Theoretical Methods in Hadron Spectroscopy

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EuNPC, 30$^{\text{th}}$ August 2015

Road Map

- Some introduction and background
- QCD
  - Theoretical details
  - Symmetries and conservation laws
  - The quark model
- Experimental motivation and connection
- Theoretical tools
  - Effective field theories
  - Potential models
  - Lattice
- Summary

HADRON SPECTROSCOPY: WHY?

- Many recently discovered hadrons have unexpected properties.
- Understand the hadron spectra to separate EW physics from strong-interaction effects
- Techniques for non-perturbative physics useful for physics at LHC energies.
- Understanding EW symmetry breaking may require nonperturbative techniques at TeV scales, similar to spectroscopy at GeV.
- Better techniques may help understand the nature of masses and transitions

Classifying Particles

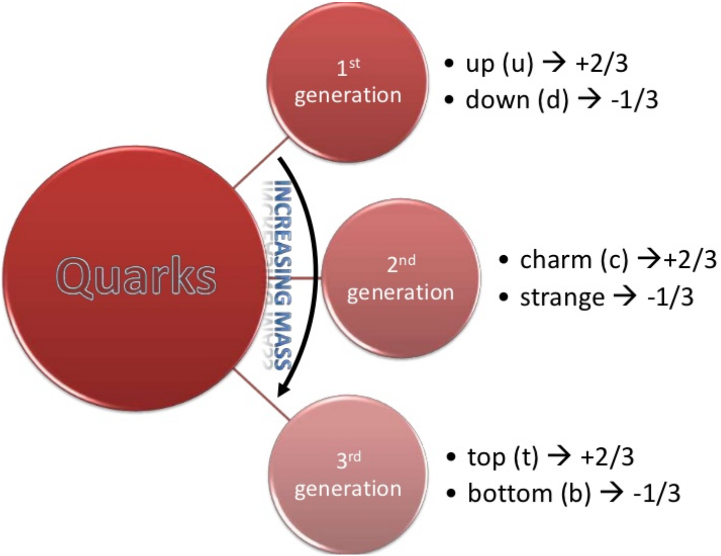
TYPES OF PARTICLE

- **Hadrons:** built from quarks, affected by strong force
- **Quarks:** fundamental particles i.e. no internal structure (not made from smaller particles). Carry fractional charge - fractions of the charge of the e^-. Combine to form hadrons.
- **Gauge Bosons:** the force carriers - $\gamma, g, W^\pm, Z$ and h.
- **Leptons:** fundamental particles, not affected by the strong force.

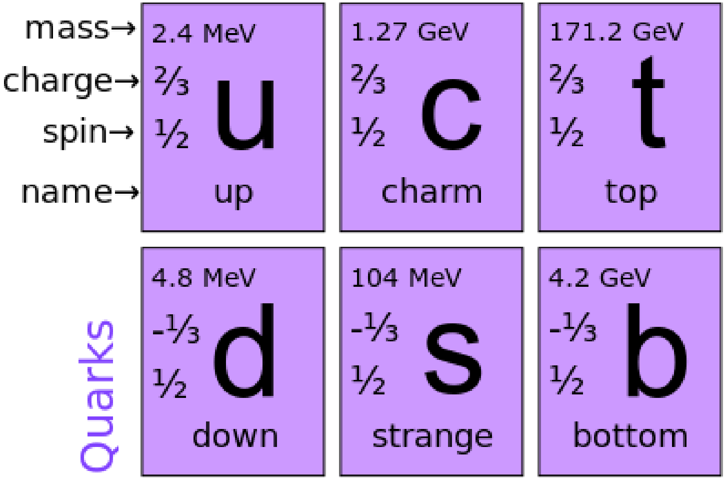
What is everything made from?

Quarks and Leptons

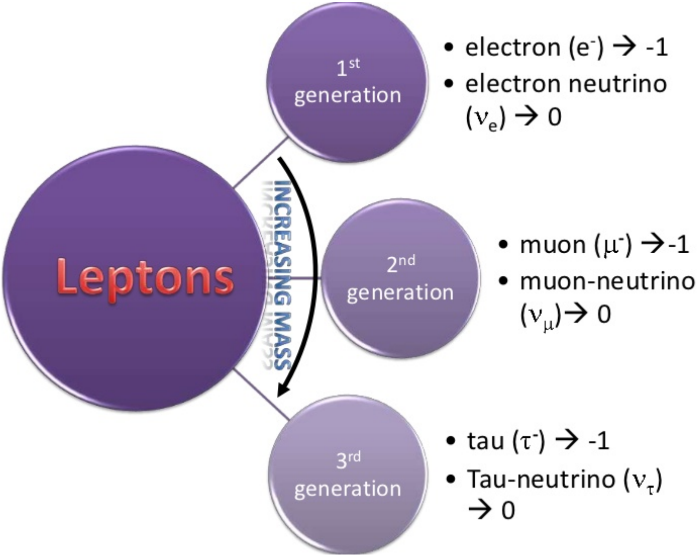
- to understand hadrons we need to understand the theory of their constituent fundamental particles.



from Alessio Bernardelli, SlideShare



mass→
charge→
spin→
name→
Quarks
2.4 MeV
⅔
½
u
up
1.27 GeV
⅔
½
c
charm
171.2 GeV
⅔
½
t
top
4.8 MeV
-⅓
½
d
down
104 MeV
-⅓
½
s
strange
4.2 GeV
-⅓
½
b
bottom



