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# Interpretation of new states in open/hidden quarkonium

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INFN BARI

## Outline

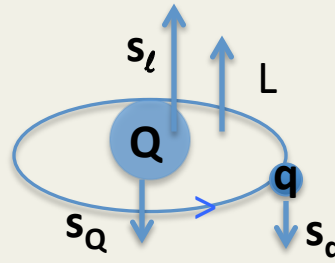
- Open flavoured heavy mesons
- Hidden flavoured quarkonia
- A brief discussion about the newly observed pentaquarks

## Mesons with open heavy flavour: $\bar{q}Q$ ( $Q=c,b$ )

- Many new states experimentally observed need to be classified
- Exploiting an effective QCD lagrangian based on heavy quark & chiral symmetries allows to predict
  - masses
  - strong couplings
  - strong decay rates
- Model independent predictions: ratios of branching fractions
- Results useful to properly classify the observed heavy mesons: the successful cases of  $D_{s3}^*(2860)$  and  $D_{s1}^*(2700)$

Hadrons with a single heavy quark:  
exploiting the heavy quark large mass limit  $m_Q \rightarrow \infty$  (Heavy Quark Effective Theory)

### Intuitive picture

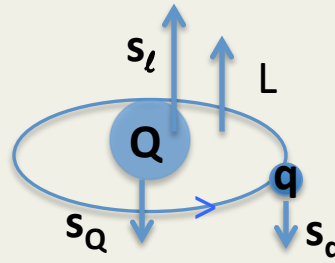


- hadrons which differ only for the HQ flavour/spin have the same configuration of the light degrees of freedom
- relations can be established among the properties of such hadrons
- heavy quark symmetry is not exact:  $1/m_Q$  corrections can be systematically included



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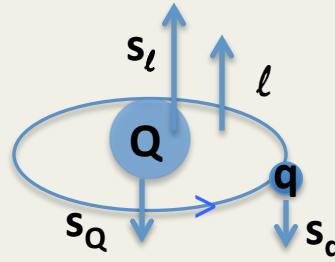


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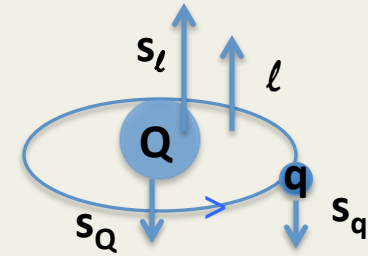
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have the same configuration of the light degrees of freedom
  - relations can be established among the properties of such hadrons
  - heavy quark symmetry is not exact:  $1/m_Q$  corrections can be systematically included
- HQ spin & flavour symmetries

Spin of the heavy quark  $s_Q$   
and total angular momentum of the light degrees of freedom  $s_l$   
separately conserved

Hadrons with a single heavy quark:  
Exploiting the heavy quark large mass limit  $m_Q \rightarrow \infty$  (HQET)



## Implications for spectroscopy

$$\vec{s}_\ell = \vec{\ell} + \vec{s}_q$$

$$P = (-1)^{\ell+1}$$

**Spin symmetry:** mesons classified in doublets with

$$J = s_\ell \pm \frac{1}{2}$$

members of the same doublet i) degenerate  
ii) same total width

**Flavour symmetry:** relations between charm and beauty hadron properties

mass splittings among the doublets independent of flavour

# Heavy-Light meson doublets

$$\vec{s}_\ell = \vec{\ell} + \vec{s}_q \quad J = s_\ell \pm \frac{1}{2}$$

| $\ell$ | $\vec{s}_\ell$ | $J^P$        | charm                     | beauty                    |
|--------|----------------|--------------|---------------------------|---------------------------|
| 0      | $\frac{1}{2}$  | $(0^-, 1^-)$ | $(D_{(s)}, D_{(s)}^*)$    | $(B_{(s)}, B_{(s)}^*)$    |
| 1      | $\frac{1}{2}$  | $(0^+, 1^+)$ | $(D_{(s)0}^*, D_{(s)1}')$ | $(B_{(s)0}^*, B_{(s)1}')$ |
|        | $\frac{3}{2}$  | $(1^+, 2^+)$ | $(D_{(s)1}, D_{(s)2}^*)$  | $(B_{(s)1}, B_{(s)2}^*)$  |
| 2      | $\frac{3}{2}$  | $(1^-, 2^-)$ | $(D_{(s)1}^*, D_{(s)2})$  | $(B_{(s)1}^*, B_{(s)2})$  |
|        | $\frac{5}{2}$  | $(2^-, 3^-)$ | $(D_{(s)2}', D_{(s)3}^*)$ |                           |

1S

1P

1D

2S

2P

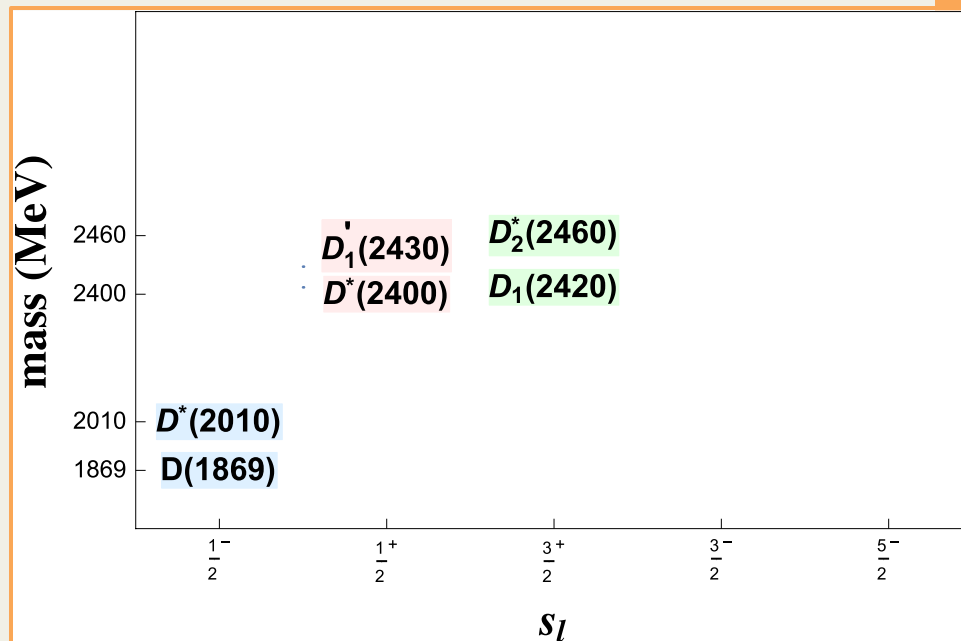
2D

radial excitations might be in the same mass range

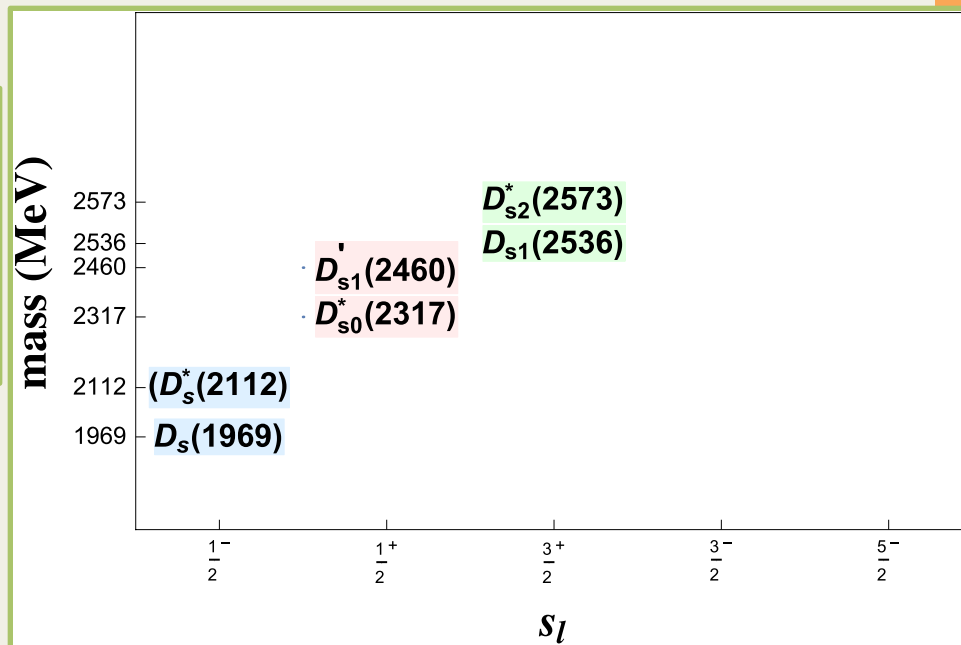


# established excited states: their place in the doublets

| $c\bar{q}$             | mass (MeV)           | $\Gamma$ (MeV)              |
|------------------------|----------------------|-----------------------------|
| $D_0^{*0}(2400)$       | $2318 \pm 29$        | $267 \pm 40$                |
| $D_0^{*\pm}(2400)$     | $2403 \pm 14 \pm 35$ | $283 \pm 24 \pm 34$         |
| $D_1^{\prime 0}(2430)$ | $2427 \pm 26 \pm 25$ | $384 \pm_{75}^{107} \pm 74$ |
| $D_1^0(2420)$          | $2421.3 \pm 0.6$     | $27.1 \pm 2.7$              |
| $D_1^{\pm}(2420)$      | $2423.4 \pm 3.1$     | $26 \pm 6$                  |
| $D_2^{*0}(2460)$       | $2462.6 \pm 0.7$     | $49.0 \pm 1.4$              |
| $D_2^{*\pm}(2460)$     | $2464.4 \pm 1.9$     | $37 \pm 6$                  |



| $c\bar{s}$       | mass (MeV)         | $\Gamma$ (MeV)           |
|------------------|--------------------|--------------------------|
| $D_{s0}^*(2317)$ | $2317.8 \pm 0.6$   | $< 3.8$                  |
| $D_{s1}'(2460)$  | $2459.6 \pm 0.6$   | $< 3.5$                  |
| $D_{s1}(2536)$   | $2535.12 \pm 0.13$ | $0.92 \pm 0.03 \pm 0.04$ |
| $D_{s2}^*(2573)$ | $2572.6 \pm 0.9$   | $20 \pm 5$               |



## Charmed mesons: recently observed states

BaBar

BaBar PRD 82 (2010) 111101

| Resonance      | Final state             | Mass (MeV)               | Width (MeV)            |
|----------------|-------------------------|--------------------------|------------------------|
| $D^0(2550)$    | $D^{*+}\pi^-$           | $2539.4 \pm 4.5 \pm 6.8$ | $130 \pm 12 \pm 13$    |
| $D^{*0}(2600)$ | $D^+\pi^-, D^{*+}\pi^-$ | $2608.7 \pm 2.4 \pm 2.5$ | $93 \pm 6 \pm 13$      |
| $D^{*+}(2600)$ | $D^+\pi^-$              | $2621.3 \pm 3.7 \pm 4.2$ | 93 (fixed)             |
| $D^0(2750)$    | $D^{*+}\pi^-$           | $2763.3 \pm 2.3 \pm 2.3$ | $60.9 \pm 5.1 \pm 3.6$ |
| $D^{*+}(2760)$ | $D^0\pi^+$              | $2769.7 \pm 3.8 \pm 1.5$ | 60.9 (fixed)           |

LHCb

LHCb JHEP 1309 (2013) 145

| Resonance      | Final state   | Mass (MeV)               | Width (MeV)               |
|----------------|---------------|--------------------------|---------------------------|
| $D^0(2580)$    | $D^{*+}\pi^-$ | $2579.5 \pm 3.4 \pm 5.5$ | $77.5 \pm 17.8 \pm 46$    |
| $D^{*0}(2650)$ | $D^{*+}\pi^-$ | $2649.2 \pm 3.5 \pm 3.5$ | $140.2 \pm 17.1 \pm 18.6$ |
| $D^0(2740)$    | $D^{*+}\pi^-$ | $2737 \pm 3.5 \pm 11.2$  | $73.2 \pm 13.4 \pm 25.0$  |
| $D^{*0}(2760)$ | $D^{*+}\pi^-$ | $2761.1 \pm 5.1 \pm 6.5$ | $74.4 \pm 3.4 \pm 37.0$   |
| $D^{*0}(2760)$ | $D^+\pi^-$    | $2760.1 \pm 1.1 \pm 3.7$ | $74.4 \pm 3.4 \pm 19.1$   |
| $D^{*+}(2760)$ | $D^0\pi^+$    | $2771.7 \pm 1.7 \pm 3.8$ | $66.7 \pm 6.6 \pm 10.5$   |
| $D^0(3000)$    | $D^+\pi^-$    | $3008.1 \pm 4.0$         | $110.5 \pm 11.5$          |
| $D^{*+}(3000)$ | $D^0\pi^+$    | 3008.1 (fixed)           | 110.5 (fixed)             |

## Charmed-strange mesons: recently observed states

BaBar & Belle

| $c\bar{s}$       | mass (MeV)              | $\Gamma$ (MeV)             |
|------------------|-------------------------|----------------------------|
| $D_{s1}^*(2700)$ | $2709 \pm_6^9$          | $125 \pm 30$               |
| $D_{sJ}(2860)$   | $2862 \pm 2 \pm 5_2$    | $48 \pm 3 \pm 6$           |
| $D_{sJ}(3040)$   | $3044 \pm 8 \pm_5^{30}$ | $239 \pm 35 \pm_{42}^{46}$ |

Belle PRL100 (08) 092001

BaBar PRL 97 (06) 222001

BaBar PRD 80 (09) 092003

LHCb

| $c\bar{s}$       | mass (MeV)                     | $\Gamma$ (MeV)             |
|------------------|--------------------------------|----------------------------|
| $D_{s1}^*(2700)$ | $2709.2 \pm 1.9 \pm 4.5$       | $115.8 \pm 7.3 \pm 12.1$   |
| $D_{sJ}(2860)$   | $2860.5 \pm 2.6 \pm 2.5 \pm 6$ | $53 \pm 7 \pm 4 \pm 6$     |
| $D_{sJ}(2860)$   | $2859 \pm 12 \pm 6 \pm 23$     | $159 \pm 23 \pm 27 \pm 72$ |

