Nuclear structure corrections for superallowed $0^+ \rightarrow 0^+$ beta decay revisited

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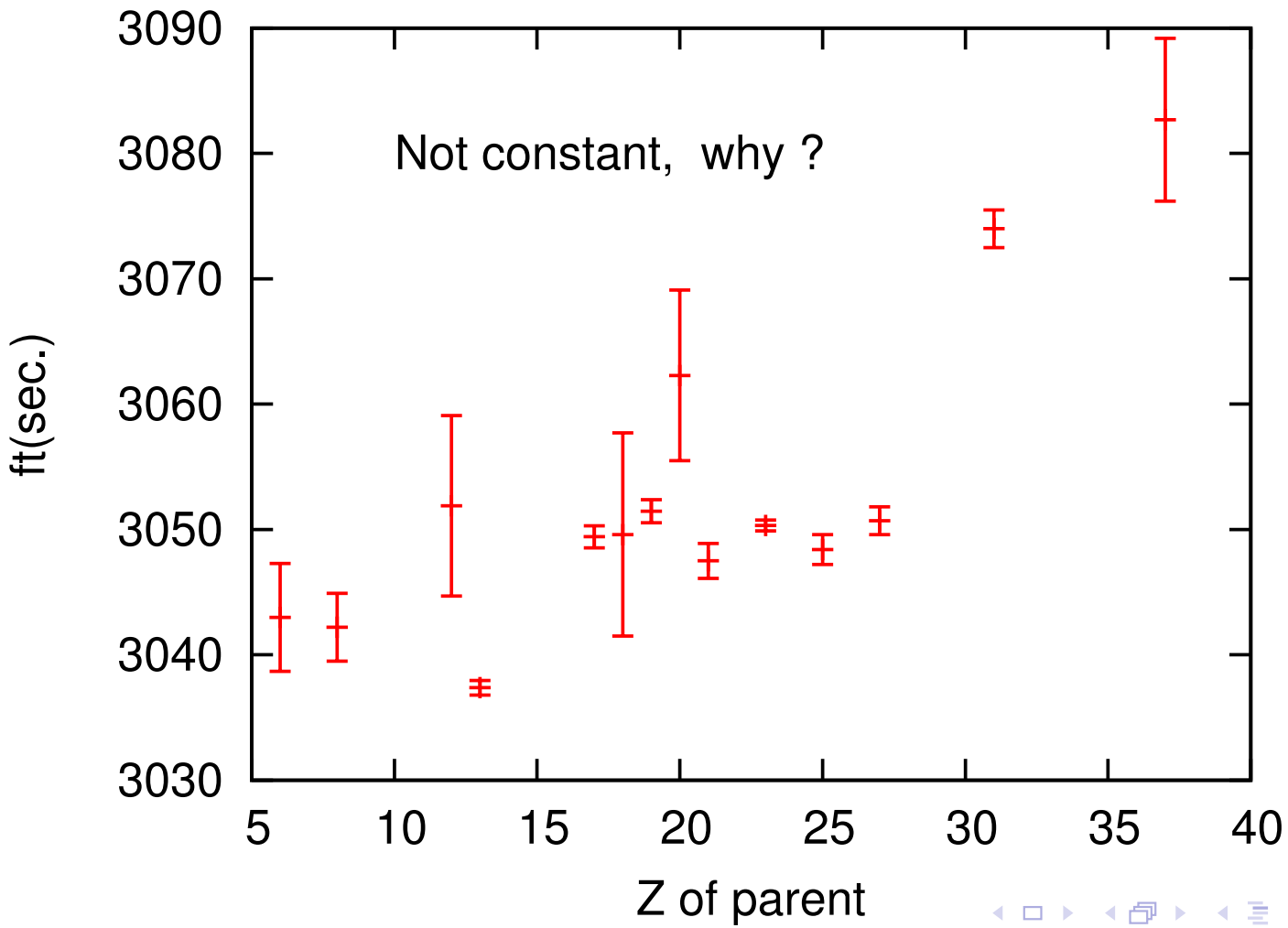
Introduction

