Evolution of Low-Energy Collective Excitations

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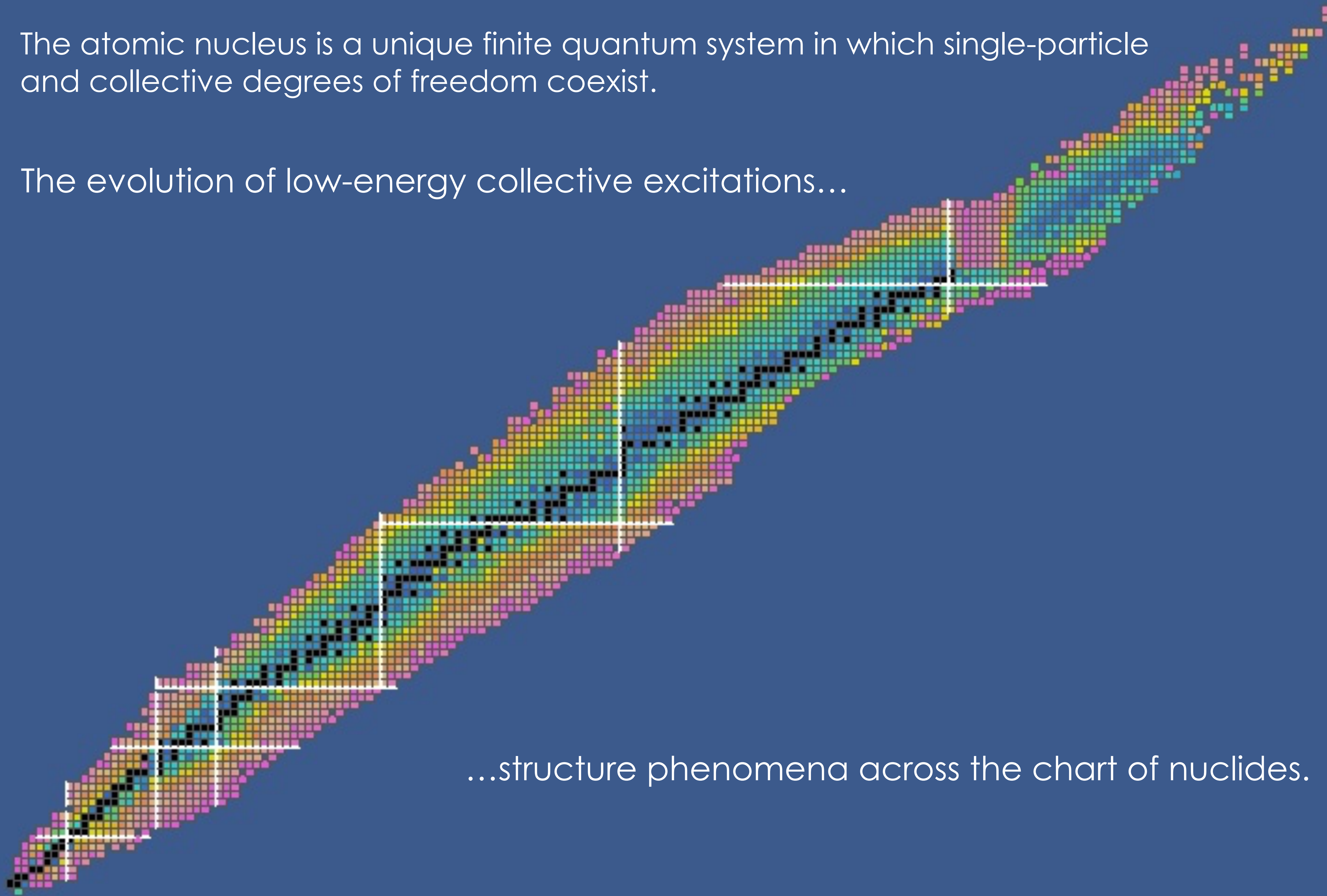
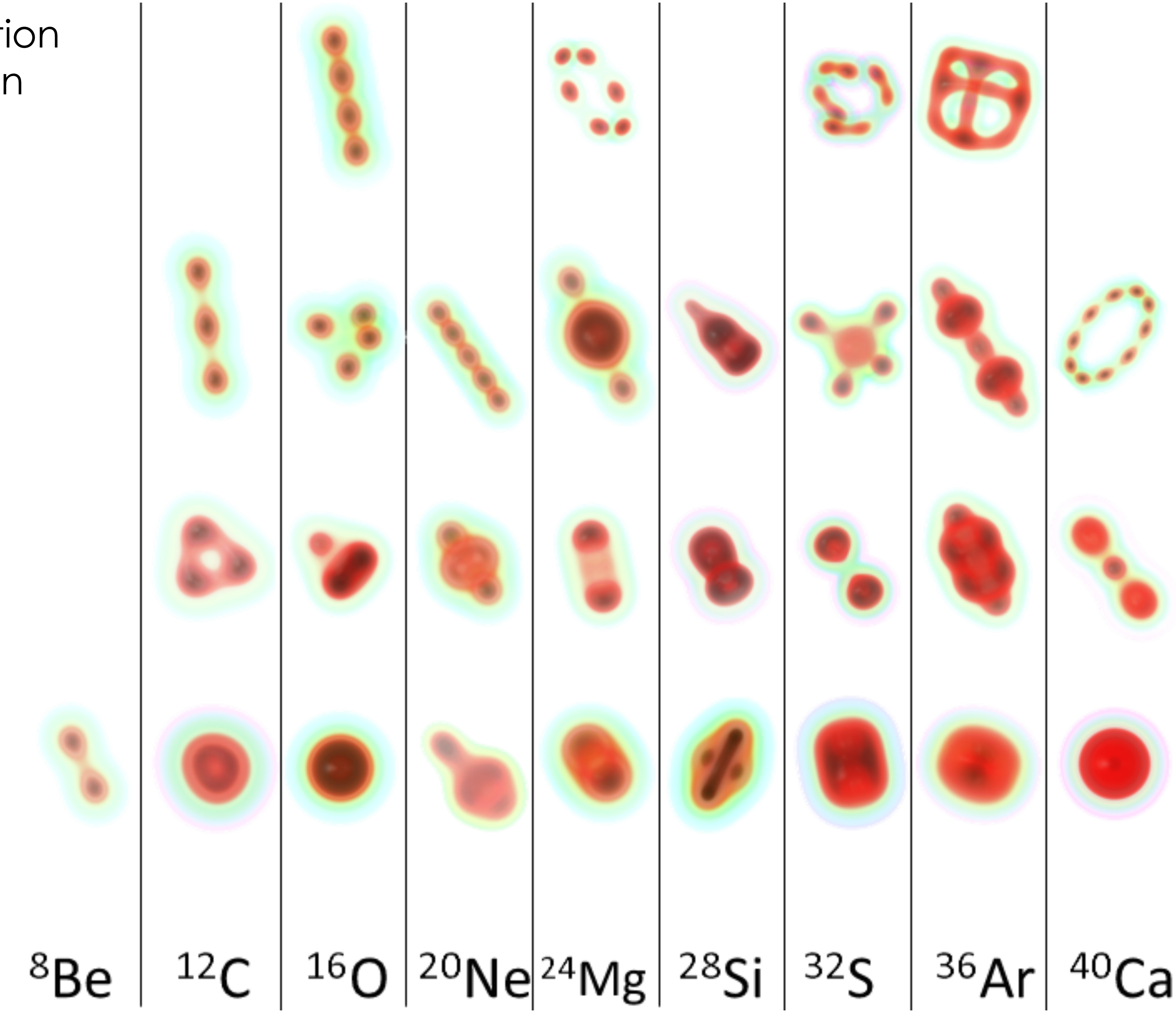
The atomic nucleus is a unique finite quantum system in which single-particle and collective degrees of freedom coexist.

The evolution of low-energy collective excitations…



…structure phenomena across the chart of nuclides.

…formation and evolution of exotic cluster states in light nuclei

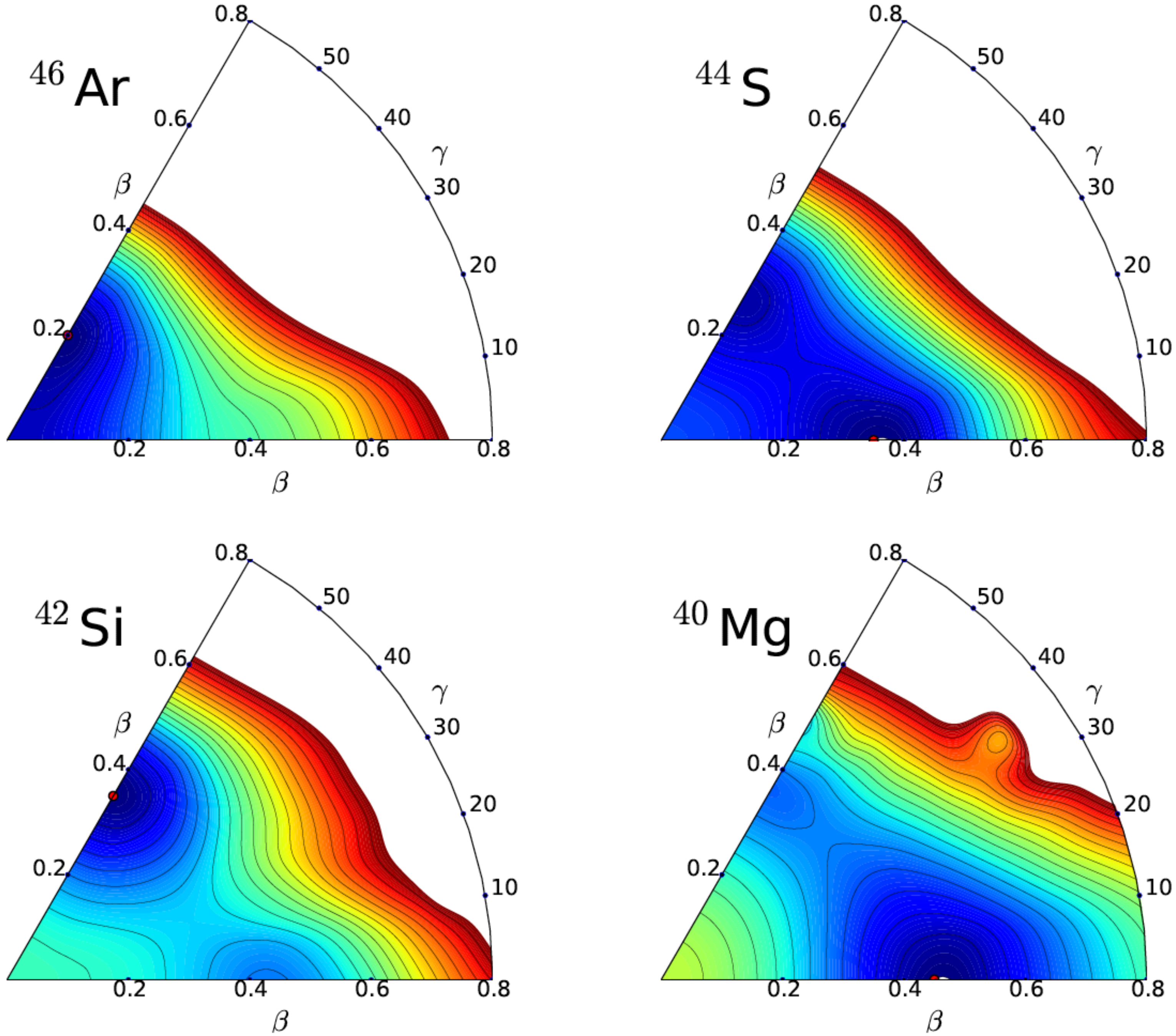


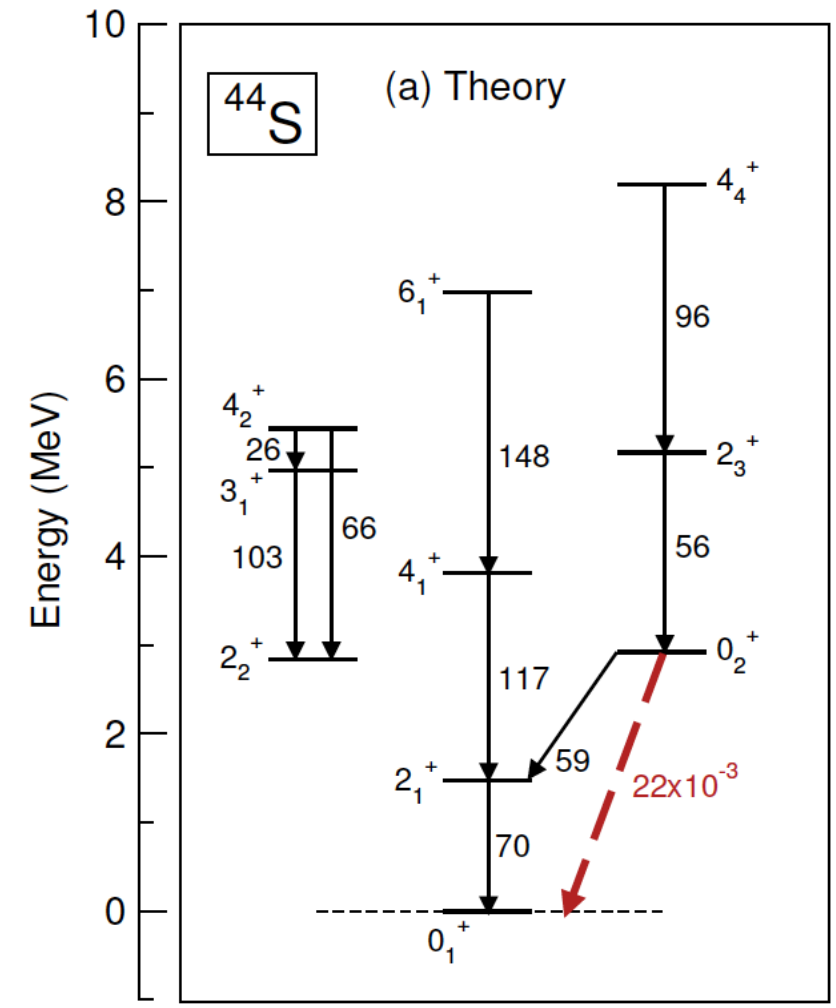
RESEARCH LETTER | VOL 487 | NATURE | 341

PHYSICAL REVIEW C **90**, 054329 (2014)

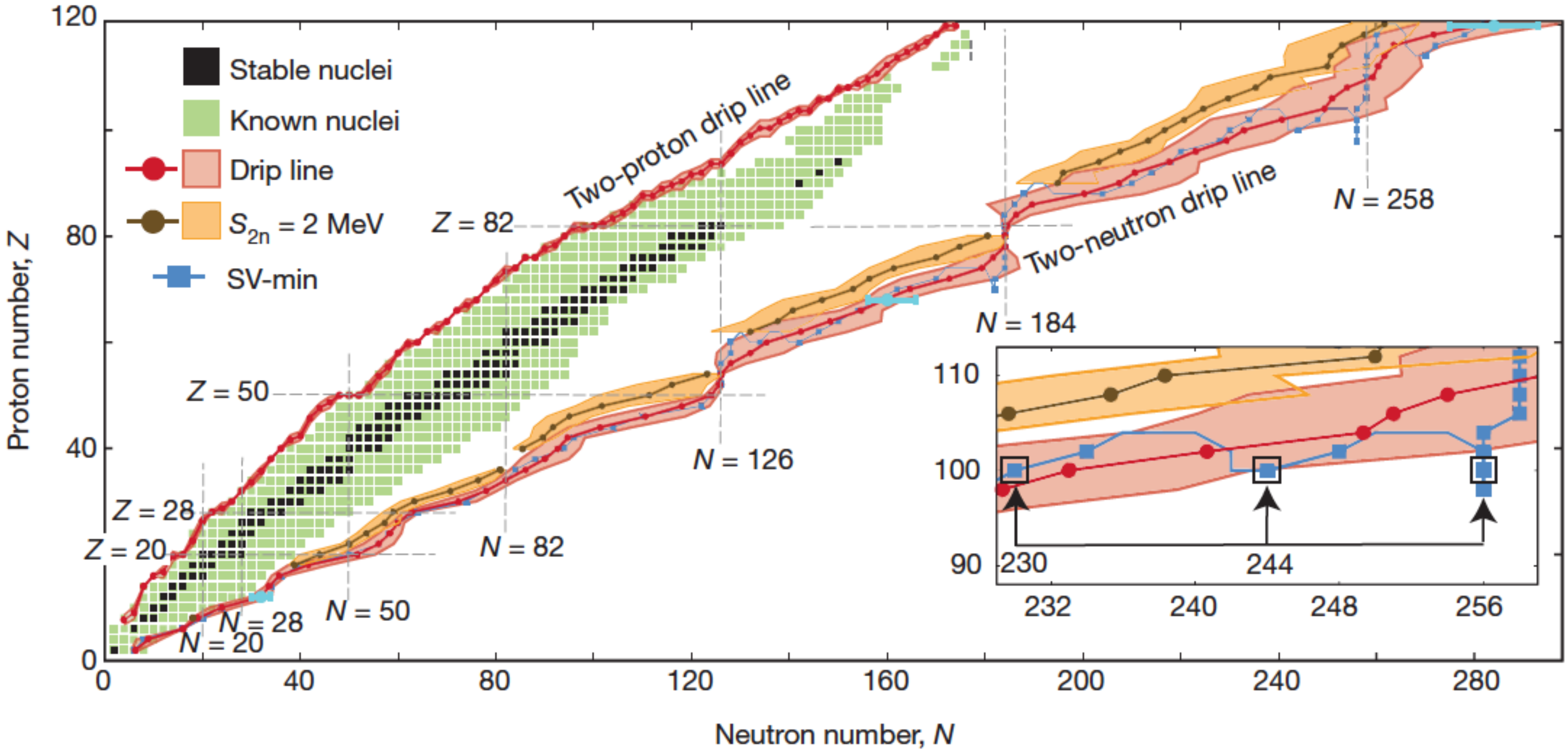
…modification of shell structure and the occurrence of deformations in closed-shell nuclei far from β-stability

