Structure of $N = Z$ nuclei in single j shell calculations

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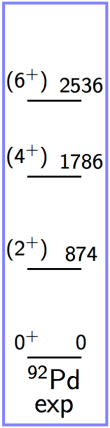
Background



Cederwall *et al.*, Nature (London) 469, 68 (2011):

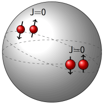
Measured excited levels of ^{92}Pd.
($N = Z = 46$. Top of $1g_{9/2}$ shell.)

Well reproduced by $1f_{5/2}2p1g_{9/2}$ shell model calculation with standard Hamiltonian.

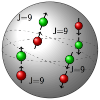
"Our results reveal evidence for a spin-aligned, isoscalar neutronproton coupling scheme [. . .] We suggest that this coupling scheme replaces normal superfluidity (characterized by seniority coupling) in the ground and low-lying excited states of the heaviest $N = Z$ nuclei."

Coupling schemes

Pertain to a single j shell.



Seniority scheme (Racah, 1940s):
Ground state composed of angular momentum $J = 0$ pairs of valence nucleons or nucleon holes.
Excitation by breaking of pairs.
"Seniority"= number of unpaired nucleons.



Stretch scheme
(Danos, Gillet, Phys. Rev. 161, 1034 (1967)):
Valence neutron proton or neutron proton hole pairs form two "chains", each with maximal angular momentum.
Ground state: Chain angular momenta are opposite.
Excitation by their bending towards each other.
Ground state configuration violates the Pauli principle. Must be antisymmetrised.

How different are these coupling schemes?

Neergård, Phys. Rev. C 88, 034329 (2013):

^{96}Cd. $1g_{9/2}$ shell model. Interaction from ^{90}Nb spectrum:[1]

Angular momentum I = isospin $T = 0$.
3-dimensional configuration space.

ψ = calculated ground state.
ϕ = seniority zero state = lowest state with the pairing interaction ($J = 0$ pairs and no others interact attractively).
χ = antisymmetrised stretch scheme ground state = lowest state when $J = 9$ pairs and no others interact attractively.
$\xi \perp \phi, \chi$.

Result: $|\langle\phi|\psi\rangle|^2 = 77\%$, $|\langle\chi|\psi\rangle|^2 = 93\%$, $|\langle\xi|\psi\rangle|^2 = 0.15\%$.

[1] Zerguine, Van Isacker, Phys. Rev. C 83, 064314 (2011). Several interactions from the literature were studied. All of them give qualitatively similar results. For clarity, I show in this talk the results for only one interaction.

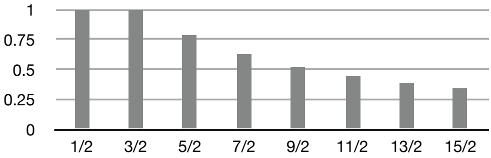
That is,

- Normal superfluidity (seniority zero) is not gone (persists to 77%).
- ψ is (essentially) a linear combination of ϕ and χ.
- ψ has large overlaps with both.

How is this possible?

Because (due to the Pauli principle) ϕ and χ are not orthogonal.

Their overlap $|\langle\phi|\chi\rangle|^2$ is a function of the single nucleon angular momentum j:

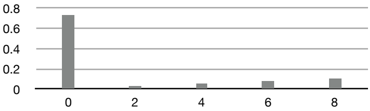


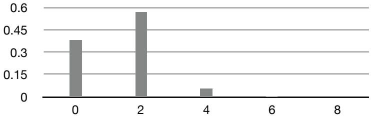
$|\langle\phi|\chi\rangle|^2 = 52\%$ for $j = 9/2$.

Other illustrations

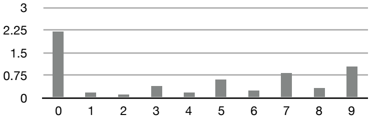
Left: ϕ. Right: χ. Abscissa: J.

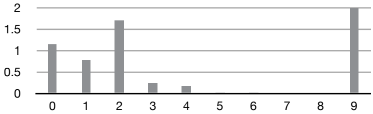
State vectors (squared amplitudes):
Basis of neutron = proton angular momentum J.
(5-dimensional because it includes $T = 0$ and 2.)





Contents of pairs with angular momentum J (total content = 6):





ϕ feels the $J = 9$ interaction and χ the $J = 0$ interaction, so these interactions cooperate.

