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# Interplay between Pairing and Quadrupole Interactions in the Microscopic Shell Model

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# Introduction:

Microscopic description of the nucleons in the valence shells

**Interactions**

- **pairing** short-range ⟺ the pairing gap; nuclei with only a few nucleons after the closed shells; spherical shape of the system.
- **quadrupole-quadrupole** long-range interactions ⟺ nuclei near the mid-shells; introduces deformation; rotational sequences

**Problems in the framework of the shell model –** computer intensive, exponential increase of the dimension of the Hilbert space

Microscopic PAIRING plus QUADRUPOLE MODEL **PQM**


S.T. Belyaev, Mat. Fys. Medd. Dan. Vid. Selsk. **31**, No. 11 (1959).
L.S. Kisslinger and R.A. Sorensen, Rev. Mod. Phys. **35** (1963) 853.
M. Baranger and K. Kumar, Nucl. Phys. **62** (1965) 113.

# Exactly solvable models – algebraic realization of the model observables

Representations

Algebras

## Dynamical Symmetry of the system -

- chains of group-subgroup structures

Classification of the basis states

Hamiltonians and interactions

**leads to exact analytic solutions for the eigenvalue problems.**

**AIM:** Dynamical symmetries of the Microscopic PAIRING plus QUADRUPOLE MODEL

P.Van Isacker, Rep. Prog. Phys. **62** (1999) 1661.

# The Microscopic Shell-Model Scheme

**AIM:** To study the dynamical symmetries in the microscopic shell model

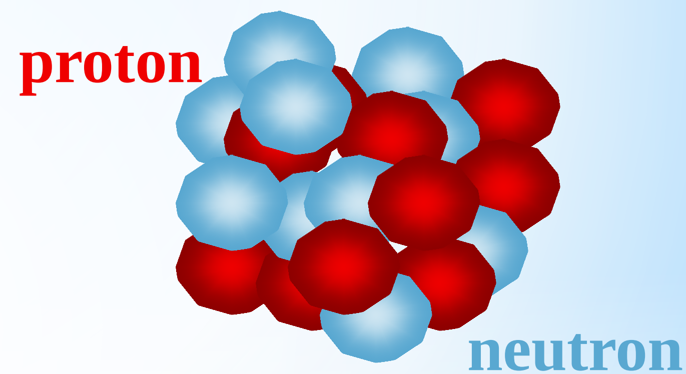
## I. LST sheme

E.P. Wigner, Phys. Rev. **51** (1937) 106.

Algebraic structure: the main chain:

$\{1^m\}$ $U(4\Omega)$

$\{f\}$ $U(\Omega)\otimes U_{ST}(4)$ $\{\tilde{f}\}$




$\Omega=\Sigma_i\,(2l_i + 1)$

$m = \sum_i \tilde{f}_i$ spatial

Spin (S), Isospin (T) -space

Number of particles

conjugated

$f_1 \geq f_2 \geq f_3 \geq f_4$

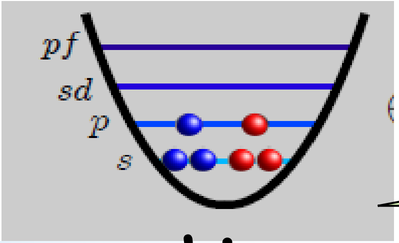
$\{\tilde{f}\} = \{f_1-f_2,\ f_2-f_3,\ f_3-f_4\}$ $\{f\} = \{f_1,\ f_2,\ f_3, f_4\}$

**filling the single-particle orbitals of the valence shells with nucleons, taking into account the Pauli principle**

# The Spin-Isospin $SU_{ST}(4)$ Symmetry

Wigner: Invariance of nuclear forces to rotations in the spin-isospin space

valid up to A~100




isomorphism

$$U_{ST}(4) \supset U_S(2) \otimes U_T(2),$$

$\{\tilde{f}\}$    S    T

$$SO_{ST}(6) \supset SO_S(3) \otimes SO_T(3),$$

$[P_1, P_2, P_3]$    S    T

$S=\Sigma_i s_i$

$T=\Sigma_i t_i$

# Reductions in the spatial part U(Ω)

## Elliott's SU(3) – Model
## Basic Symmetry in Nuclei

### Rotations and the SU(3) Symmetry

U(3) $[n_1, n_2, n_3]$

↓

SU(3) $(\lambda, \mu)$

↓

SO(3)

Relates the invariant operators of the algebras

$$Q_\mu = \sum_l \sqrt{8(2l+1)} \left( a^\dagger{}_{l\frac{1}{2}\frac{1}{2}} \times \tilde{a}_{l\frac{1}{2}\frac{1}{2}} \right)^{(200)}_{(\mu 00)}$$

$$L_\mu = \sum_l \sqrt{4l(2l+1)(l+1)/3} \left( a^\dagger{}_{l\frac{1}{2}\frac{1}{2}} \times \tilde{a}_{l\frac{1}{2}\frac{1}{2}} \right)^{(100)}_{(\mu 00)},$$

to the observables $Q_\mu$ and $L_\mu$ of the described system.

and to the harmonic oscillator $H_0 = \hbar\omega(N+3/2)$

$$H_{rot} = H_0 + \frac{1}{2}\chi Q.Q,$$

J.P. Elliott, *Proc. Roy. Soc. London*, Ser. A **245** (1958) 128; **245** (1958) 562.

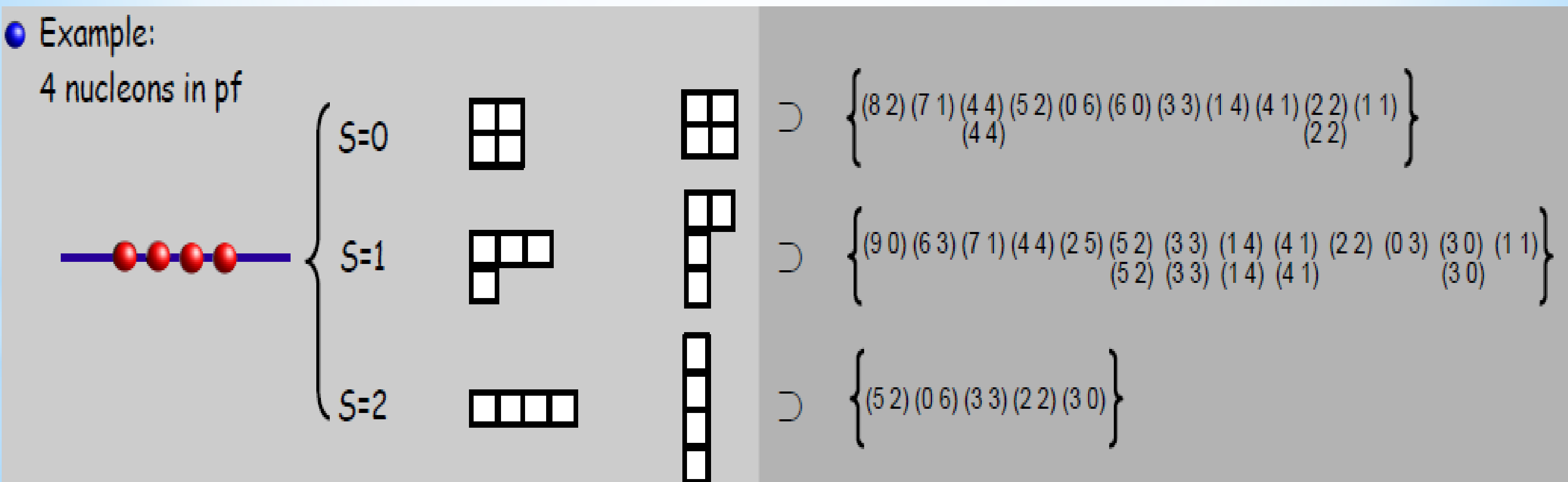
# Relates the algebra to microscopic nuclear structure

**For each set of nucleons in a HO shell determine the antysimetric representations of**

$$U(\Omega) \supset U(3) \supset SU(3) \supset SO(3),$$

$\{f\}$ $\quad \alpha \quad$ $[n_1, n_2, n_3]$ $\quad (\lambda, \mu) \quad K \quad L$

**Decompose each** $u(\Omega)$ irrep into a complete set of SU(3) irreps




J.P. Draayer, in *Algebraic Approaches to Nuclear Structure: Interacting Boson and Fermion Models*, Contemporary Concepts in Physics VI, edited by R.F. Casten (Harwood Academic, Chur, Switzerland) (1993) 423.

