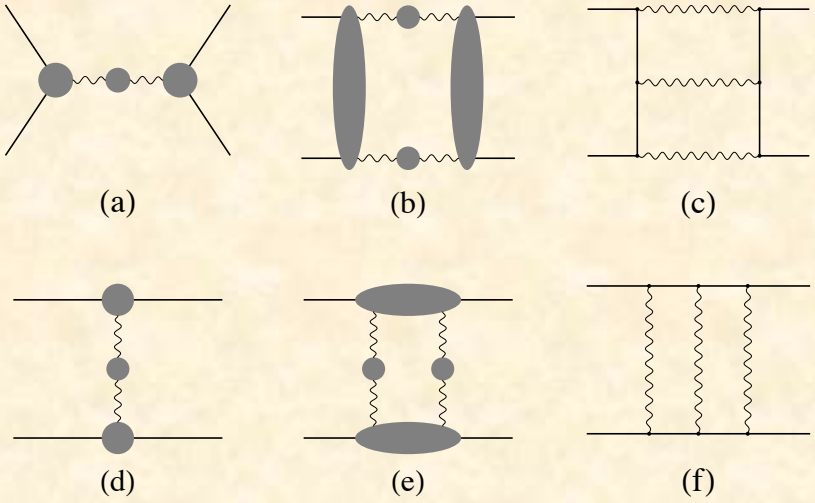
(c) Error in regularization of IR
(b) Calculational mistakes
(a) Done by two groups

(f) Three calculations at least, two incorrect.
3.12(66)MHz; 7.03(3)MHz; 1.32(7)MHz
➔ 7.021MHz



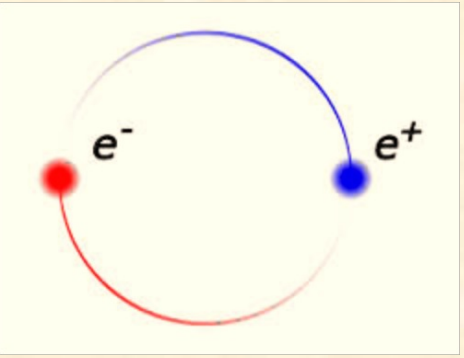
Hyperfine Structure
Three-Loop Contributions

Why should we expect to be getting the correct answers?



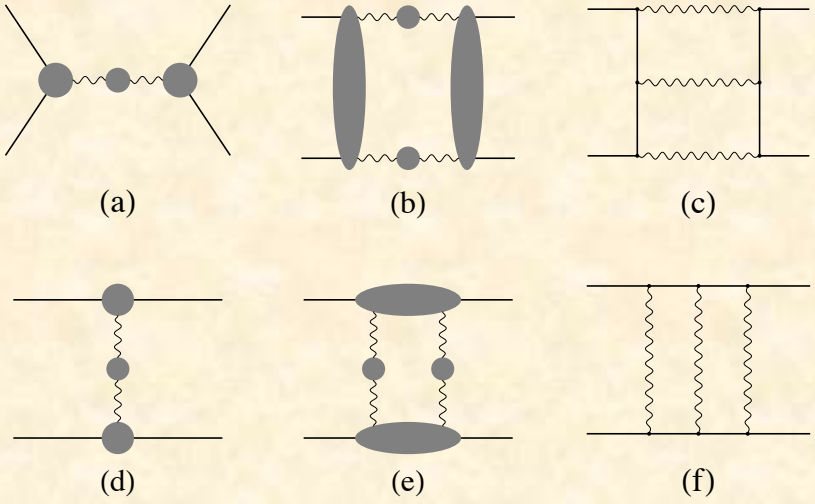
(c) Error in regularization of IR
(b) Calculational mistakes
(a) Done by two groups

(f) Three calculations at least, two incorrect.
(e) 4σ difference between numerical and exact values



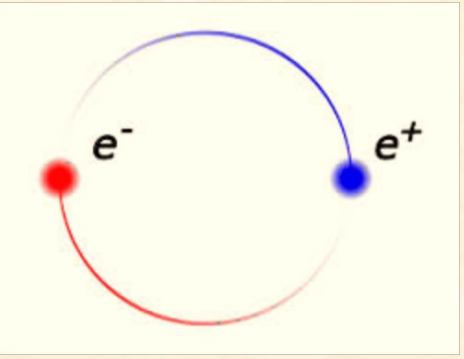
Hyperfine Structure
Three-Loop Contributions

Why should we expect to be getting the correct answers?