LHCb JHEP 10 (12) 151

LHCb PRL 113 (2014) 162001

two almost degenerate states

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LHCb

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LHCb JHEP 10 (12) 151

LHCb PRL 113 (2014) 162001

two almost degenerate states

# Heavy-Light meson doublets: effective fields

| $\ell$ | $\vec{s}_\ell$ | $J^P$        | effective field                                                                                                                                                                                                                                                                                                        |
|--------|----------------|--------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0      | $\frac{1}{2}$  | $(0^-, 1^-)$ | $H_a = \frac{1+\gamma_5}{2} [P_{a\mu}^* \gamma^\mu - P_a \gamma_5]$                                                                                                                                                                                                                                                    |
| 1      | $\frac{1}{2}$  | $(0^+, 1^+)$ | $S_a = \frac{1+\gamma_5}{2} [P_{1a}' \gamma_\mu \gamma_5 - P_{0a}^*]$                                                                                                                                                                                                                                                  |
|        | $\frac{3}{2}$  | $(1^+, 2^+)$ | $T_a^\mu = \frac{1+\gamma_5}{2} \left\{ P_{2a}^{\mu\nu} \gamma_\nu - P_{1a\nu} \sqrt{\frac{3}{2}} \gamma_5 [g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu - v^\mu)] \right\}$                                                                                                                                        |
| 2      | $\frac{3}{2}$  | $(1^-, 2^-)$ | $X_a^\mu = \frac{1+\gamma_5}{2} \left\{ P_{2a}^{*\mu\nu} \gamma_5 \gamma_\nu - P_{1a\nu}' \sqrt{\frac{3}{2}} [g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu + v^\mu)] \right\}$                                                                                                                                      |
|        | $\frac{5}{2}$  | $(2^-, 3^-)$ | $X_a^{\mu\nu} = \frac{1+\gamma_5}{2} \left\{ P_{3a}^{\mu\nu\sigma} \gamma_\sigma - P_{2a}^{*\prime\alpha\beta} \sqrt{\frac{5}{3}} \gamma_5 \left[ g_\alpha^\mu g_\beta^\nu - \frac{1}{5} \gamma_\alpha g_\beta^\nu (\gamma^\mu - v^\mu) - \frac{1}{5} \gamma_\beta g_\alpha^\mu (\gamma^\nu - v^\nu) \right] \right\}$ |

**Effective lagrangian approach based on HQET+ chiral symmetry:  
theoretical tool to compute strong decays  $F \rightarrow H M$**

$F=H,S,T,X,X'$

$M=$  light pseudoscalar meson

$$\mathcal{L}_H = g \text{Tr}[\bar{H}_a H_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu]$$

$$\mathcal{L}_S = h \text{Tr}[\bar{H}_a S_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu] + \text{H.c.}$$

$$\mathcal{L}_T = \frac{h'}{\Lambda_\chi} \text{Tr}[\bar{H}_a T_b^\mu (iD_\mu \mathcal{A} + i\not{D} \mathcal{A}_\mu)_{ba} \gamma_5] + \text{H.c.}$$

$$\mathcal{L}_X = \frac{k'}{\Lambda_\chi} \text{Tr}[\bar{H}_a X_b^\mu (iD_\mu \mathcal{A} + i\not{D} \mathcal{A}_\mu)_{ba} \gamma_5] + \text{H.c.}$$

$$\begin{aligned} \mathcal{L}_{X'} = & \frac{1}{\Lambda_\chi^2} \text{Tr}[\bar{H}_a X_b'^{\mu\nu} [k_1 \{D_\mu, D_\nu\} \mathcal{A}_\lambda \\ & + k_2 (D_\mu D_\lambda \mathcal{A}_\nu + D_\nu D_\lambda \mathcal{A}_\mu)]_{ba} \gamma^\lambda \gamma_5] + \text{H.c.} \end{aligned}$$



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contains light pseudoscalar  
meson fields

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$$\mathcal{L}_H = g \text{Tr}[\bar{H}_a H_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu] \longrightarrow \boxed{H \rightarrow H \pi}$$

$$\mathcal{L}_S = h \text{Tr}[\bar{H}_a S_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu] + \text{H.c.} \longrightarrow \boxed{S \rightarrow H \pi}$$

$$\mathcal{L}_T = \frac{h'}{\Lambda_\chi} \text{Tr}[\bar{H}_a T_b^\mu (iD_\mu \mathcal{A} + i\not{D} \mathcal{A}_\mu)_{ba} \gamma_5] + \text{H.c.} \longrightarrow \boxed{T \rightarrow H \pi}$$

$$\mathcal{L}_X = \frac{k'}{\Lambda_\chi} \text{Tr}[\bar{H}_a X_b^\mu (iD_\mu \mathcal{A} + i\not{D} \mathcal{A}_\mu)_{ba} \gamma_5] + \text{H.c.} \longrightarrow \boxed{X \rightarrow H \pi}$$

$$\begin{aligned} \mathcal{L}_{X'} = & \frac{1}{\Lambda_\chi^2} \text{Tr}[\bar{H}_a X_b'^{\mu\nu} [k_1 \{D_\mu, D_\nu\} \mathcal{A}_\lambda \\ & + k_2 (D_\mu D_\lambda \mathcal{A}_\nu + D_\nu D_\lambda \mathcal{A}_\mu)]_{ba} \gamma^\lambda \gamma_5] + \text{H.c.} \end{aligned} \longrightarrow \boxed{X' \rightarrow H \pi}$$

**Effective lagrangian approach based on HQET+ chiral symmetry:  
theoretical tool to compute strong decays  $F \rightarrow H M$**

$F=H,S,T,X,X'$

$M=$  light pseudoscalar meson

$$\mathcal{L}_H = g \text{Tr}[\bar{H}_a H_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu] \longrightarrow H \rightarrow H \pi$$

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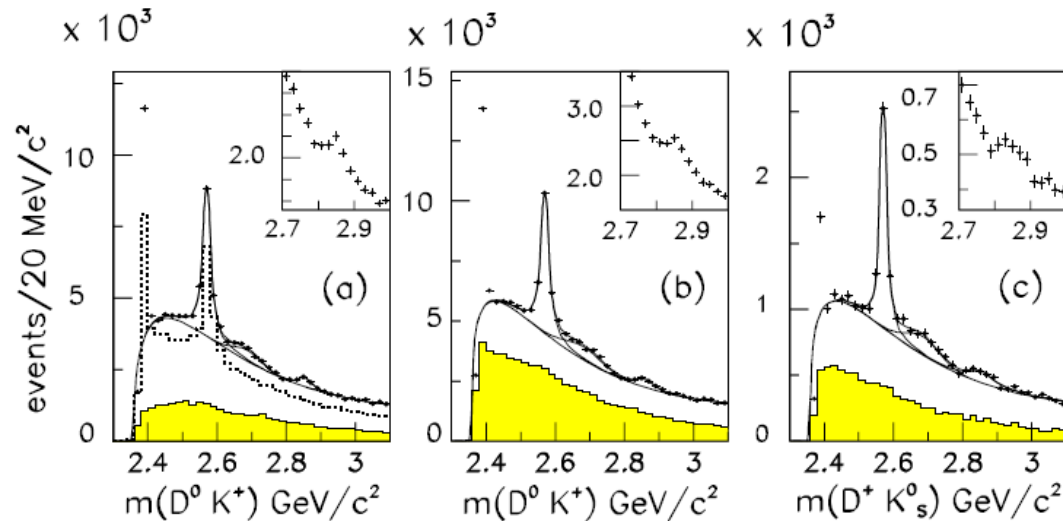
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decay rates of states in the same doublet governed by the same coupling  
 model independent predictions for ratios of Brs

## The method @ work: the case of $D_{s3}^*(2860)$

- discovered by BaBar Coll.
- reconstructed in  $D^0 K^+$  &  $D^+ K_s^0$



BaBar Collab., PRL 97 (06) 222001

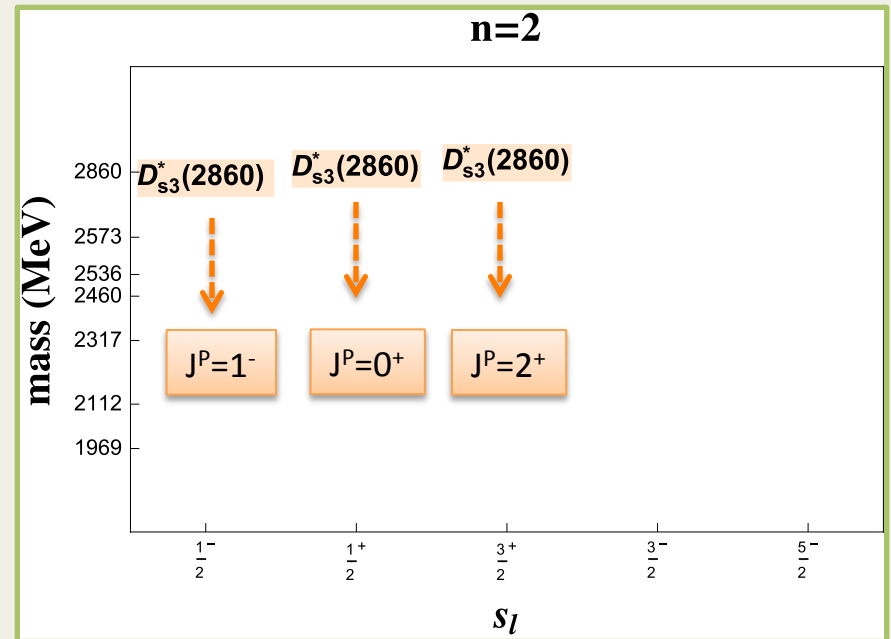
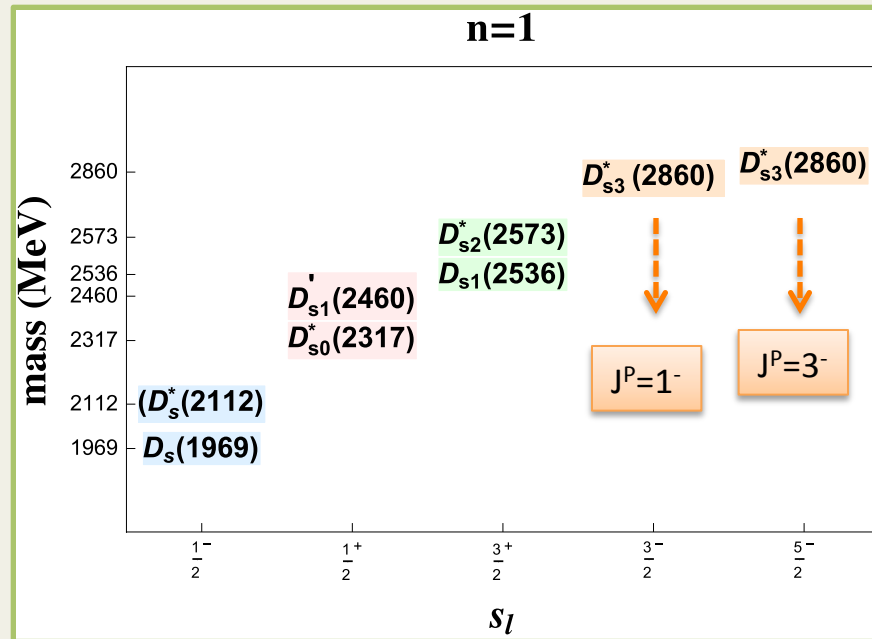
Quantum number assignment required to identify it

Possibilities: - low lying state not yet observed

- radial excitation of an already observed state

**Only states that can decay to DK allowed**

# The method @ work: the case of $D_{s3}^*(2860)$



computing strong decay rates can help discriminating the various possibilities

## The method @ work: the case of $D_{s3}^*(2860)$

P. Colangelo, S. Nicotri, FDF  
PLB 642, 48

|   | $D_{sJ}(2860)$                               | $D_{sJ}(2860) \rightarrow DK$ | $\frac{\Gamma(D_{sJ}(2860) \rightarrow D^* K)}{\Gamma(D_{sJ}(2860) \rightarrow DK)}$ | $\frac{\Gamma(D_{sJ}(2860) \rightarrow D_s \eta)}{\Gamma(D_{sJ}(2860) \rightarrow DK)}$ |
|---|----------------------------------------------|-------------------------------|--------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| 1 | $s_\ell^P = \frac{1}{2}^-, J^P = 1^-, n = 2$ | p-wave                        | 1.23                                                                                 | 0.27                                                                                    |
| 2 | $s_\ell^P = \frac{1}{2}^+, J^P = 0^+, n = 2$ | s-wave                        | 0                                                                                    | 0.34                                                                                    |
| 3 | $s_\ell^P = \frac{3}{2}^+, J^P = 2^+, n = 2$ | d-wave                        | 0.63                                                                                 | 0.19                                                                                    |
| 4 | $s_\ell^P = \frac{3}{2}^-, J^P = 1^-, n = 1$ | p-wave                        | 0.06                                                                                 | 0.23                                                                                    |
| 5 | $s_\ell^P = \frac{5}{2}^-, J^P = 3^-, n = 1$ | f-wave                        | 0.39                                                                                 | 0.13                                                                                    |

would explain the observed narrowness

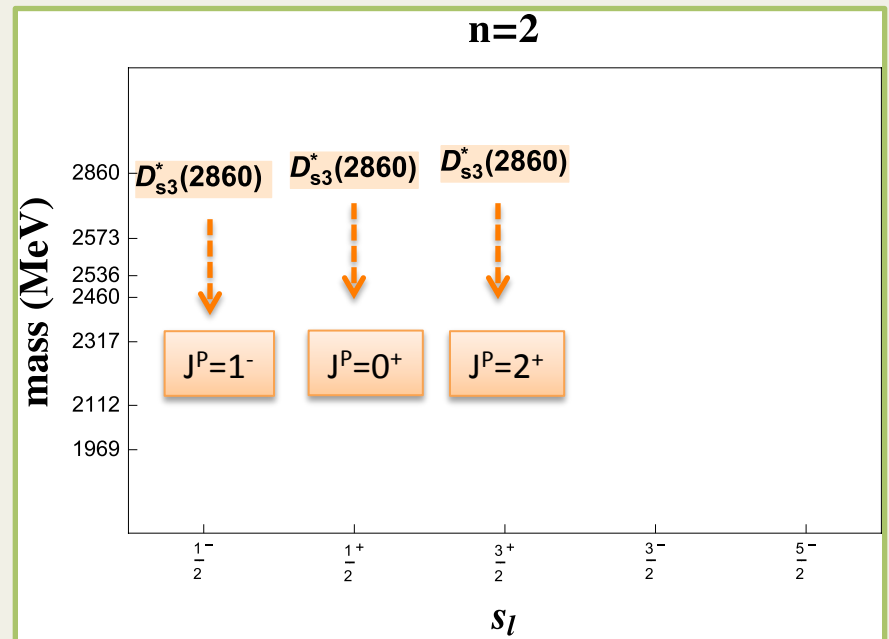
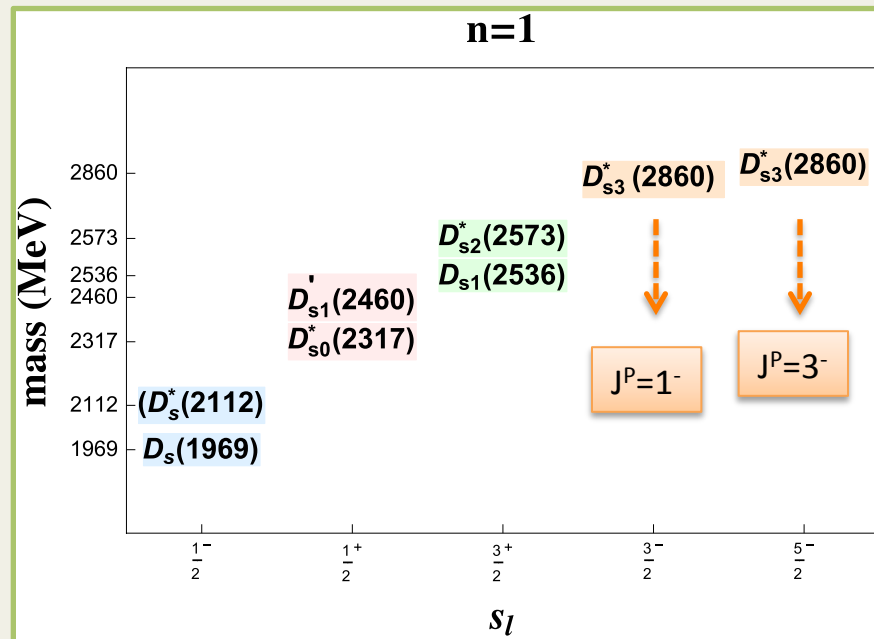
- states decaying in s-wave or p-wave expected to be much broader
- $J^P=2^+$  state could decay in p-wave as an effect of  $1/m_Q$  corrections -> broader as well

