

Test of Einstein–Cartan’s Gravity and Dark Energy Through Non–Relativistic Approximation of Dirac Equation for Ultracold Neutrons in Static Metric Spacetimes

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Einstein's Gravity

Action and Einstein's Equations of Motion

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(-\frac{1}{2} M_{\text{Pl}}^2 R + \mathcal{L} \right) \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\mu} R = -\frac{1}{M_{\text{Pl}}^2} T_{\mu\nu}$$

Reduced Planck Mass M_{Pl}
and Energy–Momentum Tensor $T_{\mu\nu}$

$$M_{\text{Pl}} = 1/\sqrt{8\pi G_N} = 2.435 \times 10^{27} \text{ eV} \quad T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}}$$

Scalar Curvature R , Ricci Tensor $R_{\mu\nu}$,
Riemann–Christoffel Tensor $R^\alpha{}_{\mu\beta\nu}$

$$R = g^{\mu\nu} R_{\mu\nu} = g^{\mu\nu} R^\alpha{}_{\mu\alpha\nu}$$

$$R^\alpha{}_{\mu\beta\nu} = \frac{\partial \Gamma^\alpha{}_{\mu\beta}}{\partial x^\nu} - \frac{\partial \Gamma^\alpha{}_{\mu\nu}}{\partial x^\beta} + \Gamma^\varphi{}_{\mu\beta} \Gamma^\alpha{}_{\varphi\nu} - \Gamma^\varphi{}_{\mu\nu} \Gamma^\alpha{}_{\varphi\beta}$$

Affine Connection or Christoffel Symbol $\Gamma^\alpha{}_{\mu\nu} = \Gamma^\alpha{}_{\nu\mu}$

$$\Gamma^\alpha{}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(\frac{\partial g_{\beta\mu}}{\partial x^\nu} + \frac{\partial g_{\beta\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta} \right) = \{\alpha{}_{\mu\nu}\}$$

Affine Connection in Einstein–Cartan's Gravity

$$\Gamma^\alpha{}_{\mu\nu} = \{\alpha{}_{\mu\nu}\} + \frac{1}{2} g^{\alpha\lambda} (\mathcal{T}_{\lambda\mu\nu} - \mathcal{T}_{\mu\lambda\nu} - \mathcal{T}_{\nu\lambda\mu})$$

Torsion Tensor

$$\mathcal{T}^\alpha{}_{\mu\nu} = g^{\alpha\lambda} \mathcal{T}_{\lambda\mu\nu} = \Gamma^\alpha{}_{\mu\nu} - \Gamma^\alpha{}_{\nu\mu}$$

- F. W. Hehl, P. van der Heyde, and G. D. Kerlick, Rev. Mod. Phys. **48**, 393 (1976)
- S. Hojman, M. Rosenbaum, M. P. Ryan, and L. C. Shepley, Phys. Rev. D **17**, 3141 (1978)

Cosmological Constant Λ_C and Dark Energy

Einstein Equations with Cosmological Constant Λ_C

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda_C = -\frac{1}{M_{\text{Pl}}^2} T_{\mu\nu}^{(\text{m})}$$

Energy–Momentum Tensor $T_{\mu\nu}^{(\text{m})}$ of Matter with Density ρ_{m} and Pressure p_{m}

$$T_{\mu\nu}^{(\text{m})} = (\rho_{\text{m}} + p_{\text{m}}) u_\mu u_\nu - p_{\text{m}} g_{\mu\nu}$$

Einstein's Equations and Dark Energy

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{1}{M_{\text{Pl}}^2} (T_{\mu\nu}^{(\text{m})} + T_{\mu\nu}^{(\Lambda)})$$

Energy–Momentum Tensor of Dark Energy $T_{\mu\nu}^{(\Lambda)}$

$$T_{\mu\nu}^{(\Lambda)} = -p_{\Lambda} g_{\mu\nu}, \quad p_{\Lambda} = -M_{\text{Pl}}^2 \Lambda_C = -\rho_{\Lambda}$$

