

Dense-Matter physics and Neutron-Star structure

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Overview

1. Introduction to Neutron Stars

2. Confronting open questions

3. Current Study and Methodology

3.1 EoS with local maxima in speed of sound

3.2 EoS with without local maxima in speed of sound

4. Conclusions - Next steps

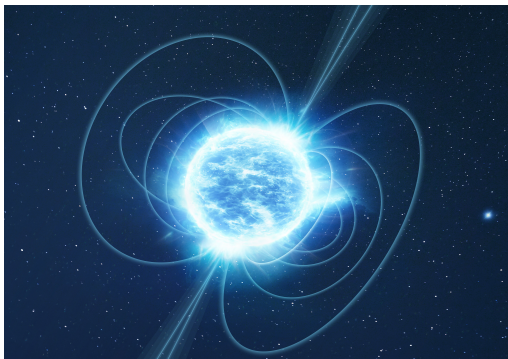
5. Citations and References

6. Appendix

Introduction to Neutron Stars

Formation of Neutron Stars

- **Neutron stars (NS)** are remnants of massive stars ($M > 8 \cdot M_{\odot}$) produced after a gravitational collapse followed by a **type II Supernova**
- Stabilized by pressure against gravity ← highly **degenerate matter** together with **strong-interaction** contributions.



Physical properties

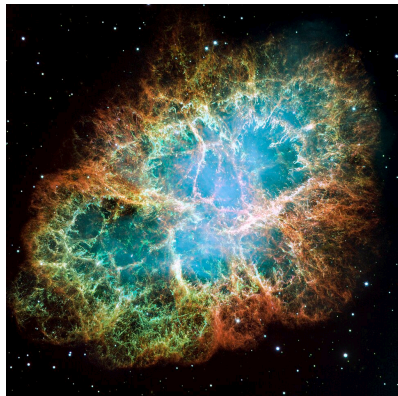
Order of magnitude

1. Mass $\sim 1.5 M_{\odot}$
2. Radius $\sim 10 \text{ km}$
3. Central density $\rho \geq \rho_0^a$
4. Temperature $< 100 \text{ MeV}$

1 teaspoon of NS matter is equivalent to
900 times the mass of Giza pyramid.

$$^a \rho_0 = 2.3 \cdot 10^{17} \frac{\text{kg}}{\text{m}^3}$$

Crab Nebula - NS remnant



Constellation in Taurus

RA: $05^{\text{h}} 34^{\text{m}} 31.8^{\text{s}}$

Dec: $+22^{\circ} 01' 03''$

Neutron Stars as relativistic objects

- Neutron stars are **relativistic** objects due to high compactness, $C = \frac{M}{R}$.
- Newtonian Physics → General Relativity

Neutron Stars as relativistic objects

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- Newtonian Physics $\rightarrow$ General Relativity
- We assume a **spherically symmetric, static compact object**¹. Einstein's field equations then reduce to two coupled equations $\rightarrow$ 2 equations (**TOV**)

$$1. \quad \frac{dm}{dr} = 4\pi r^2 \epsilon(r)$$

$$2. \quad \frac{dP}{dr} = -(\epsilon + P) \cdot \frac{m(r) + 4\pi r^3 P(r)}{r[r - 2m(r)]}$$

- $\epsilon = \frac{E}{V} = \rho c^2 + u$, where u is the internal energy density

¹in a asymptotically flat spacetime

Neutron Stars as relativistic objects

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The system of equations is not closed. Relation between pressure and energy density

Equation of State (EoS) $P(\epsilon)$

Three equations describing NS

TOV

$$(1) \frac{dm}{dr} = 4\pi r^2 \epsilon(r) \quad (2) \frac{dP}{dr} = -(\epsilon + P) \frac{m(r) + 4\pi r^3 P(r)}{r[r - 2m(r)]}$$

EoS

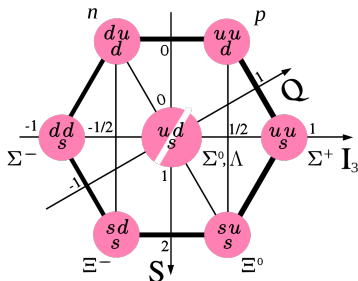
$$(3) P(\epsilon)$$

If **EoS** and central ϵ_c are defined $\rightarrow$ **Stellar parameter**.

$$P(\epsilon) \longrightarrow \boxed{\text{TOV}} \longrightarrow M, R$$

Two main problems

- The EoS, $P(\epsilon)$, in the presence of **strong interactions** and **extreme densities** has not been derived completely from first principles to this day $\rightarrow$ remains **unknown**.
- The number baryon density is so large that new particles might be produced, e.g. **hyperons**.