- Superallowed $0^+ \rightarrow 0^+$ Fermi β decay in isospin multiplet $T = 1$ is an important tool to test the Standard Model of Electroweak Interaction :
  - Analogy between the Electroweak and Electromagnetic ineteractions, known as the Conserved Vector Current (CVC) hypothesis $=>$ vector coupling constant G_V
  - Unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix, the most dominant element could be extracted from superallowed β decay : $|V_{ud}| = G_V / G_F$
  - Challenge of nuclear structure theory, this process is softly nuclear structure dependent

Introduction

- As a consequence of CVC, the transition rate
  $ft \propto 1/|M_F|^2 G_V^2 \equiv const, \quad$ where $M_F(T=1) = \sqrt{2}$



Introduction

- One of the problems is isospin symmetry violation (nuclear structure effect : *Coulomb and charge dependence*)

$$|M_F| = |<\psi_f|\hat{\tau}_{\pm}|\psi_i>| \neq \sqrt{T(T+1) \mp T_{zf}\,T_{zi}}$$

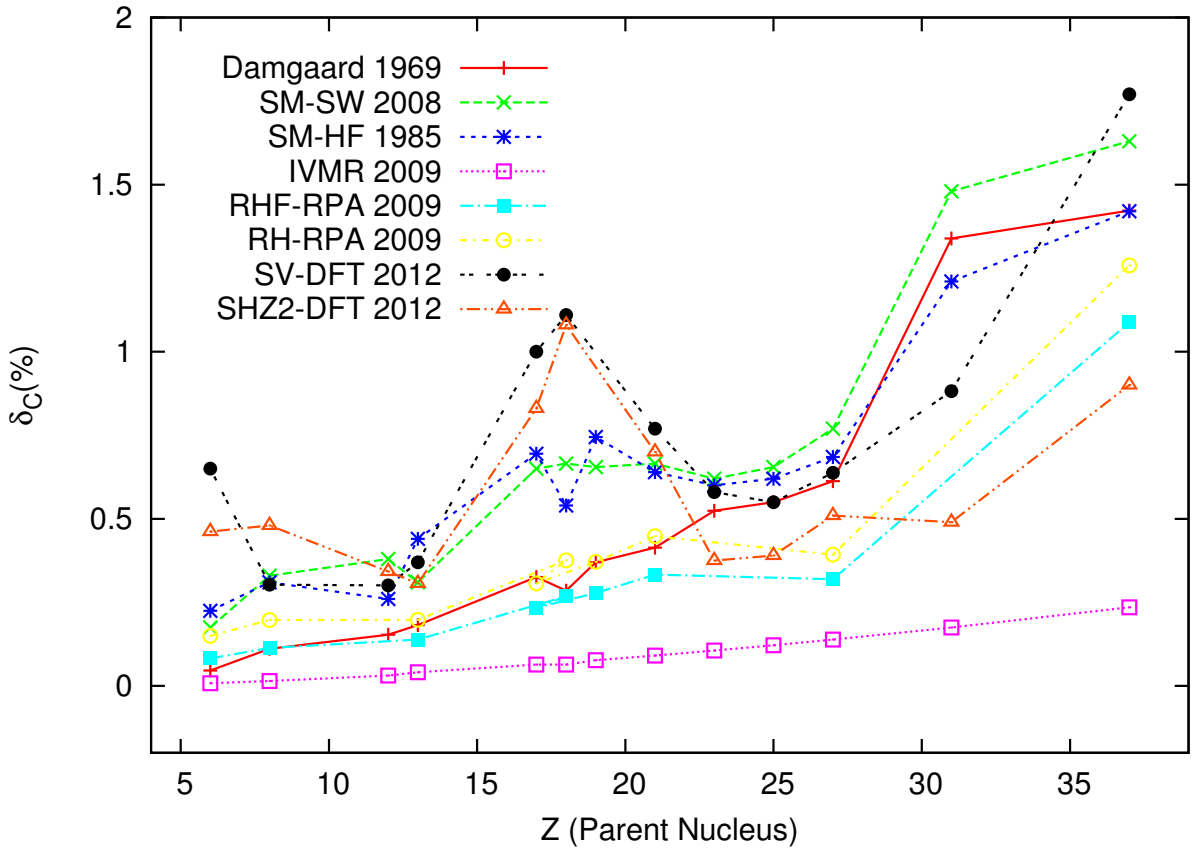
- Another problem is the radiative effect, such as bremsstrahlung effect
- The CVC hypothesis yields $Ft = const$:

$$Ft \equiv ft(1+\delta_R)(1+\delta_{NS}-\delta_C) = \frac{K}{|M_F^0|^2 G_V^2 (1+\Delta_R)}$$