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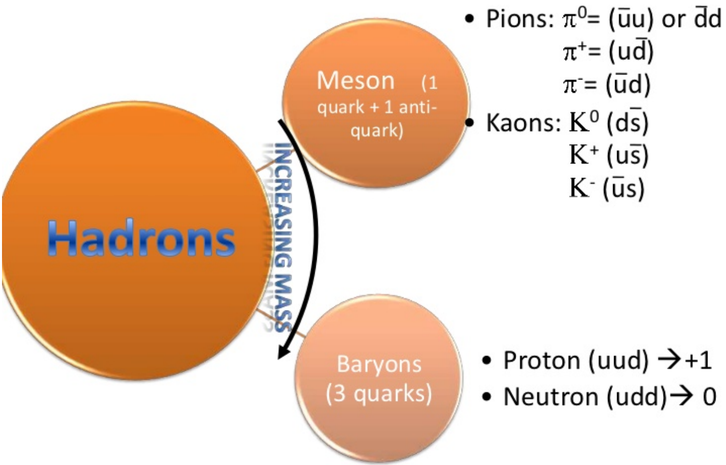

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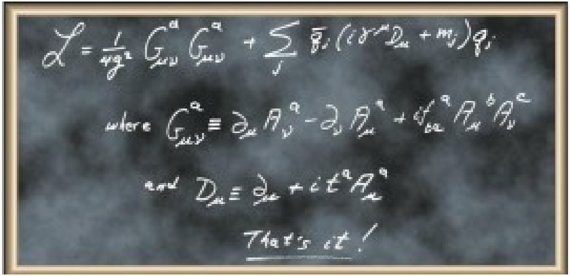
The theory of the strong force:
Quantum Chromodynamics

Quantum Chromodynamics (QCD)

The quantum field theory of the strong interaction that binds quarks and gluons to form hadrons.



from F.A. Wilczek

- this doesn't look too bad - a bit like QED which we have a well-developed toolkit to deal with

SOME MORE DETAILS

QCD is a gauge-invariant quantum field theory

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu \partial_\mu - m\right) q + g\bar{q}\gamma^\mu t_a q A^a_\mu - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

- Actually not easy at all! an enormous challenge!
- One way to see this is to note that g is not a small number so perturbation theory (an expansion in a small parameter) that works so well for QED will not be so useful for **QCD** .
  - There are some small numbers around - the quark masses $m_{u,d} \sim \mathcal{O}(1) MeV$.
- Matter: quark fields the building blocks; quark mass is input parameter in $\mathcal{L}$

$$q^f_i \begin{cases} i & \in \{\text{red}, \text{blue}, \text{green}\} \\ f & \in \{\text{u}, \text{d}, \text{s}, \text{c}, \text{b}, \text{t}\} \end{cases}$$

spin = 1/2; charge = 2/3, -1/3

- the t_a are the generators (matrices) of the group $SU(3)$
  $[t_a, t_b] = i f_{abc} t_c$
- interaction (force) carriers: 8 massless spin-1 gluons in the 8-dim representation of $SU(3)$.
- hadrons are color-singlet (ie not colored) combinations of quarks, anti-quarks and gluons

QCD vs QED

QED

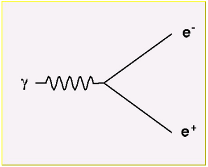
Quantum theory of electromagnetic interactions, mediated by exchange of photons.
Photon couples to electric charge e
Coupling strength $\propto e \propto \sqrt{\alpha}$

QCD

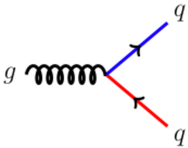
Quantum theory of strong interactions, mediated by exchange of gluons between quarks.
Gluon couples to colour charge of quark
Coupling strength is $\propto \sqrt{\alpha_s}$

Fundamental vertices

QED



QCD



Coupling constants: coupling strength of QCD $\gg$ QED

COLOUR: QUARKS

Charge of **QCD** . Conserved quantum number: “red”, “green” or “blue” Satisfies $SU(3)$ symmetry.

- **Quarks:** Come in three colours (r,g,b); anti-quarks have anti-colours.
- **Leptons and other Gauge Bosons:** Don’t carry colour charge so don’t participate in strong interaction

Believed that all free particles are colourless - never observe a free quark. Quarks always form bound states of colourless hadrons.

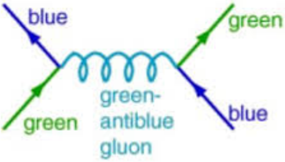
Colour Confinement Hypothesis

only colour singlet states can exist as free particles

Note: to construct colour wave-funcs for hadrons can apply maths of SU(3) uds flavour symmetry to SU(3) colour

Colour: Gluons

Massless spin-1 bosons. Emission or absorption of gluons by quarks changes colour of quark - **colour is conserved**.



Gluons carry colour charge e.g. gb gluon changes green quark to blue. Very different to QED where $q(\gamma) = 0$.

How many gluons are there?

Naively expect 9: $r\bar{b}, r\bar{g}, g\bar{b}, g\bar{r}, b\bar{r}, b\bar{g}, r\bar{r}, b\bar{b}, g\bar{g}$.