Two main problems

- The EoS, $P(\epsilon)$, in the presence of **strong interactions** and **extreme densities** has not been derived completely from first principles to this day → remains **unknown**.
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Questions

1. Is it possible by measuring astrophysical parameters to get information about the inner structure of NS?
2. Do we see a difference in NSs if they contain hyperons?

Confronting open questions

1

Introduction to
Neutron Stars

2

Confronting
open questions

3

Current Study
and Methodology

4

Conclusions
- Next steps

5

Citations and
References

6

Appendix

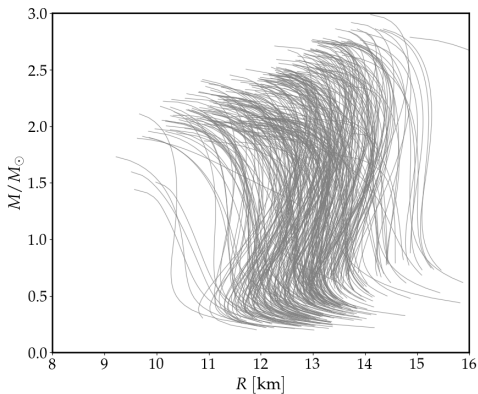
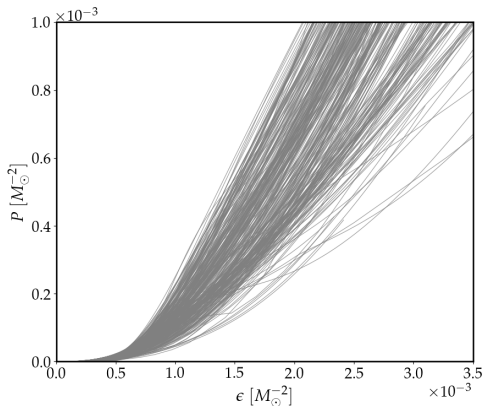


Process

- In order to address these questions we used a sample of **307 EoS** models, including hyperonic and nucleonic EoS.

Process

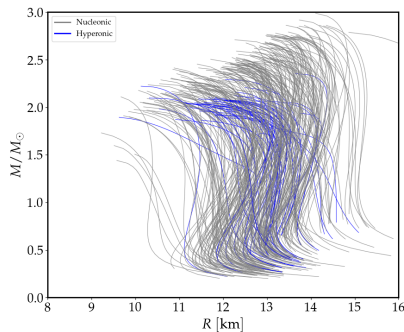
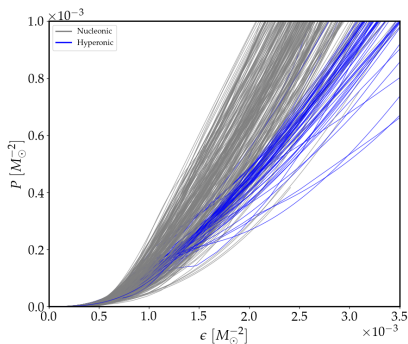
- In order to address these questions we used a sample of **307 EoS** models, including hyperonic and nucleonic EoS.
- By solving the TOV equations for each EoS $\rightarrow$ M-R curve.



Process

1. When **hyperons** are produced new degrees of freedom appear ; leading to a slower increase in $P(\epsilon)$.
2. **Nucleonic interactions** at high density could cause a slower increase in pressure

This makes the NS more **compressible**

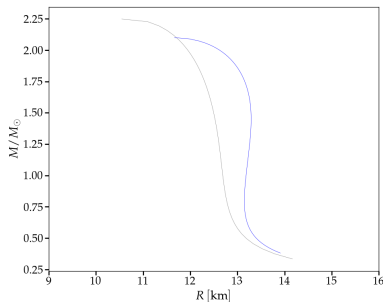


Softening of EoS

1. When **hyperons** are produced new degrees of freedom appear ; leading to a slower increase in $P(\epsilon)$.
2. **Nucleonic interactions** at high density could cause a slower increase in pressure

This makes the NS more **compressible**

- As a result the curve $M(R)$ bends more prominently → **softening of EoS**



Soft $P(\epsilon)$



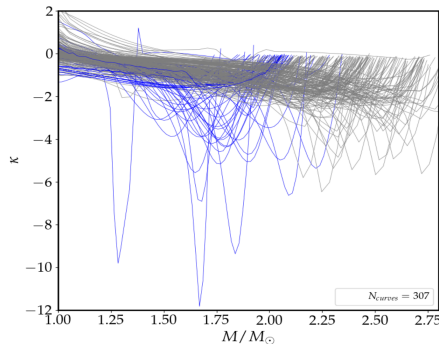
Stiff $P(\epsilon)$

Softening of EoS

- To this day there is no quantitative measure known to **discriminate** hyperonic EoSs from purely nucleonic EoSs
- One parameter to describe the bending of $M(R)$ is the **curvature radius** of the mass radius relation $M(R)$.