N=28 isotones





Location of the nucleon drip lines and the limits of the nuclear landscape:

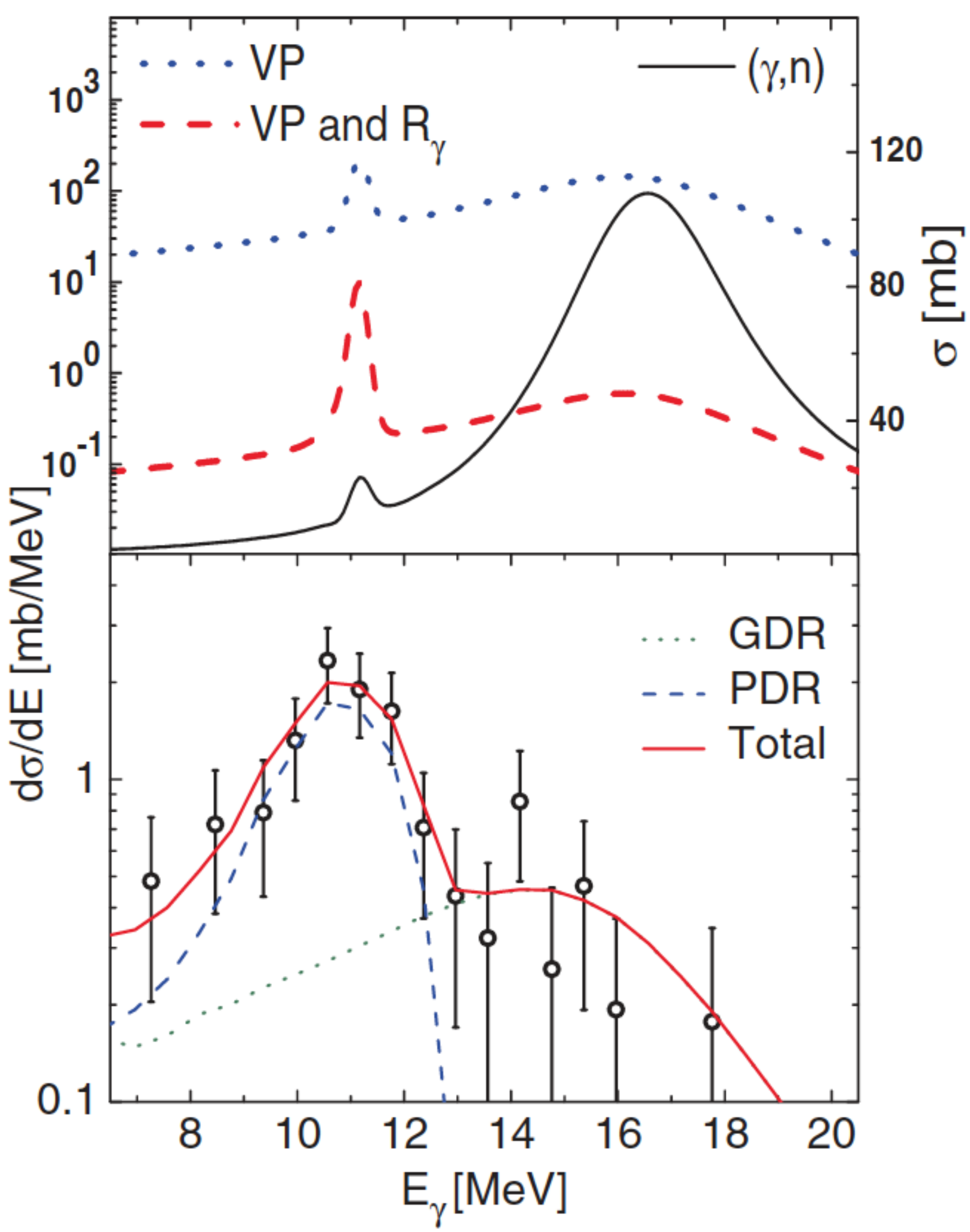


RESEARCH LETTER 510 | NATURE | VOL 486 | 28 JUNE 2012

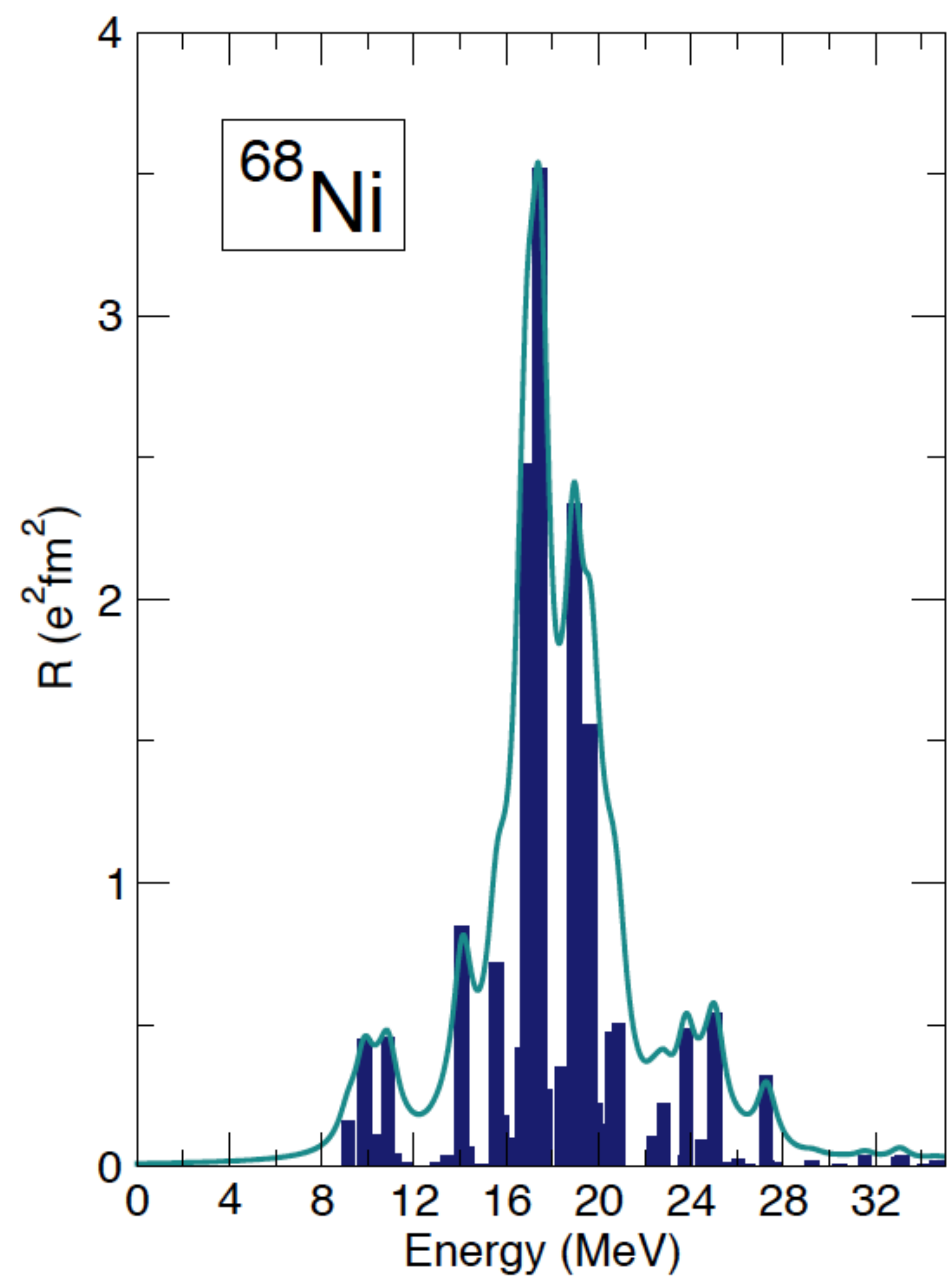
Neutron-rich nuclei → weak binding of the excess neutrons, diffuse neutron densities, formation of a neutron skin, pygmy dipole resonances (PDR) in medium-mass and heavy nuclei.

^{68}Ni photoabsorption cross section

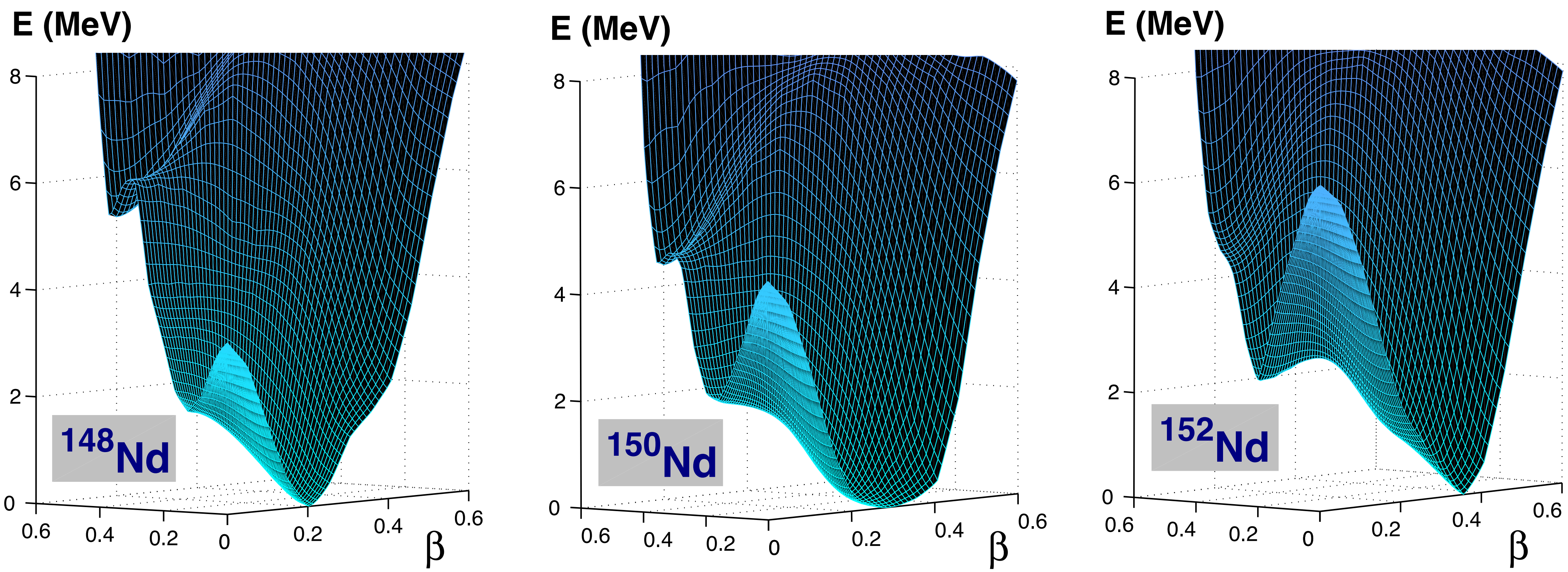
PRL **102,** 092502 (2009)



E1 strength function

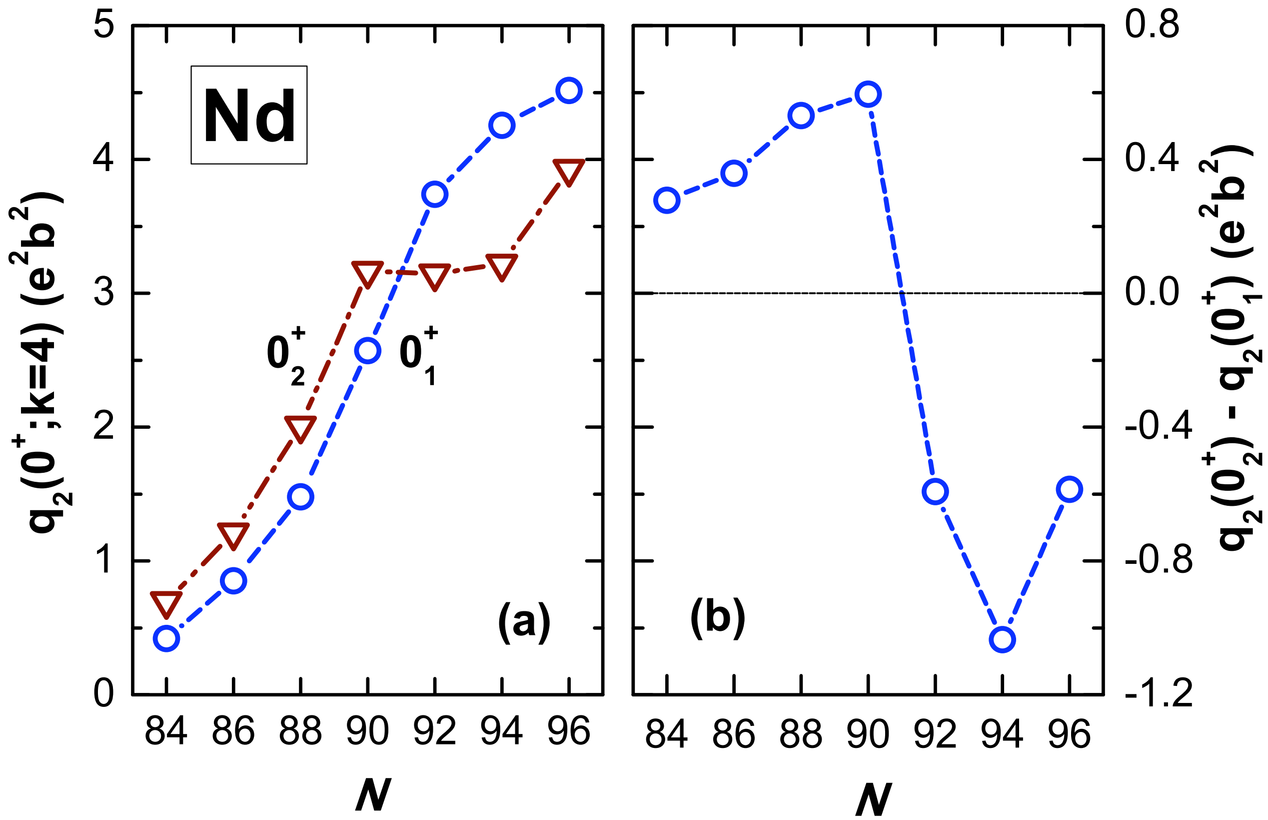


Shape coexistence and shape (phase) transitions in medium-heavy and heavy nuclei



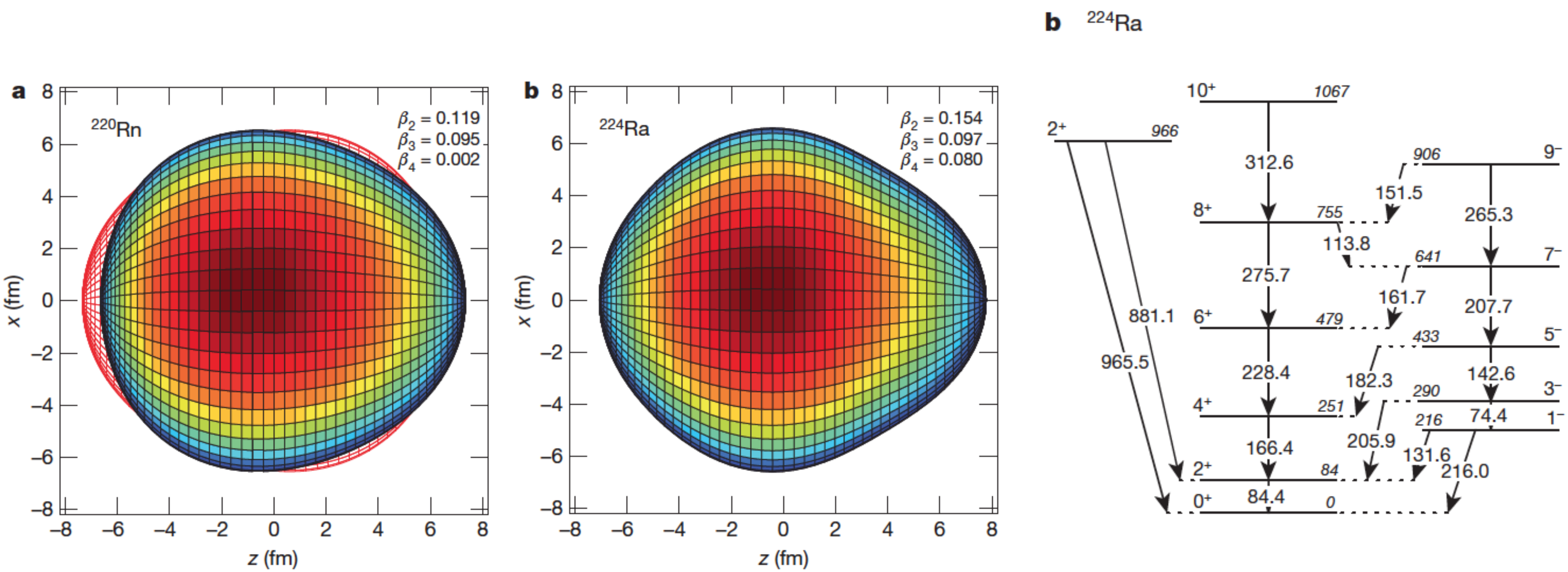
q shape invariants:

$$q_2(0_n^+;k) = \sum_{j=1}^{k} B(E2;0_n^+ \to 2_j^+)$$



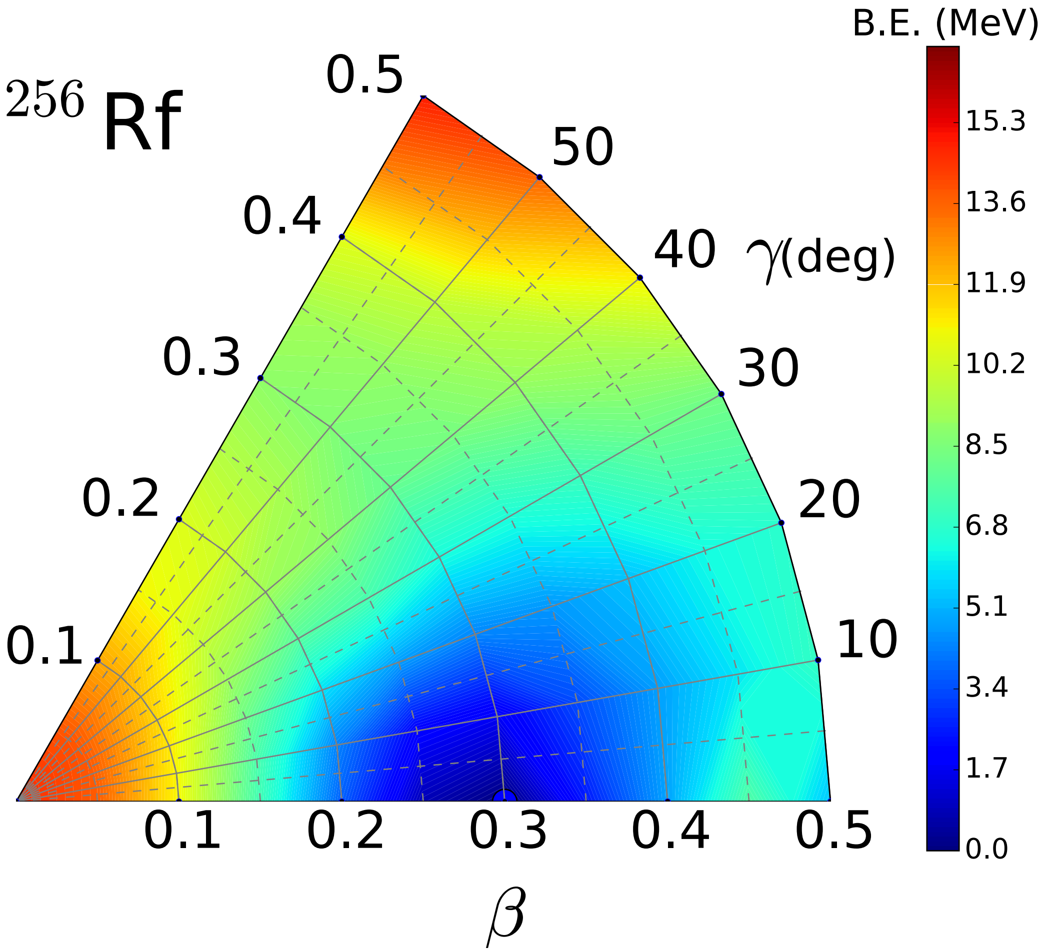
PHYSICAL REVIEW C **80**, 061301(R) (2009)

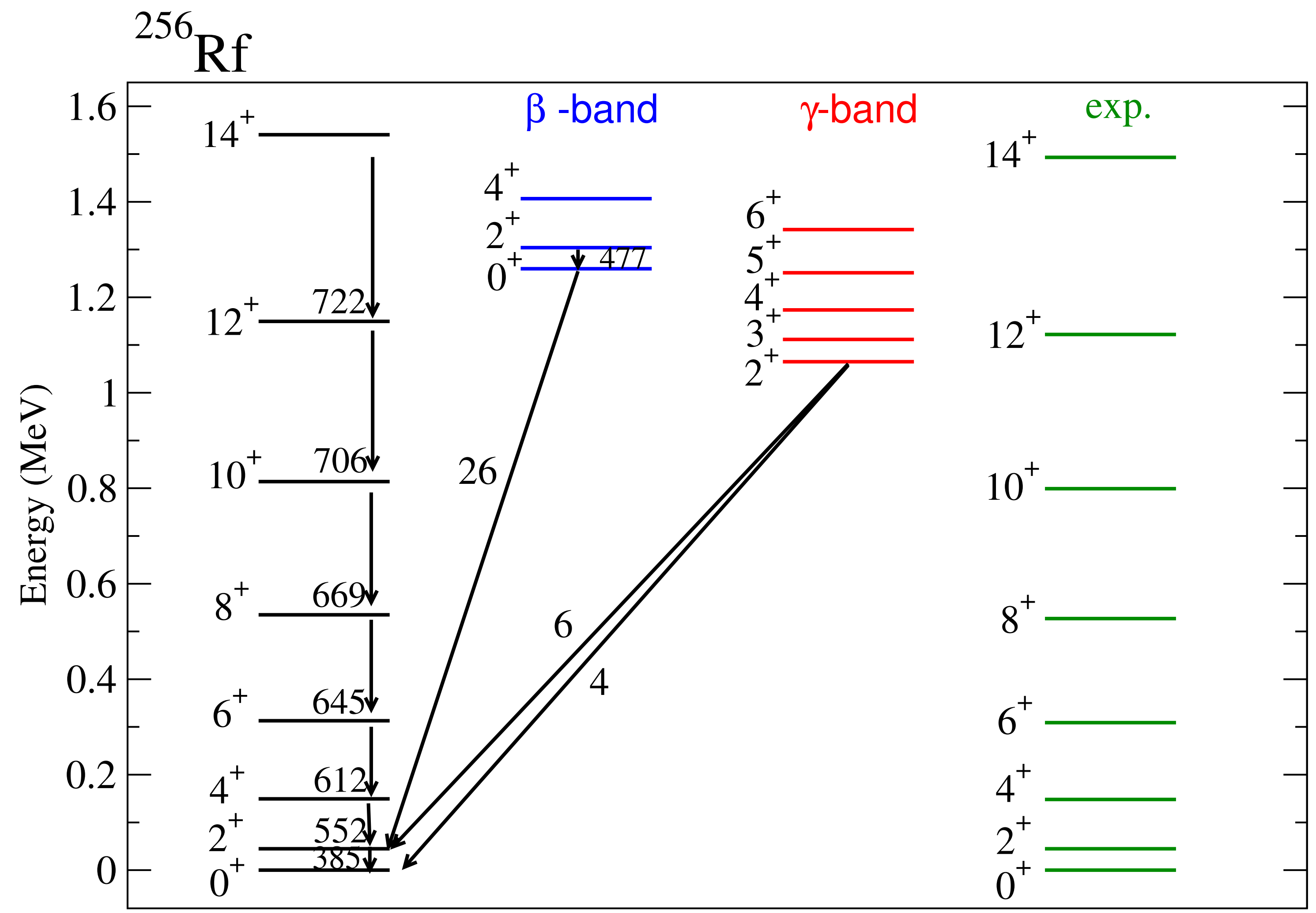
Octupole deformed (pear-shaped) heavy nuclei:



RESEARCH ARTICLE 202 | NATURE | VOL 497 | 9 MAY 2013

Subshell closures and stability of superheavy nuclei: rotational band structure





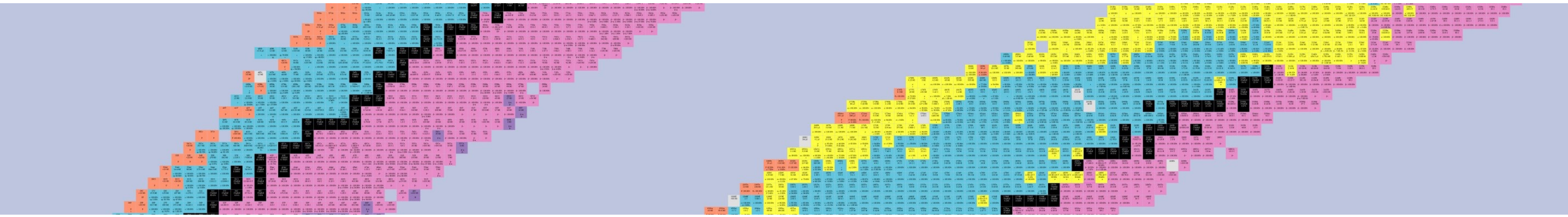
PHYSICAL REVIEW C **88**, 044324 (2013)

PRL **109**, 012501 (2012)

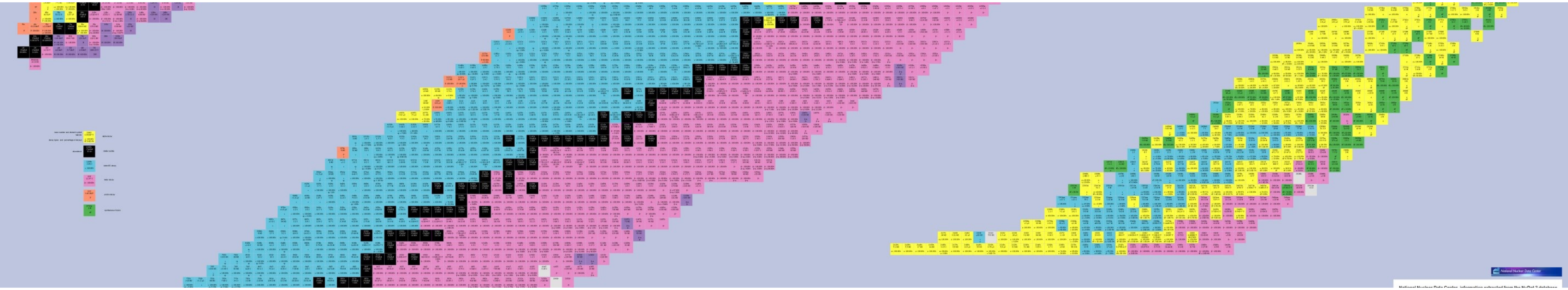
Universal collective phenomena that reflect the organisation of nucleonic matter in finite nuclei → global theory framework that can be applied to different mass regions.

Energy Density Functionals

✔ the nuclear many-body problem is effectively mapped onto a ***one-body problem*** without explicitly involving inter-nucleon interactions!



✔ the exact density functional is approximated with ***powers and gradients of ground-state densities and currents.***



✔ ***universal density functionals*** can be applied to all nuclei throughout the chart of nuclides.

Important for extrapolations to regions far from stability!

Kohn-Sham Density Functional Theory

… a universal functional:

$$F[\rho] = T_s[\rho] + U[\rho] + E_{xc}[\rho]$$

- $T_s[\rho]$: Kinetic energy of the non-interacting system
- $U[\rho]$: Hartree term
- $E_{xc}[\rho]$: Exchange-correlation energy

Self-consistent Kohn-Sham DFT: includes correlations and therefore goes beyond Hartree-Fock. It has the advantage of being a *local scheme*.

Jacob's ladder of DFT approximations for the E_{xc}.

Hartree world

Nuclear Many-Body Correlations

short-range
(hard repulsive core of the NN-interaction)

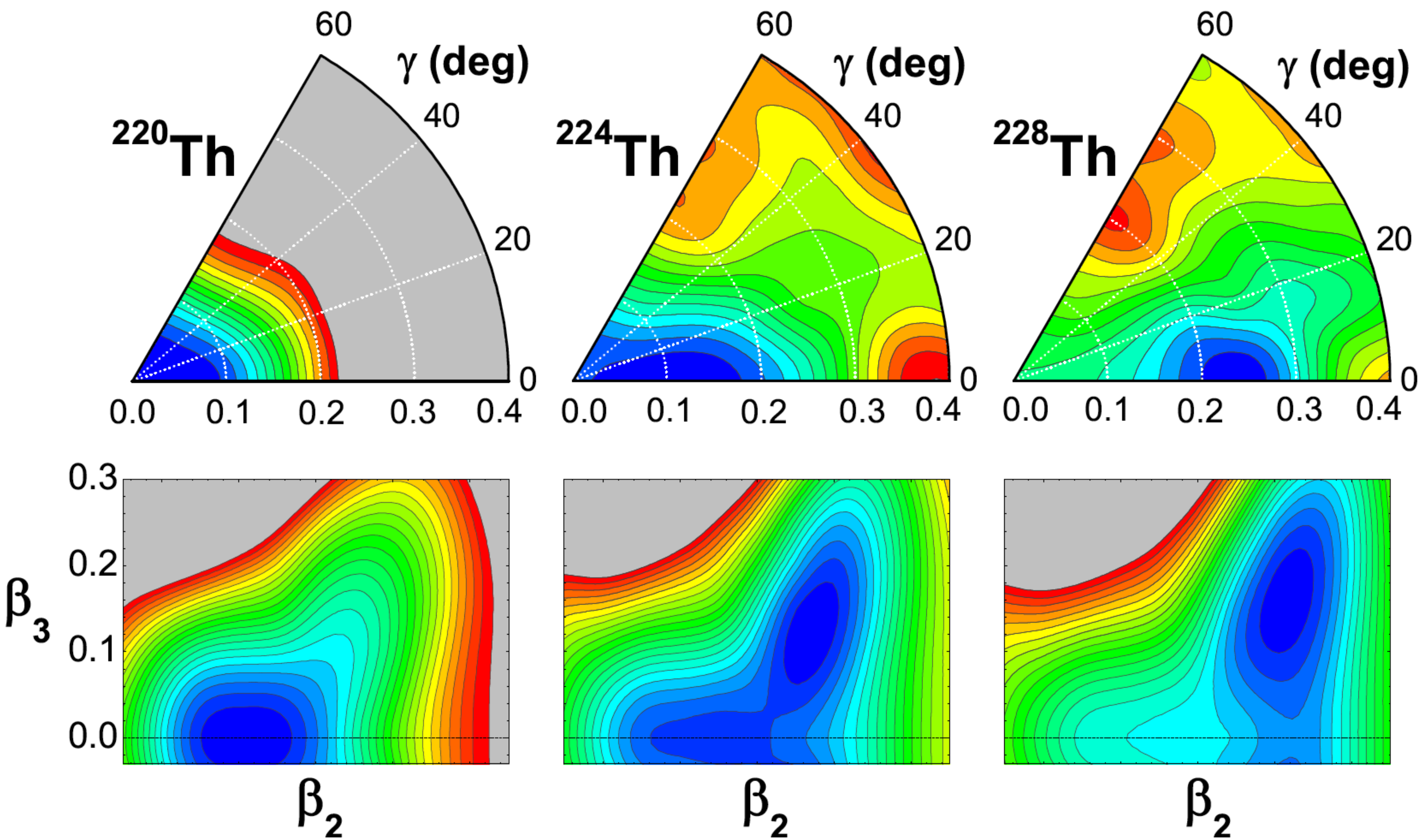
long-range
nuclear resonance modes (giant resonances)

collective correlations
large-amplitude soft modes:
(center of mass motion, rotation, low-energy quadrupole vibrations)

...vary smoothly with nucleon number! Can be included implicitly in an effective Energy Density Functional.

