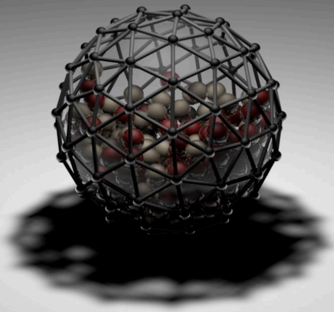
INTERPLAY OF γ-RIGID AND γ-STABLE COLLECTIVE MOTION IN THE PHASE TRANSITION FROM SPHERICAL TO DEFORMED NUCLEAR SHAPES

Radu Budaca
and Andreea-Ioana Budaca

IFIN-HH, Bucharest-Magurele, Romania



2015 European Nuclear Physics Conference, Groningen, The Netherlands, 31 August - 4 September

CONTENTS

- INTRODUCTION
- INTERPLAY BETWEEN γ-STABLE AND γ-RIGID COLLECTIVE MOTION
- MODEL'S ANALYTICAL PROPERTIES
- NUMERICAL APPLICATION
- CONCLUSIONS

The classical Hamiltonian function of the liquid drop model has 5 degrees of freedom, namely the two shape variables β and γ and the three Euler angles.

$$\mathcal{H} = \underbrace{\frac{B}{2}\left(\dot{\beta}^2 + \beta^2\dot{\gamma}^2\right)}_{T_{vib}} + \underbrace{\frac{1}{2}\sum_{k=1}^{3}\omega_k^2\mathcal{I}_k}_{T_{rot}} + V(\beta,\gamma).$$

Bohr-Mottelson Hamiltonian after quantization

Imposing a certain value for the γ shape variable, one reaches the γ-rigid version of the collective model which is interesting by itself due to its description of the basic rotation-vibration coupling.

- $\gamma \neq 0^\circ \;\; \Rightarrow$ 4 degrees of freedom $(\beta, \theta_1, \theta_2, \theta_3) \;\; \Rightarrow$ Davydov-Chaban Hamiltonian
  Davydov & Chaban NP **20** (1960) 499
- $\gamma = 0^\circ \;\; \Rightarrow$ 3 degrees of freedom $(\beta, \theta_1, \theta_2) \;\; \Rightarrow$ X(3)-type Hamiltonian
  Bonatsos *et. al.* PLB **632** (2006) 238

Although the γ-rigidity hypothesis is somewhat crude it provides simple approaches to the successful reproduction of the relevant experimental data.
Budaca EPJA **50** (2014) 87, PLB **739** (2014) 86; Buganu & Budaca PRC **91** (2015) 014306, JPG (2015);

The similarity between the β excited bands of the X(5) and X(3) solutions addresses the question about the importance of rigidity in explaining the critical collective phenomena.

INTERPLAY BETWEEN γ-STABLE AND γ-RIGID COLLECTIVE MOTION

The kinetic energy operator $\hat{T}_{vib} + \hat{T}_{rot}$ in the five-dimensional shape phase space

$$T_s = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_{k=1}^{3}\frac{Q_k^2}{\sin^2\left(\gamma - \frac{2}{3}\pi k\right)}\right]$$

In the prolate γ-rigid regime defined only by three degrees of freedom, the same operator gets a simpler form

$$T_r = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^2}\frac{\partial}{\partial\beta}\beta^2\frac{\partial}{\partial\beta} - \frac{\mathbf{Q}^2}{3\beta^2}\right]$$

The interplay between γ-stable and γ-rigid collective motion is achieved by considering the Hamiltonian:

$$H = \chi T_r + (1-\chi)T_s + V(\beta,\gamma), \quad 0 \leqslant \chi < 1$$

☞ **rigidity measure**

Budaca & Budaca JPG **42** (2015) 085103

β variable is separated from the γ-angular ones if the potential have the structure

$$v(\beta,\gamma) = \frac{2B}{\hbar^2}V(\beta,\gamma) = u(\beta) + (1-\chi)\frac{u(\gamma)}{\beta^2}$$

Factorizing the total wave function as $\Psi(\beta,\gamma,\Omega)=\xi(\beta)\varphi(\gamma,\Omega)$, the associated Schrödinger equation is separated in two parts:

$$\left[(1-\chi)\left(-\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}+\sum_{k=1}^{3}\frac{Q_k^2}{4\sin^2\left(\gamma-\frac{2}{3}\pi k\right)}+u(\gamma)\right)+\frac{\chi}{3}\mathbf{Q}^2\right]\varphi(\gamma,\Omega)=W\varphi(\gamma,\Omega)$$