SU(3) symmetry - 8 octet and 1 singlet state

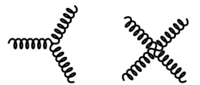
- octet $r\bar{b}, r\bar{g}, g\bar{b}, g\bar{r}, b\bar{r}, b\bar{g}, \frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g}), \frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$
- singlet $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$

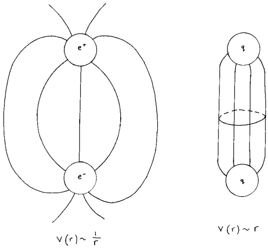
- 8 gluons realised by nature (colour octet)

What's happened to the singlet gluon?

- colour confinement hypothesis: only colour singlet states can exist as free particles.
- a colour singlet gluon would be unconfined and behave like a strongly interacting photon - infinite range Strong Force!
- empirically the strong force is short range so there are 8 gluons.
- Note this can also be understood via the group theory structure and props of SU(3).

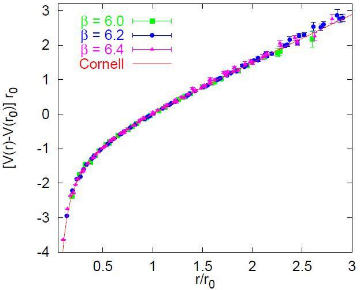
gluons attract each other - colour force lines pulled together. ⇒ very different to QED





COLOR FORCE AND QUARK POTENTIALS

Between 2 quarks at distance $r \sim O(1)\text{fm}$) define a **string** with **tension** k and a potential $V(r) = kr$.
Stored energy/unit length is constant and separation of quarks requires infinite amount of energy.
QCD Potential QED-like at short distance $r \leq 0.1fm$. String tension - potential increases linearly at large distance $r \geq 1fm$.



Force between 2 quarks at large distance is $|dV/dr| = k = 1.6 \times 10^{-10}\text{J}/10^{-15}\text{m} = 16000\text{N}$ or equivalent to the weight of a car!

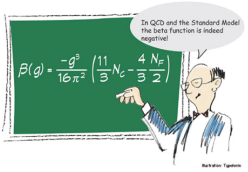
This stored energy gives the proton its mass (and not the Higgs as you sometimes hear)! Recall $m_u + m_u + m_d \sim 9\text{MeV}$ but $m_{\text{proton}} = 938\text{MeV}$

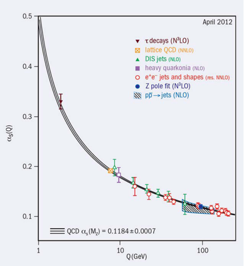
THE RUNNING QCD COUPLING

In QED, α varies with distance - running and the bare e^- is *screened* at large distances - reducing.
The same but different in QCD where *anti-screening* dominates!
$\Rightarrow$ At large distances (low energies) $\alpha_s \sim 1$ i.e. large. Higher-order diagrams - α_s increasingly larger, summation of diagrams diverges ... perturbation theory fails.

Asymptotic freedom

Coupling constant is small at high energies i.e. energetic quarks are (almost) free. QCD perturbation theory works!





Nobel prize 2004 for Gross, Politzer and Wilczek.

QCD: MAKING CALCULATIONS

There are two regimes:

Deep inside the proton

- at short distances quarks behave as free particles
- weak coupling

⇒ perturbation theory works

At "observable" (hadronic) distances

- at long distance (1fm) quarks confined
- strong coupling

⇒ perturbation theory fails: nonperturbative approach needed.

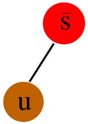
Conservation Laws

CONSERVATION: QUARKS

- Relative Charge: quarks and anti-quarks carry fractional charge. Charge is conserved in all interactions
- Baryon Number: quarks have baryon number $+\frac{1}{3}$ and anti-quarks $-\frac{1}{3}$. Baryon number is conserved in all interactions.
- Strangeness: $s = -(n_s - \bar{n}_s)$. Quarks and anti-quarks have strangeness = 0 except for the strange quark (strangeness = -1) and anti-strange (strangeness=+1). In all strong (and EM) interactions strangeness is conserved. In weak interactions strangeness *may* be conserved or may *change* by ± 1.

EXAMPLES: I

A K^+ meson is made from an up quark and an anti-strange quark. What is the relative charge, baryon number and strangeness of this particle?



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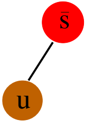
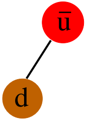


- Charge: $+\frac{2}{3}+\frac{1}{3}=1$
- Baryon Number: $+\frac{1}{3}-\frac{1}{3}=0$
- Strangeness: $0+1=+1$

EXAMPLES: II

A π^- meson is made from an anti-up quark and a down quark. What is the relative charge, baryon number and strangeness of this particle?



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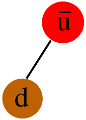


- Charge: $-\frac{2}{3} - \frac{1}{3} = -1$
- Baryon Number: $+\frac{1}{3} - \frac{1}{3} = 0$
- Strangeness: $0 - 0 = 0$

SYMMETRIES

Theorists (and Nature!) like symmetries ...
SU(2) and SU(3) play a major role in particle physics

Hadron symmetries that play a key role are:

- SU(3) uds flavour symmetry
- SU(2) and SU(3) colour symmetry
- SU(2) isospin symmetry

Explained the observed hadrons and successfully predicted others.
Define the allowed states of **QCD** :

- $q\bar{q}, qqq$ Mesons and Baryons
- $q\bar{q}q\bar{q}, qqqq\bar{q}$ Exotic states e.g. pentaquarks