  - M_F^0 is isospin symmetric Fermi matrix element $= \sqrt{2}$
  - δ_R, δ_{NS}, Δ_R are radiative effect corrections
  - δ_C is the isospin symmetry breaking correction

Introduction

- Present status of isospin symmetry breaking correction,



- Damgaard Model
- Shell Model (WS or HF basis)
- Relativistic HF
- Random phase approximation
- Density Functional Theory
- Isovector Monopole Resonance

Shell model description of isospin symmetry breaking

- In the shell model approach, the isospin symmetry breaking correction can be separated in two terms :

$$\delta_C \approx \delta_{RO} + \delta_{IM}$$

  - δ_{RO} is radial overlap correction, defined by :

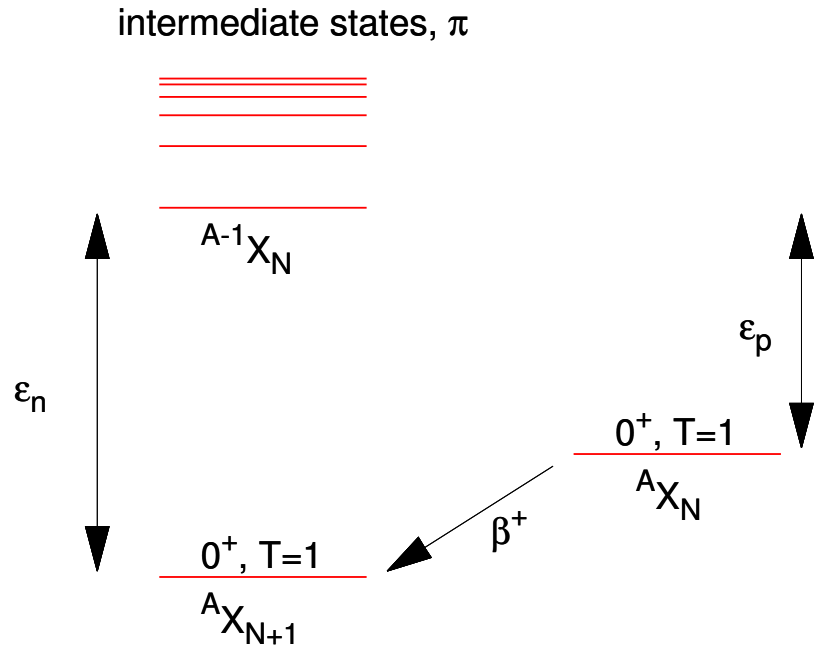
$$\delta_{RO} = \frac{2}{M_F^0} \sum_{\pi,\alpha} (1 - \Omega_\alpha^\pi)(-1)^{J_f + J_\pi + \lambda + j_\alpha} \left\{ \begin{array}{ccc} J_i & J_f & \lambda \\ j_\alpha & j_\alpha & J_\pi \end{array} \right\}$$
$$\times A(f\pi k_\alpha) A(i\pi k_\alpha) \sqrt{(2J_f + 1)(2J_i + 1)(2j_\alpha + 1)}$$

  - δ_{IM} is isospin mixing correction, defined by :

$$\delta_{IM} = \frac{1}{M_F^0} \sum_{\alpha} (2j_\alpha + 1)^{1/2} [OBTD^T(fik_\alpha\lambda) - OBTD(fik_\alpha\lambda)]$$

Shell model description of isospin symmetry breaking

- We use a parentage expression for the matrix element, which takes into account a large number of the intermediate $(A-1)$-nucleus excited states



we assume that the single particle state is fragmented over intermediate states

  - Computational cost expensive : need to account a big number of intermediate states (spectroscopic sum rule must be satisfied)

Shell model description of isospin symmetry breaking

- Radial overlap correction

$$\delta_{RO} = \frac{2}{M_F^0} \sum_{\pi,\alpha} (1 - \Omega_\alpha^\pi)(-1)^{J_f + J_\pi + \lambda + j_\alpha} \left\{ \begin{array}{ccc} J_i & J_f & \lambda \\ j_\alpha & j_\alpha & J_\pi \end{array} \right\}$$
$$\times A(f\pi k_\alpha) A(i\pi k_\alpha) \sqrt{(2J_f + 1)(2J_i + 1)(2j_\alpha + 1)}$$