# Hamiltonian and eigenstates in the rotational phase

**Rotational basis states**

$$|\Psi_r\rangle \equiv |\{f\}, \alpha(\lambda,\mu)KLM\rangle \equiv |m, \alpha(\lambda,\mu)KLM\rangle.$$

**Hamiltonian**

$$H_{rot} = H_0 + \frac{1}{2}\chi Q.Q,$$

$$Q.Q = 4C^2_{SU(3)} - 3L^2$$

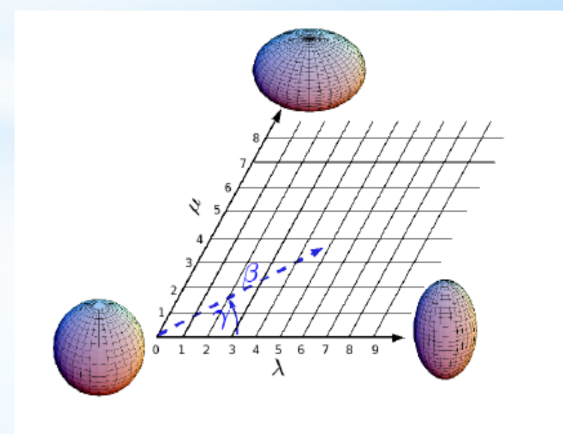
**Decompose each u(Ω) irrep into a complete set of SU(3) irreps**

$$C^2_{SU(3)} = \lambda^2 + \lambda\mu + \mu^2 + 3(\lambda+\mu)$$

## Relates the geometry to algebra

**Study shell-model dynamics against the background of an elegant algebraic/geometric picture**

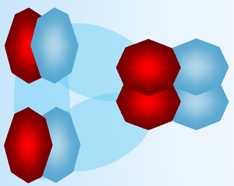
$$\beta^2 \sim \lambda^2 + \lambda\mu + \mu + 3(\lambda + \mu + 1)$$
$$\gamma = \tan^{-1}[\sqrt{3}\,\mu / (2\lambda + \mu + 3)]$$



# The Pairing Interaction

SO(8) Describes all types of pairing correlations

pp, nn, pn



the spectrum generating algebra for the isoscalar (T=0) and isovector (T=1) pair creation and annihilation operators within the nuclear shell model

**Generators:**

isoscalar (T=0)

$$S_\mu^\dagger = \sum_l \beta_l \sqrt{\frac{2l+1}{2}} \left( {a^\dagger}_{l\frac{1}{2}\frac{1}{2}} \times \tilde{a}^\dagger_{l\frac{1}{2}\frac{1}{2}} \right)^{(010)}_{(0\mu 0)}$$

I st SO$_T$(3)

isovector (T=1)

$$P_\mu^\dagger = \sum_l \beta_l \sqrt{\frac{2l+1}{2}} \left( {a^\dagger}_{l\frac{1}{2}\frac{1}{2}} \times \tilde{a}^\dagger_{l\frac{1}{2}\frac{1}{2}} \right)^{(001)}_{(00\mu)},$$

SO$_T$(5)

Non-number conserving generators

V.K.B. Kota, Symmetries in Science XI, eds. B. Gruber, G. Marmo, N. Yoshinaga, Kluwer Academic Publishers, Dordrecht (2004) 265.

# Spectrum generating algebra SO(8)

Relation to the ST spase

$SO(8) \rightarrow SO_{ST}(6) \rightarrow SO_S(3) \otimes SO_T(3)$ (A)

$SO(8) \rightarrow SO_S(5) \otimes SO_T(3) \rightarrow SO_S(3) \otimes SO_T(3)$ (B)

$SO(8) \rightarrow SO_S(3) \otimes SO_T(5) \rightarrow SO_S(3) \otimes SO_T(3)$ (C)

The phases corresponding to the dynamical symmetries

X – control parameter

$$V_{pair} = -\frac{1}{2}\mathbf{G}\{(1-x)S_\mu^\dagger.S_\mu + (1+x)P_\mu^\dagger.P_\mu\},$$

X=0 "total" pairing (A)

$$V_{pair} = \mathbf{G}\{S_\mu^\dagger.S_\mu + P_\mu^\dagger.P_\mu\}$$

X=-1 isoscalar pairing (B)

$$V_{pair(isosc)} \equiv V_0 = \mathbf{G}_0 S_\mu^\dagger.S_\mu$$

X=1 isovector pairing (C)

$$V_{pair(isov)} \equiv V_1 = \mathbf{G}_1 P_\mu^\dagger.P_\mu$$

# The number conserving shell model algebras within U(4Ω)

1.X=0 "total" pairing

(A)

$\{f\} = \{f_1, f_2, f_3, f_4\}$

$f_1 \geq f_2 \geq f_3 \geq f_4$

$[p] = v[p_1, p_2, p_3]$

$v = \Sigma_i \mu_i$

$[\mu] = [\mu_1, \mu_2, \mu_3, \mu_4]$

$\mu_1 \geq \mu_2 \geq \mu_3 \geq \mu_4$

Complementary to the limits of the SO(8) model

$U(\Omega) \otimes U_{ST}(4)$ ⟷ SO(6)

one or several orbits.

$SO(\Omega)$ ⟷ SO(8)