Exercise: use the SU(3) colour symmetry to explain e.g. why only $q\bar{q}$ *mesons and not* qq *are allowed; and why* $qqq(\bar{q}\bar{q}\bar{q})$ *baryons (antibaryons) allowed and not* $qq\bar{q}$*.*

CONSEQUENCES OF STRONG DYNAMICS

The strong-coupling and nature of gluons $\Rightarrow$ interesting particles can appear

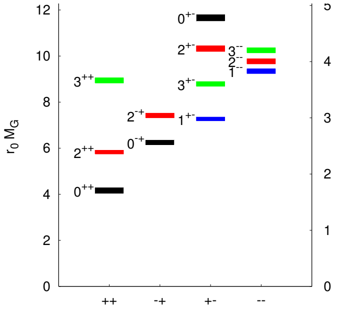
- quark condensates
- glueballs
- hybrids

GLUEBALLS

Gluons couple strongly to each other

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a, \quad F_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- expect a spectrum of gluonic excitations
- possible even in a theory without quarks i.e. "pure Yang-Mills"
- particles are called glueballs
- lattice predictions ...



Morningstar & Peardon

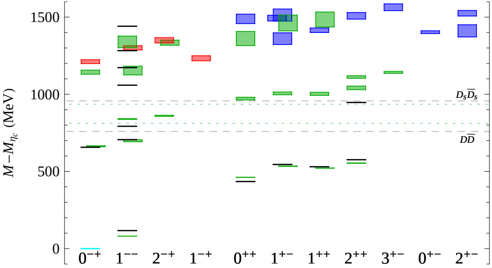
In full QCD glueballs much more complicated.

- same quantum numbers as isospin 0 mesons
- mix with lots of things!

HYBRID MESONS

States with quarks and excited gluonic field content $[q\bar{q}g]$.

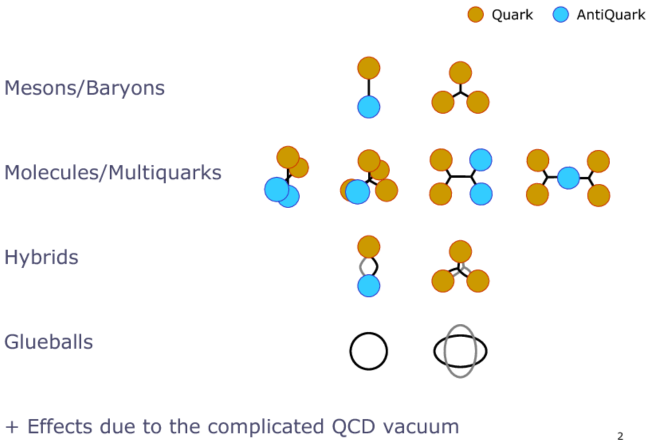
- a better chance to see gluonic excitations at experiments
- the signal is exotic: $J^{PC}_{q\bar{q}} \otimes J^{PC}_{\text{glue}} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$
- lattice is providing model-independent simulations now ...
- on the shopping list at GlueX and PANDA



HadSpec Collab

Quark Models

Objects of interest



A CONSTITUENT MODEL

- QCD has fundamental objects: quarks (in 6 flavours) and gluons
- Fields of the lagrangian are combined in colorless combinations: the mesons and baryons. Confinement.

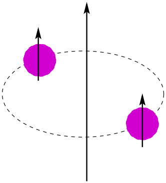
quark model object	structure
meson	$3 \otimes \bar{3} = 1 \oplus 8$
baryon	$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$
hybrid	$\bar{3} \otimes 8 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 8 \oplus 10 \oplus 10$
glueball	$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 10$
$\vdots$	$\vdots$

- This is a model. QCD does not always respect this constituent picture! There can be strong mixing.

CLASSIFYING STATES: MESONS

- Recall that continuum states are classified by J^{PC} multiplets (representations of the poincare symmetry):
  - Recall the naming scheme: $n^{2S+1}L_J$ with $S = \{0, 1\}$ and $L = \{0, 1, \ldots\}$
  - J, hadron angular momentum, $|L - S| \leq J \leq |L + S|$
  - $P = (-1)^{(L+1)}$, parity
  - $C = (-1)^{(L+S)}$, charge conjugation. Only for $q\bar{q}$ states of same quark and antiquark flavour. So, not a good quantum number for eg heavy-light mesons $(D_{(s)}, B_{(s)})$.

MESONS



- two spin-half fermions ${}^{2S+1}L_J$
- $S = 0$ for antiparallel quark spins and $S = 1$ for parallel quark spins;

- States in the natural spin-parity series have $P = (-1)^J$ then $S = 1$ and $CP = +1$:
  - $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{+-}, 2^{--}, 2^{-+}, \ldots$ allowed
- States with $P = (-1)^J$ but $CP = -1$ forbidden in $q\bar{q}$ model of mesons:
  - $J^{PC} = 0^{+-}, 0^{--}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$ forbidden (by quark model rules)
  - These are **EXOTIC** states: not just a $q\bar{q}$ pair ...