  - Two ingredients :
    - Spectroscopic amplitudes $A(...)$ (can be obtained from shell model calculation, *experimental opportunities*)

$$A(f\pi k_\alpha) = \frac{< \psi_f || \hat{a}^\dagger_{k_\alpha} || \pi >}{\sqrt{2J_f + 1}}$$

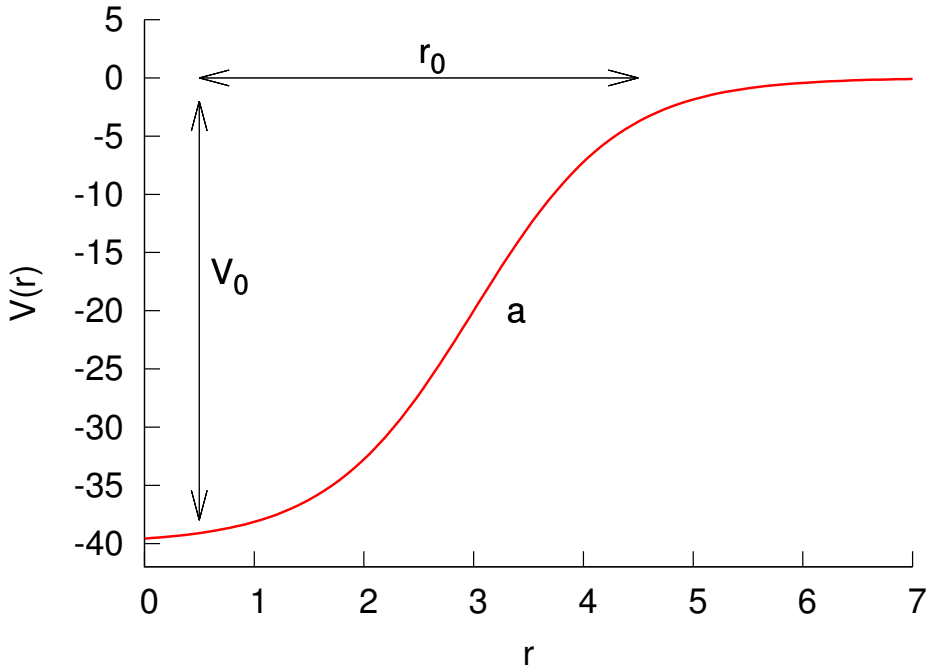
    - Overlap integeral Ω_α^π (can be obtained by using realistic single particle potentials)

$$\Omega_\alpha^\pi = \int_0^\infty R^\pi_{\alpha_p}(r) R^\pi_{\alpha_n}(r) r^2 dr$$

Application

- Phenomenological Woods-Saxon potential

$$\frac{\hbar^2}{2m}\left\{-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}\right\}u_\alpha(r)+V(r)u_\alpha(r)=\epsilon_\alpha u_\alpha(r)$$



- The potential is controlled by three parameters : V_0, r_0 and a
- We use charge uniform approximation for the Coulomb term

Application

- Phenomenological Woods-Saxon potential (cont.)

$$V(r) = \frac{-V}{1+\exp\left(\frac{r-R}{a}\right)} - V_{ls}\frac{r_s^2}{r}\frac{d}{dr}\left[\frac{V}{1+\exp\left(\frac{r-R_s}{a_s}\right)}\right] < \vec{l}.\vec{\sigma} >$$

$$+(Z-1)e^2\begin{cases} \frac{1}{r} & \text{if } r > R_c; \\ \frac{1}{R_c}\left(\frac{3}{2} - \frac{r^2}{2R_c^2}\right) & \text{otherwise.} \end{cases}$$

where $V = V_0 \pm (V_1/4)(N-Z)/A, \quad R = r_0(A-1)^{1/3}, \quad R_s = r_s(A-1)^{1/3},$

$V_1 = 146.368 MeV,\ r_s = 1.16 fm,\ a = a_s = 0.662 fm,\ V_{ls} = 0.22$

  - V_0 and r_0 are kept to adjust to reproduce separation energies and charge radii respectively