# The method @ work: the case of $D_{s3}^*(2860)$

P. Colangelo, S. Nicotri, FDF  
PLB 642, 48

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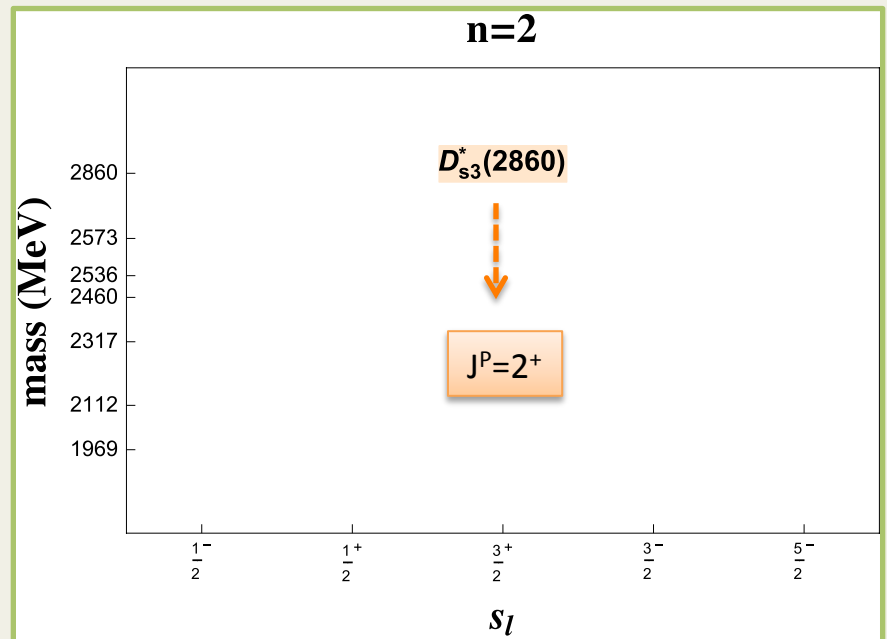
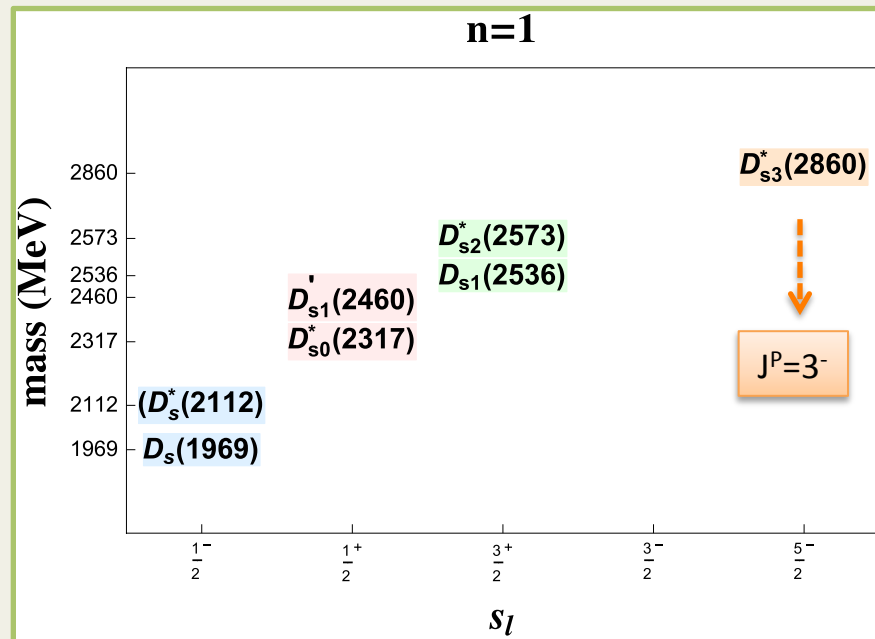


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| 3 | $s_\ell^P = \frac{3}{2}^+, J^P = 2^+, n = 2$ | d-wave                        | 0.63                                                                                 | 0.19                                                                                    |
| 4 | $s_\ell^P = \frac{3}{2}^-, J^P = 1^-, n = 1$ | p-wave                        | 0.06                                                                                 | 0.23                                                                                    |
| 5 | $s_\ell^P = \frac{5}{2}^-, J^P = 3^-, n = 1$ | f-wave                        | 0.39                                                                                 | 0.13                                                                                    |

states decaying in s-wave or p-wave expected to be much broader

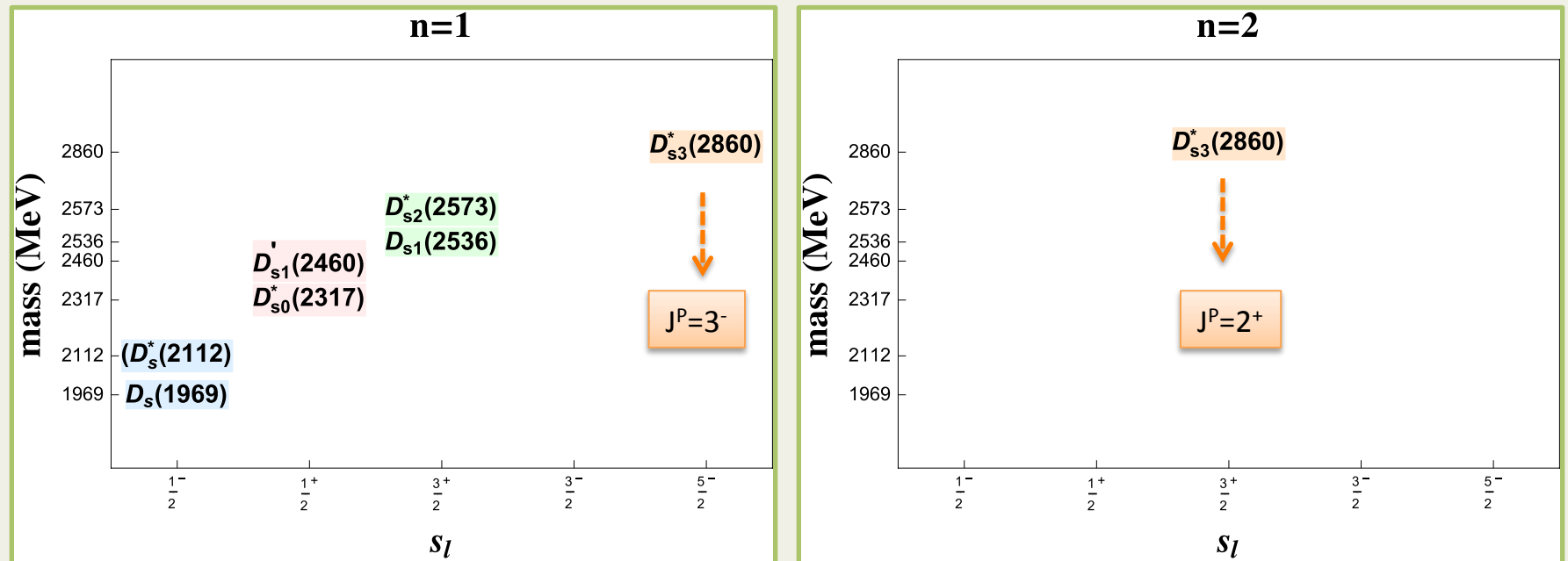


# The method @ work: the case of $D_{s3}^*(2860)$

P. Colangelo, S. Nicotri, FDF  
PLB 642, 48

|   | $D_{sJ}(2860)$                               | $D_{sJ}(2860) \rightarrow DK$ | $\frac{\Gamma(D_{sJ}(2860) \rightarrow D^* K)}{\Gamma(D_{sJ}(2860) \rightarrow DK)}$ | $\frac{\Gamma(D_{sJ}(2860) \rightarrow D_s \eta)}{\Gamma(D_{sJ}(2860) \rightarrow DK)}$ |
|---|----------------------------------------------|-------------------------------|--------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| 1 | $s_\ell^P = \frac{1}{2}^-, J^P = 1^-, n = 2$ | p-wave                        | 1.23                                                                                 | 0.27                                                                                    |
| 2 | $s_\ell^P = \frac{1}{2}^+, J^P = 0^+, n = 2$ | s-wave                        | 0                                                                                    | 0.34                                                                                    |
| 3 | $s_\ell^P = \frac{3}{2}^+, J^P = 2^+, n = 2$ | d-wave                        | 0.63                                                                                 | 0.19                                                                                    |
| 4 | $s_\ell^P = \frac{3}{2}^-, J^P = 1^-, n = 1$ | p-wave                        | 0.06                                                                                 | 0.23                                                                                    |
| 5 | $s_\ell^P = \frac{5}{2}^-, J^P = 3^-, n = 1$ | f-wave                        | 0.39                                                                                 | 0.13                                                                                    |

$J^P=2^+$  state could decay in p-wave as an effect of  $1/m_Q$  corrections -> broader as well

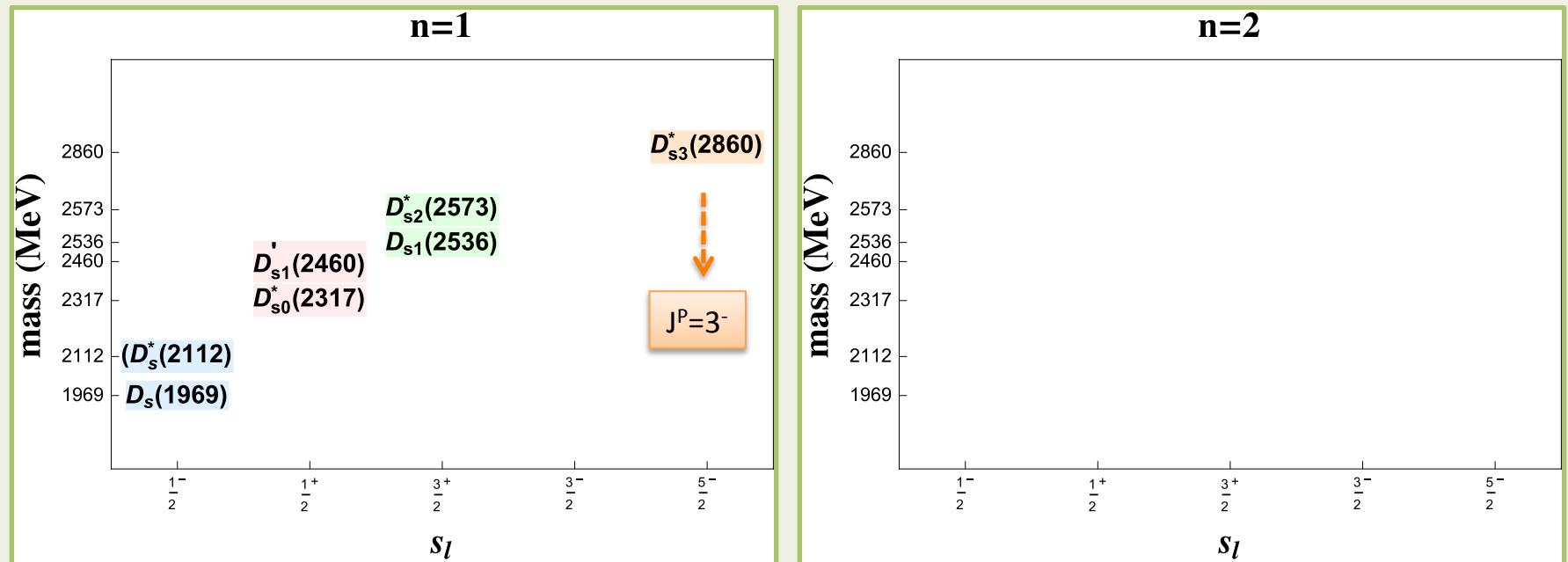


# The method @ work: the case of $D_{s3}^*(2860)$

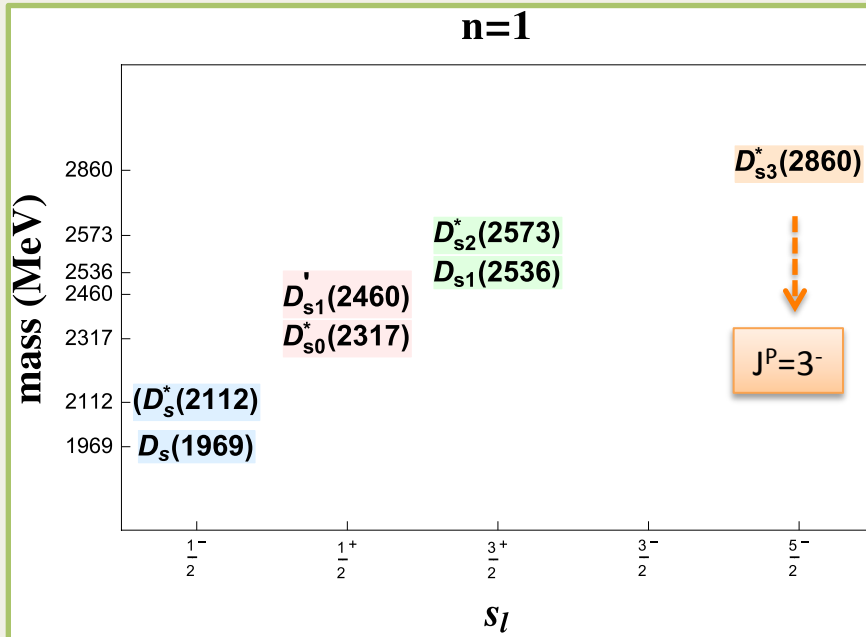
P. Colangelo, S. Nicotri, FDF  
PLB 642, 48

|   | $D_{sJ}(2860)$                               | $D_{sJ}(2860) \rightarrow DK$ | $\frac{\Gamma(D_{sJ}(2860) \rightarrow D^* K)}{\Gamma(D_{sJ}(2860) \rightarrow DK)}$ | $\frac{\Gamma(D_{sJ}(2860) \rightarrow D_s \eta)}{\Gamma(D_{sJ}(2860) \rightarrow DK)}$ |
|---|----------------------------------------------|-------------------------------|--------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| 1 | $s_\ell^P = \frac{1}{2}^-, J^P = 1^-, n = 2$ | p-wave                        | 1.23                                                                                 | 0.27                                                                                    |
| 2 | $s_\ell^P = \frac{1}{2}^+, J^P = 0^+, n = 2$ | s-wave                        | 0                                                                                    | 0.34                                                                                    |
| 3 | $s_\ell^P = \frac{3}{2}^+, J^P = 2^+, n = 2$ | d-wave                        | 0.63                                                                                 | 0.19                                                                                    |
| 4 | $s_\ell^P = \frac{3}{2}^-, J^P = 1^-, n = 1$ | p-wave                        | 0.06                                                                                 | 0.23                                                                                    |
| 5 | $s_\ell^P = \frac{5}{2}^-, J^P = 3^-, n = 1$ | f-wave                        | 0.39                                                                                 | 0.13                                                                                    |