Therefore the softening of EoS can be quantified by the value of κ . This is the main idea of the paper **arxiv 2507.10372**

$$\kappa = \frac{\frac{d^2 R}{dM^2}}{\left[1 + \left(\frac{dR}{dM}\right)^2\right]^{3/2}} \quad (1)$$



Softening of EoS

One parameter to describe the bending of $M(R)$ is the **curvature radius** of the function $R(M)$. This is the main idea of the paper **arxiv 2507.10372**

Stellar properties indicating the presence of hyperons in neutron stars

Andreas Bauswein^{1,2}, Aristeidis Nikolaïdis^{1,3}, Georgios Lioutas⁴, Hristijan Kochankovski⁵, Prasanta Char^{6,7}, Chiranjib Mondal⁸, Micaela Oertel^{9,10}, Laura Tolos^{11,12}, Nicolas Chamel⁴, and Stephane Goriely⁴

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¹²Institute of Space Sciences (ICE, CSIC), Campus UAB, Carrer de Can Magrans, 08193 Barcelona, Spain

¹³Institut d'Estudis Espacials de Catalunya (IEEC), 08860 Castelldefels (Barcelona), Spain

(Dated: January 28, 2026)

We describe distinctive stellar features indicating the presence of hyperons in neutron stars as compared to purely nucleonic systems. A strongly negative curvature of the mass-radius relation $R(M)$ is characteristic of hyperons, which can be determined from measurements of neutron stars with three different masses. Similarly, a reduced second derivative of the tidal deformability as function of mass $\lambda(M)$ points to hyperonic degrees of freedom in NS matter. The slopes of such curves $R(M)$ and $\lambda(M)$ can distinguish a hyperonic equation of state from purely nucleonic models if they appear increased (decreased for $\lambda(M)$) relative to the maximum mass of neutron stars.

INTRODUCTION

Non-nucleonic degrees of freedom such as hyperons are widely discussed as possible components of neutron star (NS) matter. Since the constituents of high-density matter and their interactions are not precisely known, the equation of state (EoS) of NSs is subject to uncertainties [1–7]. The EoS of cold NSs in beta equilibrium is a

result in a reduction of NS radii and the maximum mass of NSs [6, 17–23]. However, to the best of our knowledge there is to date no clear and distinctive *quantitative* measure known based on which a discrimination between purely nucleonic and hyperon-admixed matter in NSs is possible employing measurements of stellar structure properties. Only a statistical approach considering the slope of the mass-radius relation has been discussed

*“A **strongly negative curvature radius** of the mass-radius relation $R(M)$ is characteristic of **hyperons**, which can be determined from measurements of neutron stars with three different masses.”*

Motivation for Further Research

This paper does not correlate directly the curvature radius with microphysical variables of EoS.

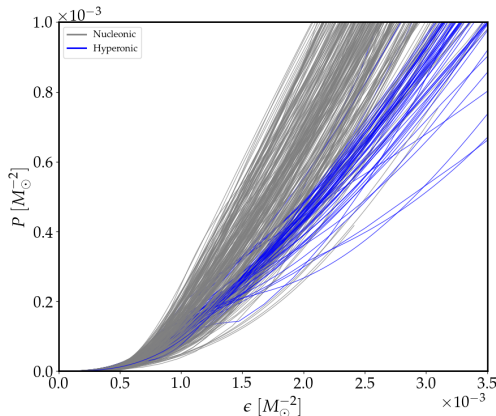
This is attempted during this summer program

Current Study and Methodology

Extension of past literature

GOAL: Correlate bending of $M(R)$ with the softening of EoS

- Quantify the softening in the **EoS**.
- Use the derivative of $P(\epsilon) \rightarrow$ assess the behavior of EoS



Speed of sound

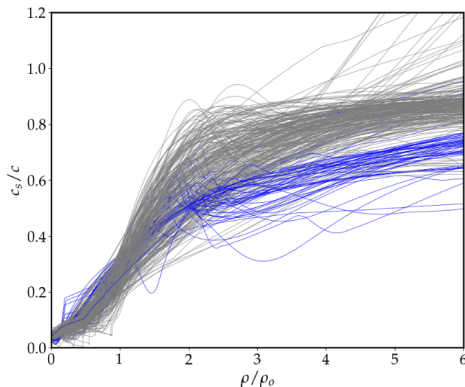
GOAL: Correlate bending of $M(R)$ with the softening of EoS