complementarity

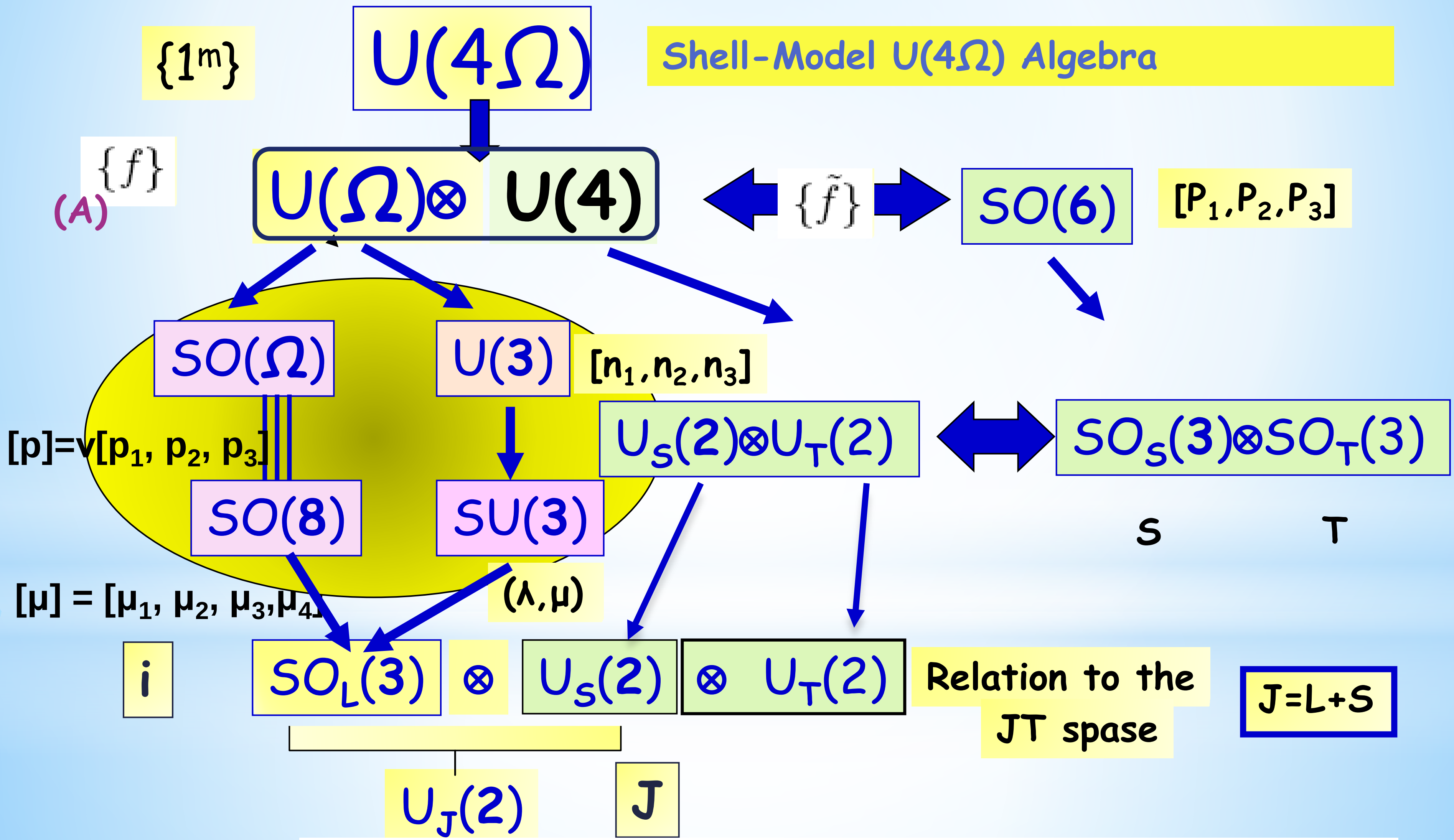
SO(3) L

**Total pairing basis states**

$$|\Psi_p\rangle \equiv |\{f\}, v, [p_1, p_2, p_3], \beta LM\rangle \equiv |m, v, [p_1, p_2, p_3], \beta LM\rangle$$

V.K.B. Kota, J.A. Castilho Alcaras, Nucl. Phys. A **764** (2006) 181.

# Unifying the Reductions of Shell-Model U(4Ω) Algebra



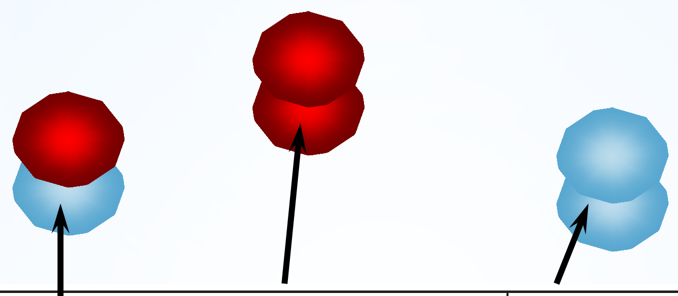

K.P. Drumev, A.I. Georgieva, NUCLEAR THEORY, Vol. **32** (2013) 151

# Complete Classification of the eigenstates in the generalized reduction scheme

## $U(\Omega)\otimes U_{ST}(4)$

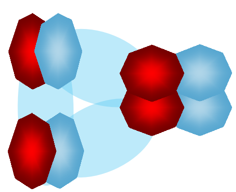
**Example: 2 particles in the ds shell ($\Omega$=6)**

U(24)



| $U(6)$ | $SO(6)$ | $SO_p(6)$ | $SU(3)$ | | $SO(3)$ | $U_{ST}(4)$ | $SO_P(6)$ | $SU_S(2)\times SU_T(2)$ |
|---|---|---|---|---|---|---|---|---|
| $\{\tilde{f}\}$ | $[\mu]$ | $\nu[p]$ | $(\lambda,\mu)$ | $K$ | $L$ | $\{f\}$ | $[P]$ | $(ST)$ |
| $\{1^2\}_{15}$ | $[1^2]_{15}$ | $2[1]$ | $(2,1)_{15}$ | $1$ | $1,2,3$ | $\{2\}_{10}$ | $[1^3]_{10}$ | $(0,0)_1$ |
| | | | | | | | | $(1,1)_9$ |
| $\{2\}_{21}$ | $[0]_1$ | $0[0]$ | $(4,0)_{15}$ | $0$ | $0,2,4$ | $\{1^2\}_6$ | $[1]_6$ | $(1,0)_3$ |
| | $[2]_{20}$ | $2[1^3]$ | $(0,2)_6$ | $0$ | $0,2$ | | | $(0,1)_3$ |

| $U(6)$ $\{\tilde{f}\}$ | $SO(6)$ $[\mu]$ | $SO_p(6)$ $\nu[p]$ | $SU(3)$ $(\lambda,\mu)$ | $K$ | $SO(3)$ $L$ | $U_{ST}(4)$ $\{f\}$ | $SO_P(6)$ $[P]$ | $SU_S(2)\times SU_T(2)$ $(ST)$ |
|---|---|---|---|---|---|---|---|---|
| $\{1^4\}_{15}$ | $[2]$ | $2[1^3]$ | $(1,2)_{15}$ | 1 | $1,2,3$ | $\{4\}_{35}$ | $[2^3]_{35}$ | $(0,0)_1$ |
| | | | | | | | | $(1,1)_9$ |
| | | | | | | | | $(2,2)_{25}$ |
| $\{21^2\}_{105}$ | $[31]$ | $4[21^2]$ | $(0,1)_3$ | 0 | $1$ | $\{31\}_{45}$ | $[21^2]_{45}$ | $(1,0)_3$ |
| | | | $(2,3)_{42}$ | 0 | $1,3,5$ | | | $(0,1)_3$ |
| | | | | 2 | $2,3,4$ | | | $(2,1)_{15}$ |
| | | | $(5,0)_{21}$ | 0 | $1,3,5$ | | | $(1,2)_{15}$ |
| | | | $(3,1)_{24}$ | 1 | $1,2,3,4$ | | | $(1,1)_9$ |
| | $[2]$ | $2[1^3]$ | $(1,2)_{15}$ | 0 | $1,2,3$ | | | |
| $\{2^2\}_{105}$ | $[0]$ | $0[0]$ | $(0,4)_{15}$ | 0 | $0,2,4$ | $\{2^2\}_{20}$ | $[2]_{20}$ | $(2,0)_5$ |
| | $[1^2]$ | $2[1]$ | $(2,0)_6$ | 0 | $0,2$ | | | $(1,1)_9$ |
| | $[2^2]$ | $4[2]$ | $(4,2)_{60}$ | 0 | $0,2,4,6$ | | | $(0,2)_5$ |
| | | | | 2 | $2,3,4,5$ | | | $(0,0)_1$ |
| | | | $(3,1)_{24}$ | 1 | $1,2,3,4$ | | | |
| $\{31\}_{210}$ | $[2]$ | $2[1^3]$ | $(1,2)_{15}$ | 1 | $1,2,3$ | $\{21^2\}_{15}$ | $[1^2]_{15}$ | $(1,0)_3$ |
| | $[1^2]$ | $2[1]$ | $(6,1)_{63}$ | 1 | $1,2,3,4,5,$ $6,7$ | | | $(0,1)_3$ |
| | $[21^2]$ | $4[1^2]$ | $(4,2)_{60}$ | 0 | $0,2,4,6$ | | | $(1,1)_9$ |
| | | | | 2 | $2,3,4,5$ | | | |
| | | | $(2,3)_{42}$ | 0 | $1,3,5$ | | | |
| | | | | 2 | $2,3,4$ | | | |
| | | | $(3,1)_{24}$ | 1 | $1,2,3,4$ | | | |
| | | | $(2,0)_6$ | 0 | $0,2$ | | | |
| $\{4\}_{126}$ | $[1^4]$ | $4[0]$ | $(8,0)_{45}$ | 0 | $0,2,4,6,8$ | $\{1^4\}_1$ | $[0]_1$ | $(0,0)_1$ |
| | | | $(4,2)_{60}$ | 0 | $0,2,4,6$ | | | |
| | | | | 2 | $2,3,4,5$ | | | |
| | $[0]$ | $0[0]$ | $(0,4)_{15}$ | 0 | $0,2,4$ | | | |
| | $[1^2]$ | $2[1]$ | $(2,0)_6$ | 0 | $0,2$ | | | |