Application

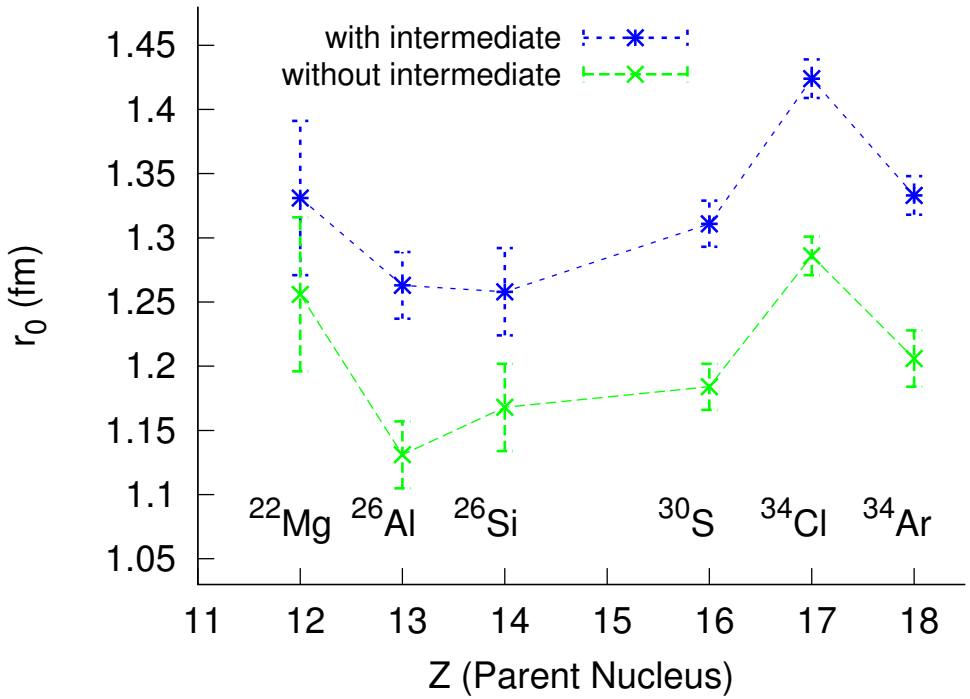
- Phenomenological Woods-Saxon potential (cont.)
  - A remark on charge radius fits

$$R_{ch}^2 = \frac{1}{Z}\sum_{\pi\alpha} S(i\pi k_\alpha)\int_0^\infty r^2 |u_\alpha^\pi(r)|^2 dr \quad \text{with intermediate}$$

$$R_{ch}^2 = \frac{1}{Z}\sum_{\alpha} n(\alpha)\int_0^\infty r^2 |u_\alpha(r)|^2 dr \quad \text{without intermediate}$$

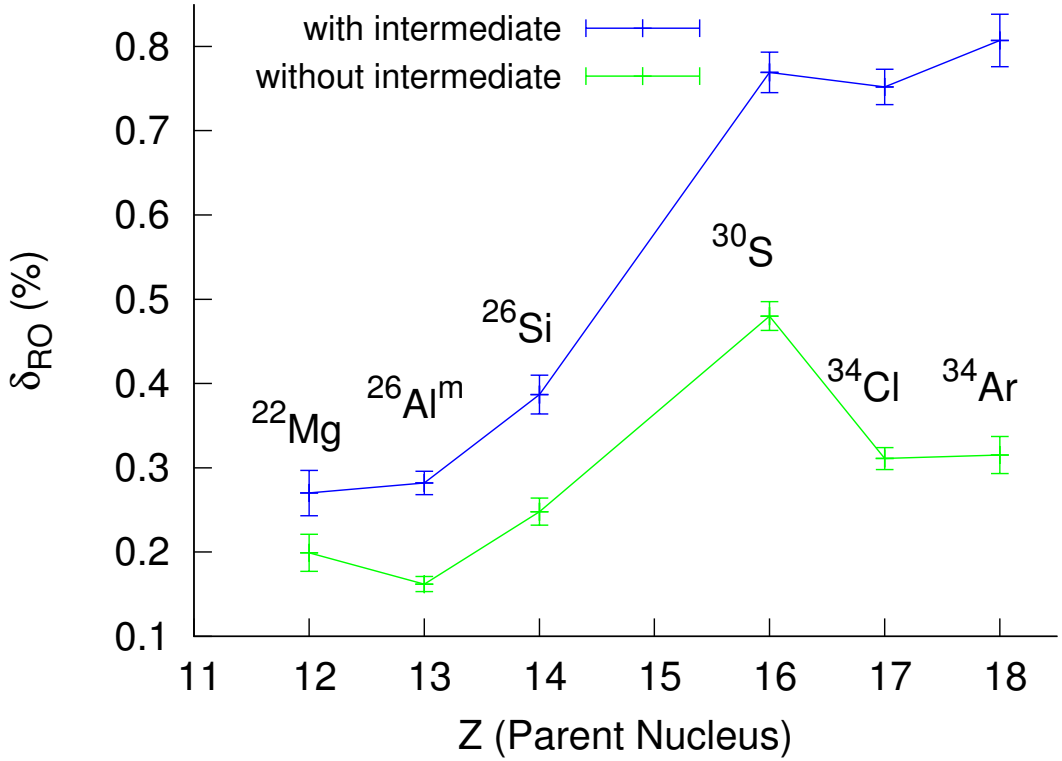
The blue is systematically overestimating the green