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Speed of sound c_s

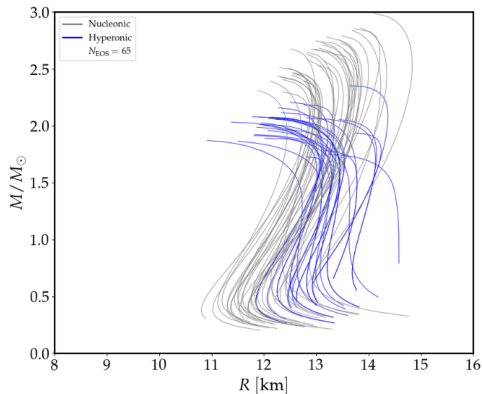
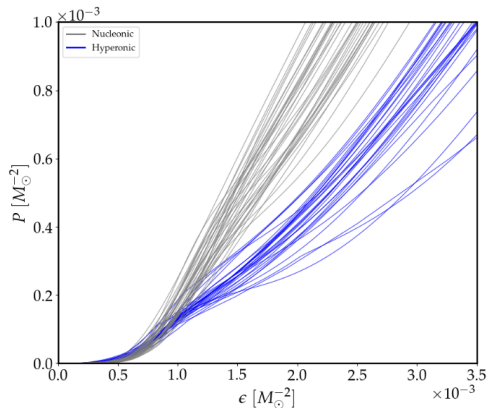
$$c_s^2 = \frac{dP}{d\epsilon} \quad (2)$$

Speed of Sound c_s



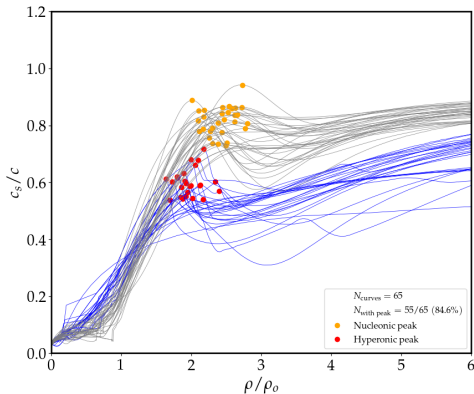
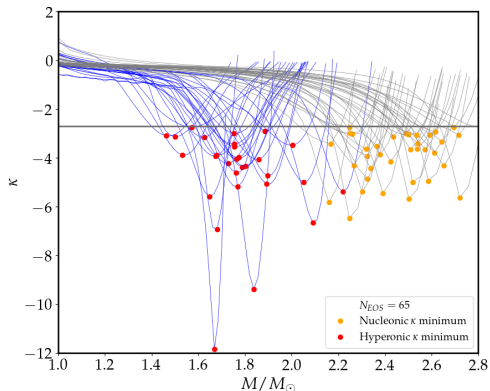
Reducing the sample of EoS

- We are interested in a **strong curvature radius**; we set a limit at κ below -2.7
- Thus from **307 EoS** $\rightarrow$ **65 EoS**

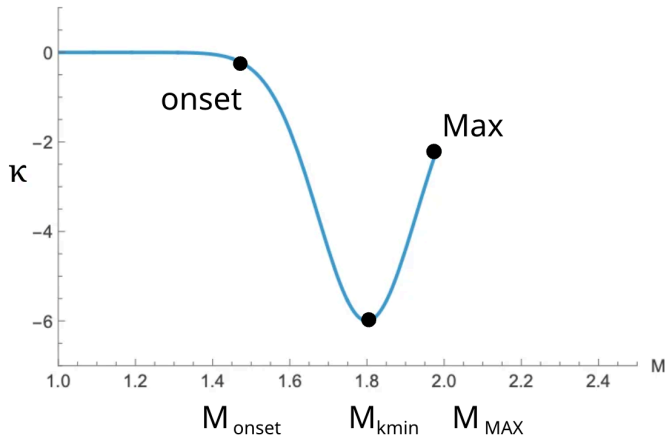


Reducing the sample of EoS - Cuts

- “Hyperons exhibit a **stronger curvature radius** in the κ at **lower masses** compared to nucleons”, arxiv 2507.10372
- The **peak** in speed of sound indicates the **softening** in the EoS



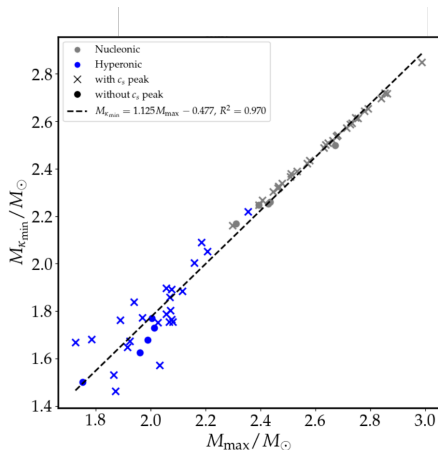
Important points in κ



- **Onset point:** $\propto$ minimum of the $\kappa(M)$ second derivative.