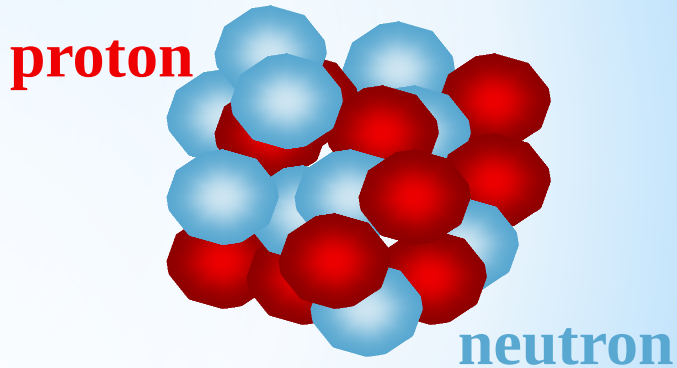


**4 particles in the ds shell ($\Omega$=6)**

# The Microscopic Shell-Model Scheme

**AIM:** To study the dynamical symmetries in the microscopic shell model

**II.** spin-isospin sheme




$\{1^m\}$ $U(4\Omega)$

$\{f\}$ $U(2\Omega) \otimes U_S(2)$ $\supset \{\tilde{f}\}$

$U(\Omega) \otimes U_T(2) \otimes U_S(2)$

$\Omega = \Sigma_i (2l_i + 1)$

$m = \sum_i \tilde{f}_i$ spatial

Number of particles

Spin (S), Isospin (T) -space

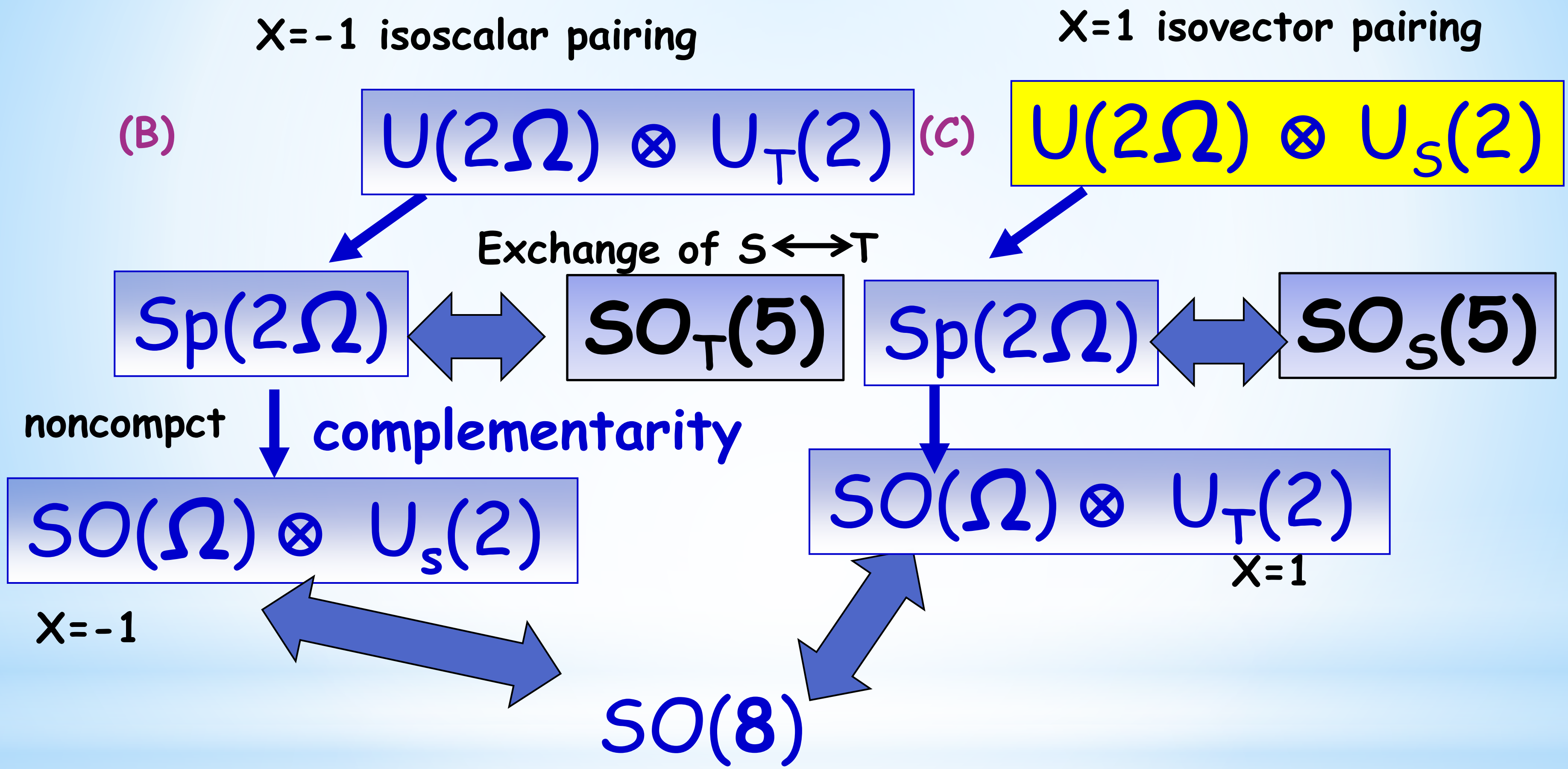
conjugated

$f_1 \geq f_2$

$S = ½ \{f_1 - f_2\}$ $\{f\} = \{f_1, f_2\}$

filling the single-particle orbitals of the valence shells with nucleons, taking into account the Pauli principle