Inclusion of intemediate dependence => smaller charge radius



Application

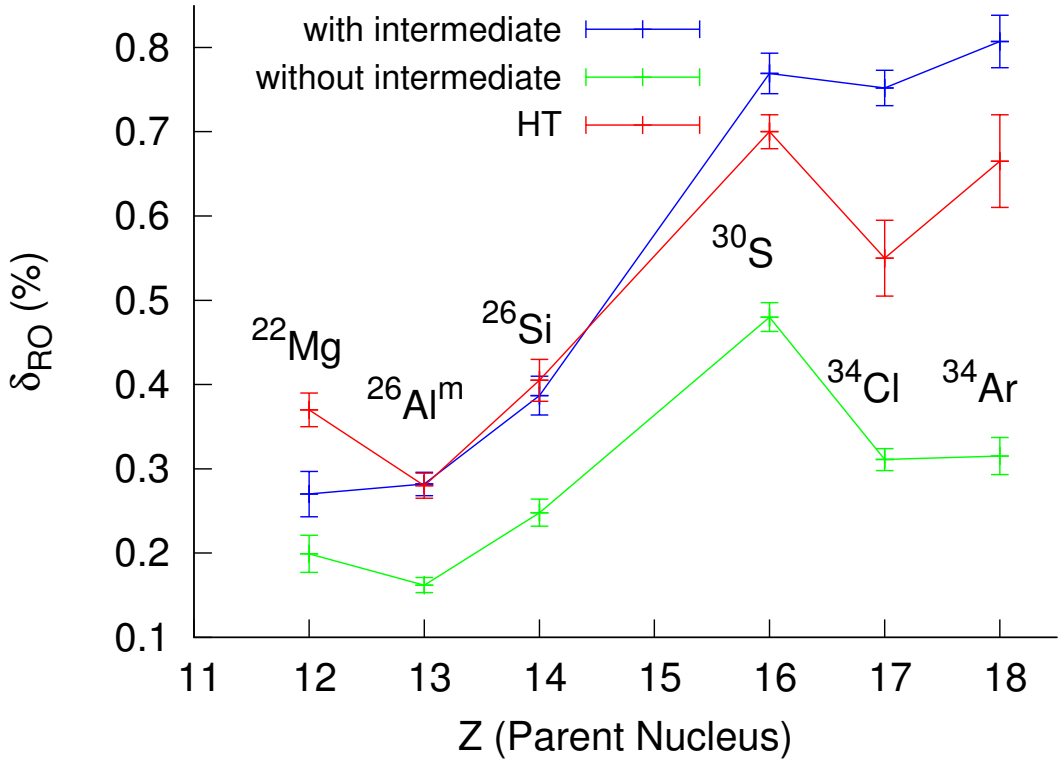
- Phenomenological Woods-Saxon potential (cont.)
  - Result for radial overlap correction



Inclusion of intemediate dependence => increasing redial overlap correction

Application

- Phenomenological Woods-Saxon potential (cont.)
  - Result for radial overlap correction *compare with Hardy & Towner, PRC 91, 025501 (2015)*