Estimation for the maximum mass of NSs

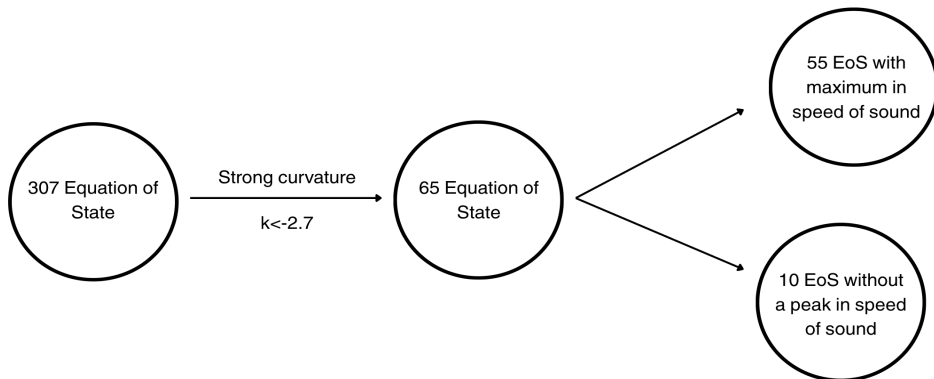
- $M_{\kappa_{min}} = 1.125M_{max} - 0.477M_{\odot}, R^2 = 0.97$



- Given sufficient measurements (M,R) to identify $M_{\kappa_{min}} \rightarrow$ estimate the M_{max} of the NS.

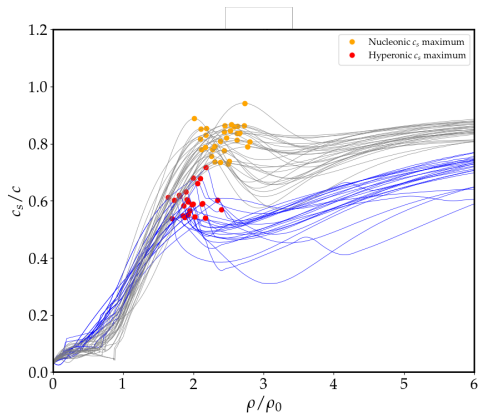
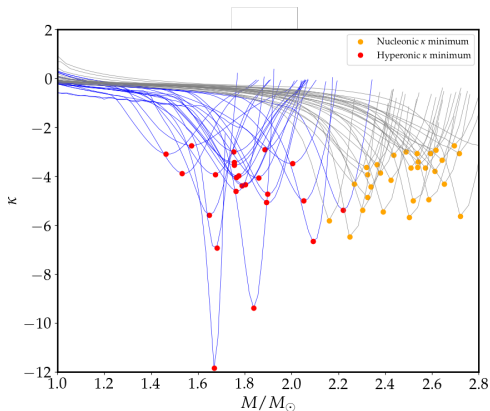
Outline

- Study both c_s **with** and **without** maximum.



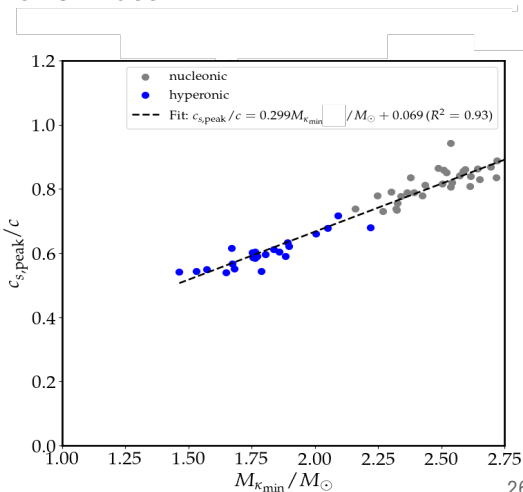
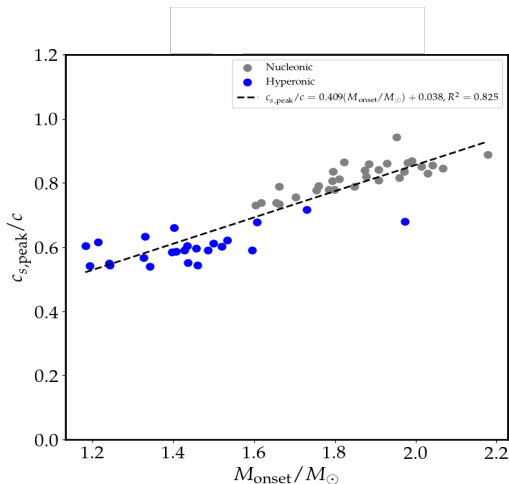
κ and c_s plots for EoS with peak (55)