Dark Energy and its Characteristic Energy Scale

Hubble Constant H_0

$$H_0 = 73.8(2.4) \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1} = 1.574(51) \times 10^{-33} \text{ eV}$$

Critical Density in Universe

$$\rho_{\text{crit}} = 3 H_0^2 M_{\text{Pl}}^2$$

Fraction of Dark Energy in Universe $\approx 70\%$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\text{crit}}} = 0.73(3)$$

Density of Dark Energy in Universe

$$\rho_\Lambda = 3 \Omega_\Lambda H_0^2 M_{\text{Pl}}^2 \approx 3.2 \times 10^{-11} \text{ eV}^4 \approx 7.5 \times 10^{-30} \text{ g/cm}^3$$

Characteristic Energy Scale of Dark Energy

$$\Lambda = \sqrt[4]{\rho_\Lambda} \approx 2.4 \times 10^{-3} \text{ eV}$$

- J. Beringer *et al.*, (PDG) PRD **86**, 010001 (2012).

Einstein's Gravity, Chameleon Field and Dark Energy

Action for Einstein's Gravity with Chameleon Field

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(-\frac{1}{2} M_{\text{Pl}}^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V_{\text{eff}}(\phi) + \mathcal{L}_m[\phi] \right)$$

Ratra–Peebles Potential

$$V_{\text{eff}}(\phi) = V(\phi) + \beta \frac{\rho}{M_{\text{Pl}}} \phi = \left(\Lambda^4 + \frac{\Lambda^{4+n}}{\phi^n} \right) + \beta \frac{\rho}{M_{\text{Pl}}} \phi$$

Minimum of Chameleon Field

$$\frac{dV_{\text{eff}}(\phi)}{d\phi} \Big|_{\phi=\phi_{\min}} = 0 \longrightarrow \phi_{\min} = \Lambda \left(\frac{nM_{\text{Pl}}\Lambda^3}{\beta\rho} \right)^{\frac{1}{n+1}}$$

Effective Mass of Chameleon Field

$$m_{\text{eff}}^2 = \frac{d^2 V_{\text{eff}}(\phi)}{d\phi^2} \Big|_{\phi=\phi_{\min}} = n(n+1) \frac{\Lambda^{4+n}}{\phi_{\min}^{n+2}} = n(n+1)\Lambda^2 \left(\frac{\beta\rho}{nM_{\text{Pl}}\Lambda^3} \right)^{\frac{n+2}{n+1}}$$

- J. Khoury and A. Weltman, PRD **69**, 044026 (2004)

Einstein's Equations with Chameleon Field

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{1}{M_{\text{Pl}}^2} \left(T_{\mu\nu}^{(\text{m})} + T_{\mu\nu}^{(\phi)} \right)$$

Energy–Momentum Tensor of Chameleon Field

$$T_{\mu\nu}^{(\phi)} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi - V_{\text{eff}}(\phi) \right)$$

Density ρ_ϕ and Pressure p_ϕ
of Chameleon Field Distribution

$$p_\phi = \frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi - V_{\text{eff}}(\phi) \quad , \quad \rho_\phi = \frac{1}{2} \partial_\alpha \phi \partial^\alpha \phi + V_{\text{eff}}(\phi)$$

- E. Rebhan, in *Theoretische Physik: Relativitätstheorie und Kosmologie*, Springer – Verlag, Berlin Heidelberg 2012.

Chameleon Field, Equations of Motion and Interaction with Matter Fields

Equations of Motion of Chameleon Field

$$\square\phi = \frac{dV_{\text{eff}}(\phi)}{d\phi} = \frac{dV(\phi)}{d\phi} + \beta \frac{\rho}{M_{\text{Pl}}} = \frac{dV(\phi)}{d\phi} - \frac{dV(\phi)}{d\phi} \Big|_{\phi=\phi_{\min}}$$
$$\square\phi = \frac{n\Lambda^{4+n}}{\phi_{\min}^{n+1}} - \frac{n\Lambda^{4+n}}{\phi^{n+1}}$$