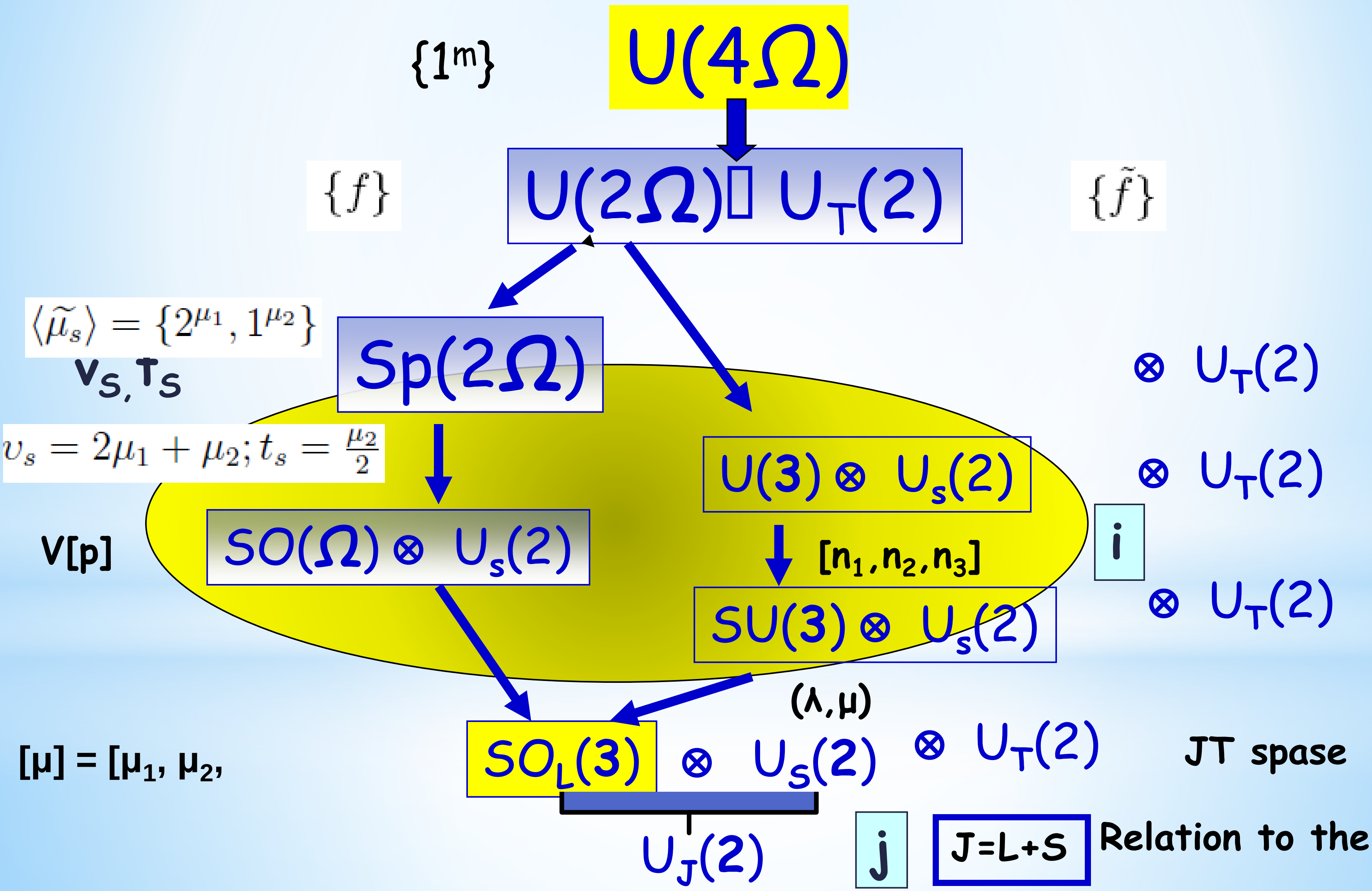
# The number conserving shell model algebras within U(4Ω)




$$|\Psi_{V_0}\rangle \equiv |m, S, v_S, t_S, T, \nu, [p]\beta L M\rangle$$

V.K.B. Kota, J.A. Castilho Alcaras, Nucl. Phys. A **764** (2006) 181.

# Unifying the Reductions of Shell-Model U(4Ω) Algebra



# Classification of states in the generalized reduction scheme $U(2\Omega) \supset U_T(2)$

**Examples: 2 particles in the p shell ($\Omega$=3)**

| $U(6)$ | $Sp(6)$ | $SO(3) \times SU_S(2)$ | $SU(3) \times SU_S(2)$ | | $SO_L(3)$ | $U_T(2)$ | $SU_T(2)$ |
|---|---|---|---|---|---|---|---|
| $\{\tilde{f}\}$ | $<\tilde{\mu}>$ | $[\tilde{\mu}] \times S$ | $(\lambda,\mu) \times S$ | $K$ | $L$ | $\{f\}$ | $T$ |
| $\{1^2\}_{15}$ | $<1^2>_{14}$ | $[2]_5 \times \{0\}_1$ | $(2,0)_6 \times 0_1$ | 0 | 0, 2 | $\{1\}_3$ | $\{1\}_3$ |
| | | $[0]_1 \times \{0\}_1$ | | | | | |
| | $<0>_1$ | $[1]_3 \times \{1\}_3$ | $(0,1)_3 \times 1_3$ | 0 | 1 | | |
| $\{2\}_{21}$ | $<2>_{21}$ | $[2]_5 \times \{1\}_3$ | $(2,0)_6 \times 1_3$ | 0 | 0, 2 | $\{1^2\}_1$ | $\{0\}_1$ |
| | | $[0]_1 \times \{1\}_3$ | | | | | |
| | | $[1]_3 \times \{0\}_1$ | $(0,1)_3 \times 0_1$ | 0 | 1 | | |

**2 particles in the ds shell ($\Omega$=6)**

| $U(12)$ | $Sp(12)$ | $SO(6) \times SU_S(2)$ | $SU(3) \times SU_S(2)$ | | $SO_L(3)$ | $U_T(2)$ | $SU_T(2)$ |
|---|---|---|---|---|---|---|---|
| $\{\tilde{f}\}$ | $<\tilde{\mu}>$ | $[\tilde{\mu}] \times S$ | $(\lambda,\mu) \times S$ | $K$ | $L$ | $\{f\}$ | $T$ |
| $\{1^2\}_{66}$ | $<1^2>_{65}$ | $[2]_{20} \times \{0\}_1$ | $(4,0)_{15} \times 0_1$ | 0 | 0, 2, 4 | $\{1\}_3$ | $\{1\}_3$ |
| | | $[1^2]_{15} \times \{1\}_3$ | $(2,1)_{15} \times 1_3$ | 1 | 1, 2, 3 | | |
| | $<0>_1$ | $[0]_1 \times \{0\}_1$ | $(0,2)_6 \times 0_1$ | 0 | 0, 2 | | |
| $\{2\}_{78}$ | $<2>_{78}$ | $[2]_{20} \times \{1\}_3$ | $(4,0)_{15} \times 1_3$ | 0 | 0, 2, 4 | $\{1^2\}_1$ | $\{0\}_1$ |
| | | $[0]_1 \times \{1\}_3$ | $(2,1)_{15} \times 0_1$ | 1 | 1, 2, 3 | | |
| | | $[1^2]_{15} \times \{0\}_1$ | $(0,2)_6 \times 1_3$ | 0 | 0, 2 | | |