The difference between bleu and red would signal parameter dependence

Application

- Self consistent Hartree-Fock potential
  - With zero range Skyrme interaction :

$$V_{Sk} = t_0(1+x_0P_\sigma)\delta + \frac{1}{2}t_1(1+x_1P_\sigma)(\mathbf{k'}^2\delta + \delta\mathbf{k}^2) + t_2(1+x_2P_\sigma)\mathbf{k'}.\delta\mathbf{k}$$
$$+\frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\mathbf{R})\delta + iW_0(\sigma_i+\sigma_j).\mathbf{k'}\times\delta\mathbf{k} + V_{coul}$$

we can transform the non local HF equation into the local energy dependent equation (*C. B. Dover and Nguyen Van Giai, NPA. 190 (1972) 373-400*) :

$$\frac{\hbar^2}{2m}\left\{-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}\right\}u_\alpha^L(r) + V(r,\epsilon_\alpha)u_\alpha^L(r) = \epsilon_\alpha u_\alpha^L(r)$$

With non local function : $u_\alpha(r) = [m^*/m]^{1/2}u_\alpha^L(r)$ and,

$$V(r,\epsilon_\alpha) = \frac{m^*}{m}\left[U(r) + \frac{1}{2}\frac{d^2}{dr^2}\left(\frac{\hbar^2}{2m^*}\right) - \frac{m^*}{2\hbar^2}\left(\frac{d}{dr}\frac{\hbar^2}{2m^*}\right)^2\right]$$
$$+\left[1-\frac{m^*}{m}\right]\epsilon_\alpha$$

Application

- Self consistent Hartree-Fock potential (cont.)

  and

$$U(r) = U^0(r) + U^{so}(r) < \vec{l}.\vec{\sigma} > + V_C(r)$$

  - The effective mass m^*, the central term $U^0(r)$ and the spin-orbit term $U^{so}(r)$ depend on the choice of Skyrme parametrization
  - The Coulomb term is given by :

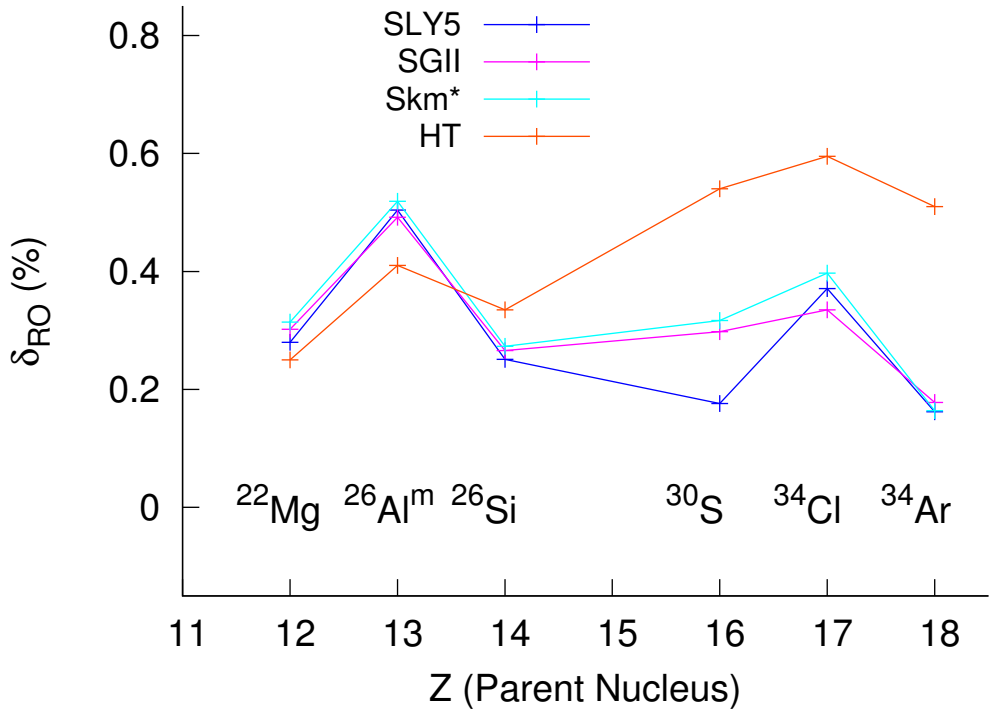
$$V_C(\vec{r}) = \frac{e^2}{2} \int d^3\vec{r'} \frac{\rho_p(\vec{r'})}{|\vec{r} - \vec{r'}|} - e^2 \left(\frac{3}{\pi}\right)^{1/3} \rho_p^{1/3}(\vec{r})$$