Interaction of Chameleon Field with Matter Fields (Ultracold Neutrons)

$$\mathcal{L}_m[\phi] \rightarrow V_{n-\phi} = \beta \frac{m}{M_{\text{Pl}}} \phi$$

- P. Brax and G. Pignol, PRL **107**, 111301 (2011)
- A. N. Ivanov, R. Höllwieser, T. Jenke, M. Wellenzohn, and H. Abele, PRD **87**, 105013 (2013)

Acceleration of Universe Expansion

Friedmann–Lemaître–Robertson–Walker Metric

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \quad 0 \leq r \leq 1$$

Universe:

Open ($k = -1$), Flat ($k = 0$), Closed ($k = +1$)

Einstein's Equations for Acceleration of Universe Expansion

$$\dot{a}^2(t) = \frac{\rho}{3M_{\text{Pl}}^2} a^2(t) - k \quad , \quad \ddot{a}(t) = -\frac{1}{6M_{\text{Pl}}^2} (\rho + 3p) a(t)$$

$$\rho_\phi(t) = \frac{1}{2} \dot{\phi}^2(t) + V_{\text{eff}}(\phi) \quad , \quad p_\phi(t) = \frac{1}{2} \dot{\phi}^2(t) - V_{\text{eff}}(\phi)$$

$$\dot{\rho} + 3(\rho + p) \frac{\dot{a}}{a} = 0 \longrightarrow \ddot{\phi}(t) + 3H\dot{\phi}(t) + \frac{dV_{\text{eff}}(\phi)}{d\phi} = 0$$

- E. Rebhan, in *Theoretische Physik: Relativitätstheorie und Kosmologie*, Springer – Verlag, Berlin Heidelberg 2012.

qBounce Experiments with Ultracold Neutrons (UCNs)

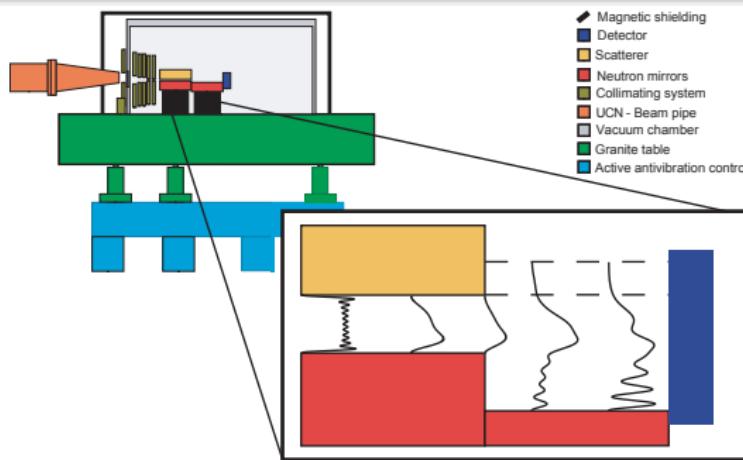


Figure: The experimental setup of the qBounce Experiments with UCNs

- T. Jenke, G. Cronenberg, J. Bürgdorfer, L. A. Chizhova, P. Geltenbort, A. N. Ivanov, T. Lauer, T. Lins, S. Rotter, H. Saul, U. Schmidt, and H. Abele, *PRL* **112**, 115105 (2014)

Schrödinger Equation for UCNs Between Two Mirrors:

$$\left(-\frac{1}{2m} \frac{d^2}{dz^2} + mg\left(\frac{d}{2} + z\right) \right) \psi_k(z) = E_k \psi_k(z)$$

Wave Functions of UCNs Between Two Mirrors:

$$d = 30.1 \text{ } \mu\text{m} \text{ and } \ell_0 = (2m^2g)^{-1/3} = 5.87 \text{ } \mu\text{m}$$

$$\psi_k(z) \sim \text{Ai}\left[\frac{1}{\ell_0}\left(\frac{d}{2} + z\right) + \xi_k\right] \text{Bi}(\xi_k) - \text{Ai}(\xi_k) \text{Bi}\left[\frac{1}{\ell_0}\left(\frac{d}{2} + z\right) + \xi_k\right]$$