...sensitive to shell-effects and strong variations with nucleon number! Cannot be included in a simple EDF framework.

The constrained self-consistent mean field method produces semi-classical energy surfaces as functions of intrinsic deformation parameters.



→ include static correlations: deformations & pairing

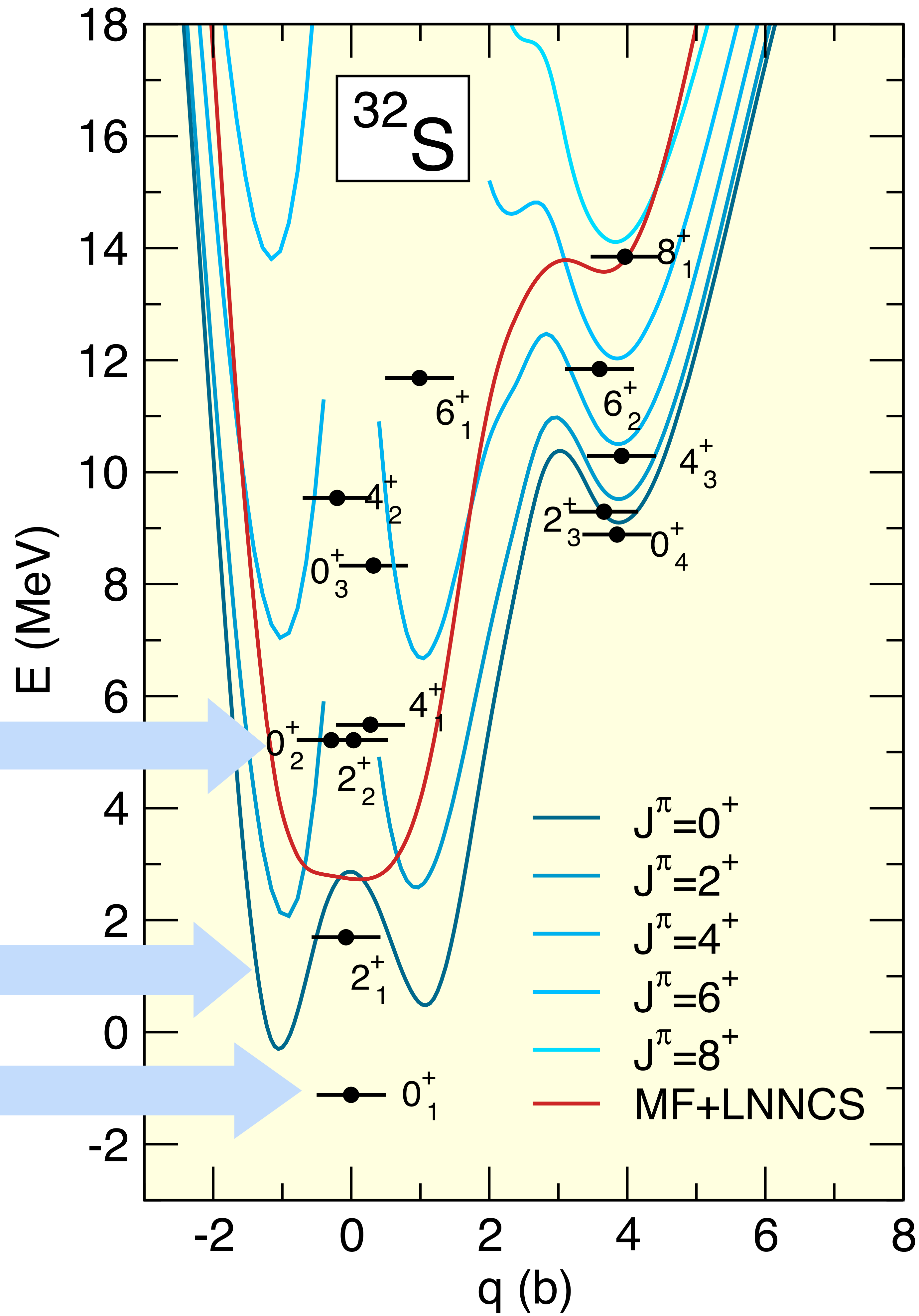
→ do not include dynamic (collective) correlations that arise from symmetry restoration and quantum fluctuations around mean-field minima

Restoration of broken symmetries (rotational, particle number) and fluctuations of collective variables (quadrupole deformation).

1. Mean-field calculations, with a constraint on the quadrupole moment.

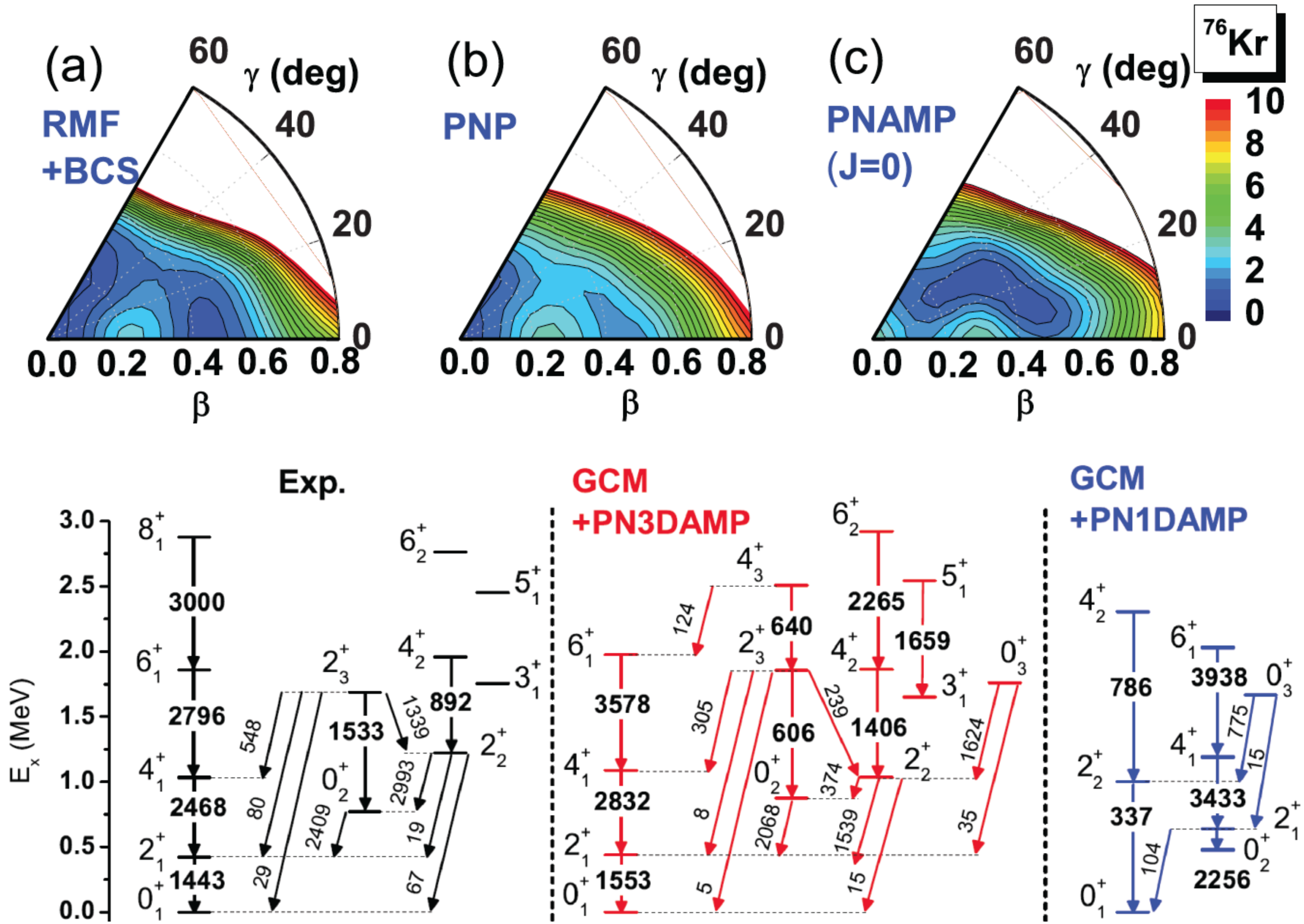
2. Angular-momentum and particle-number projection.

3. Generator Coordinate Method ⇒ configuration mixing



... larger variational space for projected GCM calculations!

Particle-number projected 3D AMP + GCM model



Five-dimensional collective Hamiltonian

Phys. Rev. C **79**, 034303 (2009).
Prog. Part. Nucl. Phys. **66**, 519 (2011).

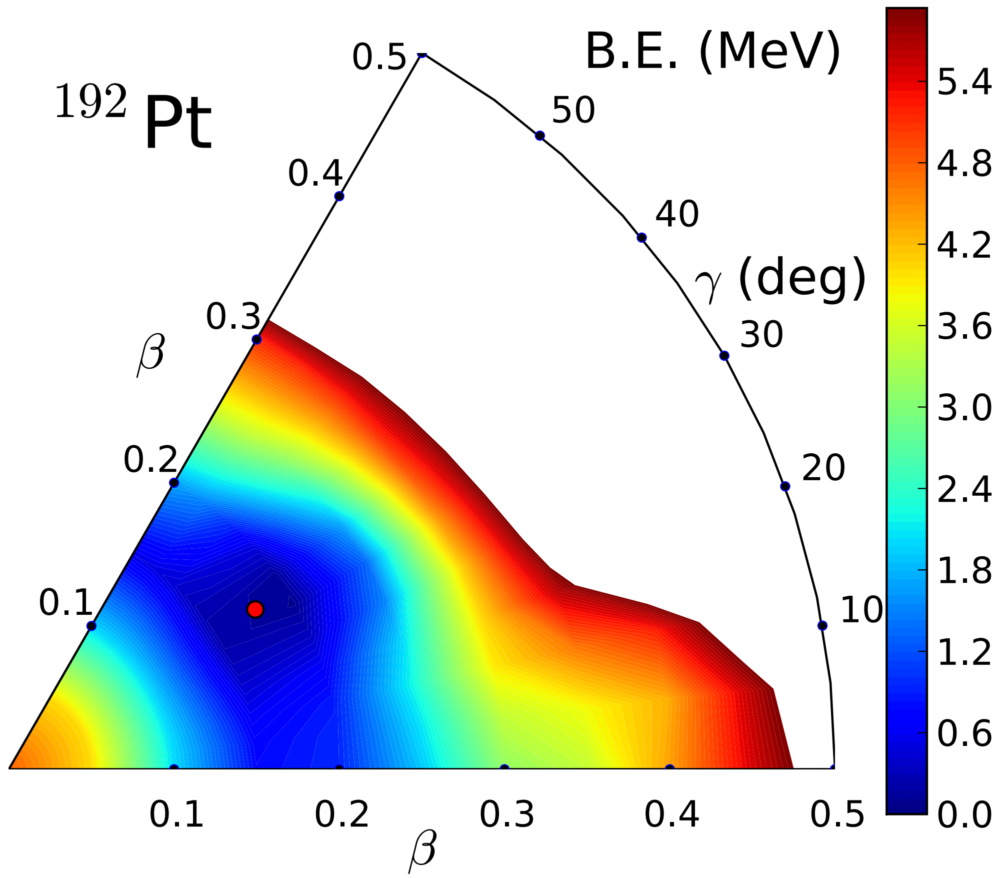
... nuclear excitations determined by quadrupole vibrational and rotational degrees of freedom

$$H_{\rm coll} = \mathcal{T}_{\rm vib}(\beta,\gamma) + \mathcal{T}_{\rm rot}(\beta,\gamma,\Omega) + \mathcal{V}_{\rm coll}(\beta,\gamma)$$

$$\mathcal{T}_{\rm vib} = \frac{1}{2}B_{\beta\beta}\dot{\beta}^2 + \beta B_{\beta\gamma}\dot{\beta}\dot{\gamma} + \frac{1}{2}\beta^2 B_{\gamma\gamma}\dot{\gamma}^2$$

$$\mathcal{T}_{\rm rot} = \frac{1}{2}\sum_{k=1}^{3}\mathcal{I}_k\omega_k^2$$

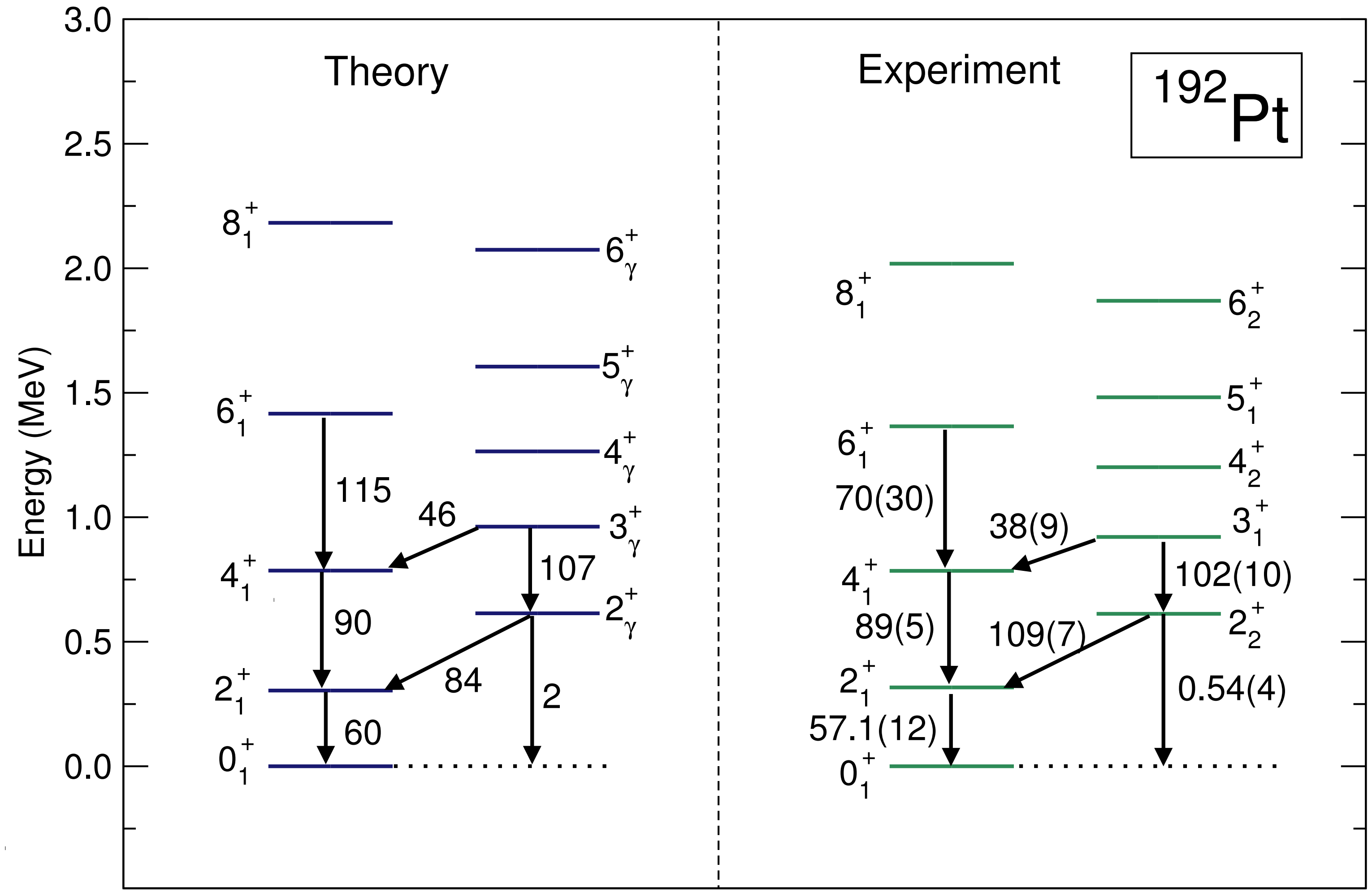
The entire dynamics of the collective Hamiltonian is governed by the seven functions of the intrinsic deformations β and γ: the collective potential, the three mass parameters: $B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$, and the three moments of inertia I_k.