- EoS with $\kappa < -2.7$ and local maximum in c_s



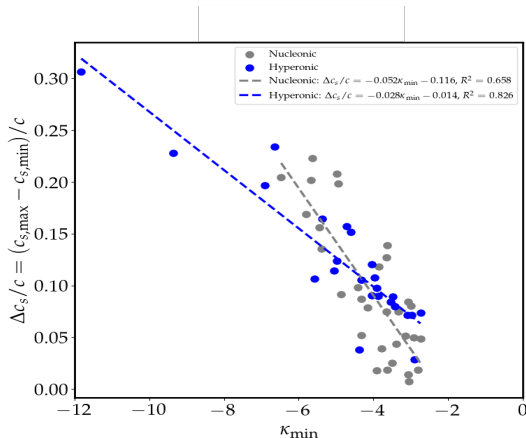
Linear relations between c_s and mass

- The peak of c_s correlates with the $M_{\text{onset}} \rightarrow$ **softening begins** $R^2 = 0.825$
- Hyperonic EoS express that feature at **lower mass**



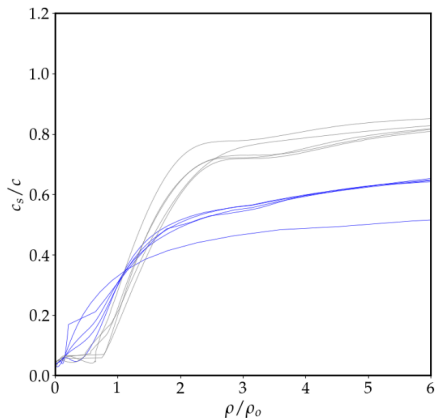
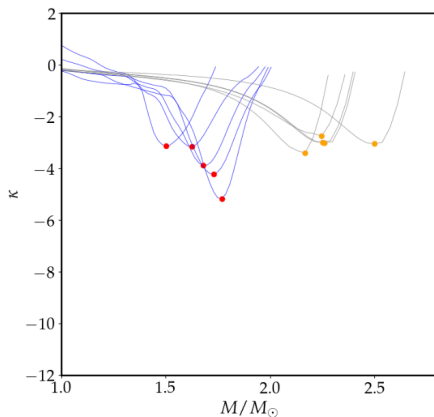
Linear relations between c_s and mass

- The peak of c_s correlates with the $M_{onset} \rightarrow$ **softening begins** $R^2 = 0.825$
- The **drop** in the c_s , Δc_s , correlates with the value of k_{min}



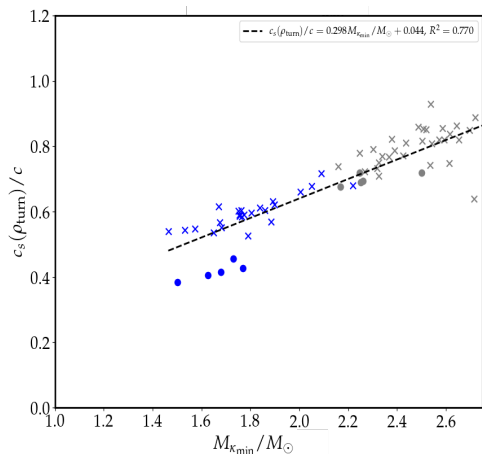
κ and c_s plots for EoS without peak c_s (10)

- Guess of no peak $\rightarrow$ low-density behavior
- We try to unify all 65 EoS



Linear relations between c_s and mass

Universal criterion for all 65 EoS



- Instead of peak in c_s we use the point at which the **second derivative of c_s is minimized**.
- **Linearity:** $\frac{c_s}{c} = 0.29 \frac{M_{\kappa_{\text{min}}}}{M_{\odot}} + 0.044$, $R^2 = 0.77$

Conclusions - Next steps

Conclusions

1. An estimation for the maximum mass of NS can be made, given a **sufficient set of astrophysical measurements** $S = \{(M_i, R_i)\}_{i=1}^N$ and a **strong curvature radius**.
2. Correlation between the **local maximum** c_s and **M at $\kappa_{\min}$** was established $R^2 = 0.93$.
3. The **drop** in the c_s also correlates with the value of $k_{\min}$, **bending of $M(R)$** ².
4. These features appear in hyperonic EoS at lower mass.