Energy Spectrum of UCNs Between Two Mirrors:

$$E_k = -mg\ell_0\xi_k = -0.602 \xi_k \text{ peV} = -0.602 \xi_k 10^{-12} \text{ eV}$$

$$\text{Ai}(\xi_k) \text{Bi}\left(\frac{d}{\ell_0} + \xi_k\right) - \text{Ai}\left(\frac{d}{\ell_0} + \xi_k\right) \text{Bi}(\xi_k) = 0$$

$$\xi_1 = -2.33875, \quad E_1 = 1.40888 \text{ peV}$$

$$\xi_2 = -4.14571, \quad E_2 = 2.49740 \text{ peV}$$

$$\xi_3 = -6.07242, \quad E_3 = 3.65806 \text{ peV}$$

- A. N. Ivanov *et al.*, PRD 87, 105013 (2013)

Quantum Gravitational States of UCNs

Densities of Quantum Gravitational States of UCNs in Spatial Region $z^2 \leq d^2/4$

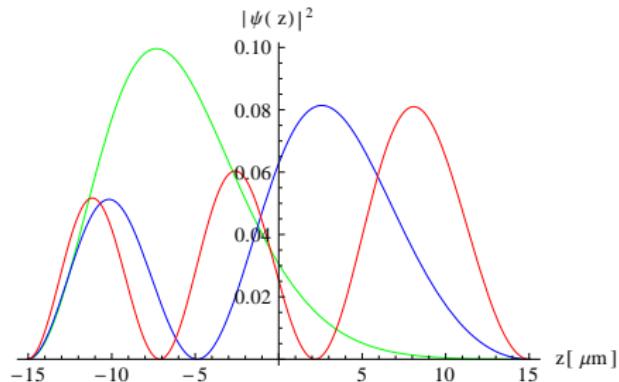


Figure: The densities $|\psi_k(z)|^2$ of the quantum gravitational states with the *principal* quantum numbers ($k = 1$ - green line), ($k = 2$ - blue line) and ($k = 3$ - red line)

- A. N. Ivanov, R. Höllwieser, T. Jenke, M. Wellenzohn, and H. Abele, PRD **87**, 105013 (2013)

Chameleon Field Between Two Mirrors

Non-linear Equation of Motion

$$\frac{d^2\phi(z)}{dz^2} = \frac{n\Lambda^{4+n}}{\phi_{\min}^{n+1}} - \frac{n\Lambda^{4+n}}{\phi^{n+1}}$$

Boundary Conditions

$$\phi(z) \Big|_{z \rightarrow \pm \frac{d}{2}^-} = \phi(z) \Big|_{z \rightarrow \pm \frac{d}{2}^+} = \phi_d, \quad \frac{d\phi(z)}{dz} \Big|_{z \rightarrow \pm \frac{d}{2}^-} = \frac{d\phi(z)}{dz} \Big|_{z \rightarrow \pm \frac{d}{2}^+}$$

Profile of Chameleon Field $\beta > 10^5$

$$\phi(z) = \phi_0 \left(1 - \frac{4z^2}{d^2}\right)^{\frac{2}{n+2}}, \quad \phi_0 = \Lambda \left(\frac{n+2}{4\sqrt{2}} \Lambda d\right)^{\frac{2}{n+2}}$$

- A. N. Ivanov, R. Höllwieser, T. Jenke, M. Wellenzohn, and H. Abele, PRD **87**, 105013 (2013)

Profile of Chameleon Field Between Two Mirrors

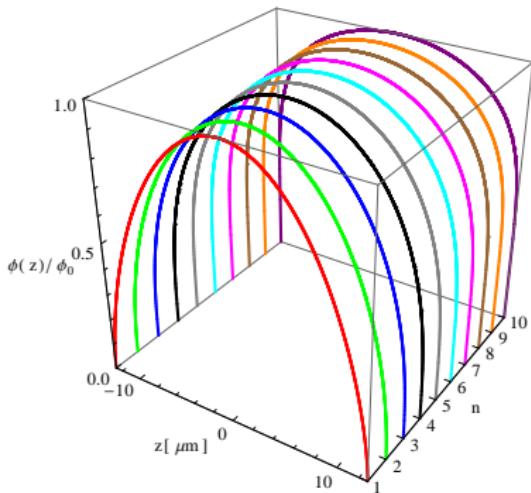


Figure: Profiles of chameleon field, calculated in the strong coupling limit $\beta \geq 10^5$ in the spatial region $z^2 \leq \frac{d^2}{4}$ and $n \in [1, 10]$