(c) Error in regularization of IR
(b) Calculational mistakes
(a) Done by two groups

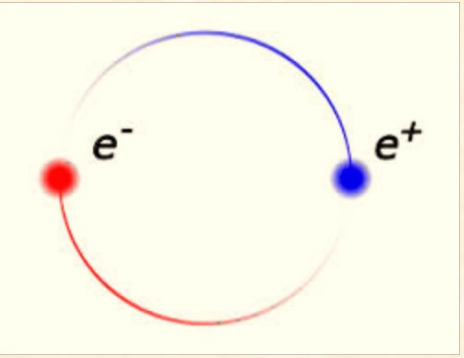
(f) Three calculations at least, two incorrect.
(e) 4σ difference between numerical and exact values
(d) Trivial



Hyperfine Structure
Three-Loop Contributions

Why should we expect to be getting the correct answers?

- Cancellation of IR and UV divergences
- Gauge invariance (terms proportional to the gauge parameter vanish)
- Trivial internal mistakes will usually cause integrals to diverge
- Comparisons to related calculations:
  - no-recoil limit of exchange terms must agree with known results (muonium, hydrogen)
  - imaginary part of annihilation terms should reproduce known decay rate corrections (orthopositronium, parapositronium)
- Computer assistance as much as possible
- Every contribution should eventually be evaluated by more than one group—preferable using at least somewhat different methods.

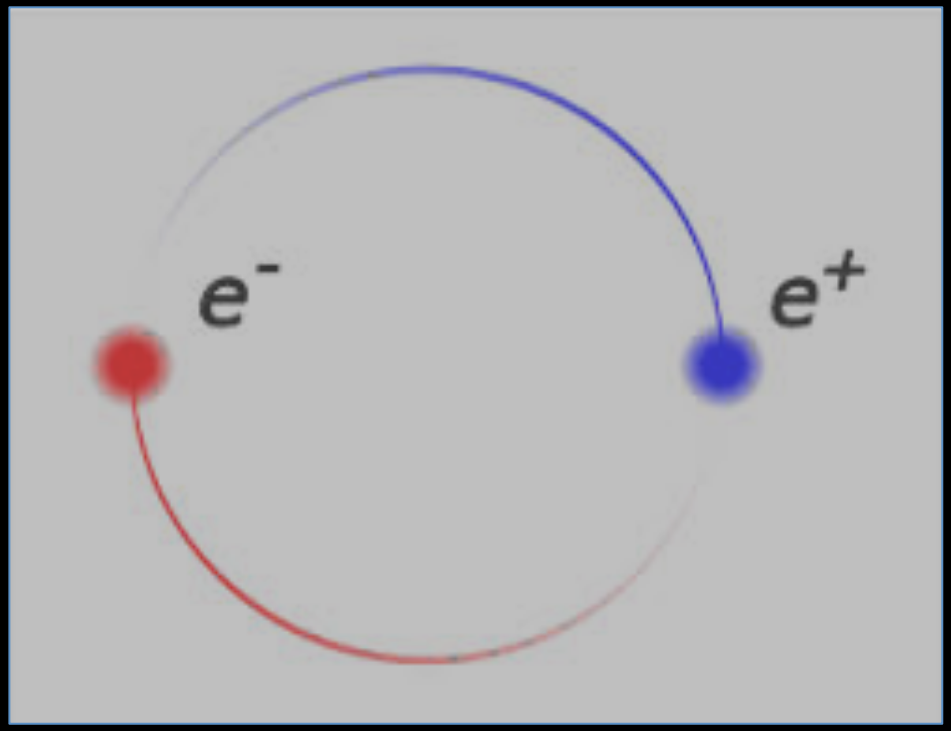


Hyperfine Structure
Three-Loop Contributions

Summary

Ultrasoft: exchange channel	272.5 kHz
Ultrasoft: annihilation channel	204.4 kHz
Additional one-photon-annihilation contribution	12.8(1.3) kHz
Anomalous moment in one-photon-exchange	3.0 kHz
Light-by-light: exchange channel	-1.0 kHz
Other two-photon-exchange contributions	-4.7 kHz
Light-by-light: annihilation channel	-6.9 kHz
Total (very incomplete)	480.1 kHz

Thank you!



Greg Adkins
Franklin & Marshall College