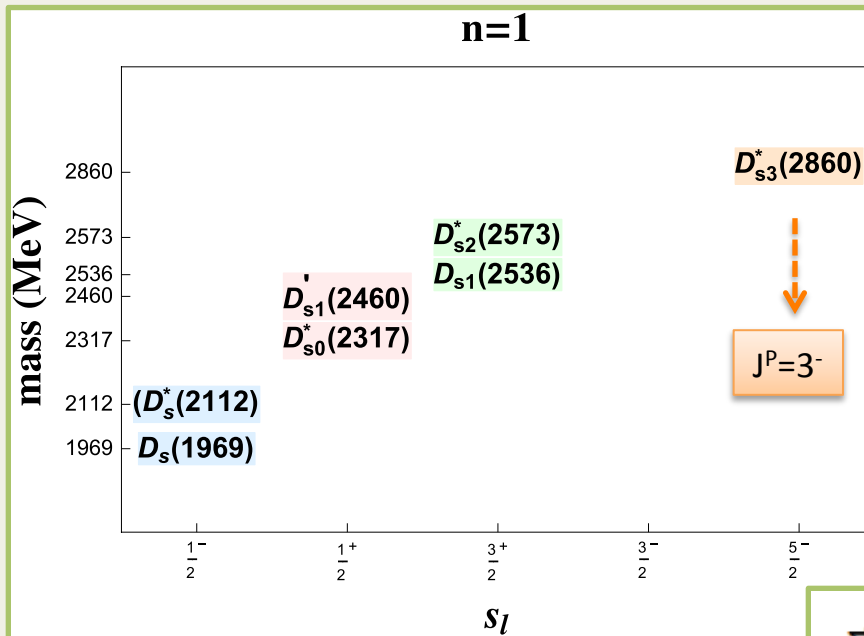
$J^P=2^+$  state could decay in p-wave as an effect of  $1/m_Q$  corrections  $\rightarrow$  broader as well



The method @ work: the case of  $D_{s3}^*(2860)$   
Here it is!



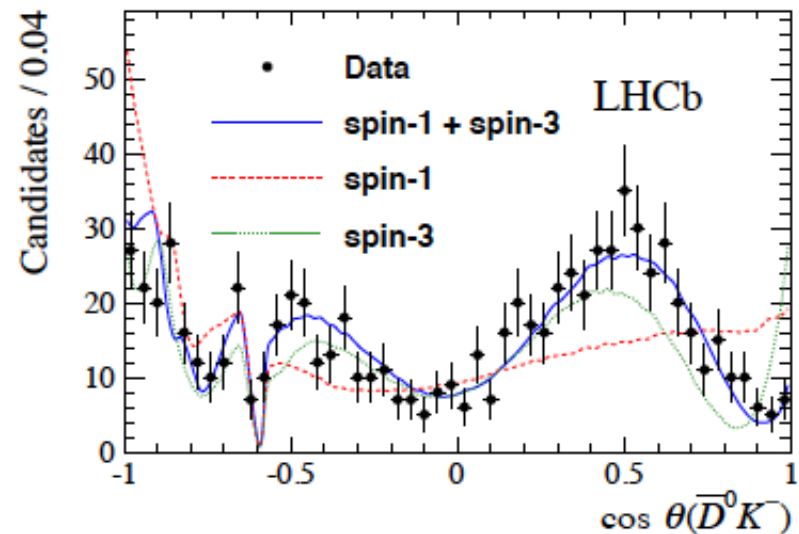
The method @ Work: the case of  $D_{s3}^*(2860)$   
Here it is!



from Dalitz plot analysis of  
 $B_s \rightarrow \bar{D}^0 K^- \pi^+$

LHCb confirms.

a second broader state  
with same mass also suggested by data

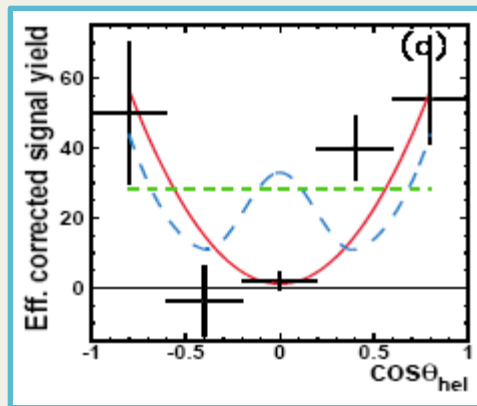


## The method @ work: the case of $D_{s1}^*(2700)$

Belle PRL100 (08) 092001  
BaBar PRD 80 (09) 092003

- observed by Belle & BaBar in  $B^+ \rightarrow \bar{D}^0 D^0 K^+$  decaying to  $D^0 K^+$
- later observed by BaBar decaying to  $D^* K$
- confirmed by LHCb
- observed by BaBar also in the Dalitz Plot analysis of  $B^0 \rightarrow D^- D^0 K^+$ ,

- the decay  $1 \rightarrow 0-0-$  implies negative parity
- spin 1 favoured



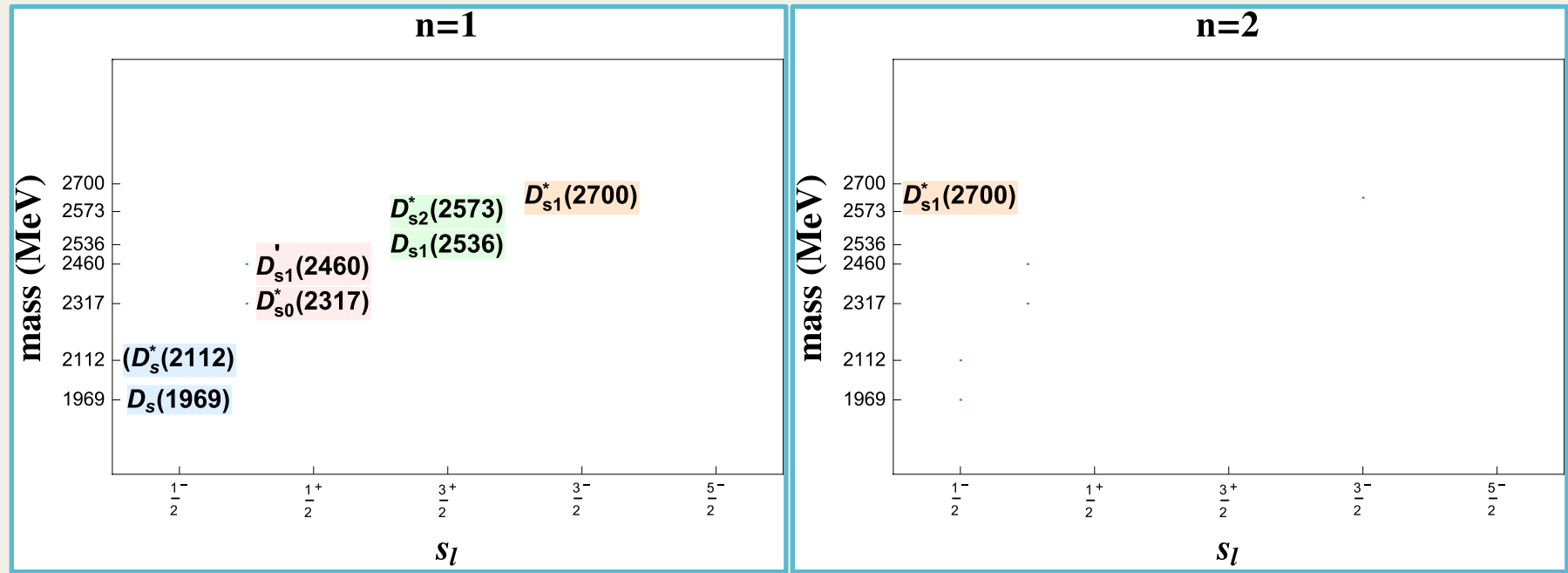
---  $J=0$   
—  $J=1$   
---  $J=2$



**$J^P=1^-$  favoured**

BaBar Dalitz Plot analysis of  $B^0 \rightarrow D^- D^0 K^+$  confirms  $J^P=1^-$

The method @ work: the case of  $D_{s1}^*(2700)$   
Where can we place it?



computing strong decay rates discriminates the two possibilities

$$R_1 = \frac{\Gamma(D_{s1}^*(2700) \rightarrow D^*K)}{\Gamma(D_{s1}^*(2700) \rightarrow DK)} \quad R_2 = \frac{\Gamma(D_{s1}^*(2700) \rightarrow D_s\eta)}{\Gamma(D_{s1}^*(2700) \rightarrow DK)} \quad R_3 = \frac{\Gamma(D_{s1}^*(2700) \rightarrow D_s^*\eta)}{\Gamma(D_{s1}^*(2700) \rightarrow DK)}$$

results

|            | $R_1 \times 10^2$ | $R_2 \times 10^2$ | $R_3 \times 10^2$ |
|------------|-------------------|-------------------|-------------------|
| $D_s^*/$   | $91 \pm 4$        | $20 \pm 1$        | $5 \pm 2$         |
| $D_{s1}^*$ | $4.3 \pm 0.2$     | $16.3 \pm 0.9$    | $0.18 \pm 0.07$   |



$$R_1 = \frac{\Gamma(D_{s1}^*(2700) \rightarrow D^*K)}{\Gamma(D_{s1}^*(2700) \rightarrow DK)} \quad R_2 = \frac{\Gamma(D_{s1}^*(2700) \rightarrow D_s\eta)}{\Gamma(D_{s1}^*(2700) \rightarrow DK)} \quad R_3 = \frac{\Gamma(D_{s1}^*(2700) \rightarrow D_s^*\eta)}{\Gamma(D_{s1}^*(2700) \rightarrow DK)}$$

results

|            | $R_1 \times 10^2$ | $R_2 \times 10^2$ | $R_3 \times 10^2$ |
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| $D_s^*/$   | $91 \pm 4$        | $20 \pm 1$        | $5 \pm 2$         |
| $D_{s1}^*$ | $4.3 \pm 0.2$     | $16.3 \pm 0.9$    | $0.18 \pm 0.07$   |



most sensitive ratio

$$R_1 = \frac{\Gamma(D_{s1}^*(2700) \rightarrow D^*K)}{\Gamma(D_{s1}^*(2700) \rightarrow DK)} \quad R_2 = \frac{\Gamma(D_{s1}^*(2700) \rightarrow D_s\eta)}{\Gamma(D_{s1}^*(2700) \rightarrow DK)} \quad R_3 = \frac{\Gamma(D_{s1}^*(2700) \rightarrow D_s^*\eta)}{\Gamma(D_{s1}^*(2700) \rightarrow DK)}$$

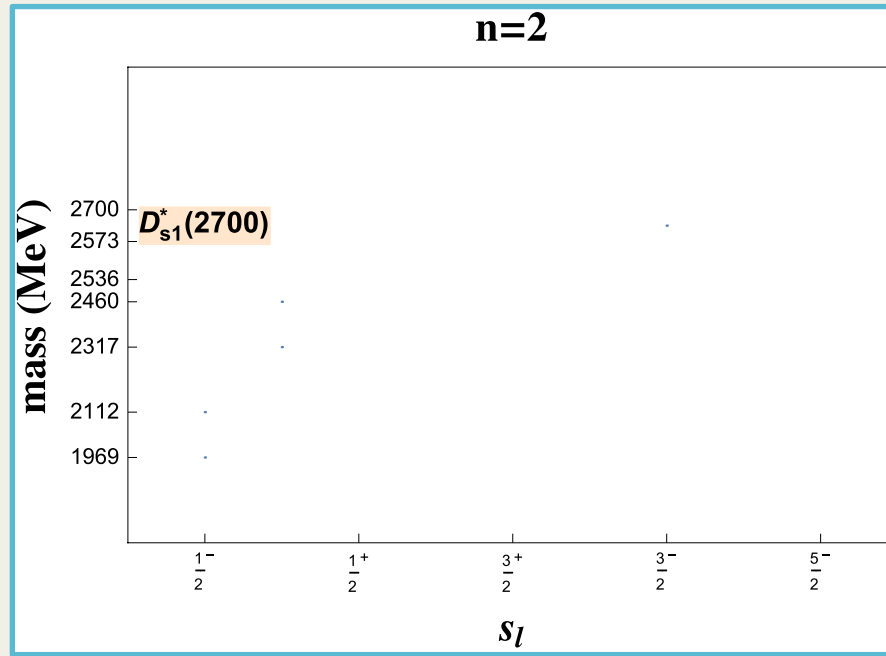
results

|            | $R_1 \times 10^2$ | $R_2 \times 10^2$ | $R_3 \times 10^2$ |
|------------|-------------------|-------------------|-------------------|
| $D_s^*/$   | $91 \pm 4$        | $20 \pm 1$        | $5 \pm 2$         |
| $D_{s1}^*$ | $4.3 \pm 0.2$     | $16.3 \pm 0.9$    | $0.18 \pm 0.07$   |