- A. N. Ivanov, R. Höllwieser, T. Jenke, M. Wellenzohn, and H. Abele, PRD **87**, 105013 (2013)

Contribution of Chameleon Field to Transition Frequencies of Quantum Gravitational States of UCNs

Stationary Schrödinger Equation for UCNs

$$\left(-\frac{1}{2m} \frac{d^2}{dz^2} + \Phi_{\text{eff}}(z) \right) \Psi_k(z) = E_k \Psi_k(z)$$

Effective Gravitational Potential $\Phi_{\text{eff}}(z) = m U_{\text{eff}}(z)$

$$\Phi_{\text{eff}}(z) = mg \left(\frac{d}{2} + z \right) + \beta \frac{m}{M_{\text{Pl}}} \phi(z)$$

Contribution of Chameleon Field to Transition Frequencies

$$\omega_{k'k} = \beta \frac{m}{M_{\text{Pl}}} \int_{-d/2}^{+d/2} dz \phi(z) \left(|\psi_{k'}(z)|^2 - |\psi_k(z)|^2 \right)$$

Experimental Upper Bound $\beta < 5.8 \times 10^8$ at 95 % C.L. with
sensitivity $\Delta\omega \simeq 10^{-14}$ eV

- T. Jenke et al., PRL 112, 115105 (2014)

Contribution of Chameleon Field to Transition Frequencies of Quantum Gravitational States of UCNs

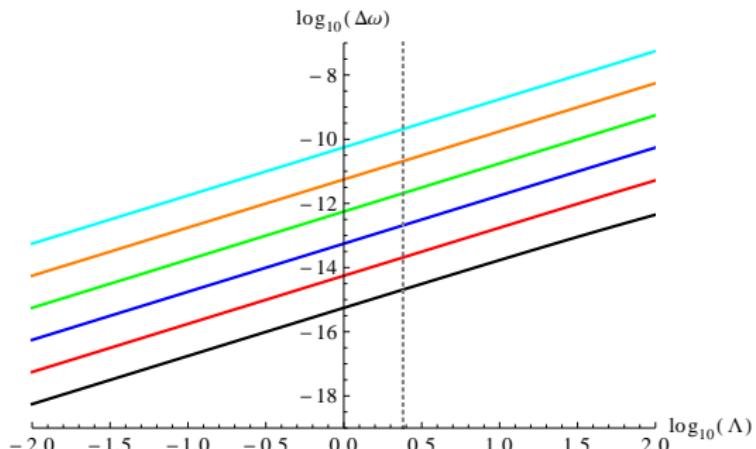


Figure: The sensitivity of the qBounce experiments $\Delta\omega$, measured in meV, as a function of the parameters Λ , measured in meV, and β of the chameleon field theory for $10^{-2} \leq \Lambda \leq 10^2$ and $\beta = 10^5$ (black), 10^6 (red), 10^7 (blue), 10^8 (green), 10^9 (gold) and 10^{10} (cyan).