    - the first term is the Coulomb direct term
    - the second is the Coulomb exchange term in Slater approximation

  - To reproduce experimental data, we will adjust the Hartree-Fock field by scaling the central potential $U^0(r)$

Application

- Self consistent Hartree-Fock potential (cont.)
  - Result for redial overlap correction (*preliminary*)

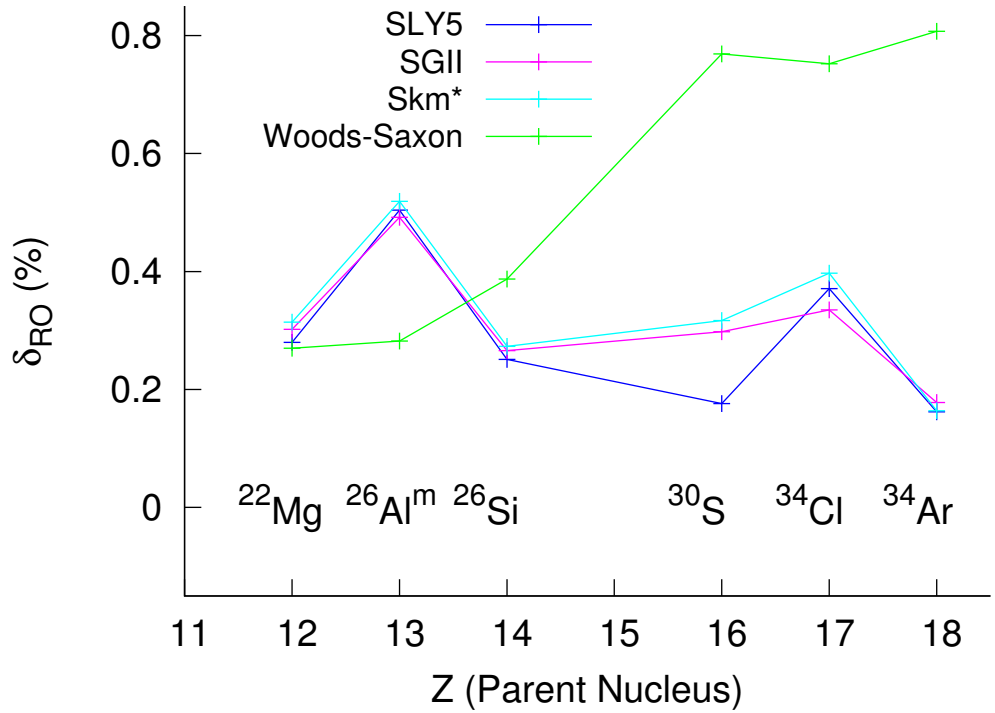


HT : Hardy & Towner PRC 79, 055502 (2009)

- Why they are so different ?
  - HT use another approach for Coulomb terms

Application

- Self consistent Hartree-Fock potential (cont.)
  - Result for redial overlap correction (*preliminary*)



  - Why are there big disagreements between WS and HF ?
    - If the Coulomb is the main source of isospin symmetry breaking, the correction δ_{RO} should increase with Z
    - from this argument, the HF results might be unreasonable

Summary

- We have calculated the radial overlap correction for six transitions in the middle of sd shell, using shell model with realistic Woods-Saxon and Hartree-Fock basis
- The spectroscopic amplitudes were obtained from large scale shell model calculation, using the USD, $USDA$ and $USDB$ interactions
- To ensure that the spectroscopic sum rule is satisfied, we have accounted 500 intermediate states

Perspective

- We have adjusted the potential to reproduce exp. data, even these data contain the contribution of some effects beyond the single particle model, such as pairing and Wigner effects ($we\ study\ N \approx Z\ nuclei$)

Summary

These results will be submitted to PRC :
L. Xayavong, N. Smirnova,
Radial Overlap Correction from the Shell Model with Woods-Saxon Potential.

Thank you for your attention !