Small angle approximation $\Rightarrow\quad u(\gamma)=(3a)^2\frac{\gamma^2}{2}$ $\Downarrow$

$$W=3a(1-\chi)(n_\gamma+1)+\frac{L(L+1)-(1-\chi)K^2}{3},\quad \varphi(\gamma,\Omega)=\eta(\gamma)D^L_{MK}(\Omega)$$

$$\eta_{n_\gamma,|K|}(\gamma)=N_{n,|K|}\gamma^{|K/2|}\exp\left(-3a\frac{\gamma^2}{2}\right)L_n^{|K/2|}(3a\gamma^2),\quad n=\frac{1}{2}\left(n_\gamma-\left|\frac{K}{2}\right|\right)$$

$$\left[-\frac{\partial^2}{\partial\beta^2}-\frac{2(2-\chi)}{\beta}\frac{\partial}{\partial\beta}+\frac{W}{\beta^2}+u(\beta)\right]\xi(\beta)=\epsilon\xi(\beta),\quad \text{Flat potential}\ \ u(\beta)=\begin{cases}0, & \beta\leqslant\beta_W,\\ \infty, & \beta>\beta_W.\end{cases}$$

$$\Longrightarrow\qquad \epsilon_{L,K,s,n_\gamma}(\beta_W)=\left(\frac{x_{s,\nu}}{\beta_W}\right)^2,\quad \xi_{L,K,s,n_\gamma}(\beta)=N_{s,\nu}\beta^{\chi-\frac{3}{2}}J_\nu\left(\frac{x_{s,\nu}\beta}{\beta_W}\right)$$

$x_{s,\nu}$ is s-th zero of the Bessel function $J_\nu(x_{s,\nu}\beta/\beta_W)$ and $n_\beta=s-1$.

$$\nu=\left[\frac{L(L+1)-(1-\chi)K^2}{3}+\left(\frac{3}{2}-\chi\right)^2+(1-\chi)3a(n_\gamma+1)\right]^{\frac{1}{2}}.$$

The excitation energy of the whole system in respect to the ground state

$$E_{K,n_\beta,n_\gamma}(L) = \frac{\hbar^2}{2B}\left[\epsilon_{L,K,n_\beta+1,n_\gamma}(\beta_W) - \epsilon_{0,0,1,0}(\beta_W)\right]$$

The full solution after proper normalization and symmetrization reads:

$$\Psi_{LMKn_\beta n_\gamma}(\beta,\gamma,\Omega) = \xi_{L,K,n_\beta,n_\gamma}(\beta)\eta_{n_\gamma,|K|}(\gamma)\sqrt{\frac{2L+1}{16\pi^2(1+\delta_{K,0})}}\left[D^L_{MK}(\Omega) + (-)^L D^L_{M-K}(\Omega)\right]$$

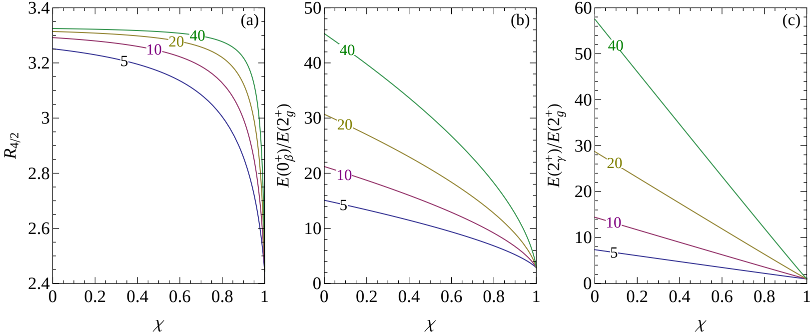
The transition rates are calculated by employing the general expression for the quadrupole transition operator:

$$T^{(E2)}_\mu = t\beta\left[D^2_{\mu 0}(\Omega)\cos\gamma + \frac{1}{\sqrt{2}}\left(D^2_{\mu 2}(\Omega) + D^2_{\mu -2}(\Omega)\right)\sin\gamma\right]$$

The final result for the $E2$ transition probability is given in a factorized form:

$$B(E2; LKn_\beta n_\gamma \to L'K'n'_\beta n'_\gamma) = \frac{5t^2}{16\pi}\left(C^{L\ 2\ L'}_{K K'-K K'} B^{LKn_\beta n_\gamma}_{L'K'n'_\beta n'_\gamma} G^{Kn_\gamma}_{K'n'_\gamma}\right)^2$$

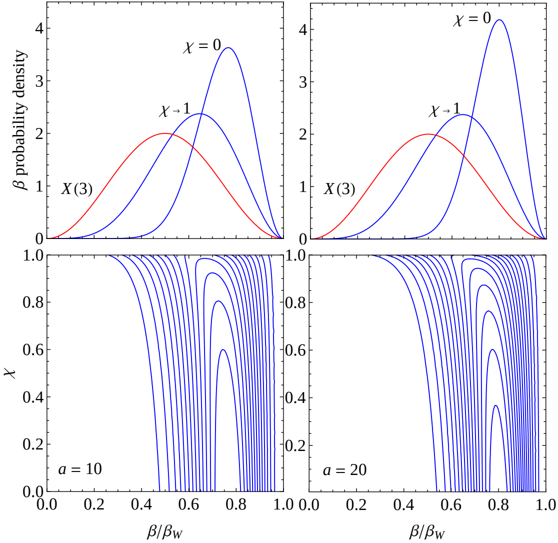
The evolution as function of χ and a of theoretically evaluated spectral observables such as $R_{4/2} = E(4_g^+)/E(2_g^+)$ (a) ratio and the β (b) and γ (c) band heads normalized to the energy of the first excited state.



