Dipole toroidal resonance: vortical properties, relation to pygmy mode, deformation impact

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EuNCP2015, Groningen, 31.08-04.09.2015
★ Exotic isoscalar E1 resonances:
  - toroidal (TR),
  - compressional (CR)
  - pygmy (PDR)

★ TR: the most accurate measure of the nuclear vorticity

J. Kvasil, V.O.N., W. Kleinig, P.-G. Reinhard, P. Vesely, 
PRC 84, 034303 (2011)

★ TR: anomalous deformation splitting

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard, 

★ TR: origin of PDR

A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil, 

★ TR: experimental status
Exotic dipole resonances

V.M. Dubovik (1975)
S.F. Semenko (1981)
M.N. Harakeh (1977)
S. Stringari (1982)

Dominate in E1(T=0) channel (after exclusion of spurious E1(T=0) c.m. motion)

irrotational vortical irrotational

TR and CR constitute low- and high-energy ISGDR branches

Experiment: \((\alpha, \alpha')\)

D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

Theory:


Skyrme RPA, SLy6
Toroidal E1 operator:

\[ \hat{M}_{\text{tor}}(E1\mu) = \frac{1}{10\sqrt{2}c} \int d\vec{r} \left[ r^3 + \frac{5}{3} r < r^2 >_0 \right] \hat{Y}_{11\mu}(\vec{r}) \cdot \left[ \vec{\nabla} \times \hat{j}_{\text{nuc}}(\vec{r}) \right] \]

- second-order part of the electric operator

Compression E1 operator:

\[ \hat{M}_{\text{com}}(E1\mu) = -\frac{i}{10c} \int d\vec{r} \left[ r^3 - \frac{5}{3} r < r^2 >_0 \right] Y_{1\mu} \left[ \vec{\nabla} \cdot \hat{j}_{\text{nuc}}(\vec{r}) \right] \]

irrotational flow

\[ \hat{M}'_{\text{com}}(E1\mu) = \int d\vec{r} \, \hat{\rho}(\vec{r}) \left[ r^3 - \frac{5}{3} r < r^2 >_0 \right] Y_{1\mu} \quad \hat{M}_{\text{com}}(E1\mu) = -k\hat{M}'_{\text{com}}(E1\mu) \]

\[ \dot{\rho} + \vec{\nabla} \cdot \vec{j}_{\text{nuc}} = 0 \]

- TR and CR are ideal examples of the vortical and irrotational motion

- to be used below a main test cases
Toroidal motion as the measure of the nuclear vorticity
Two familiar conceptions of nuclear vorticity: HD, RW

1. Hydrodynamical vorticity:

\[ \vec{\omega}(\vec{r}) = \nabla \times \vec{V}(\vec{r}) \]
\[ \delta \vec{V}(\vec{r}) = \frac{\delta \vec{j}_{\text{nuc}}(\vec{r})}{\rho_0(\vec{r})} \]

2. RW vorticity


\[ \delta \hat{j}_{i(i)}(\vec{r}) = \left\langle j_f m_f \mid \hat{j}_{\text{nuc}}(\vec{r}) \mid j_i m_i \right\rangle = \sum_{\lambda \mu} \frac{(j_i m_i \lambda \mu | j_f m_f)}{\sqrt{2j_f + 1}} \left[ \hat{j}^{(fi)}_{\lambda \lambda - 1}(r) \hat{Y}^*_{\lambda \lambda - 1 \mu} + \hat{j}^{(fi)}_{\lambda \lambda + 1}(r) \hat{Y}^*_{\lambda \lambda + 1 \mu} \right] \]

\[ \delta \hat{j}^{\nu}_{i(\mu)}(\vec{r}) = \left\langle \nu \mid \hat{j}_{\text{nuc}}(\vec{r}) \mid 0 \right\rangle = -\frac{i}{\sqrt{3}} \left[ \hat{j}^{\nu}_{10}(r) \hat{Y}^*_{10 \mu} + \hat{j}^{\nu}_{12}(r) \hat{Y}^*_{12 \mu} \right] \]

\[ j^{\nu}(r) \]

- independent part of charge-current distribution,
- decoupled from CE in the integral sense
- may be the measure of the vorticity