BARYONS

Baryon number $B = 1$: three quarks in colourless combination

- J is half-integer, C not a good quantum number: states classified by J^P
- spin-statistics: a baryon wavefunction must be antisymmetric under exchange of any 2 quarks.
- totally antisymmetric combinations of the colour indices of 3 quarks
- the remaining labels: flavour, spin and spatial structure must be in totally symmetric combinations

$$|qqq\rangle_A = |\text{color}\rangle_A \times |\text{space, spin, flavour}\rangle_S$$

With three flavours, the decomposition in flavour is

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

Many more states predicted than observed: missing resonance problem

EXERCISE

Verify the following statements, using the quark model

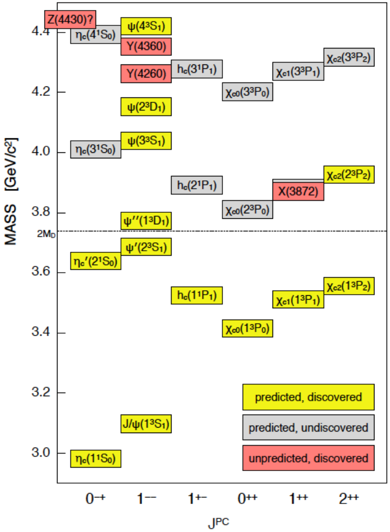
- Baryons won't have spin 1
- What is the quark combination of an antibaryon of electric charge +2
- Why are mesons with charge +1 and strangeness -1 not possible?

Experiments: motivating hadron spectroscopy

A RENAISSANCE IN CHARMONIUM SPECTROSCOPY

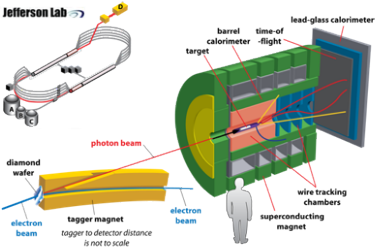
- Early in the noughties, new narrow structures were seen by Belle and BaBar above the open-charm threshold.
- This led to substantial renewed interest in spectroscopy. Were these more quark-anti-quark states, or something more?
  - $X(3872)$: very close to $D\bar{D}$ threshold - a molecule?
  - $Y(4260)$: a 1^{--} hybrid?
  - $Z^{\pm}(4430)$: charged, can't be $\bar{c}c$.
- Very little is known and no clear picture seems to be emerging...
- Lattice calculations have a role to play

KNOWN UNKNOWNS IN CHARMONIUM



from D. Bettoni CIPANP2015

THE GLUEX EXPERIMENT AT JLAB



- 12 GeV upgrade to CEBAF ring
- New experimental hall: Hall D
- New experiment: GlueX

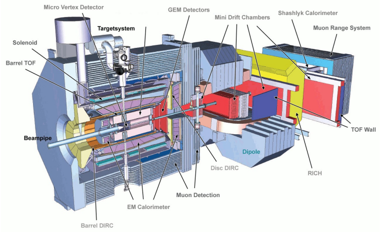
- Aim: photoproduce mesons, in particular the hybrid meson (with intrinsic gluonic excitations)
- Expected to start taking data 2015

PANDA@FAIR, GSI

- Extensive new construction at GSI Darmstadt

PANDA: Anti-Proton ANnihilation at DArmstadt

- Anti-proton beam from FAIR on fixed-target.
- Physics goals include searches for hybrids and glueballs (as well as charm and baryon spectroscopy).



Methods for calculating in QCD

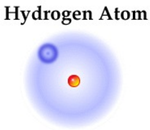
EFFECTIVE THEORIES AND HADRON PHYSICS

Assume that scales much larger or smaller than those of interest shouldn't matter - and can be integrated out *[made systematic through renormalisation]*

Long distance dynamics doesn't depend on short distances or low-energy interactions don't see the details of high-energy interactions.

One way to think of it is to remember that classical physics is an effective theory: don't need to know nuclear physics to build a bridge ...

EXAMPLE I



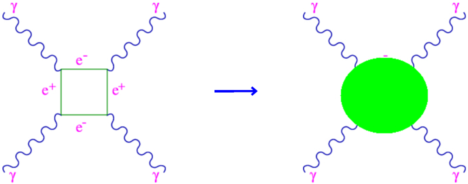
A quantum bound state of an electron and a proton

- the spectrum of the hydrogen atom is precisely determined without knowing e.g. the top quark mass
- at lowest order need the mass & charge of the electron, the charge of the proton and the static Coulomb interaction:

$$E = E_0 = -\frac{m_e}{2n^2}\left(\frac{e^2}{4\pi}\right)^2$$

- an approximate answer that can be improved with systematic expansion: $E = E_0\left[1 + \mathcal{O}\left(\alpha, \frac{m_e}{m_p}\right)\right]$.
- corrections come from the em interaction, the proton structure ...

EXAMPLE II: LIGHT-BY-LIGHT SCATTERING



- energy scales: photon energy ω electron mass m_e
- $\omega \ll m_e$

fermions (as the massive dofs) integrated out: $\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{\text{eff}}[A_\mu]$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2) + \frac{e^4}{360\pi^2 m_e^4}\left[(\vec{E}^2 - \vec{B}^2) + 7(\vec{E} \cdot \vec{B})^2\right] + \ldots$$

- energy expansion in small parameter: $(\omega/m_e)^{2n}$
- leads to a x-section $\sigma(\omega) = \frac{1}{16\pi^2}\frac{973}{10125\pi}\frac{e^8}{m_e^2}(\omega/m_e)^6$ in good agreement with experimental measurement via PW lasers. *see e.g. PRD78 (2008) 032006*

HQET AND CHPT

Exploit the symmetries of QCD and existing hierarchies of scales to write down effective lagrangians that are appropriate for the problem at hand.