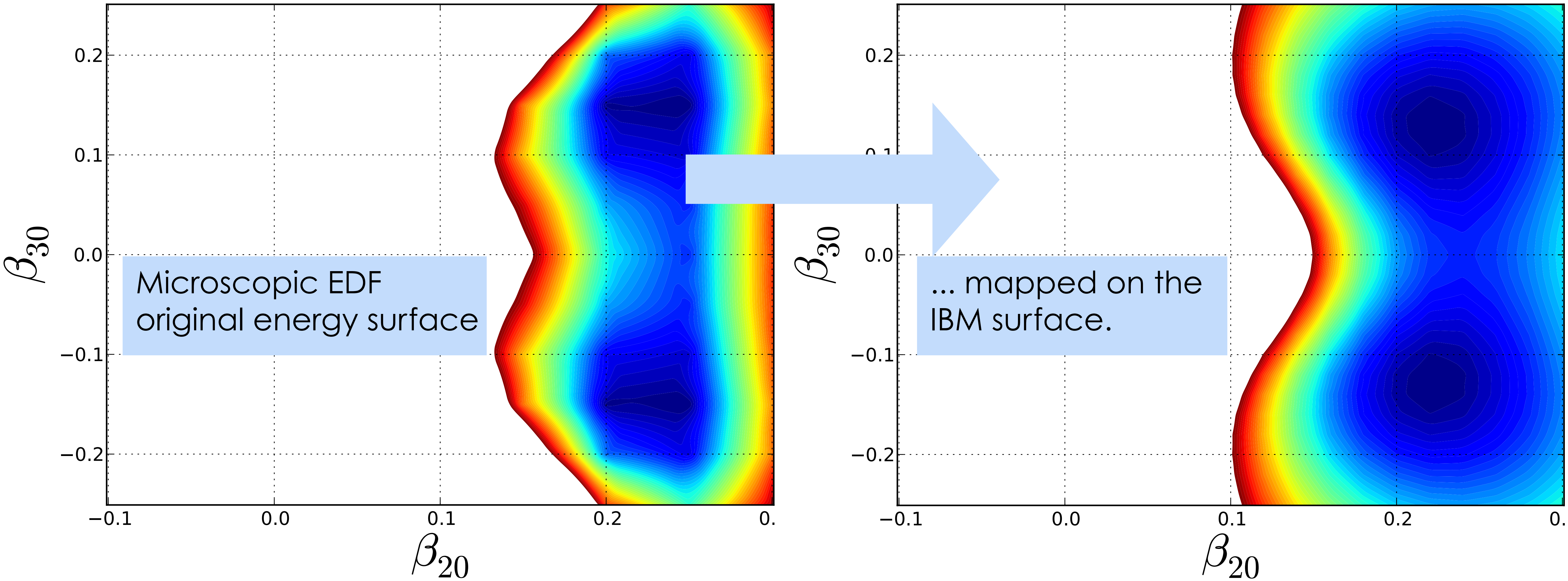
Prog. Part. Nucl. Phys. **66**, 519 (2011).

$$E^{th}_{4_1^+}/E^{th}_{2_1^+} = 2.58$$

$$E^{exp}_{4_1^+}/E^{exp}_{2_1^+} = 2.48$$



More complex shapes: additional (octupole) degrees of freedom



Mapping the microscopic PES on the expectation value of the IBM Hamiltonian in the sdf-boson condensate state:

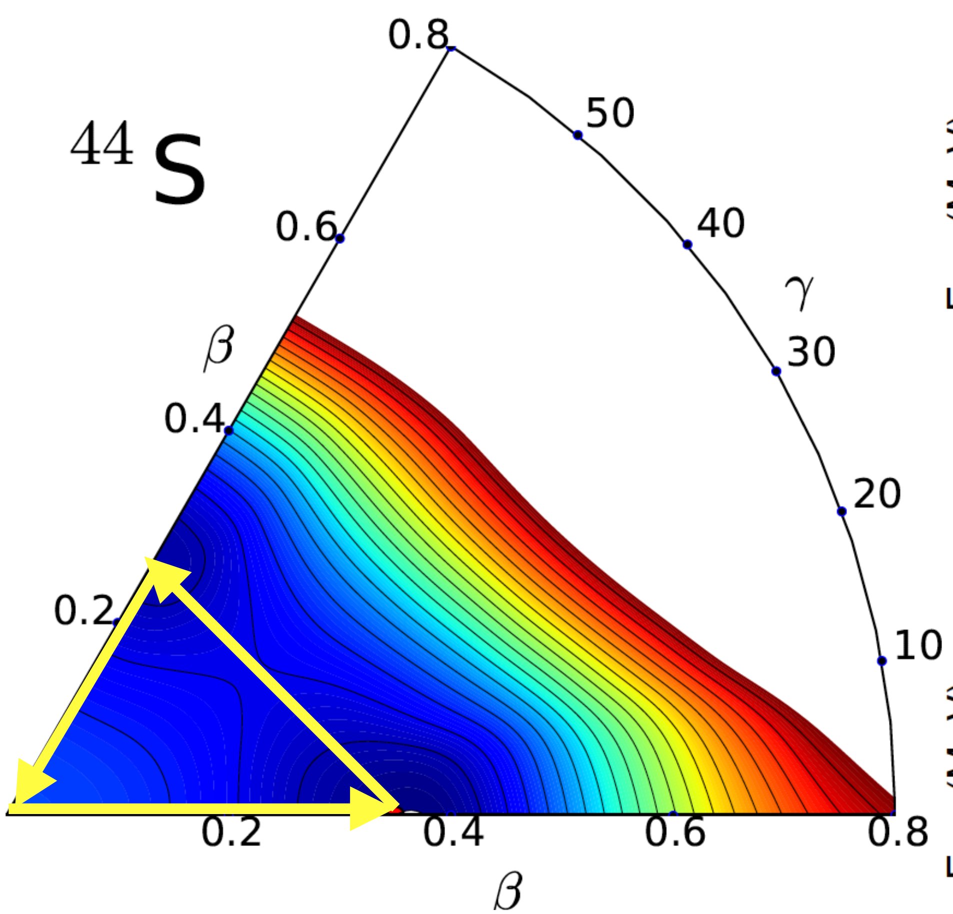
$$\hat{H} = \epsilon_d \hat{n}_d + \epsilon_f \hat{n}_f + \kappa_2 \hat{Q} \cdot \hat{Q} + \alpha \hat{L}_d \cdot \hat{L}_d + \kappa_3 : \hat{V}_3^{\dagger} \cdot \hat{V}_3 :$$

$$|\phi\rangle = \frac{1}{\sqrt{N!}} (\lambda^{\dagger})^N |-\rangle \qquad \lambda^{\dagger} = s^{\dagger} + \beta_2 d_0^{\dagger} + \beta_3 f_0^{\dagger}$$

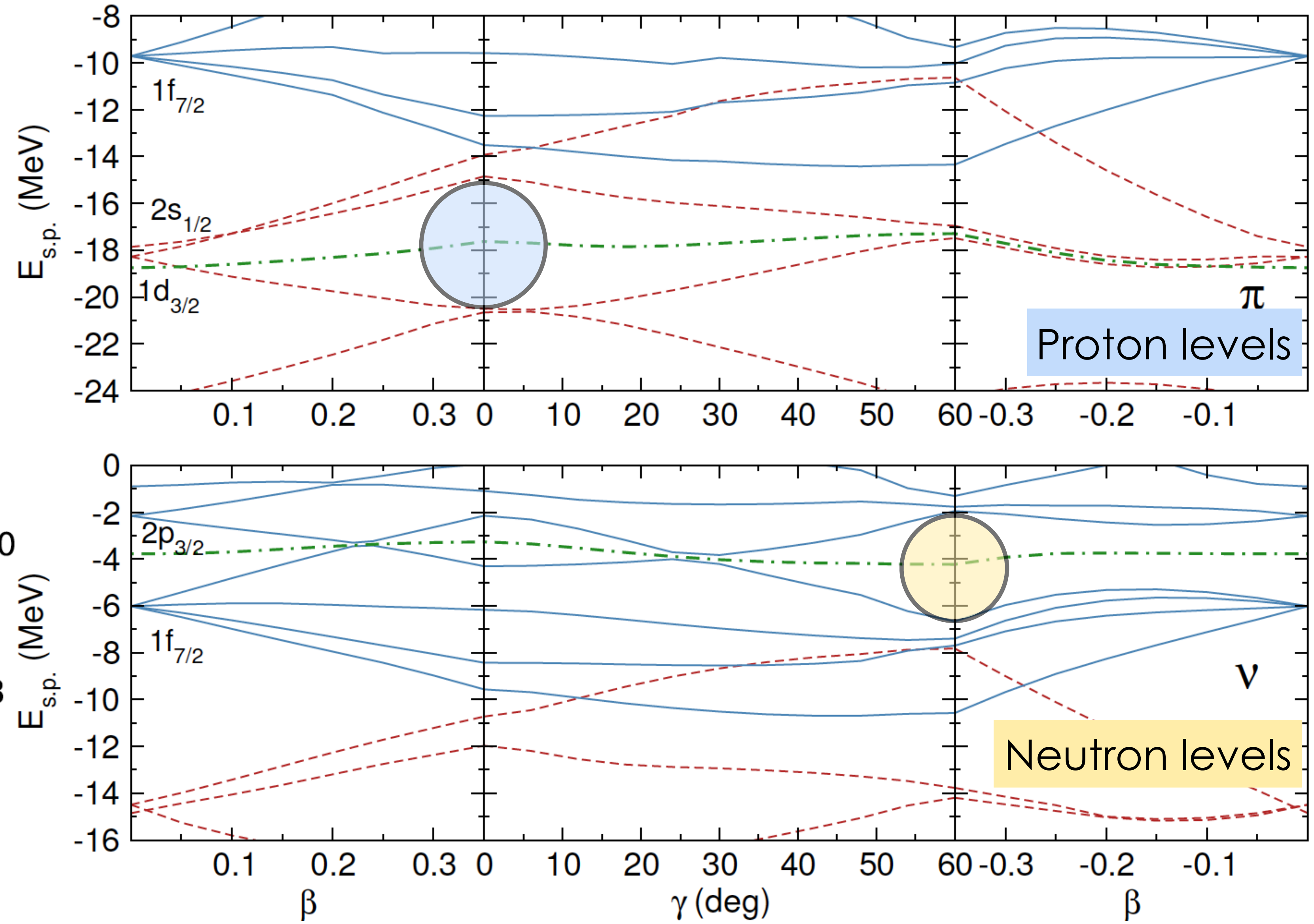
Phys. Rev. C 88, 021303(R) (2013).

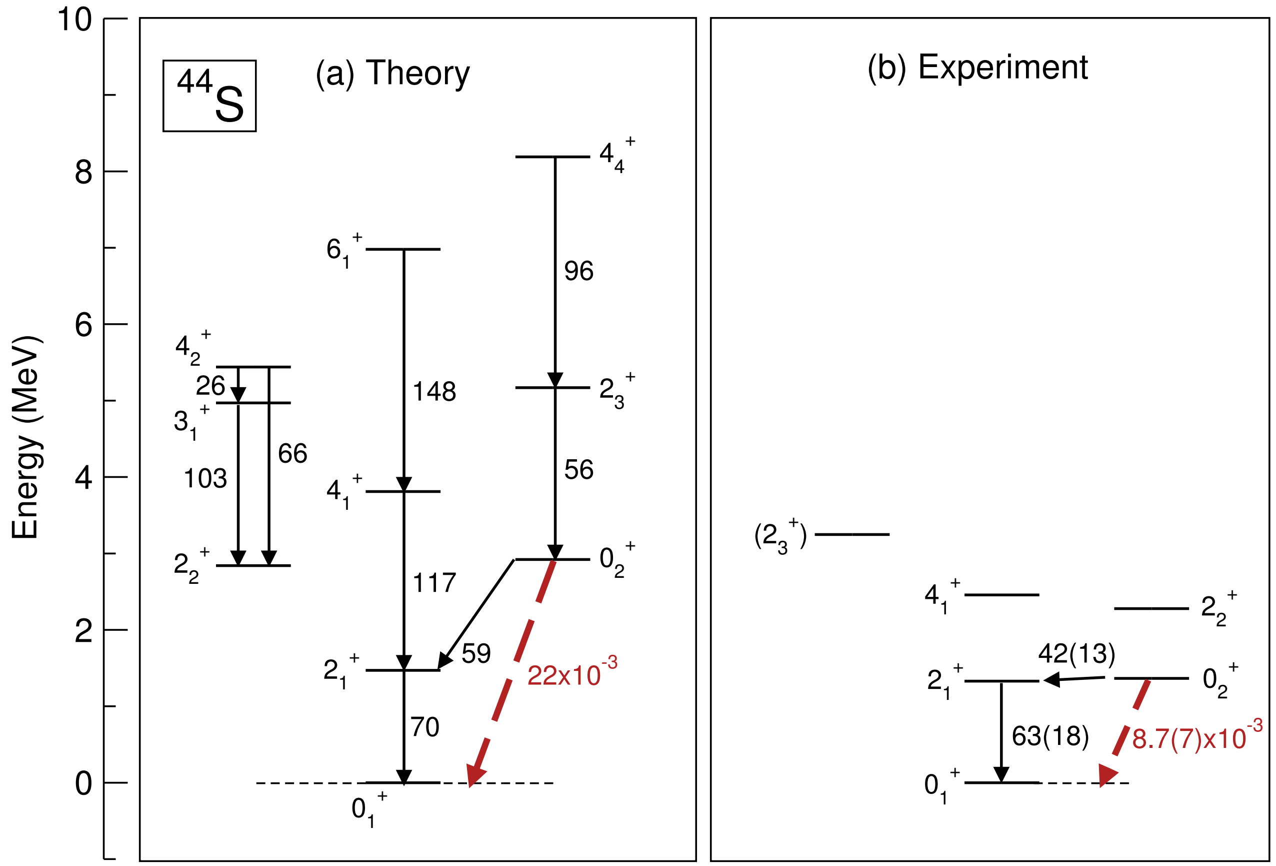
Phys. Rev. C 89, 024312 (2014).