Dirac Equation for UCNs in Static Metric Spacetime

Static Spacetime (Schwarzschild $\gamma = 1$) Metric

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu = g_{00}(x)dt^2 + g_{ij}(x)dx^i dx^j = V^2 dt^2 - W^2(d\vec{r})^2$$

$$V^2 = 1 + 2U \quad W^2 = 1 - 2\gamma U$$

$$U = \vec{g} \cdot \vec{r} + \beta \frac{m}{M_{\text{Pl}}} \phi(\vec{r})$$

Dirac Equation and Hamilton Operator

$$i \frac{\partial \psi(t, \vec{r})}{\partial t} = \hat{H}(\vec{r}, \vec{\nabla}, \vec{\sigma}) \psi(t, \vec{r})$$

$$\hat{H}(\vec{r}, \vec{\nabla}, \vec{\sigma}) = \gamma^0 m V - i \frac{V}{W} \gamma^0 \vec{\gamma} \cdot \left(\vec{\nabla} + \frac{\vec{\nabla} V}{2V} + \frac{\vec{\nabla} W}{W} \right)$$

- E. Fischbach, B. S. Freeman, and W.-K. Cheng, PRD **23**, 2157 (1981).
- Y. N. Obukhov, PRL **86**, 192 (2001).
- A. N. Ivanov and M. Pitschmann, PRD **90**, 045040 (2014)

Nonrelativistic Dirac Equation for UCNs

Matrix Elements of Hamilton Operator $g = \det\{g_{ij}\} = -W^6$

$$\begin{aligned}\langle f | \hat{H} | i \rangle &= \int d^3x \sqrt{-g} \psi_f^\dagger(t, \vec{r}) \hat{H} \psi_i(t, \vec{r}) = \\ &= \int d^3x W^3 \psi_f^\dagger(t, \vec{r}) \hat{H} \psi_i(t, \vec{r})\end{aligned}$$

Hermitian Hamilton Operator $\psi(t, \vec{r}) = W^{-3/2} \Psi(t, \vec{r})$

$$\langle f | \hat{H} | i \rangle = \int d^3x \psi_f^\dagger(t, \vec{r}) \hat{H} \psi_i(t, \vec{r})$$

$$\hat{H} = W^{3/2} \hat{H} W^{-3/2} = \gamma^0 m V - i \frac{V}{W} \gamma^0 \vec{\gamma} \cdot \vec{\nabla} - \frac{i}{2} \gamma^0 \vec{\gamma} \cdot \vec{\nabla} \left(\frac{V}{W} \right)$$

- E. Fischbach, B. S. Freeman, and W.-K. Cheng, PRD **23**, 2157 (1981).
- Y. N. Obukhov, PRL **86**, 192 (2001).
- A. N. Ivanov and M. Pitschmann, PRD **90**, 045040 (2014)

Foldy–Wouthuysen Transformation

$$\hat{\mathbf{H}}_1 = e^{i\hat{S}_1} \hat{\mathbf{H}} e^{-i\hat{S}_1} \longrightarrow \hat{\mathbf{H}}_2 = e^{i\hat{S}_2} \hat{\mathbf{H}}_1 e^{-i\hat{S}_2}$$

$$\hat{\mathbf{H}}_2 = -\frac{1}{2m} \Delta + \Phi(\vec{r}, \vec{\nabla}, \vec{\sigma})$$

$$\begin{aligned} \Phi(\vec{r}, \vec{\nabla}, \vec{\sigma}) &= m(V - 1) - \frac{1}{4m} \left(\vec{\nabla} \frac{V}{W^2} \right) \cdot \vec{\nabla} + \frac{i}{4m} \left(\vec{\nabla} \frac{V}{W^2} \right) \cdot (\vec{\sigma} \times \vec{\nabla}) \\ &\quad + \frac{W^2 - V}{2mW^2} \Delta - \frac{1}{4mW} \left(\Delta \frac{V}{W} \right) - \frac{1}{2mW} \left(\vec{\nabla} \frac{V}{W} \right) \cdot \vec{\nabla} \\ &\quad + \frac{1}{4mVW} (\vec{\nabla} V) \cdot \left(\vec{\nabla} \frac{V}{W} \right) - \frac{1}{8mV} \left(\vec{\nabla} \frac{V}{W} \right)^2 - \frac{1}{8mW^3} (\vec{\nabla} W) \cdot (\vec{\nabla} V) \\ &\quad + \frac{1}{4mW^2} (\vec{\nabla} V) \cdot \vec{\nabla} + \frac{1}{8mW^2} (\Delta V) - \frac{1}{8mVW^2} (\vec{\nabla} V)^2 \end{aligned}$$