- Using hadronic degrees of freedom:
  - Chiral perturbation theory, an EFT for light hadrons. Expansion parameter is the pion energy/momentum.
- Using quark and gluon degrees of freedom:
  - HQET an EFT for hadrons with 1 heavy quark. Expansion in powers of the quark mass. Spin and flavour symmetries emerge.
  - NRQCD an EFT for hadrons with 2 heavy quarks. Expansion in relative velocity of the heavy quarks.
- many others ... and effective theories can be a useful tool in combination with other methods e.g. LQCD

EXAMPLE: HEAVY HADRONS AT FINITE TEMPERATURE

The hierarchy of scales $M \geq T > \dots$

hierarchy of scales:

- heavy quark mass, M
- temperature, T
- inverse size $g^2 M$
- Debye mass, gT
- binding energy, $g^4 M$

weak coupling $\longrightarrow$

corresponding EFT:

- NRQCD
- NRQCD + HTL
- pNRQCD
- pNRQCD + HTL
- ...

Laine, Philipsen, Romatschke and Tassler 07; Laine 07/08; Burnier, Laine and Vepsalainen 08/09; Beraudo, Blaizot and Ratti 08, Escobedo and Soto 08; Brambilla, Ghiglieri, Vairo and Petreczky 08; Brambilla, Escobedo, Ghiglieri, Soto and Vairo 10; Escobedo, Soto and Mannarelli 11; ...

EFT SUMMARY

- The basic ideas underpinning EFTs: separate physics at different scales; identify approprite degrees of freedom
- Implement the consequences of symmetries
- EFT allows you to compute using dimensional analysis - even if the underlying theory is unknown
- EFT a powerful tool for probing QCD and hadron spectroscopy

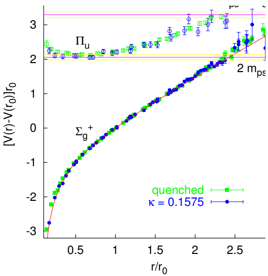
Keep in mind ...

- in some cases the full theory (QCD) cannot be formally recovered i.e. the EFT is nonrenormalisable e.g. lattice NRQCD .
- the effective theory is a good description of some regime in QCD of interest but cannot predict/describe beyond that.
- accuracy/precision physics needs a robust expansion as well as a reliable estimate of systematic uncertainties.

POTENTIAL MODELS

$$V(r) = \underbrace{\frac{4}{3}\frac{\alpha_s}{r}}_{vectorpart} + \underbrace{kr}_{scalarpart} + \underbrace{V_{LS}}_{spin-orbit} + \underbrace{V_{SS}}_{spin-spin} + \underbrace{V_T}_{tensorterm}$$

Many models exist, most have a similar set of ingredients:
The (confining) potl assumed from phenomenological arguments and might be extracted from data or a lattice.
With EFTs gives a useful tool.
Particularly effective for understanding particular regimes (e.g. quarkonia) or states (e.g. XYZ)



Keep in mind

Relies on an assumed potential. There are many choices and some discrimination is needed.
Not a systematic approach to full QCD

LATTICE QCD

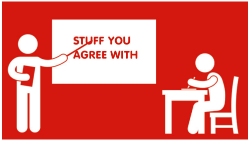
The only systematically-improvable non-perturbative formulation of QCD .

In principle non-perturbative observables can be computed precisely.

In practice, calculations are made using stochastic estimation $\Rightarrow$ statistical errors. Systematic errors result from algorithmic and field theoretic restrictions.

The precision achievable depends on the quantity and the details of the numerical approach.

I will spend a little bit more time here (biased!) but hope to give a flavour of the difficulties (and triumphs!).



LQCD and Path Integrals

a QFT can be expressed as a path integral

$$Z = \int \mathcal{D}\phi(x)\, e^{i\int d^4x \mathcal{L}[\phi(x)]}$$

with $\int \mathcal{D}\phi(x)$ *a functional integral over all possible field configurations and* $\mathcal{L}$ *the lagrangian of our theory.*

Observables can be expressed in terms of these path integrals.

$$\langle \phi(y)\phi(x)\rangle = Z^{-1}\int \mathcal{D}\phi\phi(x)\phi(y)e^{i\int d^4x \mathcal{L}[\phi]}$$

is the propagator in the free scalar theory and in this case the functional (Gaussian) integral can be done exactly

In general, while we can write down the functional integral we can't solve it exactly

What does it look like for **QCD** ?

In **QCD**

$$Z_{\mathbf{QCD}} = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A_\mu e^{i\int d^4x \bar{q}(i\gamma^\mu\partial_\mu - m)q + g\bar{q}\gamma^\mu t_a q A^a_\mu - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a}$$

and now $\mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}A_\mu$ represent an infinite number of d.o.f. that is the field strength at every point in continuous spacetime.

- make the number of degrees of freedom finite then the integral is tractable
  - this is Lattice **QCD**
  - discretise spacetime on a grid of points of finite extent (L), with finite grid spacing (a).

What symmetries are lost and what is the effect?