²if the EoS presents a local maximum in the speed of sound

Next Steps

1. Investigate why some EoSs with strong negative curvature exhibit a local maximum in c_s , while others do not.
2. Find alternative definitions of the onset point in $\kappa(M)$.
3. Investigate possible relations concerning the radius of NSs.

Thank you for your attention

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Citations and References

References I



Bauswein, A., Nikolaidis, A., Lioutas, G., Kochankovski, H., Char, P., Mondal, C., Oertel, M., Tolos, L., Chamel, N., & Goriely, S. (2026). **Stellar properties indicating the presence of hyperons in neutron stars.** Physical Review Research, 8(1), 013253. <https://doi.org/10.1103/ygtr-ktqk>

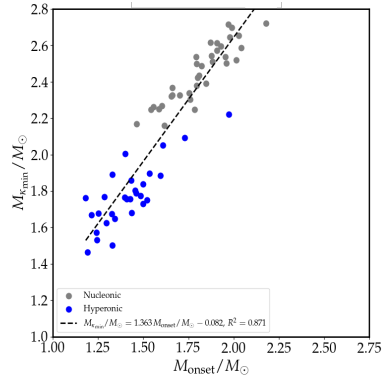
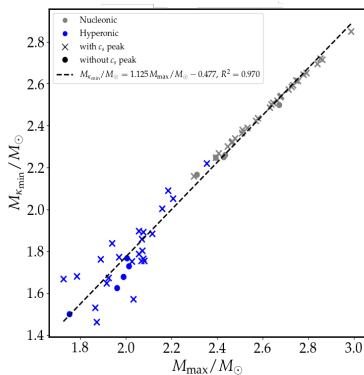
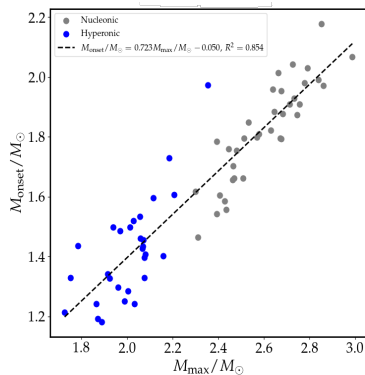


Schaffner-Bielich, J. (2020). Compact star physics. Cambridge University Press. <https://doi.org/10.1017/9781316848357>

Appendix

Linear relation between characteristic masses

Linear relations between masses were discovered, $R_1^2 = 0.854$, $R_2^2 = 0.97$, $R_3^2 = 0.871$



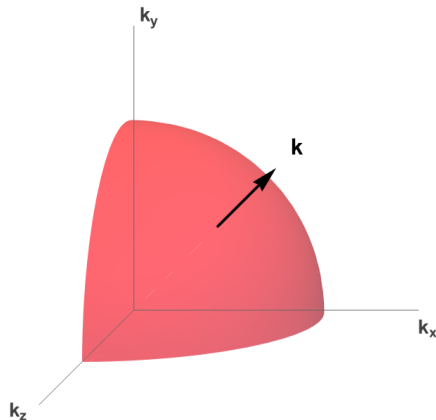
Appendix A: Number density of states

The chemical potential of a particle state is given by $\mu_i = E_{F,i} = \sqrt{k_{F,i}^2 + m_i^2}$. For a group of particles occupying a certain region there is a direct relation between number density and energy states, based on the phase space $k^i = \{k_x^i, k_y^i, k_z^i\}$.

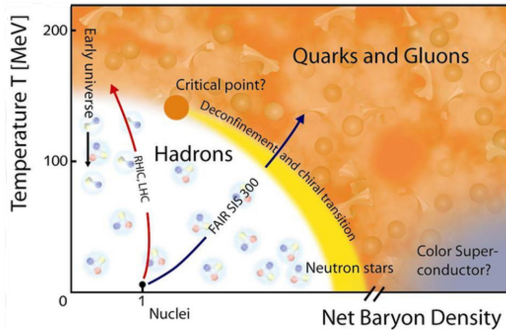
$$dN = \frac{1}{8} \frac{g_i}{(\pi)^3} 4\pi k^2 dk \Rightarrow dN = \frac{g_i}{2\pi^2} V k^2 dk \Rightarrow$$