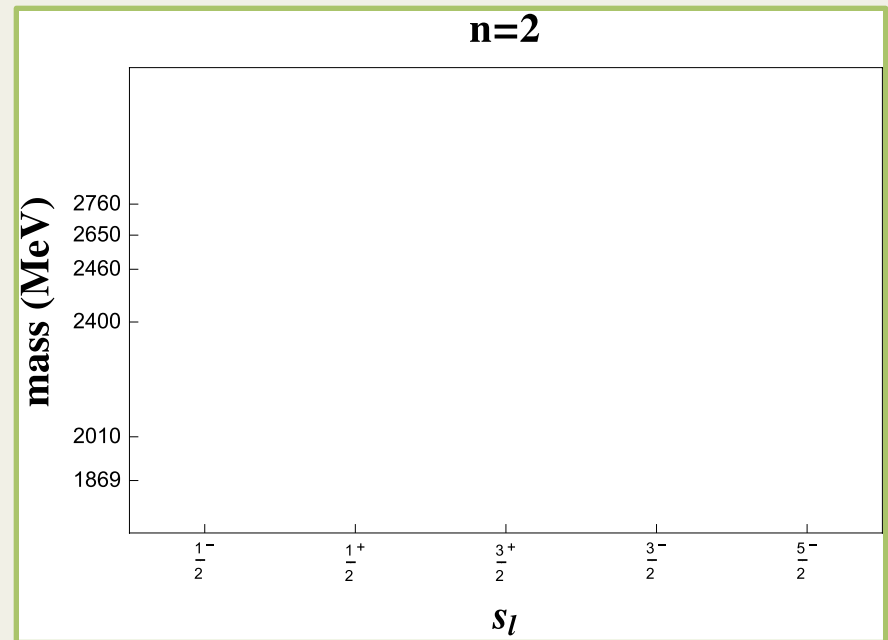
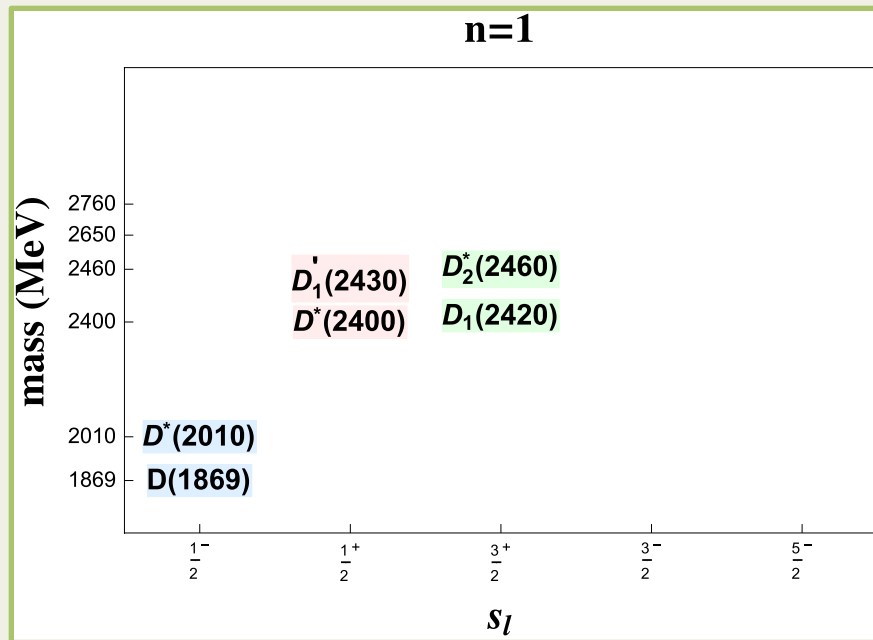
Babar :

$$R_1 = 0.91 \pm 0.13_{stat} \pm \pm 0.12_{syst}$$

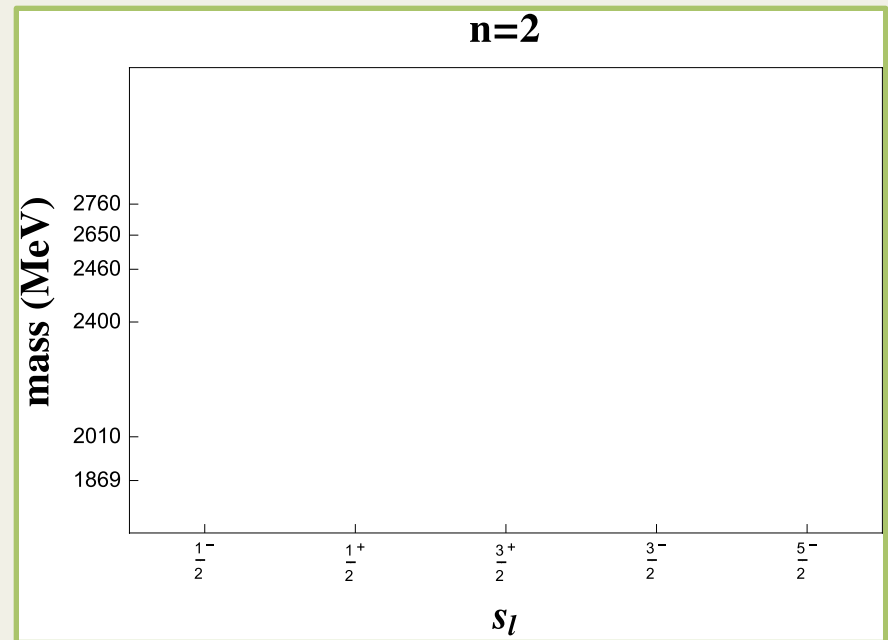
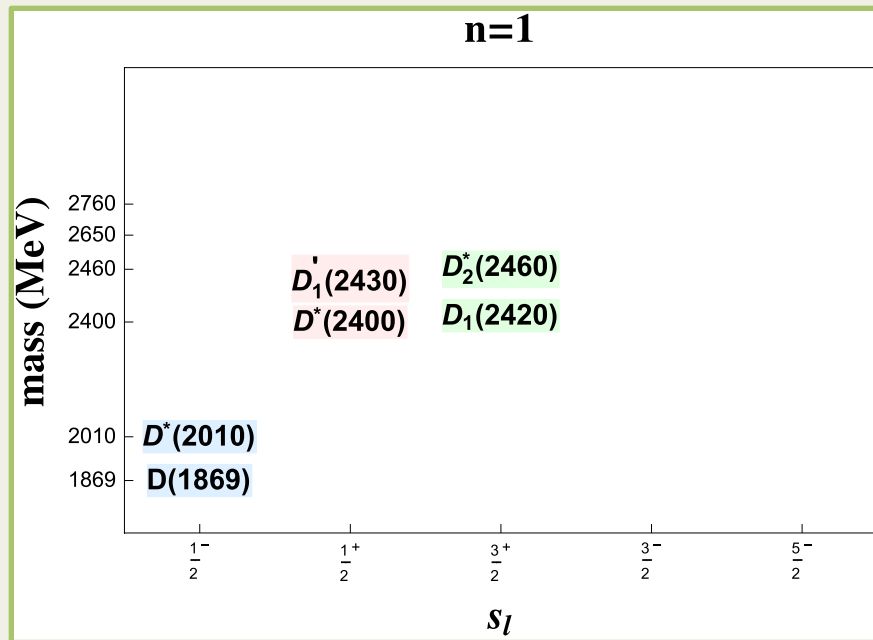
The method @ work: the case of  $D_{s1}^*(2700)$   
Here it is!



Charmed mesons:  
what about the new states?

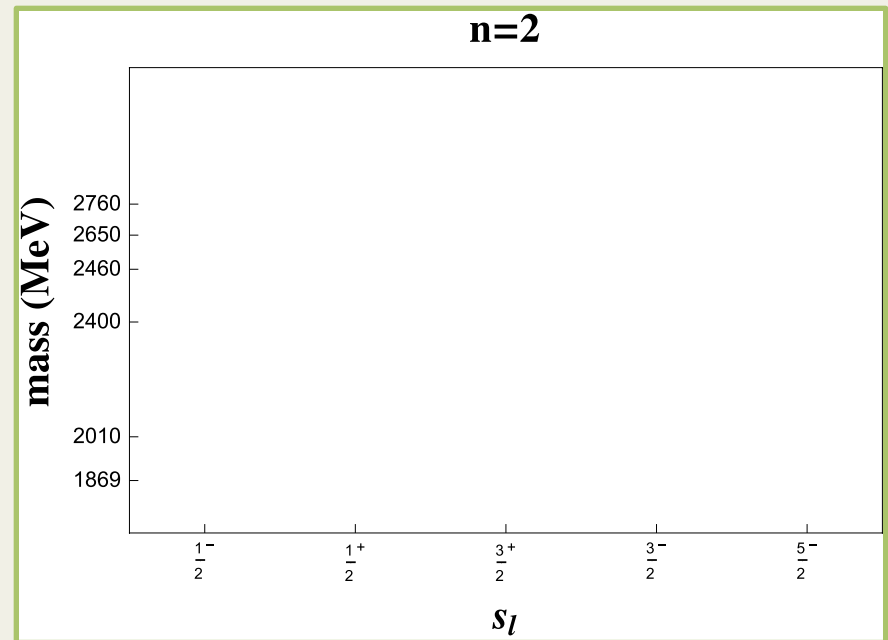
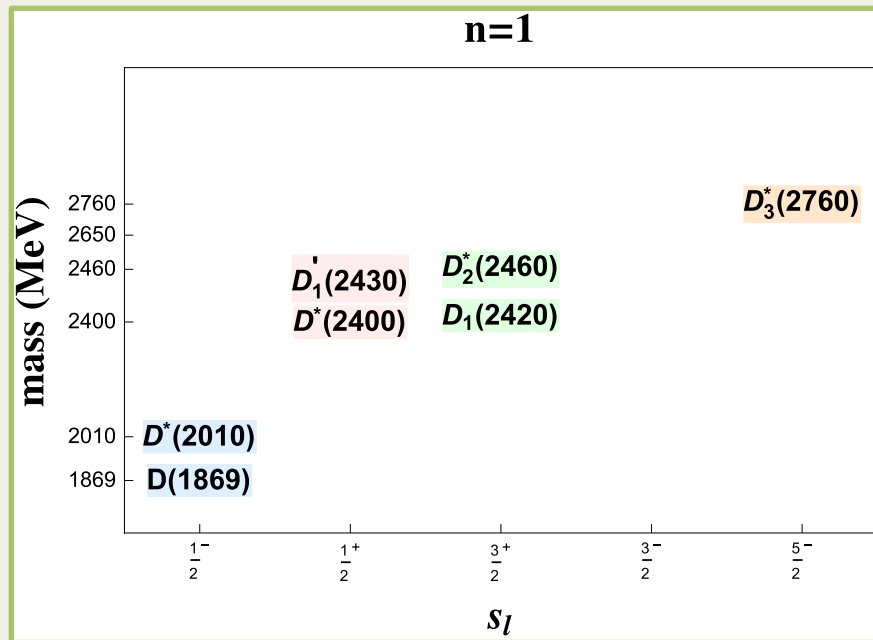


Charmed mesons:  
what about the new states?



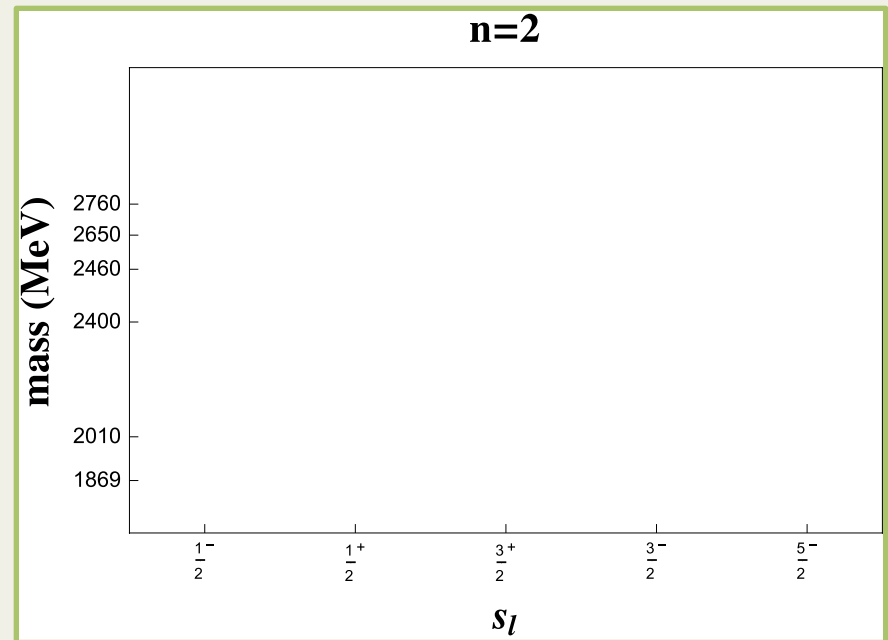
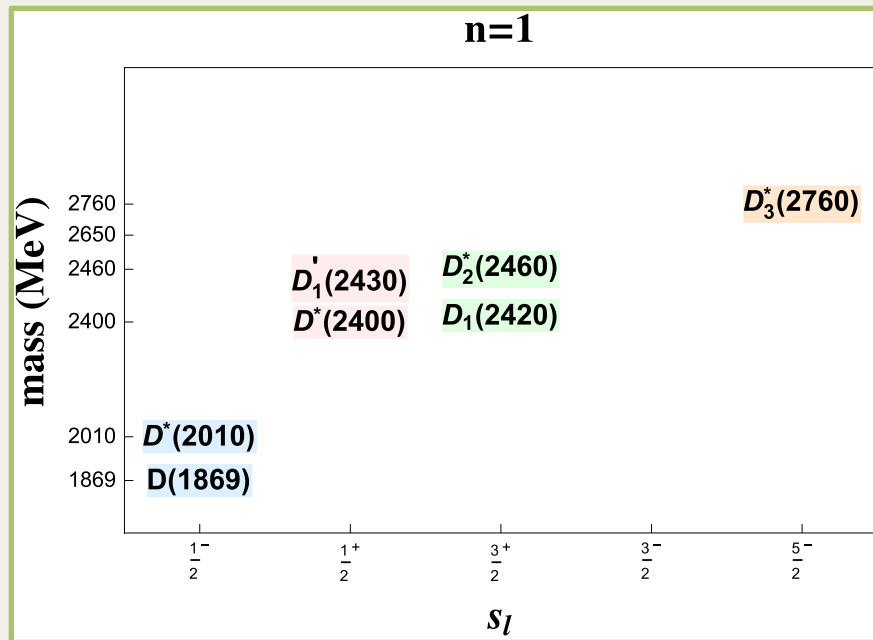
LHCb determines:  $D_J^*(2760) \rightarrow J^P=3^-$

Charmed mesons:  
what about the new states?



LHCb determines:  $D_J^*(2760) \rightarrow J^P=3^-$

Charmed mesons:  
what about the new states?