Bonatsos *et. al.* PLB **649** (2007) 394

$$\begin{array}{ccc} \text{ES-}X(5) & \xrightarrow{\chi} & X(3) \\ \chi = 0 & & \chi = 1 \end{array}$$

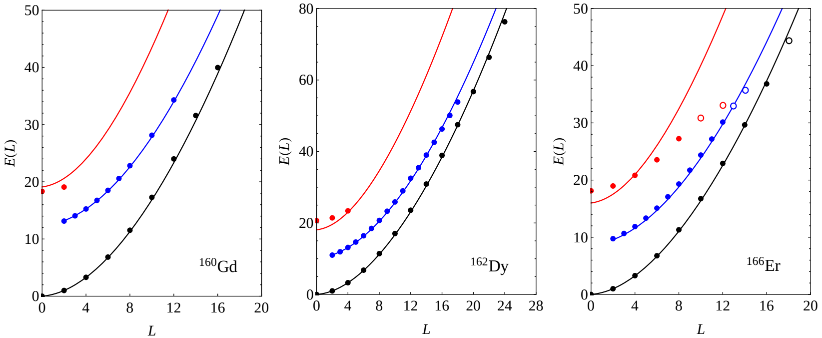
Bonatsos *et. al.* PLB **632** (2006) 238



- The ground state β probability density in respect to the $d\beta$ integration measure.

NUMERICAL APPLICATION

- ● ● ● Experimental ground, γ and β band states
- ○ ○ ○ Experimental ground, γ and β band states with uncertain asignment
- / / / Theoretical ground, γ and β band predictions



χ	0.948	0.269	0.848
a	168.899	10.309	41.191
σ	0.567	0.845	1.359
Nr. of states	19	32	24

Comparison of theoretical results with experiment and rigid rotor (R. R.) predictions for several $E2$ transition probabilities.

Nucleus	$\frac{4_g^+ \to 2_g^+}{2_g^+ \to 0_g^+}$	$\frac{6_g^+ \to 4_g^+}{2_g^+ \to 0_g^+}$	$\frac{8_g^+ \to 6_g^+}{2_g^+ \to 0_g^+}$	$\frac{10_g^+ \to 8_g^+}{2_g^+ \to 0_g^+}$	$\frac{12_g^+ \to 10_g^+}{2_g^+ \to 0_g^+}$	$\frac{14_g^+ \to 12_g^+}{2_g^+ \to 0_g^+}$	$\frac{2_\gamma^+ \to 2_g^+}{2_\gamma^+ \to 0_g^+}$	$\frac{2_\gamma^+ \to 4_g^+}{2_\gamma^+ \to 0_g^+}$
^{160}Gd							1.87(12)	0.189(29)
	1.45	1.62	1.74	1.83	1.90	1.97	1.441	0.073
^{162}Dy	1.42(6)	1.48(9)	1.70(9)	1.72(11)	1.62(20)	1.62(20)	1.78(16)	0.137(12)
	1.45	1.64	1.77	1.87	1.96	2.04	1.444	0.074
^{166}Er	1.44(6)	1.71(10)	1.72(8)	1.80(9)	1.71(10)	1.84(23)	1.86(14)	0.151(10)
	1.45	1.64	1.77	1.87	1.95	2.03	1.445	0.074
R. R.	1.43	1.57	1.65	1.69	1.72	1.74	1.429	0.071

- Very good agreement with experiment for $\Delta K = 0$ transitions.
- The $\Delta K = 2$ experimental transitions rates are slightly underestimated.
- Very weak dependence on the two parameters.
- $\Delta K = 0$ theoretical predictions are overestimated in respect to the R. R., while the $\Delta K = 2$ ones are almost identical.

The last result is related to the quasi-rigid description of the γ degree of freedom.

CONCLUSIONS

- A simple exactly separable model was constructed by taking the kinetic energy of the Bohr Hamiltonian as a combination of prolate γ-rigid and γ-stable rotation-vibration kinetic operators.
- The relative weight of these two components is managed through a so called rigidity parameter which bridges the $X(3)$ γ-rigid solution to its γ-stable counterpart represented by ES-$X(5)$ model when an infinite square well potential in β is adopted.
- The best experimental realization of the model is found in few rare earth nuclei around $N = 96$, namely ^{160}Gd and ^{162}Dy, while ^{166}Er is another suitable candidate.

The proposed hybrid formalism unveils

alternative features of the collective motion in the vicinity of the critical point of a spherical to deformed shape phase transition.