$$n = \frac{N}{V} = \int_0^{k_F^i} \frac{g_i}{2\pi^2} k^2 dk \Rightarrow$$

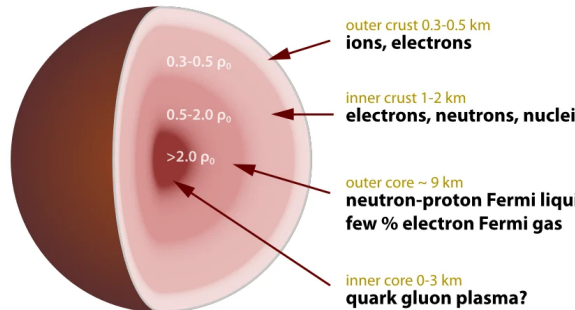
$$n_i = \frac{g_i}{6\pi^2} k_{F,i}^3$$



QCD Phase Diagram



Section of Neutron Star



Outer crust: Gas of electrons

Inner crust: Liquid neutrons and electrons

Outer core: Up to this point the EOS are known, densities $\rho \sim 1\rho_0$

Inner core: This region covers most of the NS, Hyperons maybe present

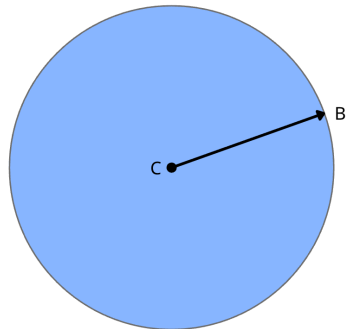
Solving the EoS

The process is straightforward. Boundary conditions are defined accordingly to star properties and a supercomputer is used in order to solve numerically the TOV equations.

Boundary conditions

At the center of NS :

$m(0) = 0$, $\epsilon(0) = \epsilon_c$, $P(0) = P(\epsilon_c) = P_c$. By numerical integration till point B where $P(R) = 0$. This procedure is done for one $\epsilon_c \rightarrow \{M, R\}$, for one EoS. The outcome if we try with different ϵ_c is the M-R curve.



This is a Neutron star

Solving the EoS

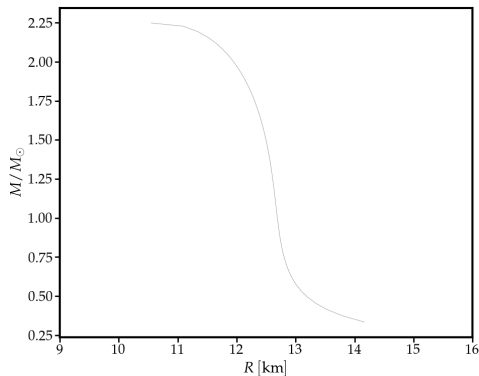
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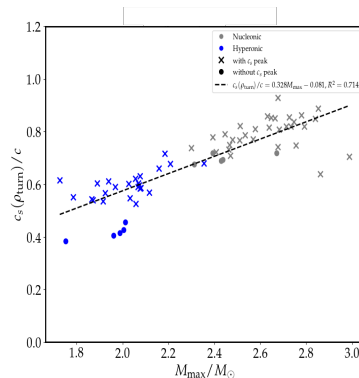
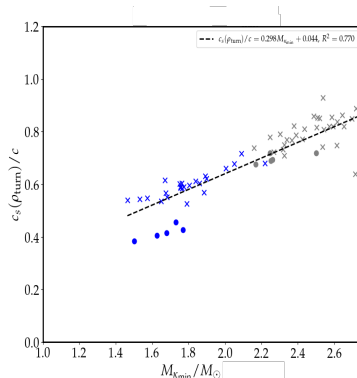
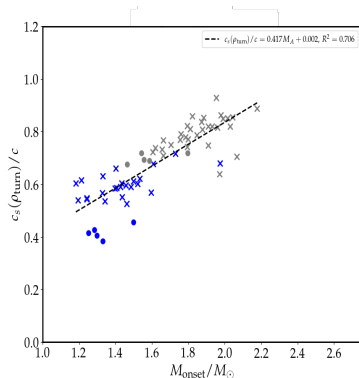
By numerical integration till point B where $P(R) = 0$. This procedure is done for one $\epsilon_c \rightarrow \{M, R\}$, for one EoS. The outcome if we try with different ϵ_c is the M-R curve.



Each point is a Neutron Star $\rightarrow \{M, R\}$

Linear relations between c_s and mass

Universal criterion for all 65 EoS



Second derivative of κ

Second derivative of κ . Let's assume that $\kappa = -6 \cdot e^{-(M-1.8)^2/(0.18^2)}$