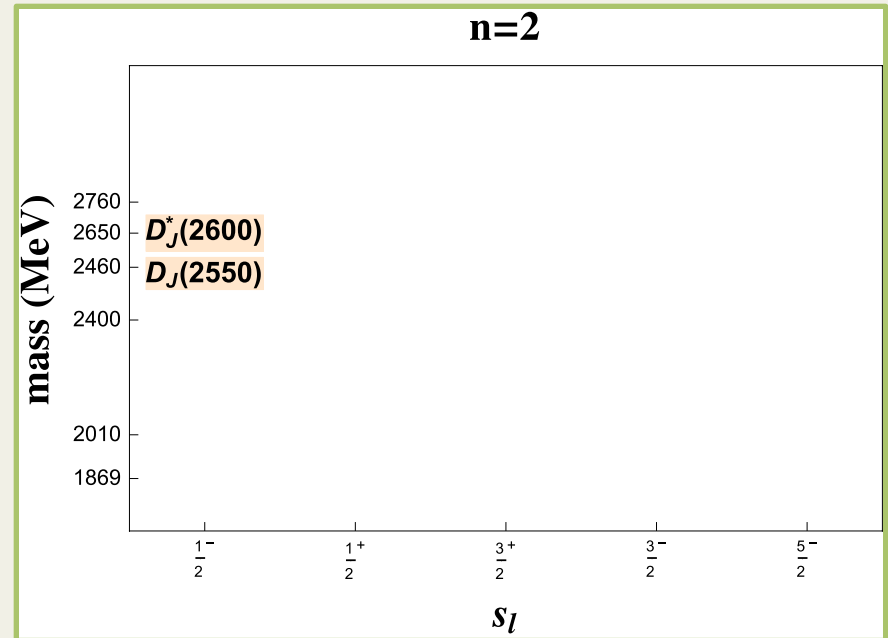
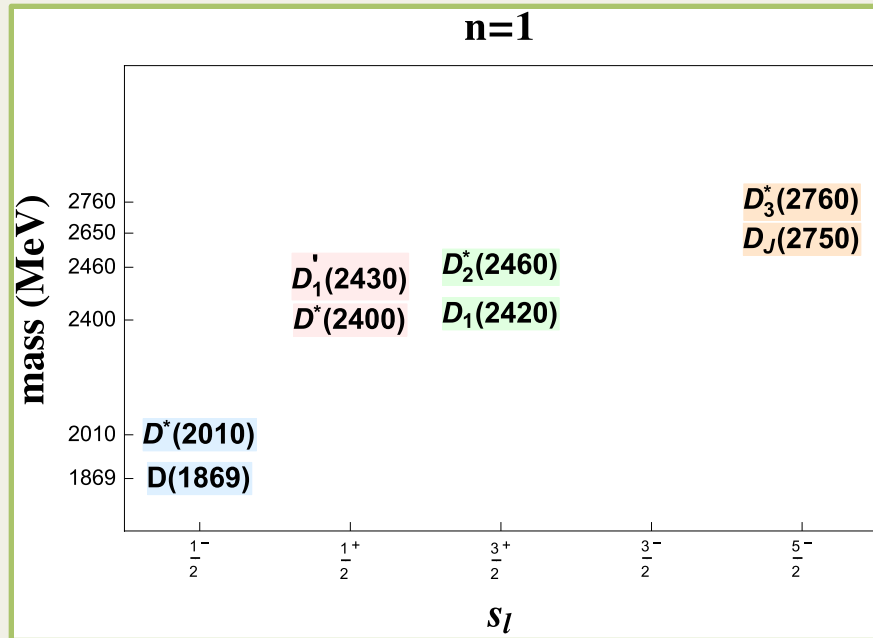


LHCb determines:  $D_J^*(2760) \rightarrow J^P=3^-$

BaBar and LHCb: no complete agreement on the parameters of the other observed states

Charmed mesons:  
what about the new states?

tentative assignment proposed by BaBar





## From charm to beauty

### Observed excited mesons with open beauty

| $b\bar{q}$       | mass (MeV)       | $\Gamma$ (MeV)                          |
|------------------|------------------|-----------------------------------------|
| $B_1^0(5721)$    | $5723.4 \pm 2.0$ |                                         |
| $B_2^{*0}(5747)$ | $5743 \pm 5$     | $22.7 \pm_{3.2}^{3.8} \pm_{10.2}^{3.2}$ |

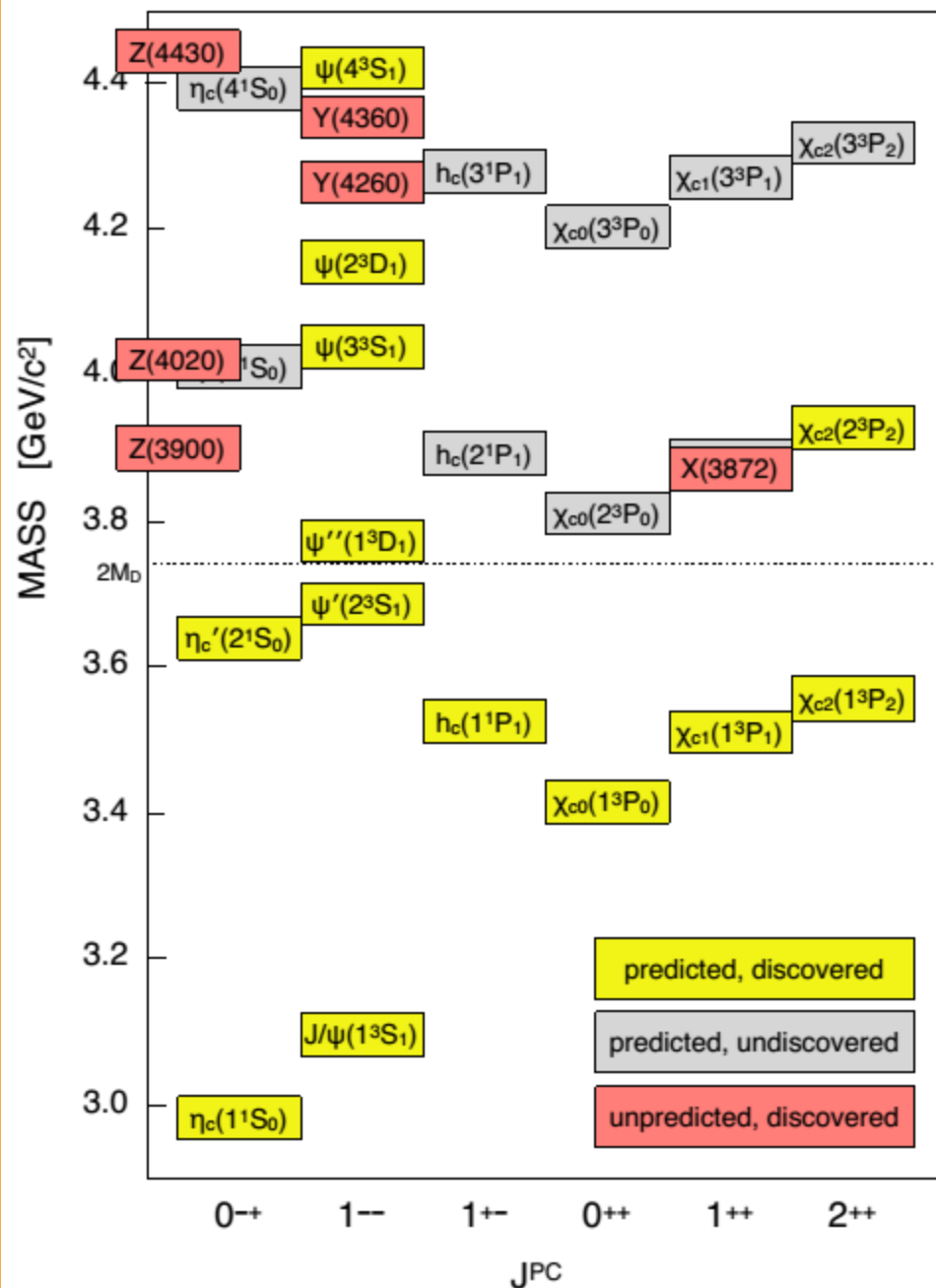
| $b\bar{s}$          | mass (MeV)       | $\Gamma$ (MeV) |
|---------------------|------------------|----------------|
| $B_{s1}^0(5830)$    | $5829.4 \pm 0.7$ |                |
| $B_{s2}^{*0}(5840)$ | $5839.7 \pm 0.6$ |                |

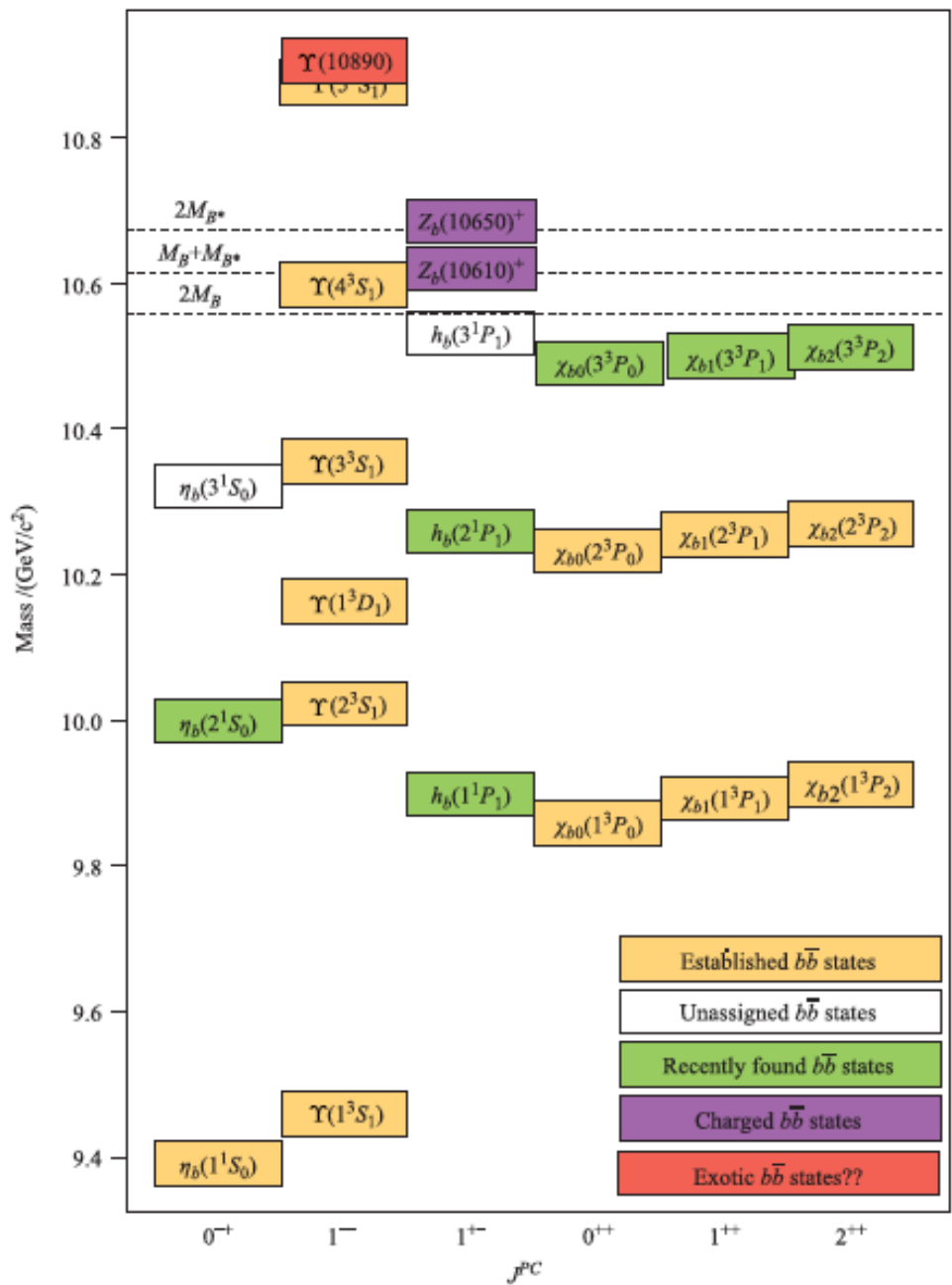
New states observed  
candidates for identification with radial excitations of the fundamental doublet:

$$(B_J(5840), B_J(5960)) \longrightarrow n=2 \ J^P=(0^-,1^-)$$

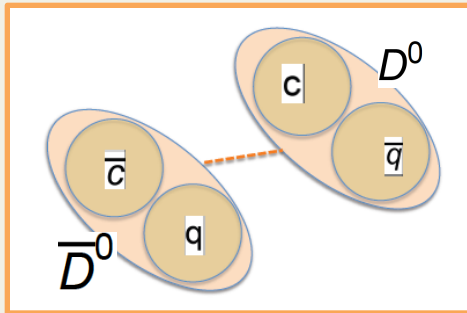
## hidden charm

- ordinary charmonia
- exotic candidates:  
X,Y,Z all charmonium like
- Y with  $J^{PC}=1^{--}$
- X with  $J^{PC} \neq 1^{--}$
- Z composed by  $c\bar{c} + q\bar{q}$

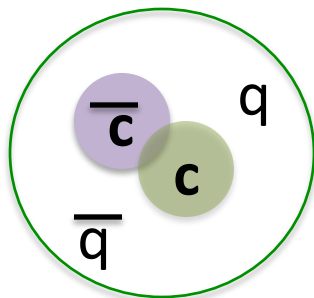




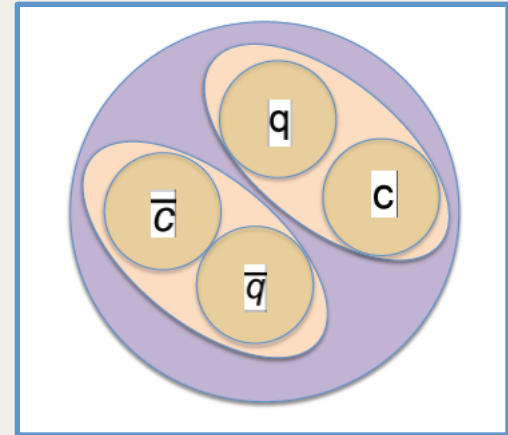
## Many theoretical proposals



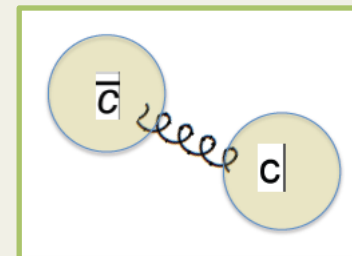
loosely bound molecule



hadrocharmonium

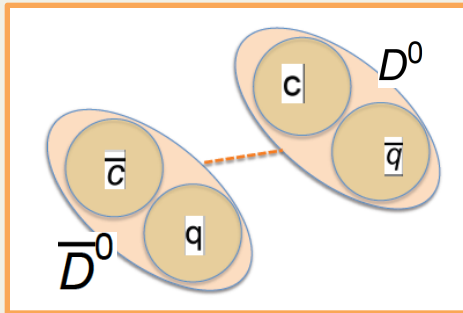


tightly bound diquark-antidiquark



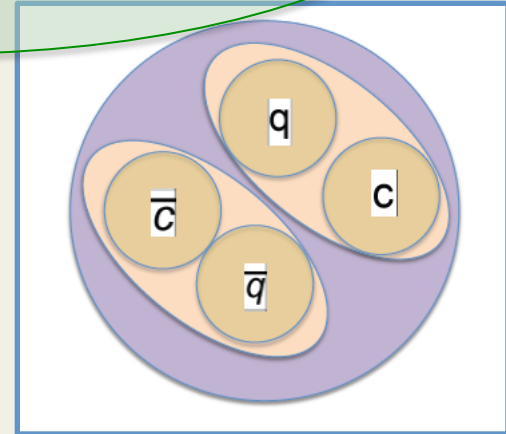
hybrid

Many theoretical proposals

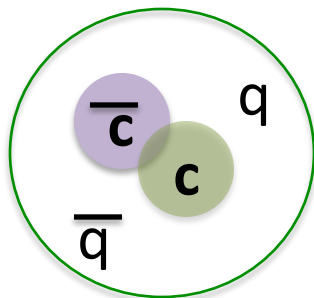


loosely bound molecule

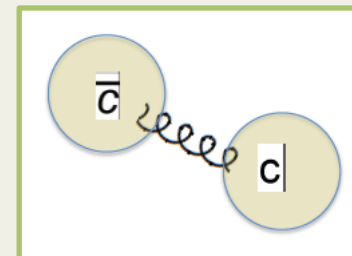
when the mass of the state is close to  
a open flavour threshold for a pair of  
 $Q\bar{q}$  mesons  
width of constituents must be  $\ll$   
than that of the state



tightly bound diquark-antidiquark

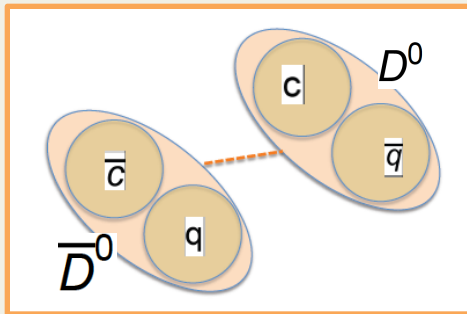


hadrocharmonium

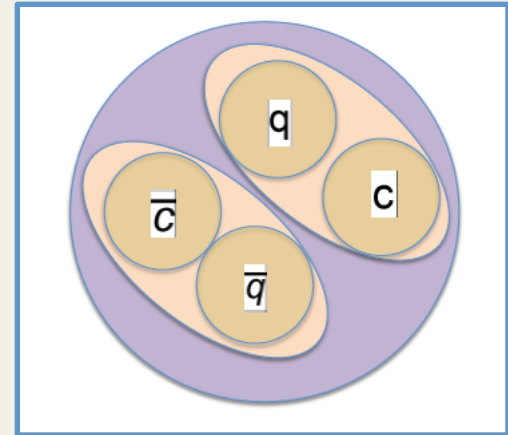


hybrid

# Many theoretical proposals



loosely bound molecule



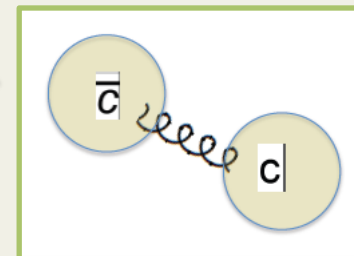
tightly bound diquark-antidiquark

$Q\bar{Q}g$  are its constituents, with  $Q\bar{Q}$  in a color octet state.