# Relation between the basis states in the pairing and quadrupole limits

## Rotational states

(B)

$$|\Psi_r\rangle \equiv |\{f\}, \alpha(\lambda,\mu)KLM\rangle \equiv |m, \underbrace{\alpha(\lambda,\mu)K}_{\{i\}}LM\rangle.$$

## Total pairing basis states

$$|\Psi_p\rangle \equiv |\{f\}, v, [p_1,p_2,p_3], \beta LM\rangle \equiv |m, \underbrace{v, [p_1,p_2,p_3], \beta}_{\{j\}} LM\rangle.$$

X=-1 (B) $\{j\}=v_S, t_S$

X=1 (C) $\{j\}=v_T, t_T$

X=0 (A)

$$|\Psi_p\rangle_i \equiv |\{f\}, i, LM\rangle = \sum_j C_{ij} |\{f\}, j, LM\rangle.$$

# Probability distribution

## of rotational states into pairing basis

$${}_i\langle\Psi_p|H_{pair}|\Psi_p\rangle_i = E_{pair}(m,i,[P],(ST)) =$$

$$= \underbrace{\sum_{j,k} C^*_{ki}C_{ij}\delta_{k,j}}_{P_{ij}} \cdot \underbrace{{}_k\langle\Psi_r|H_{pair}|\Psi_r\rangle_j}_{\text{Diagonalized Matrix of Hpair in SU(3) rotational basis}}$$

$P_{ij}$

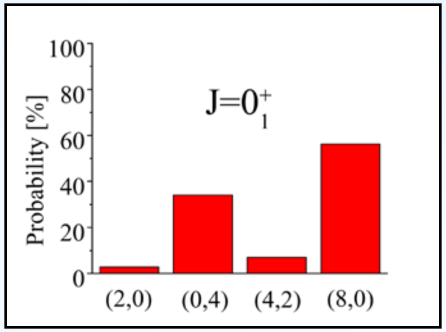
Diagonalized Matrix of Hpair
in SU(3) rotational basis

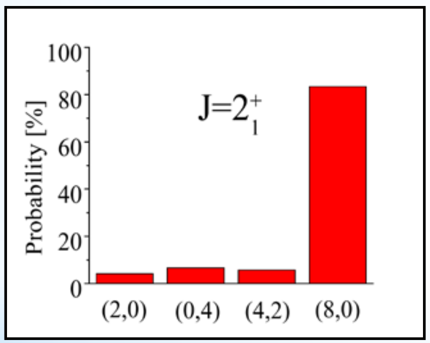
$$E_{pair}(m,v,[p],[P],(ST)) = \tfrac{1}{2}\{-\tfrac{1}{4}(m-\nu)(4\Omega+12-m-\nu) - \quad (15)$$
$$-[p_1(p_1+4)) + p_2(p_2+2) + p_3^2] + [P_1(P_1+4)) + P_2(P_2+2) + P_3^2]\},$$

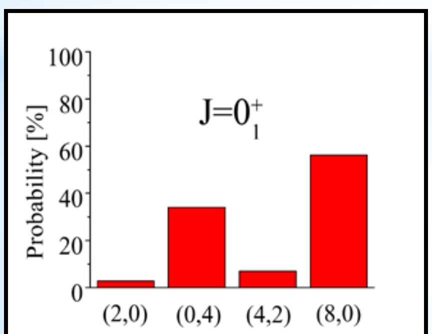
Ep(X=-1)= $-t_T(t_T+1) + T(T+1)$
$$\frac{1}{4}(m-v_T)(4\Omega+6-m-v_T)$$

Ep(X=1)= $-t_S(t_S+1) + S(S+1)$
$$-\frac{1}{4}(m-v_S)(4\Omega+6-m-v_S)$$

# Correspondence to the observable spectrum




| $\|i\rangle \equiv \{\|\nu[p][P]\rangle\}$ | Energy [MeV] | $\|\{j\} \equiv \{(\lambda,\mu)L,S>\}\rangle$ | $\|C_{ij}\|^2[\%]$ |
|---|---|---|---|
| $0_1$ | $-16$ | $(8,0)0,0$ | 56.25 |
| $\|0[0][0]\rangle$ | | $(4,2)0,0$ | 6.94 |
| | | $(0,4)0,0$ | 34.03 |
| | | $(2,0)0,0$ | 2.78 |
| $2_1$ | $-10$ | $(8,0)2,0$ | 83.45 |
| $\|2[1][0]\rangle$ | | $(4,2)2,0$ | 5.68 |
| | | $(0,4)2,0$ | 6.71 |
| | | $(2,0)2,0$ | 4.16 |
| $4_1$ | $-10$ | $(8,0)4,0$ | 40.86 |
| $\|2[1][0]\rangle$ | | $(4,2)4,0$ | 53.81 |
| | | $(0,4)4,0$ | 5.33 |
| $0_2$ | $-10$ | $(4,2)0,0$ | 77.78 |
| $\|2[1][0]\rangle$ | | $(0,4)0,0$ | 11.11 |
| | | $(2,0)0,0$ | 11.11 |
| $0_3$ | $-10$ | $(8,0)0,0$ | 1.13 |
| $\|2[1][0]\rangle$ | | $(4,2)0,0$ | 77.92 |
| | | $(0,4)0,0$ | 10.54 |
| | | $(2,0)0,0$ | 10.41 |
| $0_4$ | $-10$ | $(8,0)0,0$ | 16.87 |
| $\|2[1][0]\rangle$ | | $(4,2)0,0$ | 79.86 |
| | | $(0,4)0,0$ | 2.57 |
| | | $(2,0)0,0$ | 0.70 |
| $2_2$ | $-10$ | $(8,0)2,0$ | 1.41 |
| $\|2[1][0]\rangle$ | | $(4,2)2,0$ | 72.57 |
| | | $(0,4)2,0$ | 15.74 |
| | | $(2,0)2,0$ | 10.28 |
| $2_3$ | $-10$ | $(4,2)0,2$ | 77.78 |
| $\|2[1][0]\rangle$ | | $(0,4)0,2$ | 11.11 |
| | | $(2,0)0,2$ | 11.11 |

# PHASES AND PHASE TRANSITIONS IN THE PQM

Different phases are characterized by different symmetries

Phase transitions involve change of symmetries

## The phase transitions in the PQM

The Hamiltonian of PQM

$$H = x\, Q.Q - G_0 S^{\dagger}.S - G_1 P^{\dagger}.P$$

$V_{QQ}$ $V_P$

- Measuring the weights of the different interactions in the PQM Hamiltonian

## The interplay between the pairing and the quadrupole – quadrupole interaction

The limits:

A. Two parameter fits x- control parameter

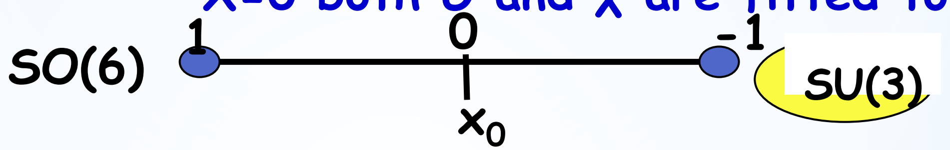
$$H = 1/2(1-x)\ \chi\ Q.Q - 1/2(1+x)G(S^{\dagger}.S - P^{\dagger}.P)$$