- L. L. Foldy and S. A. Wouthuysen, PR **78**, 29 (1950)
- A. N. Ivanov and M. Pitschmann, PRD **90**, 045040 (2014)

Hamilton Operator of UCNs to Orders $1/m$ and U

Hamilton Operator of UCNs

$$\hat{H} = -\frac{1}{2m} \Delta + \Phi(\vec{r}, \vec{\nabla}, \vec{\sigma})$$

Effective Gravitational Potential to Orders $1/m$ and U

$$\Phi(\vec{r}, \vec{\nabla}, \vec{\sigma}) = m U(\vec{r})$$

$$-\frac{1+2\gamma}{2m} \left(U \Delta + \vec{\nabla} U \cdot \vec{\nabla} + \frac{1}{4} \Delta U \right) + i \frac{1+2\gamma}{4m} \vec{\nabla} U \cdot (\vec{\sigma} \times \vec{\nabla})$$

de Sitter Precession or Torsion–Matter Interaction

$$\tilde{\Phi}(\vec{r}, \vec{\nabla}, \vec{\sigma}) = i \frac{1+2\gamma}{4m} \vec{\nabla} U \cdot (\vec{\sigma} \times \vec{\nabla})$$

- A. N. Ivanov and M. Pitschmann, PRD **90**, 045040 (2014)
- S. Weinberg, in *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, John Wiley & Sons, Inc. New York, 1972.
- T. Fliessbach, in *Allgemeine Relativitätstheorie*, B•I Wissenschaftsverlag, Mannheim Wien Zürich, 1990.

Effective Hamilton Operator of UCNs

$$\hat{H} = \hat{H}_0 + \hat{V}_G + \hat{V}_{Ch} + \hat{V}_T$$

$$\hat{H}_0 = -\frac{1}{2m} \Delta + m \vec{g} \cdot \vec{r}$$

$$\hat{V}_G = -\frac{1+2\gamma}{2m} (\vec{g} \cdot \vec{r} \Delta + \vec{g} \cdot \vec{\nabla})$$

$$\hat{V}_{Ch} = \beta \frac{m}{M_{Pl}} \phi(\vec{r}) - \frac{1+2\gamma}{2m} \frac{\beta}{M_{Pl}} \left(\phi(\vec{r}) \Delta + \vec{\nabla} \phi(\vec{r}) \cdot \vec{\nabla} + \frac{1}{4} \Delta \phi(\vec{r}) \right)$$

$$\hat{V}_T = i \frac{1+2\gamma}{4m} \left(\vec{g} + \frac{\beta}{M_{Pl}} \vec{\nabla} \phi(\vec{r}) \right) \cdot (\vec{\sigma} \times \vec{\nabla})$$

- A. N. Ivanov and M. Pitschmann, PRD **90**, 045040 (2014)

Torsion–Matter Lagrangian

$$\hat{V}_T \longrightarrow \delta \mathcal{L}_T(x) = \frac{i}{2} g_T T_\mu(x) \bar{\psi}(x) \sigma^{\mu\nu} \overleftrightarrow{\partial}_\nu \psi(x)$$

Torsion Tensor

$$g_T T_\mu(x) = \xi_6^{(5)} T_\mu(x) + \xi_7^{(5)} A_\mu(x) =$$

$$= \left(\xi_6^{(5)} \frac{1}{3} g^{\alpha\beta} \delta^\lambda_\mu + \xi_7^{(5)} \frac{1}{6} \varepsilon^{\alpha\beta\lambda}{}_\mu \right) T_{\alpha\beta\lambda}$$

$$T_{\alpha\beta\lambda} = -T_{\alpha\lambda\beta}$$

$$T^\alpha{}_{\mu\nu} = g^{\alpha\beta} T_{\beta\mu\nu} \longrightarrow T^\alpha{}_{\mu\nu} = -T^\alpha{}_{\nu\mu}$$

- V. A. Kostelecky, N. Russell, and J. D. Tasson, PRL **100**, 111102 (2008)
- A. N. Ivanov and M. Pitschmann, PRD **90**, 045040 (2014)