In QCD

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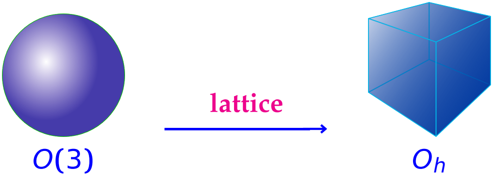
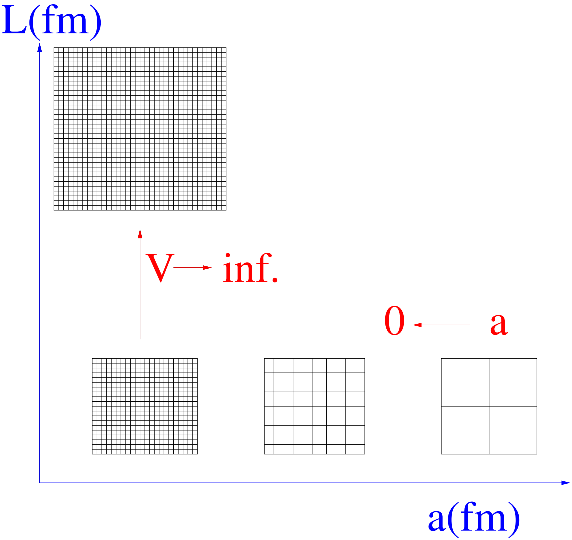


RECOVERING CONTINUUM QCD



PRACTICAL LQCD

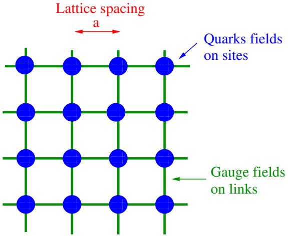
- Consider gluons on links of the lattice i.e. $U_\mu(x) = e^{-aA_\mu(x)}$. Quark fields on sites.
- Discretise derivatives with finite differences e.g. in 1-dim

$$\frac{df}{dx} = \frac{f(x+a) - f(x-a)}{2a} + \mathcal{O}(a^2)$$

Exercise: Write a 1-dim derivative correct to $\mathcal{O}(a^4)$.

- Many ways to discretise fermions and you will hear many philosophies ...

- Wilson, Clover
- Staggered, asqtad, HISQ
- Domain wall, overlap



Making calculations

- If $e^{i\int d^4x\mathcal{L}}$ real then treat as a probability and use stochastic estimation (Monte Carlo) to estimate the integral
- Rotate to Euclidean time: $t \to i\tau; i\int d^4x\mathcal{L} \to -i\int d^4x\tilde{\mathcal{L}}$
- An observable looks like

$$\langle\mathcal{O}\rangle = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}U\mathcal{O}e^{-S[q,\bar{q},U]}$$

- Fermion fields integrate exactly, $\int \mathcal{D}\bar{q}\mathcal{D}qe^{-\bar{q}_iQ_{ij}q_j} = \det Q$ leaving something like

$$\langle\bar{q}_x(t')\Gamma'q_x(t')\cdot\bar{q}_y(t)\Gamma q_y(t)\rangle = \int \mathcal{D}UQ^{-1}_{x,y}\Gamma'Q'_{y,x}\Gamma\det Q[U]e^{-S_{\text{gauge}}[U]}$$

- Notice $\det Q[U]e^{-S_{\text{gauge}}[U]}$ looks like a probability weight so generate gauge field configurations according to this and save them.
- An observable (two point function) is then $\sum_{\{U\}} Q^{-1}_{xy}\Gamma'Q^{-1}_{y,x}\Gamma$

WHY DOES LQCD NEED BIG COMPUTERS??

- once the gauge configurations are generated just have to invert the Dirac matrix Q to get the fermion propagators ... how hard can that be?
- let's calculate:
  - a lattice might have $24 \times 24 \times 24 \times 48 = 663,552$ sites
  - a fermion (quark) has 4 Dirac components
  - 3 colours in $SU(3)$
- $\Rightarrow Q$ is easily $10^6 \times 10^6$!!

Keep in mind in addition to statistical errors:

- Lattice artefacts

$$\left.\frac{m_N}{m_\Omega}\right|_{lat} = \left.\frac{m_N}{m_\Omega}\right|_{cont} + \mathcal{O}(a^p),\ p \geq 1$$

  requires extrapolation to the continuum limit, $a \to 0$
- Finite volume effects
  - Energy measurements can be distorted by the finite box
  - Rule of thumb: $m_\pi L > 3$ ok for many things ...
- Unphysically heavy pions
  - Simulations at physical pion mass started but most calculations rely on chiral extrapolation to reach physical m_u, m_d
  - Use Chiral Perturbation Theory to guide the extrapolations. Are chiral corrections reliably described by ChPT?
- Fitting
  - Uncertainties from the choice of fit range, t_0 etc.

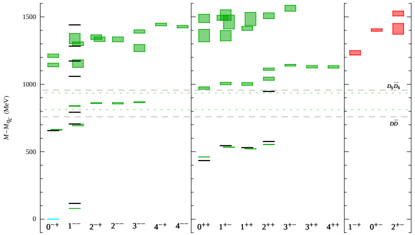
LQCD AND SPECTROSCOPY

Huge progress in the last 5 years. (With the caveats mentioned)

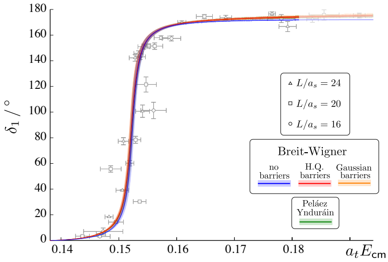
- Understood how to determine the excited and exotic (hybrid) spectra of states from light to heavy; including isoscalars and up to spin 4.
- First results from studies of the XYZ states in charmonium and $D\pi$, DK scattering.
- Huge strides made on scattering and resonance calculations. $\rho \rightarrow \pi\pi$ phase shift determined; partial wave mixing analyses ...
- Understood how to tackle coupled-channels: results for two coupled channels, theory and proof of concept for three ...