Control parameter - x

X=1 only pairing with strenght $G$

X=-1 only Q.Q with strengt $\chi$

X=0 both $G$ and $\chi$ are fitted to experiment




$$H = 1/2(1-x)\ \chi\ Q.Q - 1/2(1+x)G_0 S^{\dagger}.S$$

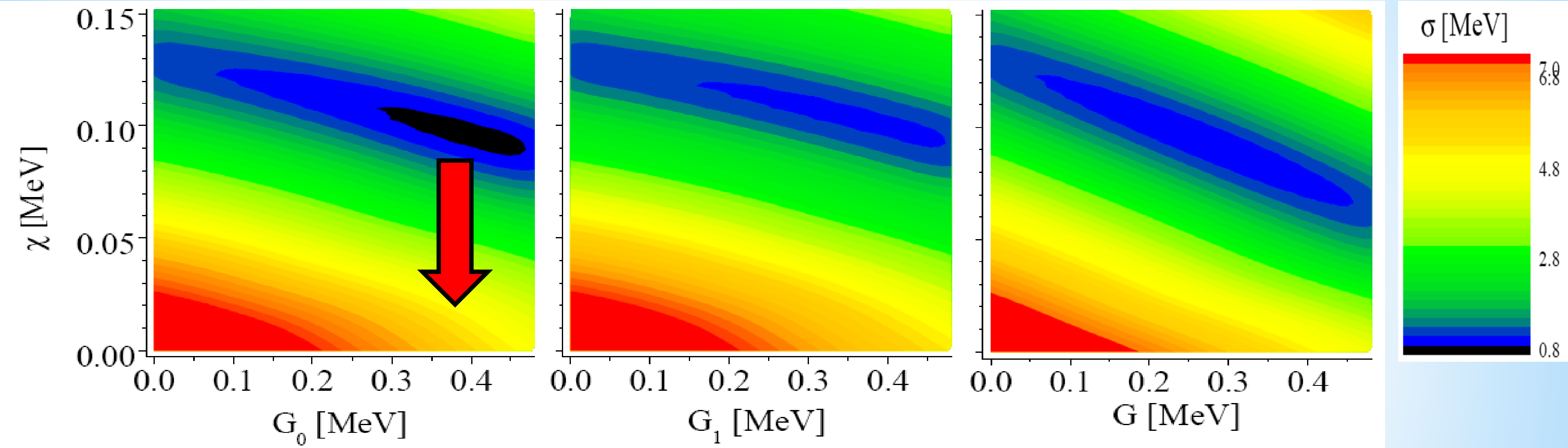

isoscalar

$$H = 1/2(1-x)\ \chi\ Q.Q - 1/2(1+x)G_1 P^{\dagger}.P$$

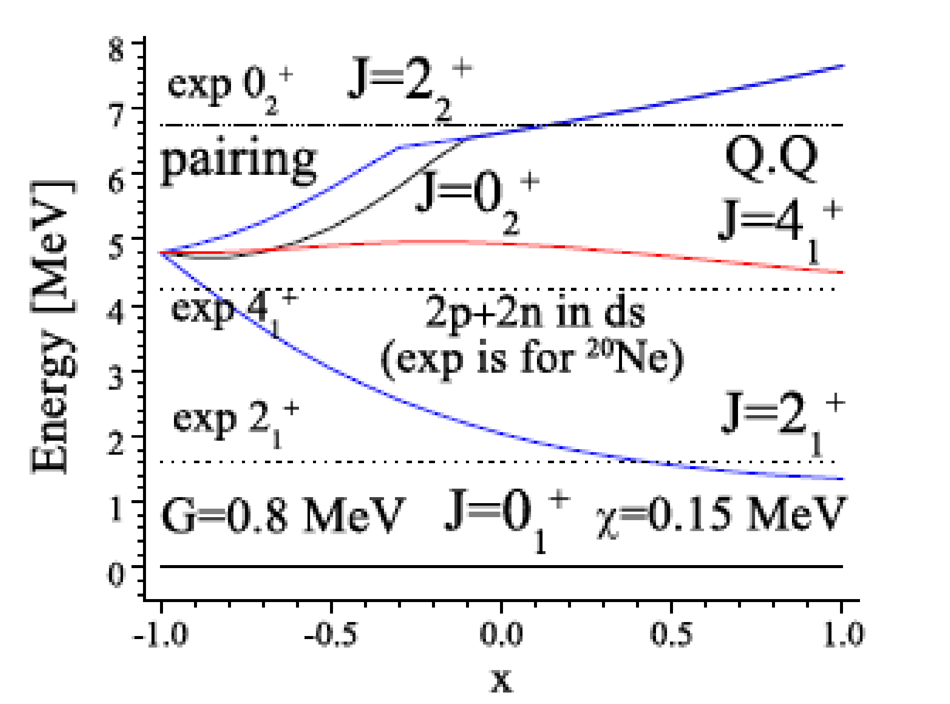
isovector

# Two-Parameter Results: $^{20}$Ne

$$\sigma = \sqrt{\sum_i \left(E^i_{\rm Th} - E^i_{\rm Exp}\right)^2 / d}$$




## Energies: From Pairing to Quadrupole



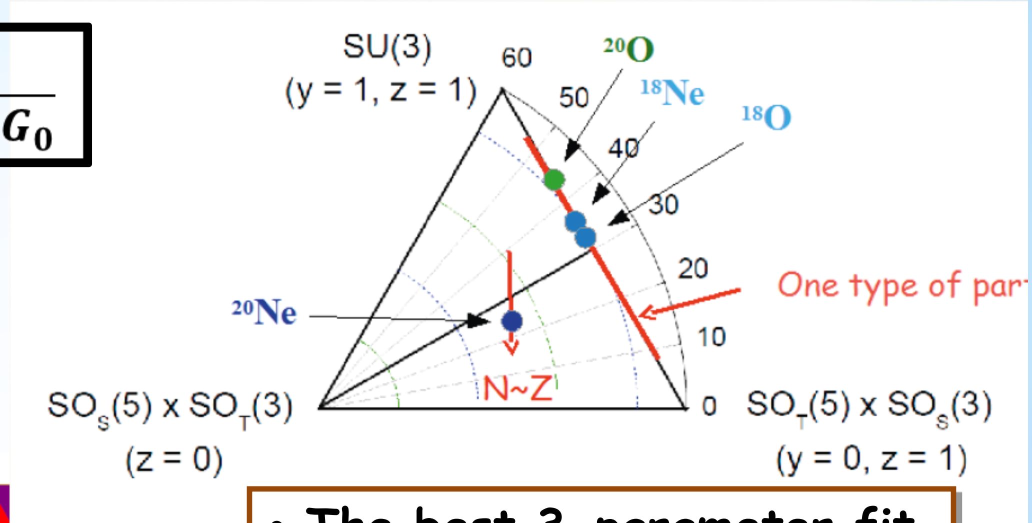
## A. Three parameter fits z and y- control parameters