HD and j+ prescriptions give opposite conclusions on CM vorticity!
208Pb:
all RPA states
at E=6-9 MeV

\[\langle \mathbf{v} / \hat{M}_{\text{tor}} (E1\mu) / 0 \rangle = -\frac{1}{6c} \int \, dr \, r^2 \left[ \frac{\sqrt{2}}{5} r^2 j^+ (r) + (r^2 - \langle r^2 \rangle_0) j^- (r) \right]\]

\[\langle \mathbf{v} / \hat{M}_{\text{com}} (E1\mu) / 0 \rangle = -\frac{1}{6c} \int \, dr \, r^2 \left[ \frac{2\sqrt{2}}{5} r^2 j^+ (r) - (r^2 - \langle r^2 \rangle_0) j^- (r) \right]\]

\[\jmath^+ \text{, } \jmath^-:\]
- both have strong curl's and div's
- there is no any advantage of \(\jmath^+\) over \(\jmath^-\) to represent the vorticity

The vortical or irrotational character of the flow is provided not by \(\jmath^+\) or \(\jmath^-\) components separately but by their proper superposition
Finally:

- RW conception of the vorticity is not relevant:
  - CE-unrestricted in integral sense,
  - failure for CM,
  - j+ has no advantages over j-.

- TR conception is more correct:
  - vortical by construction,
  - locally CE-unrestricted,
  - close to HD conception,
  - gives visually vortical image,
  - correct for both TR and CR.

So just the toroidal current and strength are the best measure of the nuclear vorticity.
Deformation effects in the toroidal resonance


Deformation effects in the toroidal mode

RPA

GDR: $E(\mu = 0) < E(\mu = 1)$

TM: $E(\mu = 0) > E(\mu = 1)$

Unusual sequence of $\mu = 0$ and $\mu = 1$ branches

Deformation (not resid. Interaction) effect

Non-Tassie mode!

Relation of E1 toroidal and pygmy resonances


Review:
Strength functions

SLy6

Two peaks at 7.5 and 10.3 MeV in agreement to RMF calculations (D. Vretenar, N. Paar, P. Ring, PRC, 63, 047301 (2001))

\((\alpha, \alpha')\) experiment of Uchida et al (2003)

PDR region hosts TR and CR!

Typical PDR transition density:

\(-n\)

\(-p\)
RPA vs 1ph

-both isoscalar and isovector
- toroidal flow mainly from neutrons

1ph

- mainly isoscalar
- toroidal flow from both n/p

So the toroidal flow is basically formed already by the mean-field. But residual interaction makes it collective and more impressive.
Does the **vortical** toroidal flow contradict the **irrotational** PRD picture?
- PDR can be viewed as a local peripheral part of TR and CR
- Our calculations demonstrate the TR flow in PDR energy region also in Ni, Zr, Sn, ...

132Sn, SVbas, with PDR
TR: experimental status

Experiment: \((\alpha, \alpha')\)


Looks reasonable since the theory predicts only TR to form the low-energy part of ISGDR.

Anyway is it possible to propose a reaction where TR:
- could be observed alone or
- could demonstrate a particular fingerprint?

The reaction should be:
- IS (to suppress the effect of the dominant E1(T=1) modes)
- transversal but not polluted by magnetic form-factors
- sensitive to nuclear interior

\((e,e'), \quad (\alpha, \alpha') \) peripheral, not sensitive to nuclear interior, 
\((p,p') \) both IS/IV

Reactions with polarized beams/targets?

So far \((\alpha, \alpha')\) is the best option where TR can be excited:
- not directly but through the coupling with CR or PDR
- through peripheral part of TR
Conclusions

- **Toroidal current (strength)** is the most relevant fingerprint and measure of the **nuclear vorticity**.
  - It is more convenient and relevant than RW and HD prescriptions.
  - TR is the **only** known example of the **vortical collective electric motion**.

- Anomalous deformation splitting in TR

- PDR seems to be a local surface part of the **toroidal motion**.
  - PDR is a complex mixture of:
    - IS/IV,
    - collective/s-p,
    - irrotational/vortical,
    - TM / CM / GDR,
    - complex configurations

- Unambiguous experimental observation of TR: still a challenge. $(\alpha, \alpha')$ is so far best.
Thank you for attention!

QPM calculations taking into account complex configurations

**Summed** QPM velocity fields in 6.5-10.5 MeV region

Toroidal-like picture in T=1 channel.

\[
\delta \vec{V} = \frac{N}{A} \delta \vec{V}_p - \frac{Z}{A} \delta \vec{V}_n
\]
FIG. 3. Velocity distributions for the most pronounced dipole peaks in $^{116}\text{Sn}$ (see Fig. 2). The velocity fields correspond to the peaks at 8.82 MeV (a), 10.47 MeV (b), 17.11 MeV (c), and 30.97 MeV (d).
Motivation of PDR picture (2): oscillations of excess neutrons against the N=Z core

D. Vretenar, N. Paar, and P. Ring,
PRC, 63, 047301 (2001)

RMF calculations

RPA state at 7.29 MeV

neutron excess

82 \leq N \leq 126

nuclear core