Distinctive feature:  
exotic quantum numbers  
 $0^{+-}, 1^{-+}, 2^{+-}, \dots$

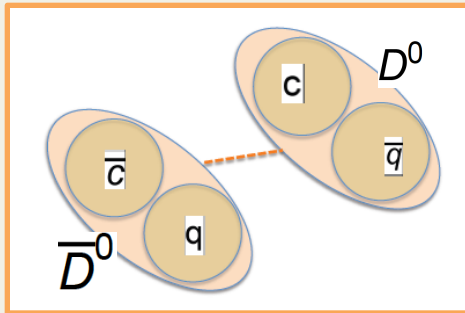
$\bar{q}$

hadrocharmonium

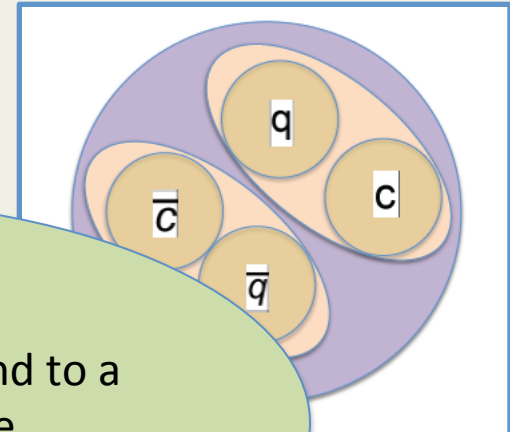


hybrid

## Many theoretical proposals

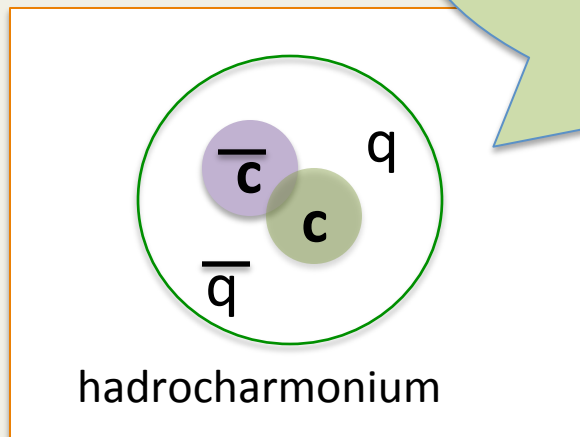


loosely bound molecule

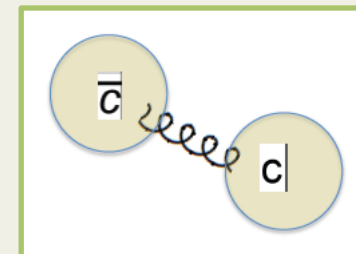


antidiquark

color-singlet  $q\bar{q}$  pair bound to a  
color-singlet  $Q\bar{Q}$  core.  
Binding: analog to van der Waals

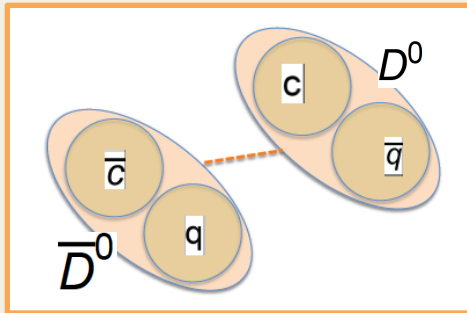


hadrocharmonium



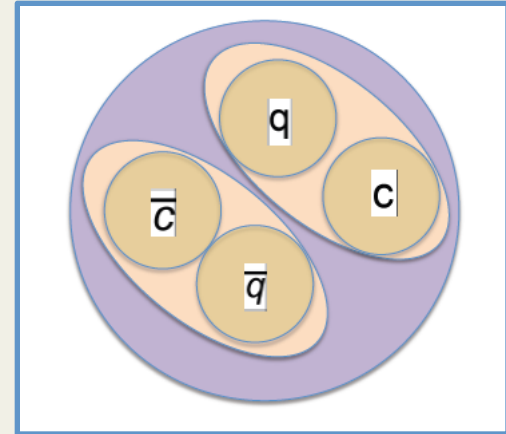
hybrid

Many theoretical proposals

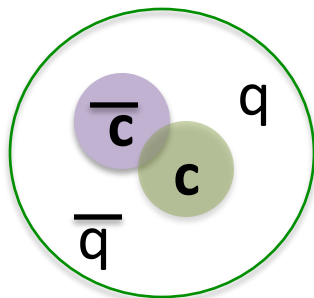


loosely bound molecule

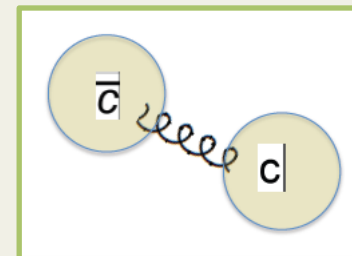
Several variants  
- compact tetraquarks  
- diquarkonium tetraquarks



tightly bound diquark-antidiquark



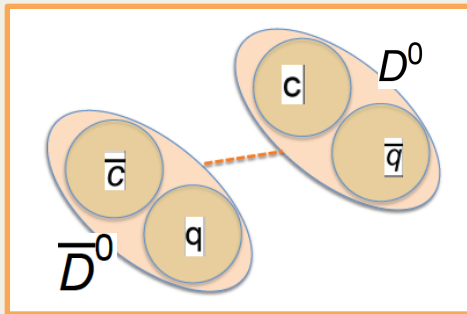
hadrocharmonium



hybrid

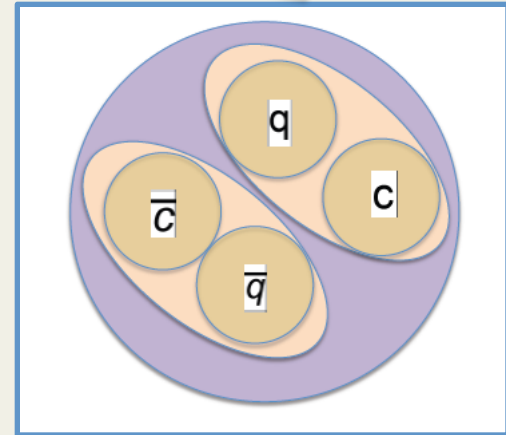


Many theoretical proposals

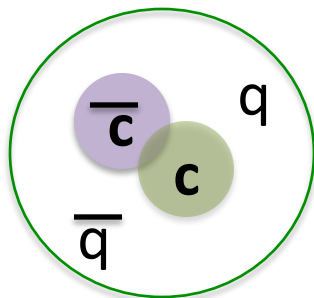


loosely bound molecule

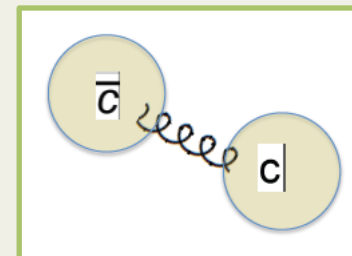
Compact  $Q\bar{Q}q\bar{q}$ : interaction potential acts between each quark pair.  
Requires  $M(Q\bar{Q}q\bar{q}) < M(\bar{Q}q) + M(Q\bar{q})$ ,  
 $M(Q\bar{Q}q\bar{q}) < M(Q\bar{Q}) + M(\bar{q}q)$ , otherwise falling apart happens



tightly bound diquark-antidiquark

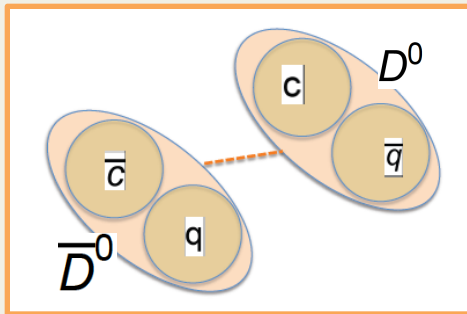


hadrocharmonium

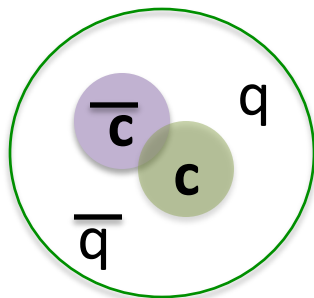


hybrid

Many theoretical proposals



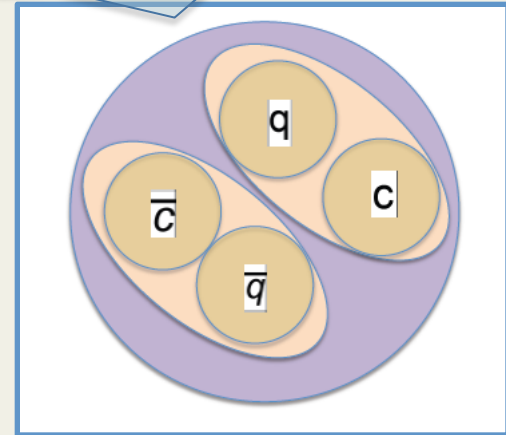
loosely bound molecule



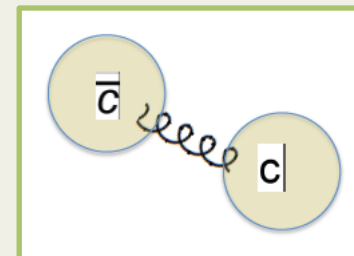
hadrocharmonium

Diquarkonium tetraquarks:  
Bound state of diquarks  $Qq$   
(color antitriplet) and  $\bar{Q}q$   
(triplet).

Very large number of predicted  
states



tightly bound diquark-antidiquark



hybrid

## The beginning of the new spectroscopy era: X(3872)

Belle PRL 91 (03) 262001

- discovered by Belle in 2003 in the channel:  $X \rightarrow J/\psi \pi^+ \pi^-$
- confirmed by BaBar, CDF, D0
- very narrow  $\Gamma < 1.2 \text{ MeV}$  (Belle)
- observed in  $J/\psi \gamma \Rightarrow C=+1$
- 2013 LHCb full angular analysis fixes  $J^{PC}=1^{++}$
- decays to  $J/\psi \pi^+ \pi^-$  and  $J/\psi \pi^+ \pi^- \pi^0$  occur with similar rates  
 $\Rightarrow$  isospin violation considering  $\rho$  and  $\omega$  as intermediate states

LHCb EPJC 72 (12) 1972

BUT - phase space effects should be considered

- $B(\rho \rightarrow \pi\pi) > B(\omega \rightarrow \pi\pi\pi)$

### Interpretations

- mass very close to  $DD^*$  threshold  $\rightarrow$  molecule?
- tetraquark interpretation: no isospin partners found yet
- mixing between molecule and ordinary charmonium  $\rightarrow$  would explain isospin violation
- ordinary charmonium  $\rightarrow \chi_{c1} (2P)$

X(3872) : ordinary charmonium possibility

If X is an ordinary charmonium state it is likely to be  $\chi_{c1}(2P)$



Searching for  $\chi_{c1}(2P)$  is a parallel strategy to understand what is X

Belle looks for the mode  $\chi_{c1}(2P) \rightarrow \chi_{c1}(1P) \pi^+ \pi^-$   
with  $M(\chi_{c1}(2P)) \approx 3920 \text{ MeV}$  (QM prediction)  $\Rightarrow$  no signal found

PDG

$$\frac{\Gamma(\gamma\psi(2S))}{\Gamma(\gamma J/\psi)}$$

VALUE

**<2.1**

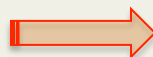
Belle

**$3.4 \pm 1.4$**

BaBar

LHCb , EPJ Web Conf. 96 (2015) 01005

$$R_\psi = 2.46 \pm 0.63(stat) \pm 0.29(syst)$$



Ratio suppressed in composite models

In the hypothesis  $X(3872) = \chi_{c1}(2P)$   
HQ spin symmetry predicts:

$$\frac{\Gamma(X(3872) \rightarrow \psi(2S)\gamma)}{\Gamma(X(3872) \rightarrow \psi(1S)\gamma)} = 1.64 \pm 0.25.$$



Plausible identification

FDF, PRD79 (09) 054015

## Z states

Z(4430) : first observed in 2008 by Belle in  $B \rightarrow K \pi^+ \psi'$  decaying to  $\pi^+ \psi'$   
not confirmed by BaBar  
confirmed in 2014 by LHCb , fixing to  $J^P=1^+$

Belle

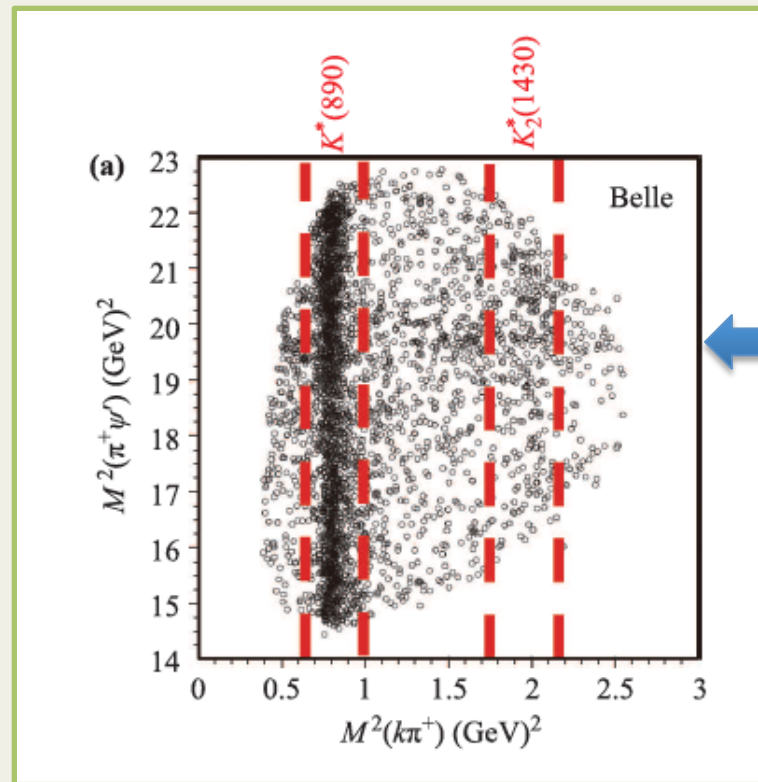
$$M=4433 \pm 5 \text{ MeV}$$

$$\Gamma=45 \pm {}^{35}_{18} \text{ MeV}$$

LHCb

$$M=4475 \pm {}^{17}_{26} \text{ MeV}$$

$$\Gamma=172 \pm {}^{39}_{36} \text{ MeV}$$



contains  $c\bar{c}$  pair  
& is charged  $\Rightarrow$   
at least 4 quarks !