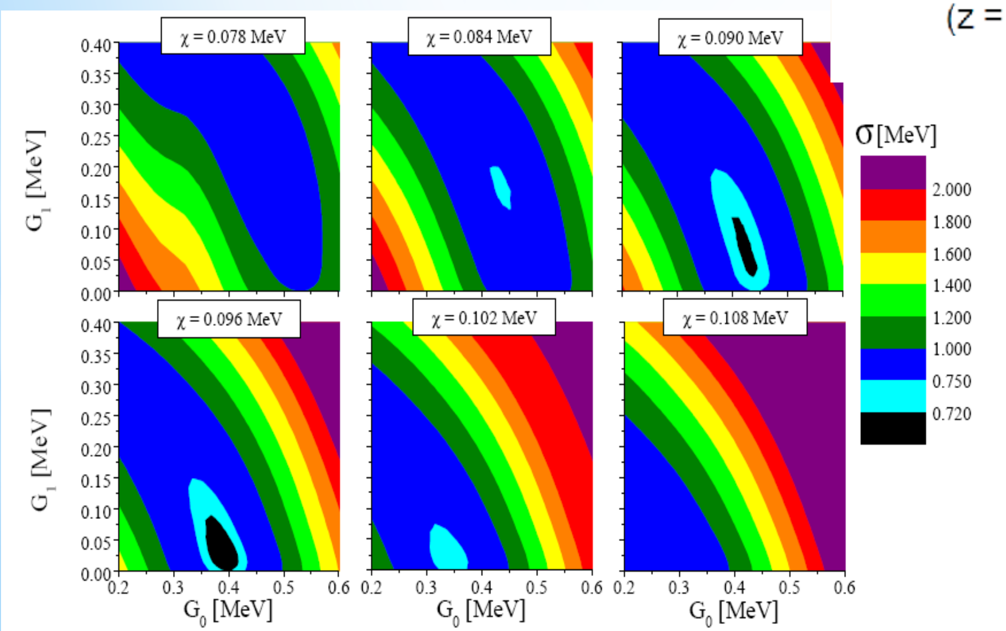
$$H = yzQ.Q - (1-z)S^{\dagger}.S - (1-y)zP^{\dagger}.P$$

$$y = \frac{\chi}{\chi + G_1}$$

$$z = \frac{\chi + G_1}{\chi + G_1 + G_0}$$

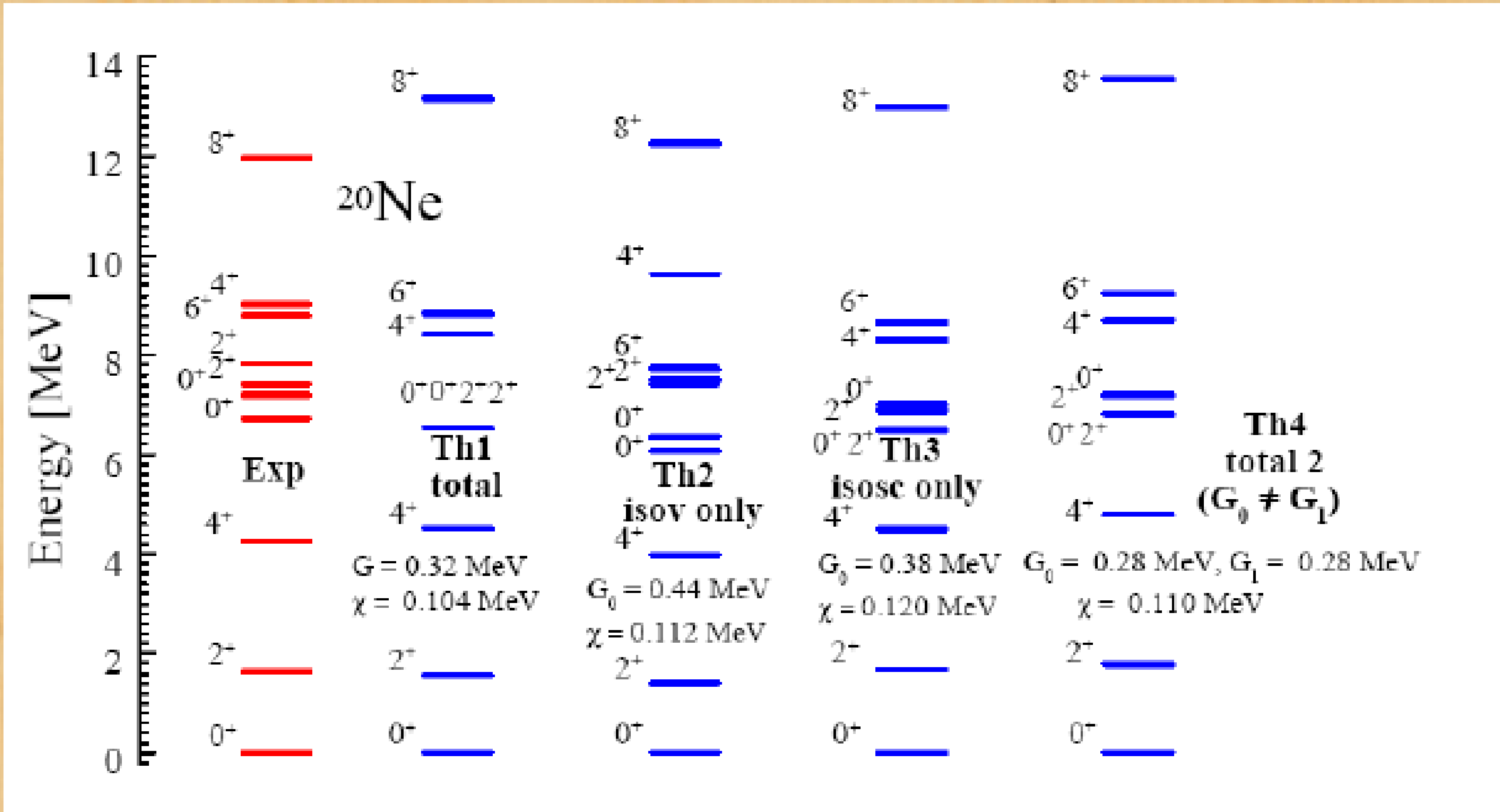
$$C = \chi + G_1 + G_0$$




- The best 3-parameter fit gives around $\chi$ = - 0.096 MeV which is the same as in the 2-parameter fit;
- The value $G_0 > G_1$ (isoscalar pairing is more important for the N~Z ds-shell nuclei).
- $G_0 / G_1 > 4$

# Energy Spectrum: $^{20}Ne$




| | | | | |
|---|---|---|---|---|
| σ [MeV] = | 1.652 | 1.675 | 1.707 | 1.630 |

# Conclusions

1. The algebraic structure of the shell-model algebra $U(4\Omega)$ is investigated to obtain its reductions through the microscopic pairing algebras, containing isoscalar (T=0, S=1), isovector (T=1, S=0) total pairing operators and Elliott's SU(3) algebra. The 4 reduction chains appear as distinct dynamical symmetries of the shell-model algebra, which allows the classification of the basis states of the system along each of them.

2. A relation between these chains is established on the basis of their complementarity to the Wigner's spin-isospin $U_{ST}(4)$ symmetry or $U_T(2) \sim SO_T(3)$, $U_S(2) \sim SO_S(3)$, .

3. This elucidates the phases – dynamical symmetries of an extended Pairing-plus-Quadrupole Model, realized in the framework of the Elliott's SU(3) scheme.

4. The phase transitions between all this limits are studied by evaluating the weights of the different interactions in the PQM Hamiltonian for a particular nuclear system .

# The use of the theory

1. We obtain numerically the probability $(Cij)^2$ **transformation brackets** with which the states of the SU(3) basis enter into the expansion of the pairing basis.

2. Evaluate the importance (weight) of the different SU(3) - states, when we need to impose restrictions on the basis because of computational difficulties.

- The parameter adjustment for a realistic nuclear system gives us the extent to which one of the two (three) modes is present; this can be addressed by introduction of one (two) control parameter(s) to explore the phase transitions;

- A comparison between the two- and three-parameter results has been made, as well as the evolution along the different number of protons and neutrons in the system;

**Thank you**