Coexisting shapes in N=28 isotones

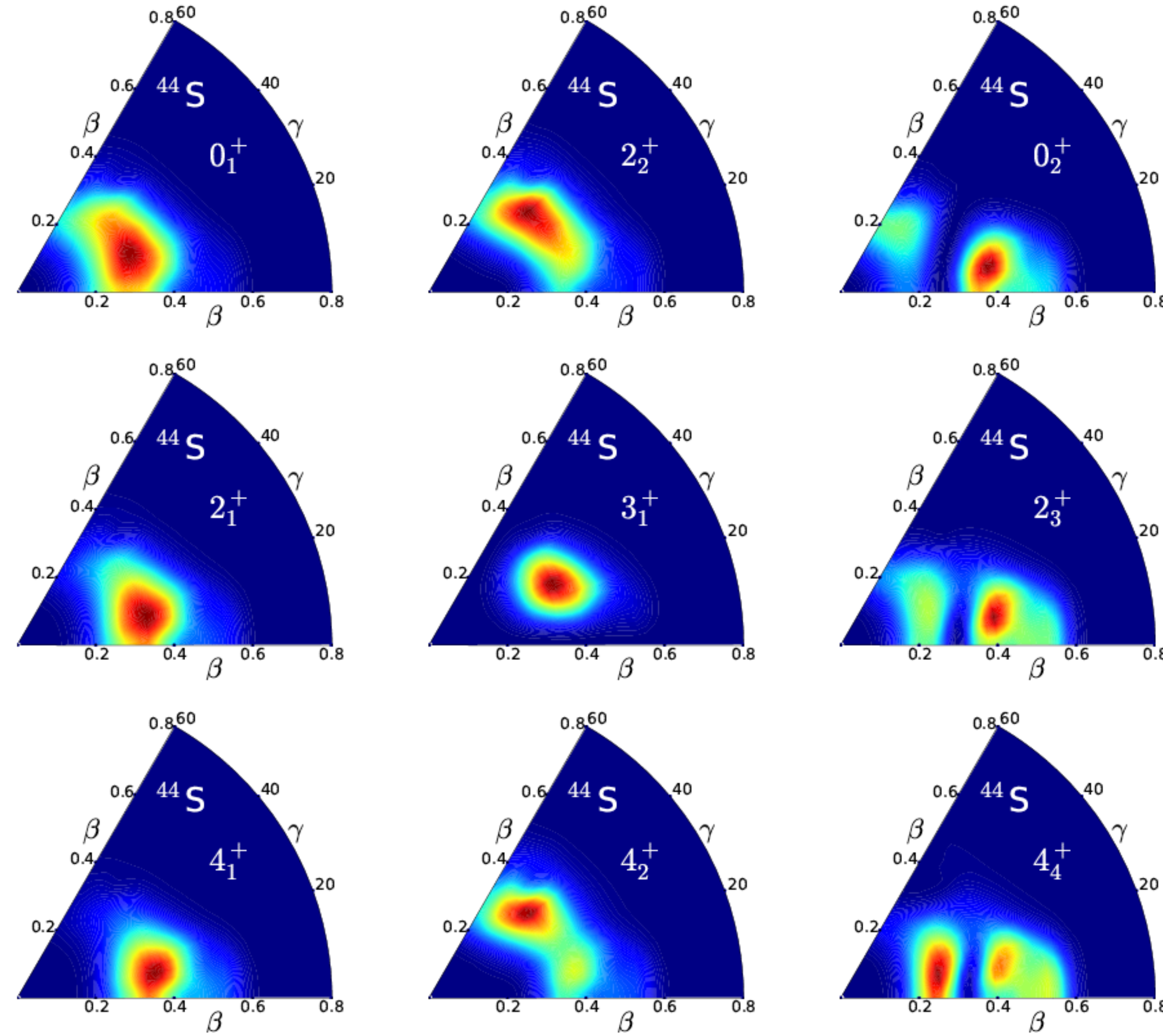


Single-particle levels





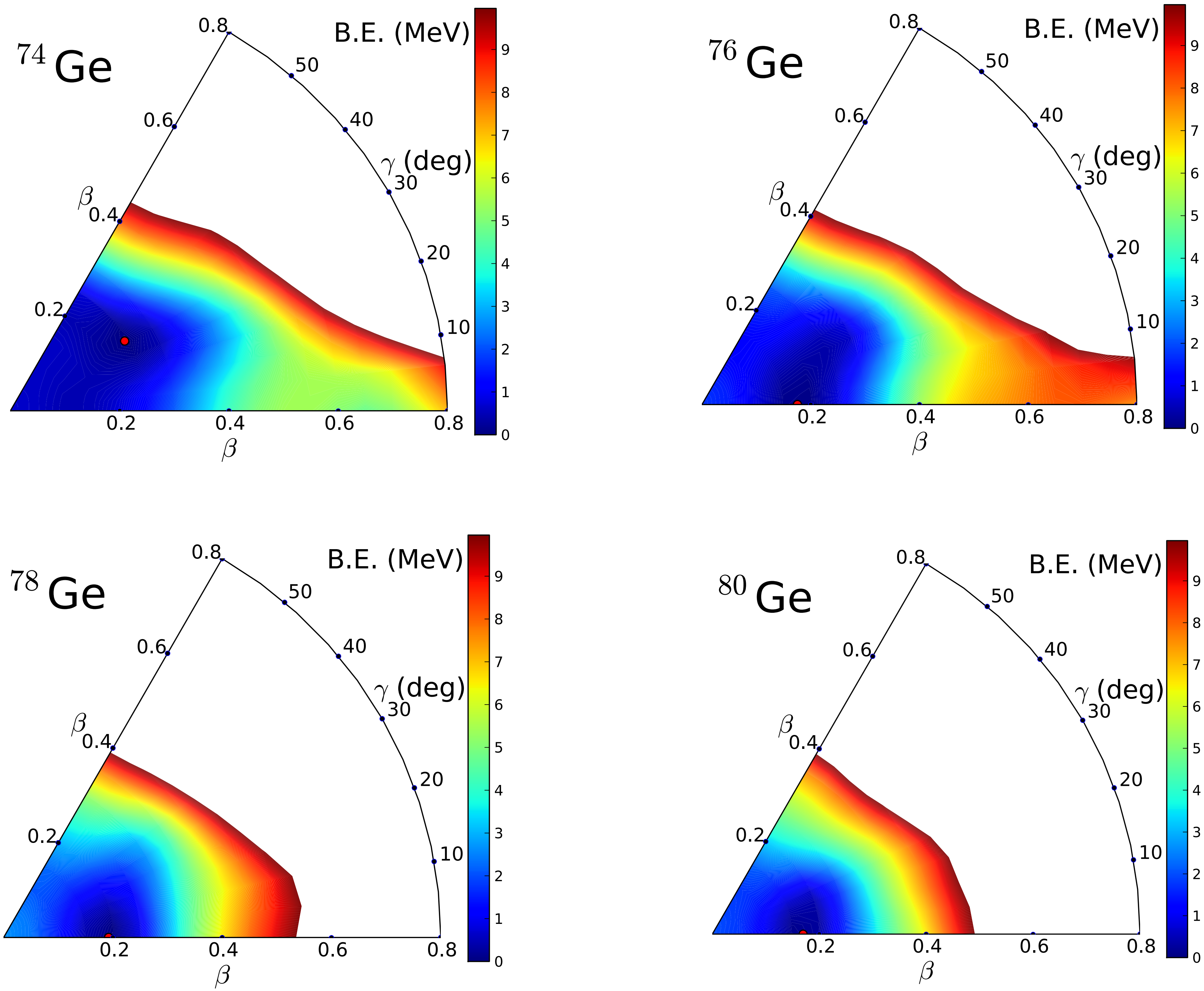
Probability density distributions:



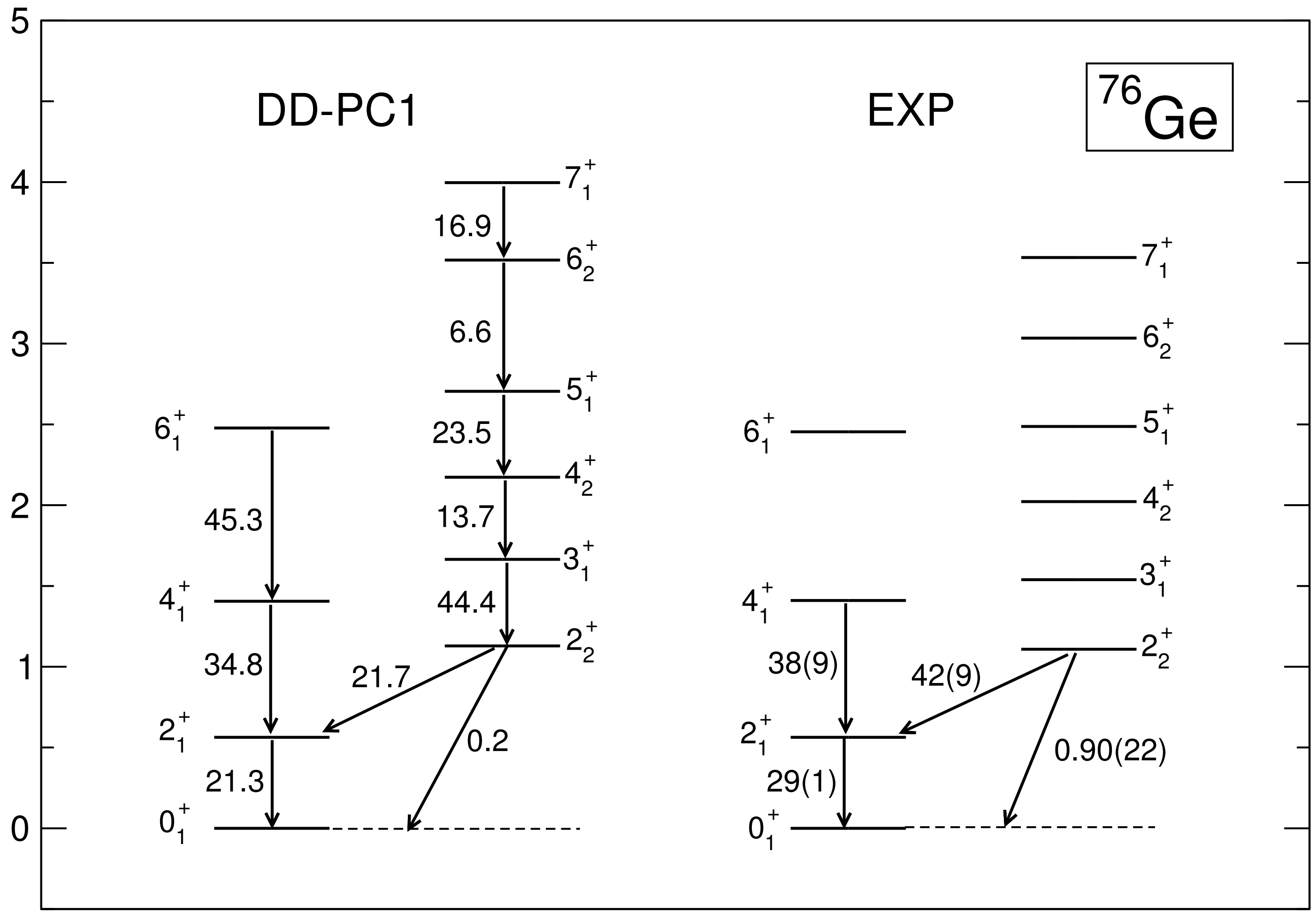
	$K=0$	$K=2$	Q_{spec}
2_1^+	89%	11%	−10.8
2_2^+	21%	79%	8.2
2_3^+	78%	22%	−7.3

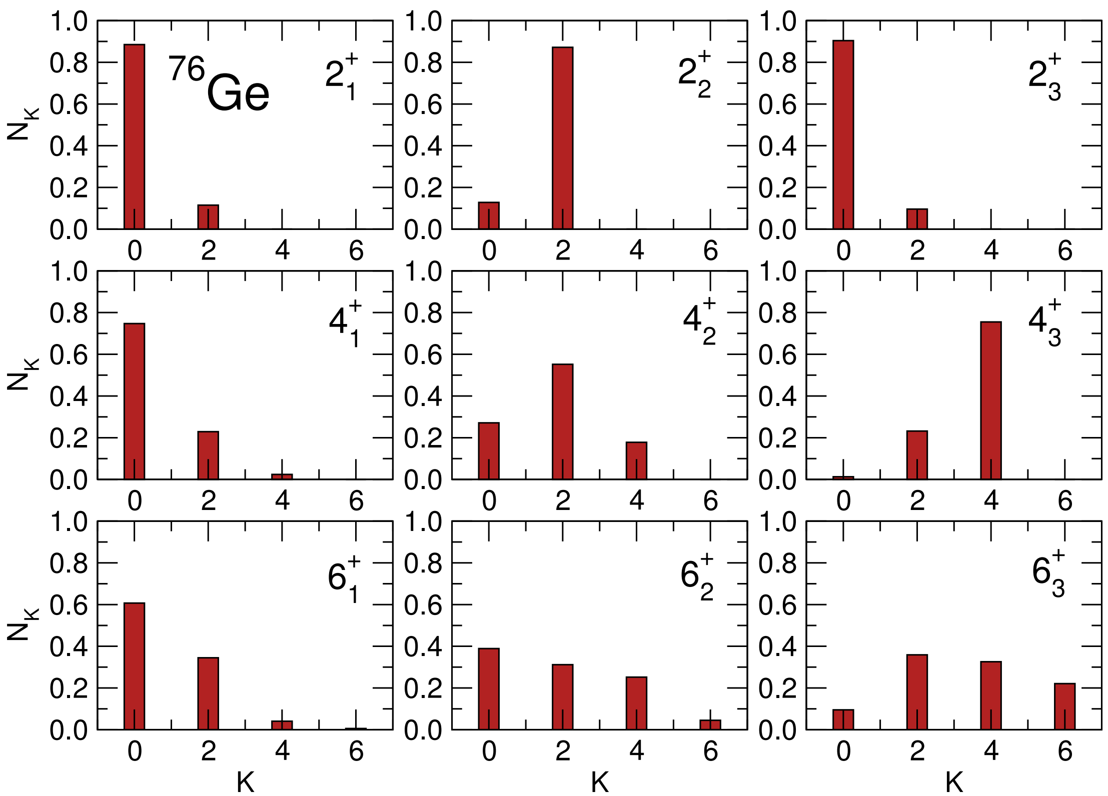
Shape evolution and triaxiality in germanium isotopes

Phys. Rev. C 89, 044325 (2014).



Quadrupole collective Hamiltonian based on the functional DD-PC1





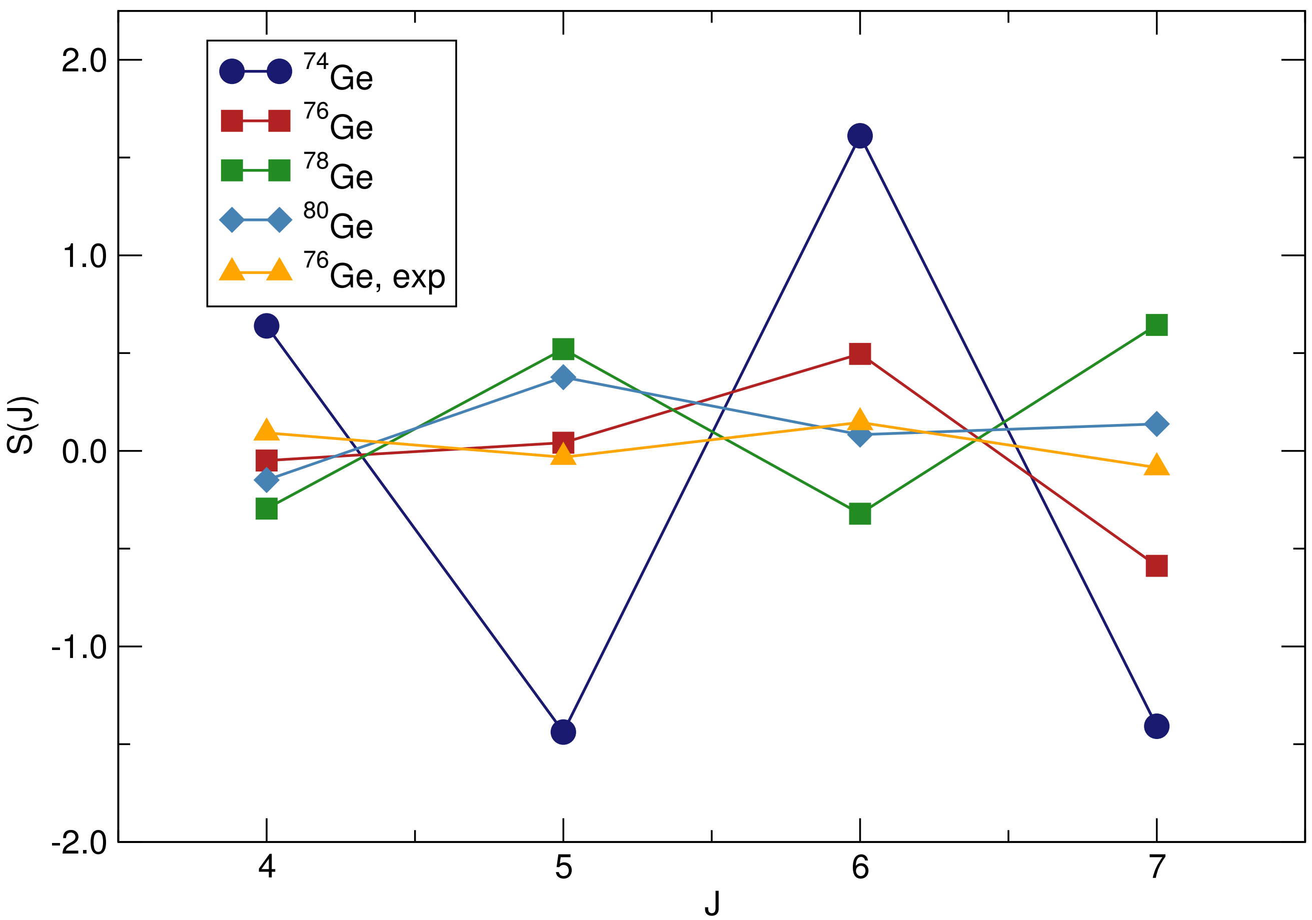
Distribution of *K* components (projection of the angular momentum on the body-fixed symmetry axis) in the collective wave functions of the nucleus ^{76}Ge.

The level of K-mixing is reflected in the staggering in energy between odd- and even-spin states in the γ band:

$$S(J) = \frac{E[J^+_\gamma] - 2E[(J-1)^+_\gamma] + E[(J-2)^+_\gamma]}{E[2^+_1]}$$

Deformed γ-soft potential $\Rightarrow$ S(J) oscillates between negative values for even-spin states and positive values for odd-spin states.