## Z states

$Z_c(3900)$  : discovered by BESIII in 2013 in  $e^+ e^- \rightarrow Y(4260) \rightarrow J/\psi \pi^+ \pi^-$  with  $Z_c \rightarrow J/\psi \pi$   
 confirmed by Belle, CLEOc  
 neutral partner observed

|         | M (Mev)                  | $\Gamma$ (MeV)     |
|---------|--------------------------|--------------------|
| Bes III | $3899.0 \pm 3.6 \pm 4.9$ | $46 \pm 10 \pm 20$ |
| Belle   | $3894.5 \pm 6.6 \pm 4.5$ | $63 \pm 24 \pm 26$ |
| CLEOc   | $3885. \pm 5 \pm 1$      | $34 \pm 12 \pm 4$  |

BESIII PRL 110 (2013) 252001;  
 PRL 111 (2013) 242001  
 PRL 112 (2014) 132001  
 Xiao et al. (CLEOc) PLB 727 (2013) 366

$Z_1(4050)$  &  $Z_2(4250)$  discovered by Belle decaying to  $\chi_{c1} \pi^+$   
 confirmed by BESIII

Belle PRD 78 (2008) 072004

## Z states: Z(4430) & Z(3900)

- molecular interpretation:  
Z(4430)  $\rightarrow$   $D^* D_1(2420)$  molecule  
Z(3900)  $\rightarrow$   $D D_1(2420)$  molecule

Rosner, PRD76 (07) 114002  
Branz et al, PRD 82 (10) 054025

Wang et al., PRL111 (13) 132003  
PLB 725 (13) 106

- type II tetraquark model:  
Z(4430) radial excitation of Z(3900)  
Indeed:  
Z(3900) $^+$   $\rightarrow$   $\pi^+ \psi(1S)$   
Z(4430) $^+$   $\rightarrow$   $\pi^+ \psi(2S)$

Maiani et al., PRD89 (14) 114010

- cusp effects  
Z(3900) very close to the threshold  $M(D)+M(D^*) \approx 3879$  MeV

E. Swanson,  
PRD 91 (2015) 034009

## $Z_b$ states

Belle Collab. 1105.4583  
1403.0992

found in 2011 by Belle in  $e^+ e^- \rightarrow \Upsilon(5S) \rightarrow \Upsilon(nS) \pi^+ \pi^-$

&  $e^+ e^- \rightarrow \Upsilon(5S) \rightarrow h_b(nP) \pi^+ \pi^-$



|              | Mass (MeV)        | $\Gamma$ (MeV) |
|--------------|-------------------|----------------|
| $Z_b(10610)$ | $10608.4 \pm 2.0$ | $15.6 \pm 2.5$ |
| $Z_b(10650)$ | $10653.2 \pm 1.5$ | $14.4 \pm 3.2$ |

Angular analysis favours  $J^P=1^+$

### Possible interpretations:

E. Swanson, PRD 91 (2015) 034009

cusps effects



$$\begin{aligned} m(Z_b(10610)^+) - m(B) - m(B^*) &= 3.69 \pm 2.05 \text{ MeV} \\ m(Z_b(10650)^+) - m(B^*) - m(B^*) &= 3.68 \pm 1.71 \text{ MeV} \end{aligned}$$

tetraquarks

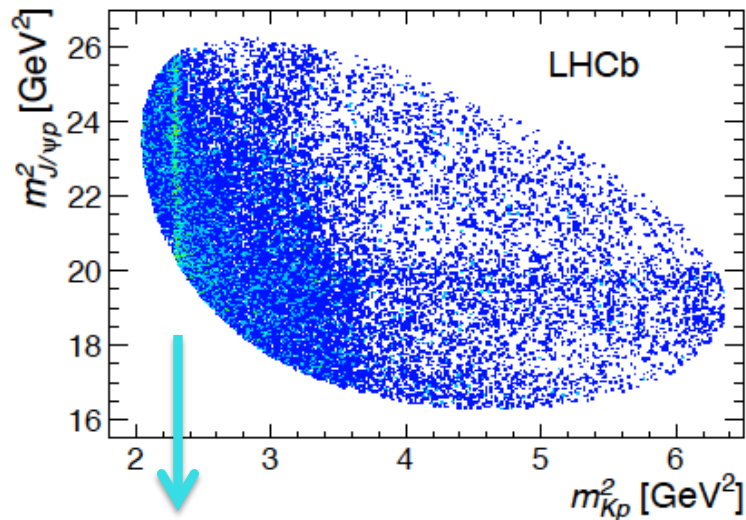
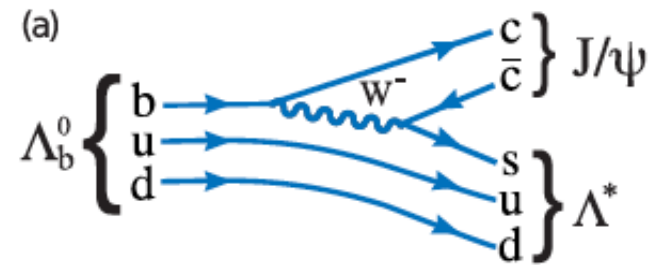
Ali et al, PLB 684 (10) 28  
PRD 85 (12) 054011



## The last surprise: Pentaquark

- LHCb looks at  $\Lambda_b \rightarrow J/\psi K^- p$  to measure  $\Lambda_b$  lifetime
- dominance of  $\Lambda^*$  resonances expected with  $\Lambda^* \rightarrow K^- p$

LHCb PRL 115 (2015) 07201

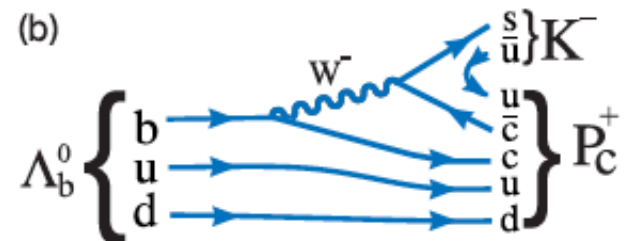


$\Lambda(1520)$

resonant structure observed in  $J/\psi p$

minimal quark content  $c\bar{c}uud$

$P_c^+$



## The last surprise: Pentaquark

best fit obtained with two states

$$\begin{aligned} M_1 &= 4380 \pm 8 \pm 29 \text{ MeV} \\ \Gamma_1 &= 205 \pm 18 \pm 86 \text{ MeV} \end{aligned}$$

$$\begin{aligned} M_1 &= 4449 \pm 1.7 \pm 2.5 \text{ MeV} \\ \Gamma_1 &= 39 \pm 5 \pm 19 \text{ MeV} \end{aligned}$$

possibilities:

$$J^P = (3/2^-, 5/2^+), (3/2^+, 5/2^-), (5/2^+, 3/2^-)$$



favourite one

Problems:

- if  $P_c = c\bar{c}uud$  and  $Z_c(4475) = c\bar{c}ud$  why both  $P_c$  are lighter than  $Z_c$ ? (for this reason  $P_c$  cannot decay to  $\psi(2S)$  and  $Z_c$  does)
- decays to  $\Lambda_c \bar{D}^{(*)0}$  should also be allowed

## What are $P_c$ states?

- molecules:  $\Sigma_c \bar{D}^{(*)0}$
- rescattering through  $\chi_{c1}$  p effect
- composite object of two compact color constituents

Karliner & Rosner, 1506.06386  
Roca et al, 1507.04249

Guo et al., 1507.04950

Maiani et al., PLB 749 (15) 289



already considered by Karliner & Lipkin to interpret “old” pentaquark  $\Theta(1535)$

## Conclusions

- many new states discovered
- many new ideas about the fundamental structure of hadrons
- an important tool to understand strong interactions
- great work for experimentalists and theorists

## BackUp Slides

## Predicting the heavy meson masses

### Spin averaged masses

$$\overline{M}_H = \frac{3M_{P^*} + M_P}{4}$$

$$\overline{M}_S = \frac{3M_{P'_1} + M_{P_0^*}}{4}$$

$$\overline{M}_T = \frac{5M_{P_2^*} + 3M_{P_1}}{8}$$

$$\overline{M}_X = \frac{5M_{P_2} + 3M_{P_1^*}}{8}$$

$$\overline{M}_{X'} = \frac{7M_{P_3} + 5M_{P_2'^*}}{12}$$

### Mass splittings higher doublets- fundamental doublet

$$\Delta_F = \overline{M}_F - \overline{M}_H$$

Inclusion of  $1/m_Q$  corrections for hyperfine splitting within the doublets

$$\mathcal{L}_{1/m_Q} = \frac{1}{2m_Q} \left\{ \lambda_H \text{Tr}[\bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}] + \lambda_S \text{Tr}[\bar{S}_a \sigma^{\mu\nu} S_a \sigma_{\mu\nu}] + \lambda_T \text{Tr}[\bar{T}_a^\alpha \sigma^{\mu\nu} T_a^\alpha \sigma_{\mu\nu}] \right. \\ \left. + \lambda_X \text{Tr}[\bar{X}_a^\alpha \sigma^{\mu\nu} X_{a\alpha} \sigma_{\mu\nu}] + \lambda_{X'} \text{Tr}[\bar{X}_a'^{\alpha\beta} \sigma^{\mu\nu} X_{a'}^{\alpha\beta} \sigma_{\mu\nu}] \right\} ,$$

$$\lambda_H = \frac{1}{8} (M_{P^*}^2 - M_P^2)$$

$$\lambda_S = \frac{1}{8} (M_{P'_1}^2 - M_{P_0^*}^2)$$

$$\lambda_T = \frac{3}{16} (M_{P_2^*}^2 - M_{P_1}^2)$$

$$\lambda_X = \frac{3}{16} (M_{P_2}^2 - M_{P_1^*}^2)$$

$$\lambda_{X'} = \frac{5}{24} (M_{P_3}^2 - M_{P_2'^*}^2)$$

HQ limit relations

$$\Delta_F^{(c)} = \Delta_F^{(b)}$$

$$\lambda_F^{(c)} = \lambda_F^{(b)}$$

allow to move from charm to beauty

strong decay rates

$H = (P, P^*)$  or  $\tilde{H} = (\tilde{P}, \tilde{P}^*)$

$$\Gamma(P^* \rightarrow PM) = C_M \frac{g^2}{6\pi f_\pi^2} \frac{M_P}{M_{P^*}} |\vec{p}_M|^3$$

$$\Gamma(\tilde{P}^* \rightarrow PM) = C_M \frac{\tilde{g}^2}{6\pi f_\pi^2} \frac{M_P}{M_{\tilde{P}^*}} |\vec{p}_M|^3$$

$$\Gamma(\tilde{P}_i^* \rightarrow P_f^* M) = C_M \frac{\tilde{g}^2}{3\pi f_\pi^2} \frac{M_{P_f^*}}{M_{\tilde{P}_i^*}} |\vec{p}_M|^3$$

$$\Gamma(\tilde{P} \rightarrow P^* M) = C_M \frac{\tilde{g}^2}{2\pi f_\pi^2} \frac{M_{P^*}}{M_{\tilde{P}}} |\vec{p}_M|^3.$$

$X = (P_1^*, P_2)$

$$\begin{aligned} \Gamma(P_1^* \rightarrow PM) \\ = C_M \frac{4k'^2}{9\pi f_\pi^2} \frac{M_P}{M_{P_1^*}} [m_M^2 + |\vec{p}_M|^2] |\vec{p}_M|^3 \end{aligned}$$

$$\begin{aligned} \Gamma(P_1^* \rightarrow P^* M) \\ = C_M \frac{2k'^2}{9\pi f_\pi^2} \frac{M_{P^*}}{M_{P_1^*}} [m_M^2 + |\vec{p}_M|^2] |\vec{p}_M|^3 \end{aligned}$$

$$\begin{aligned} \Gamma(P_2 \rightarrow P^* M) \\ = C_M \frac{2k'^2}{3\pi f_\pi^2} \frac{M_{P^*}}{M_{P_2}} [m_M^2 + |\vec{p}_M|^2] |\vec{p}_M|^3. \end{aligned}$$

$S = (P_0^*, P_1')$

$$\Gamma(P_0^* \rightarrow PM) = C_M \frac{h^2}{2\pi f_\pi^2} \frac{M_P}{M_{P_0^*}} [m_M^2 + |\vec{p}_M|^2] |\vec{p}_M|$$

$$\begin{aligned} \Gamma(P_1' \rightarrow P^* M) \\ = C_M \frac{h^2}{2\pi f_\pi^2} \frac{M_{P^*}}{M_{P_1'}} [m_M^2 + |\vec{p}_M|^2] |\vec{p}_M|. \end{aligned}$$

$T = (P_1, P_2^*)$  or  $\tilde{T} = (\tilde{P}_1, \tilde{P}_2^*)$

$$\Gamma(P_1 \rightarrow P^* M) = C_M \frac{2h'^2}{3\pi f_\pi^2} \frac{M_{P^*}}{M_{P_1}} |\vec{p}_M|^5$$

$$\Gamma(P_2^* \rightarrow PM) = C_M \frac{4h'^2}{15\pi f_\pi^2} \frac{M_P}{M_{P_2^*}} |\vec{p}_M|^5$$

$$\Gamma(P_2^* \rightarrow P^* M) = C_M \frac{2h'^2}{5\pi f_\pi^2} \frac{M_{P^*}}{M_{P_2^*}} |\vec{p}_M|^5.$$

$X' = (P_2^*, P_3)$

$$\Gamma(P_2'^* \rightarrow P^* M) = C_M \frac{4k^2}{15\pi f_\pi^2} \frac{M_{P^*}}{M_{P_2'^*}} |\vec{p}_M|^7$$

$$\Gamma(P_3 \rightarrow PM) = C_M \frac{4k^2}{35\pi f_\pi^2} \frac{M_P}{M_{P_3}} |\vec{p}_M|^7$$

$$\Gamma(P_3 \rightarrow P^* M) = C_M \frac{16k^2}{105\pi f_\pi^2} \frac{M_{P^*}}{M_{P_3}} |\vec{p}_M|^7.$$