Dependence on mass number A

In calculations for the shells $1f_{7/2}$ and $1g_{9/2}$ with interactions from the literature, $|\langle\chi|\psi\rangle|^2$ is larger in the $1f_{7/2}$ shell.

In the hypothetical case of validity of the model for ^{8}Be in the $1p_{3/2}$ shell, one would have $|\langle\chi|\psi\rangle|^2 = 1$.

The occurrence of stretch scheme structure (represented by χ) is thus not characteristic of heavy nuclei.

^{48}Cr, ^{92}Pd (four valence nucleons or holes of each kind)[2]

Neergård, Phys. Rev. C 90, 014318 (2014):

χ' = antisymmetrised product of two stretch scheme ground states of four valence nucleons or nucleon holes.

Contents of v = seniority, t = reduced isospin in %.
(*Cf.* Flowers, Proc. R. Soc. London, Ser. A 212, 248 (1952)):

	State a	$v, t = 0, 0$	$4, 0$	$4, 2$	$6, 1$	$8, 0$	$\|\langle a\|\psi\rangle\|^2$
^{48}Cr	ψ	80	20		0	0	
	χ	49	30		19	2	77
	χ'	67	26		6	1	92
^{92}Pd	ψ	70	27	1	0	2	
	χ	25	34	10	20	10	64
	χ'	47	36	0	6	10	86

[2] Interaction in the $1f_{7/2}$ shell derived from the the ^{42}Sc spectrum by Zamick, Robinson, Yad. Fiz. 65, 773 (2002).

^{48}Cr, ^{92}Pd, observations

- ψ consists of roughly 3/4 of $v = 0$ and 1/4 of $v, t = 4, 0$.
- Thus, normal superfluidity ($v = 0$) is certainly not gone.
- The content of χ in ψ is less than that of $v = 0$; that of χ' is greater.
- The compositions of both χ and χ' are very different from that of ψ.
- In particular, both have large components ($\approx$ 20% and 6 %) of $v, t = 6, 1$, which is vitually absent from ψ and in ^{48}Cr decouples exactly from the rest of the configuration space (Neergård, Phys. Rev. C 91, 044313 (2015)).
- (Not to be seen from the table above:) As ψ and χ in the 4 valence nucleon case, ψ, χ and χ' project to almost the same 1-dimensional subspace of the 2-dimensional $v, t = 4, 0$ space (and to a slightly lesser extent of the 7-dimensional $v, t = 8, 0$ space in the case of ^{92}Pd—in the case of ^{48}Cr, this space is 1-dimensional).

Symmetry and Wigner energy coefficients for $A = 48$ (1)

Bentley, Neergård, Frauendorf, Phys. Rev. C 89, 034302 (2014):

Background: Satuła, Dean, Gary, Mizutori, Nazarewicz, Phys. Lett. B 407, 103 (1997), based on $1f2p$ shell model calculations for $A = 48$: The Wigner energy stems from the isoscalar part of the effective two-nucleon interaction.

We calculated (in the $1f_{7/2}$ shell model) the lowest energy E for $T = 0, 2, 4$ in ^{48}Cr. Extracted κ and X in

$$E = E_{T=0} + \kappa T(T + X).$$

κ = symmetry energy coefficient, κX = Wigner energy coefficient, X = "Wigner X".

Like Satuła *et al.* we succesively turned off interaction channels.

Symmetry and Wigner energy coefficients for $A = 48$ (2)

J of included interactions								κ/MeV	κX/MeV	X
0	1	2	3	4	5	6	7	1.21	1.59	1.31
0		2	3	4	5	6	7	1.12	1.19	1.07
0		2		4	5	6	7	0.93	1.15	1.23
0		2		4		6	7	0.69	1.21	1.75
0		2		4		6		0.13	0.21	1.71
0		2		4				0.13	0.21	1.71
0		2						0.22	0.31	1.41
0								0.41	0.41	1.00

κX indeed decreases when the isoscalar interactions (odd J) are turned off and very much so when the $J = 7$ interaction leaves the scene. But κ also decreases and X actually increases.

Conclusion: The isoscalar interaction is responsible for the symmetry energy and not particularly the Wigner energy.

The well known result $X = 1$ is recovered for the pairing interaction.