γ-rigid triaxial potential $\Rightarrow$ S(J) oscillates between positive values for even-spin states and negative values for odd-spin states.

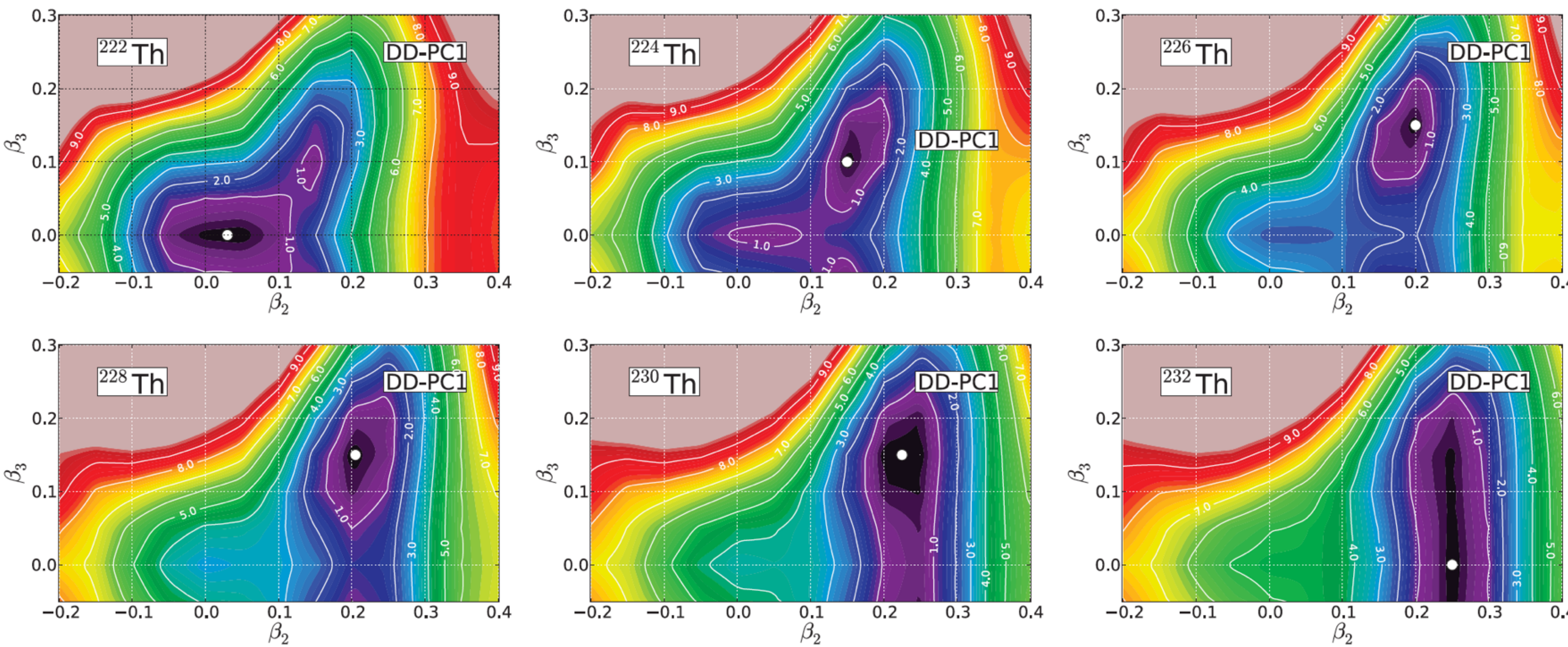


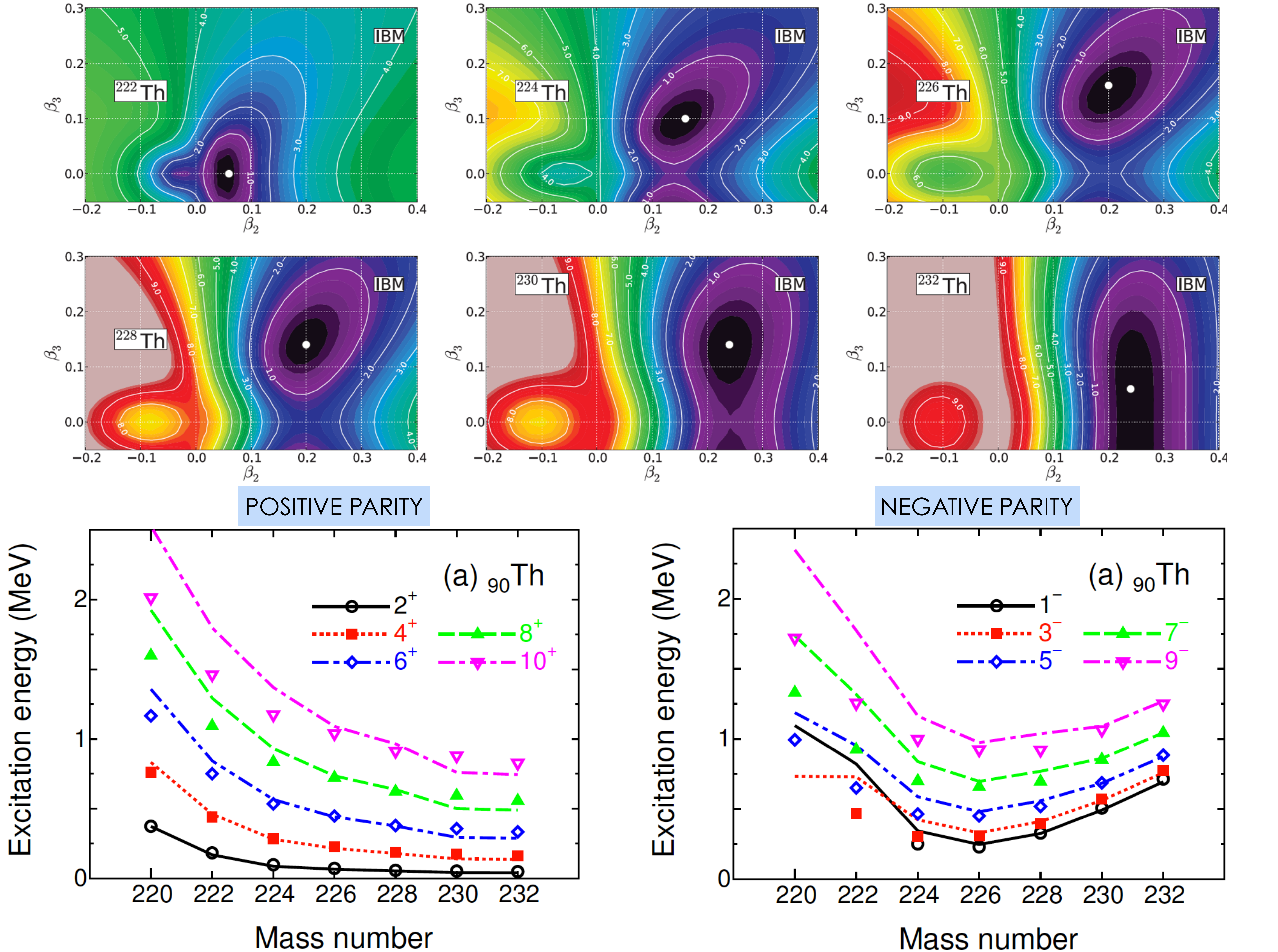
The mean-field potential of ^{76}Ge is γ soft. The inclusion of collective correlations (symmetry restoration and quantum fluctuations) drives the nucleus toward triaxiality, but not strong enough to stabilize a $\gamma \approx 30^\circ$ shape.

Octupole shape-phase transitions in light actinides

Phys. Rev. C 89, 024312 (2014).

Axially symmetric deformation energy surfaces of $^{222-232}$Th in the (β_2,β_3) plane:





222Th
224Th
226Th
228Th
230Th
232Th
IBM
β2
β3
POSITIVE PARITY
NEGATIVE PARITY
(a) 90Th
Excitation energy (MeV)
Mass number
2+
4+
6+
8+
10+
1−
3−
5−
7−
9−

Extrapolation to SHE

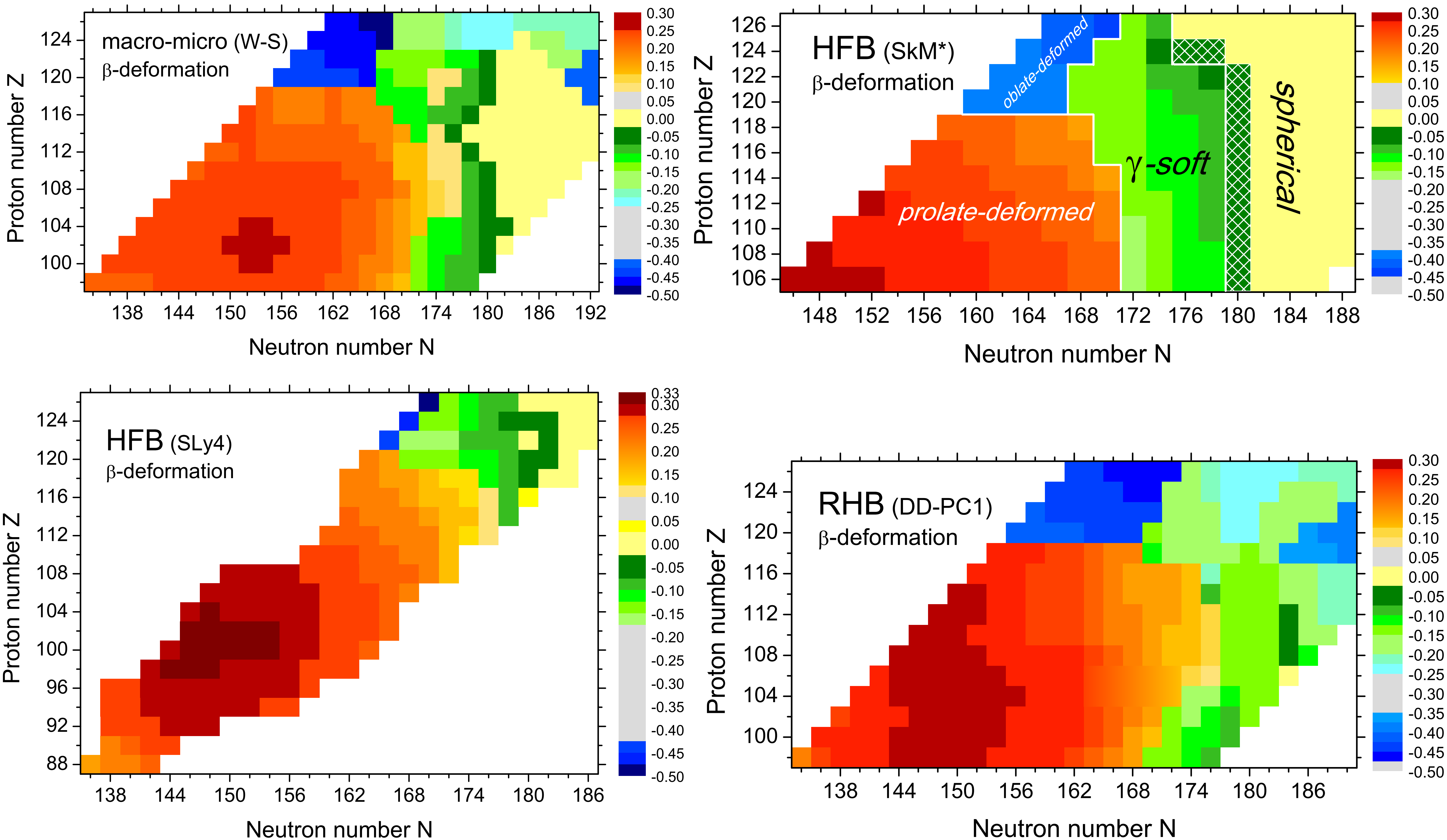
EDFs and the corresponding structure models are applied to a region far from those in which their parameters are determined by data ➠ large uncertainty in model predictions?

Much higher density of single-particle states close to the Fermi energy ➠ details of the evolution of deformed shells with nucleon number will have more pronounced effects on energy gaps, separation energies, Q_α-values, band-heads in odd-A nuclei, K-isomers …

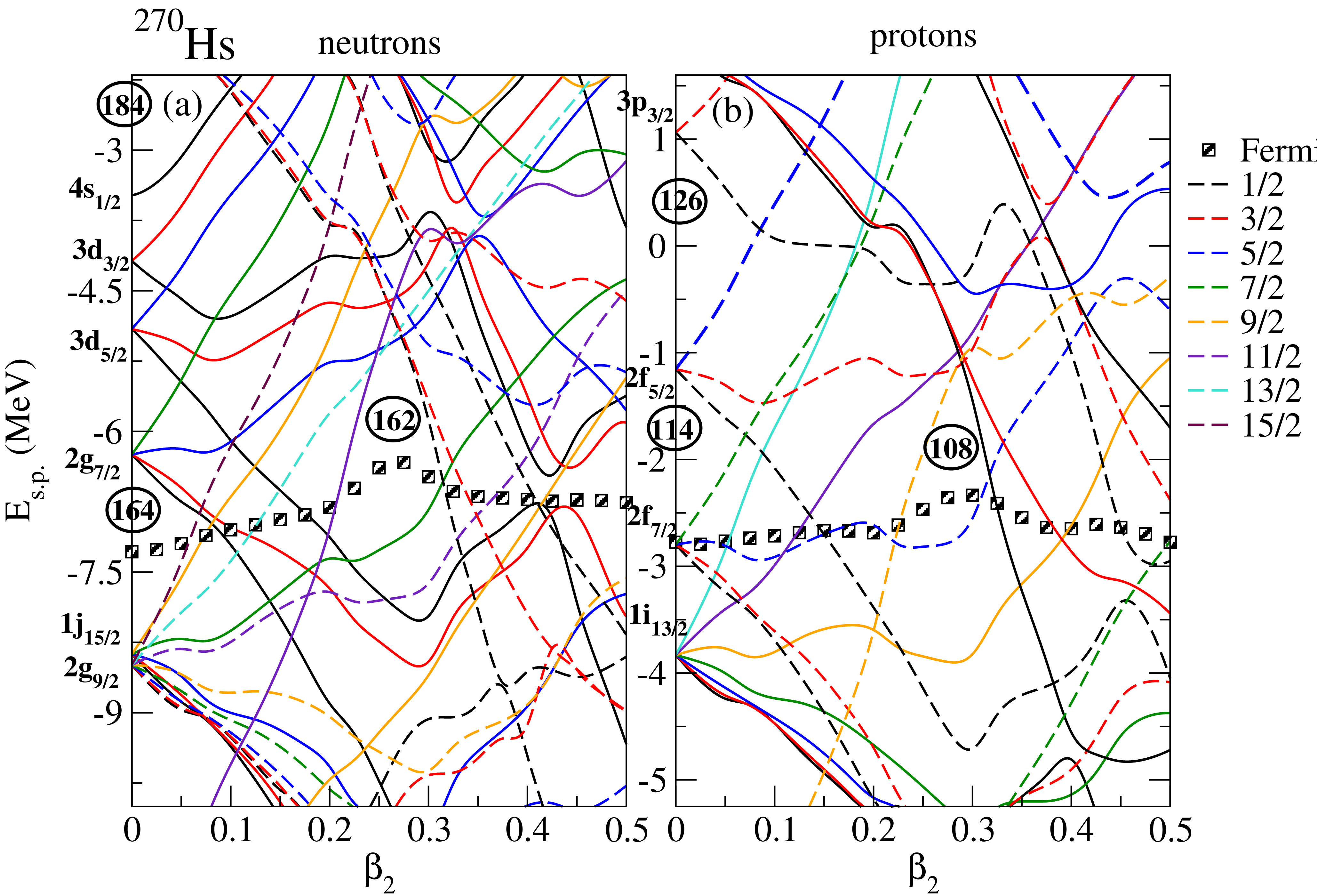
Much stronger competition between the attractive short-range nuclear interaction and the long-range electrostatic repulsion ➠ pronounced effects on the Coulomb, surface and isovector energies! Fast shape transitions! Exotic shapes!