Why was this such a problem?

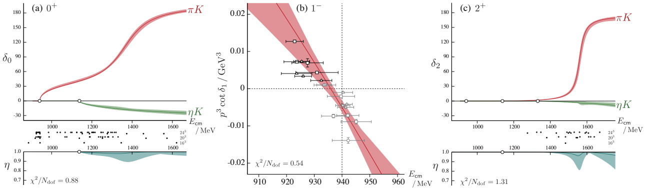
$t \rightarrow i\tau$ allows computation but loses direct info on scattering. New theoretical ideas mean now know how to retrieve this.



charmonium



$\rho \to \pi\pi$



$k\pi$ scattering

HadSpec results

Not discussed here

Many aspects of hadron spectroscopy and QCD have been omitted in these lectures.

- chiral symmetry, spontaneous symmetry breaking, ...
- nitty-gritty of modern lattice calculations
- other theoretical tools for theoretical hadronic physics
- ⋮

I hope nevertheless this has been useful
THANKS FOR LISTENING!

Brief Solutions to Exercises

Exercise

Verify the following statements, using the quark model

- Baryons won't have spin 1
  A baryon consists of 3 quarks. Since the spin of each is 1/2, they cannot combine to form a baryon of spin 1.
- What is the quark combination of an antibaryon of electric charge +2
  An antibaryon comprises 3 antiquarks. To combine 3 antiquarks to make a baryon of charge +2 you need antiquarks of electric charge +2/3.
- Why are mesons with charge +1 and strangeness -1 not possible?
  A meson consists of a quark and an antiquark. Only the strange quark as non-zero strangeness so to form a meson of strangeness -1 and electric charge 1 you would need an a strange quark and an antiquark of electric charge 4/3.

EXERCISE

Use the SU(3) colour symmetry to explain e.g. why only $q\bar{q}$ mesons and not qq are allowed; and why $qqq(\bar{q}\bar{q}\underline{a}rq)$ baryons (antibaryons) allowed and not $qq\bar{q}$.

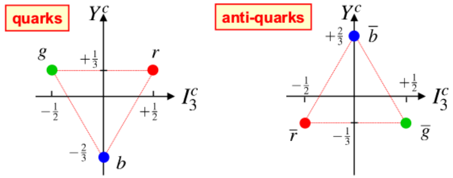
★ Represent r, g, b **SU(3)** colour states by:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

★ Colour states can be labelled by two quantum numbers:
- I_3^c colour isospin
- Y^c colour hypercharge

Exactly analogous to labelling u,d,s flavour states by I_3 and Y

★ Each quark (anti-quark) can have the following colour quantum numbers:



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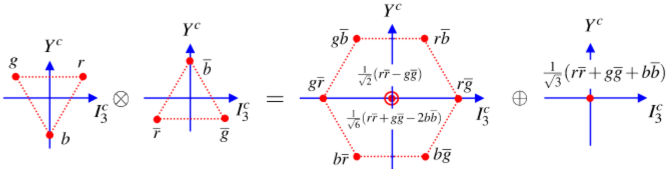
Meson Colour Wave-function

★ Consider colour wave-functions for $q\bar{q}$

★ The combination of colour with anti-colour is mathematically identical to construction of meson wave-functions with uds flavour symmetry



➡ Coloured octet and a colourless singlet

• Colour confinement implies that hadrons only exist in colour singlet states so the colour wave-function for mesons is:

$$\boxed{\psi_c^{q\bar{q}} = \frac{1}{\sqrt{3}}(r\bar{r}+g\bar{g}+b\bar{b})}$$

★ Can we have a $qq\bar{q}$ state ? i.e. by adding a quark to the above octet can we form a state with $Y^c = 0;\ I_3^c = 0$. The answer is clear no.

➡ $qq\bar{q}$ bound states do not exist in nature.

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EXERCISE

Write a 1-dim derivative correct to $\mathcal{O}(a^4)$.

$$\frac{df}{dx} = \frac{-f(x+3a) + 27f(x+a) - 27f(x-a) + f(x-3a)}{48a} + \mathcal{O}(a^4)$$

EXERCISE

What symmetries are broken by the lattice and what are the consequences?

The lattice break lorentz symmetry. One important consequence for spectroscopy is that states are classified by the irreducible representations of the cubic group, O_h on the lattice (as opposed to O_3 in the continuum).

There is an infinite number of irreps (J values) in the continuum but a finite set (just 5) on the lattice with a non-trivial mapping between them, complicating spin identification.

	A_1	A_2	E	T_1	T_2
$J=0$	1				
$J=1$				1	
$J=2$			1		1
$J=3$		1		1	1
$J=4$	1		1	1	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- In principle then to identify e.g. a $J = 2$ state, results from E and T_2 at finite a should extrapolate to the same value. However, this is a numerical costly procedure requiring multiple lattice spacings etc.
- Even then, it may not be possible to disentangle nearly-degenerate high-spin states. An example of this difficulty occurs in charmonium where a $J = 4$ state which would lie across the same irreps as an excitation of the triplet $(0^{++}, 1^{++}, 2^{++})$. The ground state of the spin 4 meson could be close in energy to this excitation making identification, even in the continuum limit, very difficult or impossible.
- A method to use the overlaps Z_n to identify spin at finite lattice spacing works well. See arXiV:0707.4162 and e.g. arXiV:1204.5425 for details.