Chameleon Field as Origin of Torsion Field

Torsion Field From Dirac Equation

$$g_T \vec{T}(x) = -\frac{1+2\gamma}{4m} \left(\vec{g} + \frac{\beta}{M_{\text{Pl}}} \vec{\nabla} \phi(\vec{r}) \right)$$

Constant Part of Torsion Field is Not Observable to First Order Perturbation Theory

$$i \frac{\partial \psi(\vec{r}, t)}{\partial t} = \left(-\frac{1}{2m} \Delta + m \vec{g} \cdot \vec{r} + i \frac{1+2\gamma}{4m} (\vec{g} \times \vec{\sigma}) \cdot \vec{\nabla} \right) \psi(\vec{r}, t)$$

$$\psi(\vec{r}, t) = \Omega \psi'(\vec{r}, t) \longrightarrow \vec{\nabla} \Omega = i \frac{1+2\gamma}{4} (\vec{g} \times \vec{\sigma}) \Omega$$

$$i \frac{\partial \psi'(\vec{r}, t)}{\partial t} = \left(-\frac{1}{2m} \Delta + m \vec{g} \cdot \vec{r} - g^2 \frac{(1+2\gamma)^2}{16m} \right) \psi'(\vec{r}, t)$$

$$E \rightarrow E' = E + g^2 \frac{(1+2\gamma)^2}{16m}$$

- D. Colladay and V. A. Kostelecky, PRD **58**, 116002 (1998)
- A. N. Ivanov and M. Pitschmann, PRD **90**, 045040 (2014)

Torsion Field

$$g_T \vec{T}(x) = -\frac{1+2\gamma}{4m} \frac{\beta}{M_{\text{Pl}}} \vec{\nabla} \phi(\vec{r})$$

Chameleon Field as Origin of Torsion

$$\mathcal{T}^\alpha{}_{\mu\nu} = \frac{\beta}{M_{\text{Pl}}} \left(\delta^\alpha{}_\nu \partial_\mu \phi - \delta^\alpha{}_\mu \partial_\nu \phi \right) \longrightarrow \mathcal{T}_\mu = \frac{1}{3} \delta^\nu{}_\alpha \mathcal{T}^\alpha{}_{\mu\nu} = \frac{\beta}{M_{\text{Pl}}} \frac{\partial \phi}{\partial x^\mu}$$

Affine Connection in Torsion Gravity with Chameleon Field and Dark Energy

$$\begin{aligned} \Gamma^\alpha{}_{\mu\nu} &= \{{}^\alpha{}_{\mu\nu}\} + \frac{1}{2} g^{\alpha\lambda} (\mathcal{T}_{\lambda\mu\nu} - \mathcal{T}_{\mu\lambda\nu} - \mathcal{T}_{\nu\lambda\mu}) = \\ &= \{{}^\alpha{}_{\mu\nu}\} + \frac{\beta}{M_{\text{Pl}}} \left(\delta^\alpha{}_\nu \frac{\partial \phi}{\partial x^\mu} - g_{\mu\nu} \frac{\partial \phi}{\partial x_\alpha} \right) \end{aligned}$$

- A. N. Ivanov and M. Pitschmann, PRD **90**, 045040 (2014)
- S. Hojman, M. Rosenbaum, M. Ryan, and L. Shepley, PRD **17**, 3141 (1978)

- On the basis of a non-relativistic approximation of the Dirac equation of ultracold neutrons, coupled to the gravitational field of the Earth and a chameleon field, we have presented a possible extension of the Einstein gravity on the Einstein–Cartan gravity with a torsion field and torsion–matter interactions, caused by a chameleon field.
- The proposed extension of the Einstein gravity incorporates i) the explanations of the current phase of the acceleration of the Universe expansion and of dark energy in terms of a chameleon field and ii) can be fairly investigated experimentally in the terrestrial laboratories in the qBounce experiments with quantum gravitational states of ultracold neutrons, bouncing either above a mirror or between two mirrors.

Thank You for Attention