Shape transitions in superheavy nuclei

Nuclear Physics A (2015)

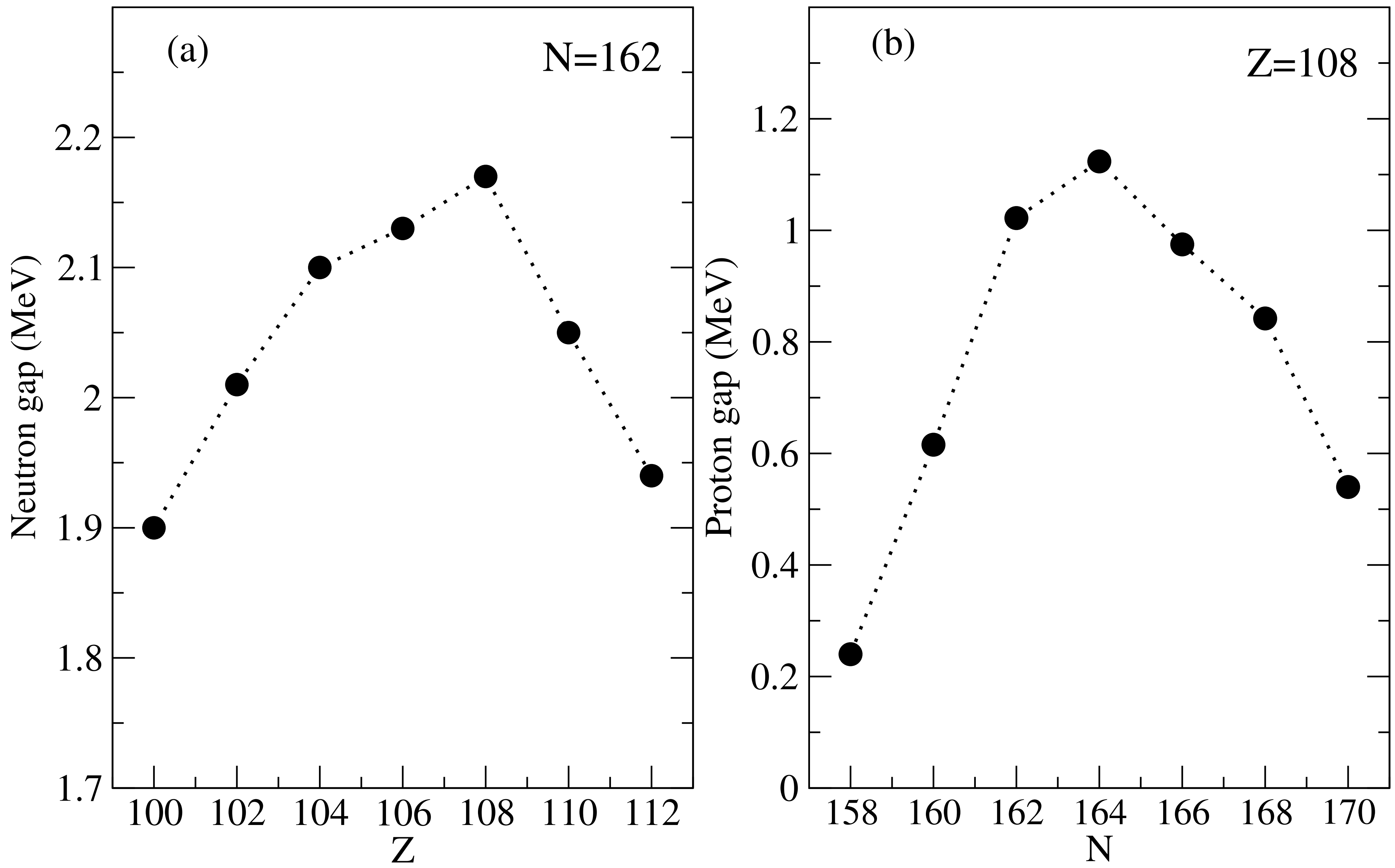


Energy gaps are small. Shape stabilisation depends on how fast the shell structures vary with deformation!

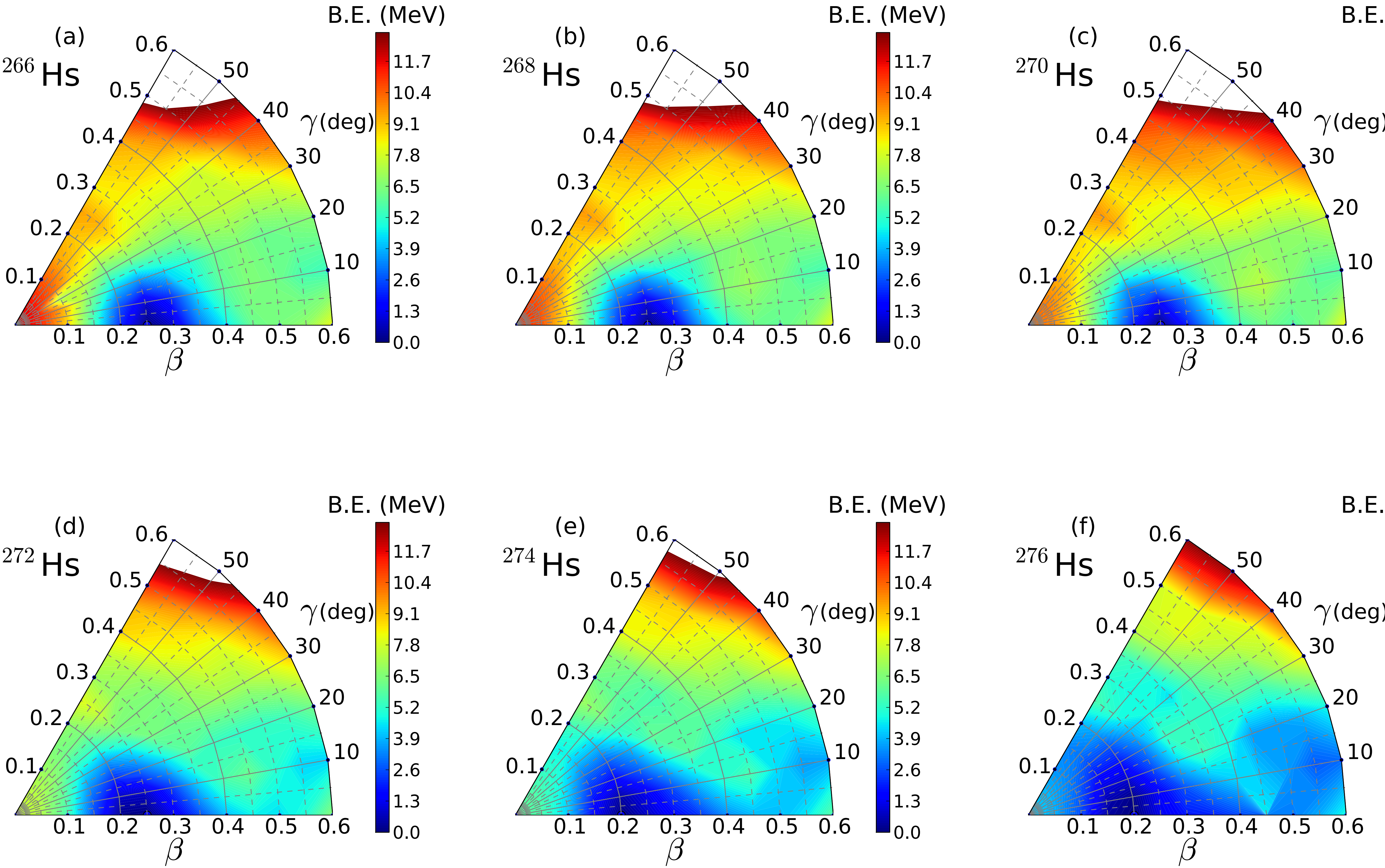


Phys. Rev. C **88**, 044324 (2013)

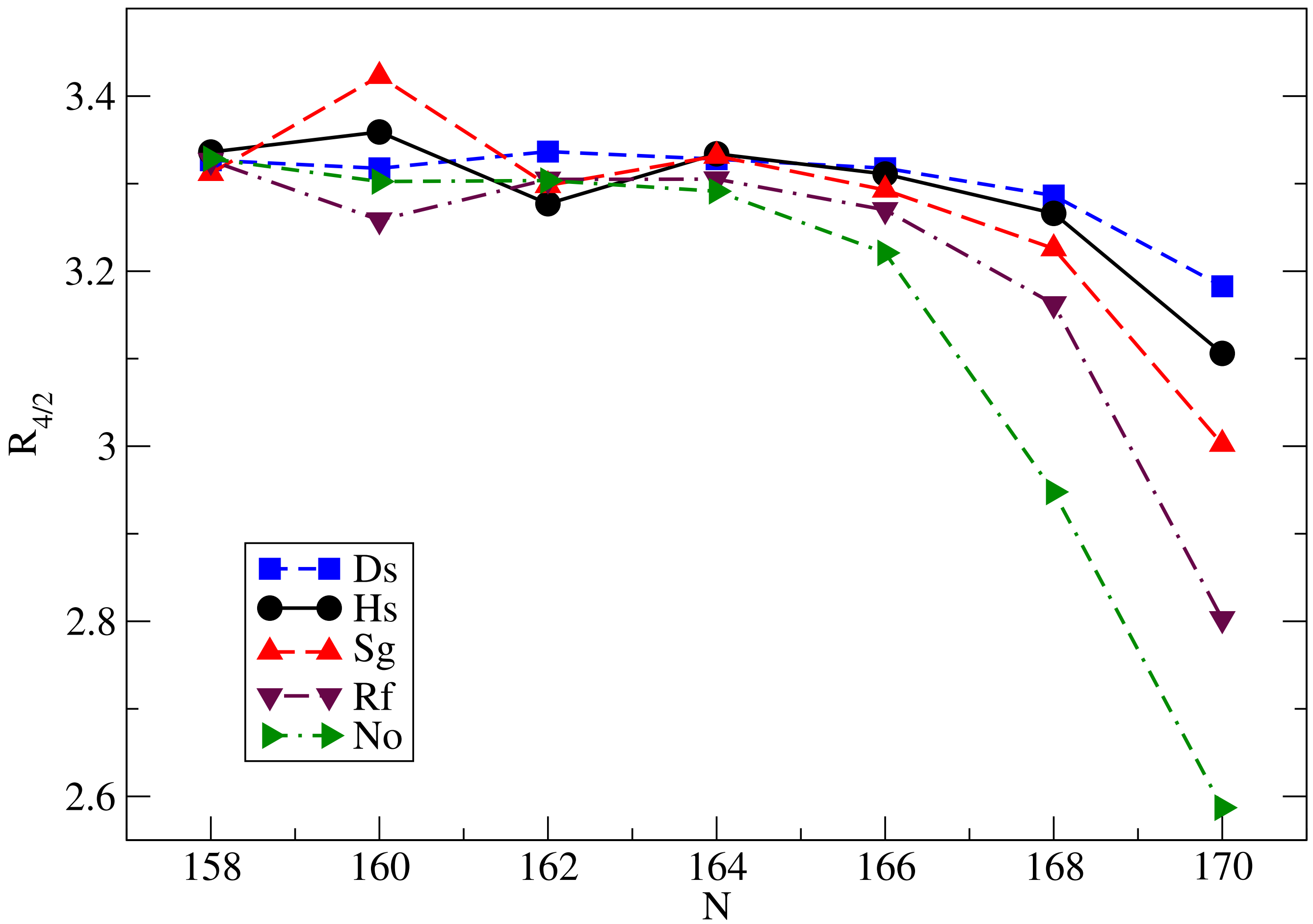
Neutron and proton shell gaps



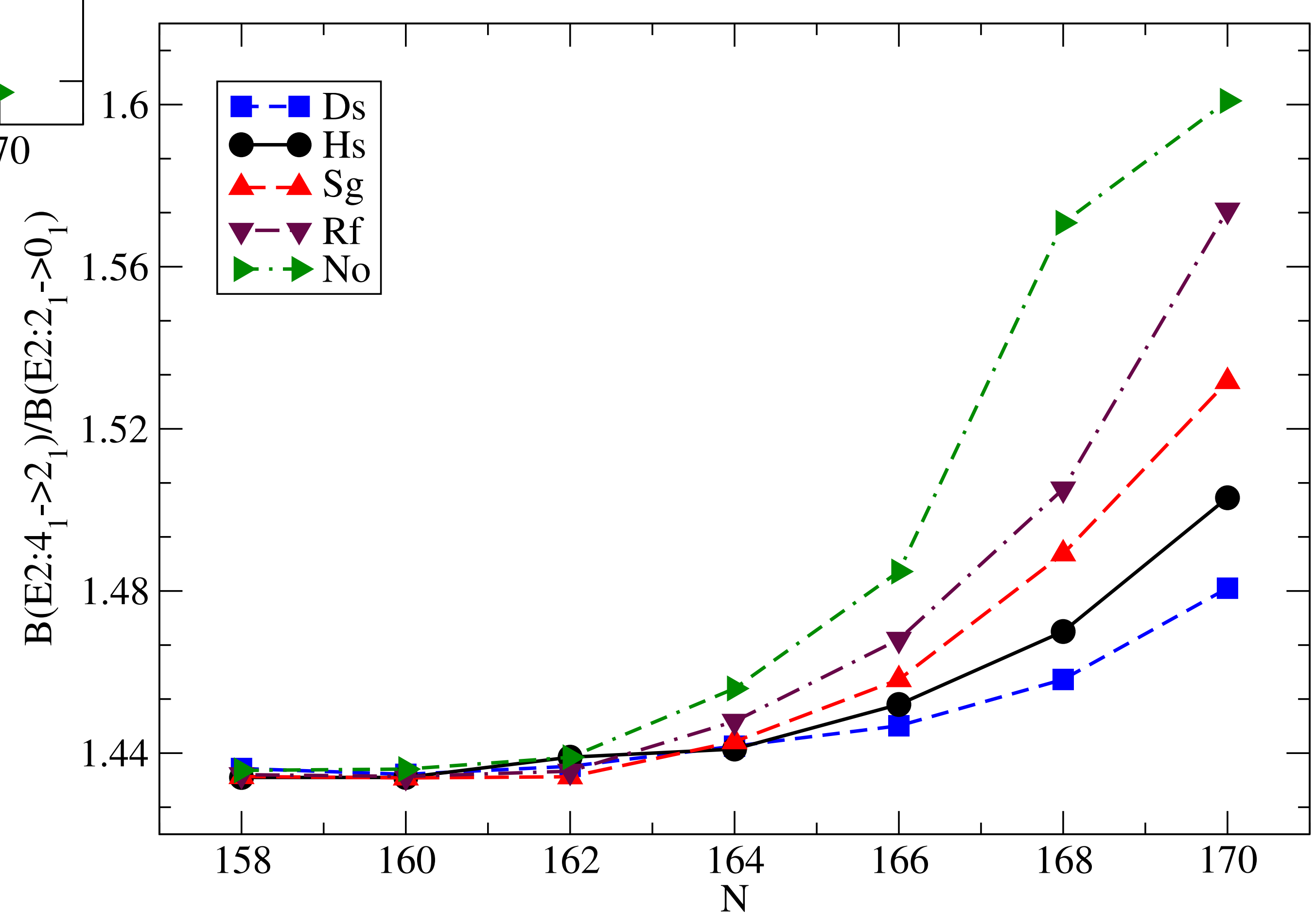
^{270}Hs ➠ deformed “doubly magic” nucleus


(a)
^{266}Hs
(b)
^{268}Hs
(c)
^{270}Hs
(d)
^{272}Hs
(e)
^{274}Hs
(f)
^{276}Hs
B.E. (MeV)
11.7
10.4
9.1
7.8
6.5
5.2
3.9
2.6
1.3
0.0
γ(deg)
10
20
30
40
50
0.1
0.2
0.3
0.4
0.5
0.6
β

Collective states



The ratio R4/2 of excitation energies of the yrast states $4^+{}_1$ and $2^+{}_1$ as a function of the neutron number for the isotopic chains of No, Rf, Sg, Hs and Ds.



The ratio of reduced transition probabilities $B(E2;\, 4^+{}_1 \rightarrow 2^+{}_1\,)/\, B(E2;\, 2^+{}_1 \rightarrow 0^+{}_1\,)$ as a function of the neutron number.

Nuclear Energy Density Functional Framework

✔ …description of universal collective phenomena that reflect the organisation of nucleonic matter in finite nuclei and the underlying inter-nucleon forces → universal theory framework that can be applied to different mass regions.

✔ NEDFs provide an economic, global and accurate microscopic approach to nuclear structure that can be extended from relatively light systems to superheavy nuclei, and from the valley of β-stability to the particle drip-lines.

✔ NEDF-based structure models that take into account collective correlations → microscopic description of low-energy observables: excitation spectra, transition rates, changes in masses, isotope and isomer shifts, related to shell evolution with nuclear deformation, angular momentum, temperature and number of nucleons.