Equation of State for Astrophysical Applications

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Outline

- **Introduction**
  Astrophysics and EoS, Thermodynamic Conditions, Objectives, Nuclear and Stellar Matter, Constraints, Models of Dense Matter, Correlations
- **Generalized Relativistic Density Functional**
  Details of gRDF Model, Effective Interaction, Degeneracy Factors of Nuclei, Mass Shifts, Particle Fractions, Low-Density Limit, Neutron Matter, Chemical Composition of Stellar Matter
- **Conclusions**

Details:

S. Typel, G. Röpke, T. Klähn, D. Blaschke, H.H. Wolter, Phys. Rev. C 81 (2010) 015803
M.D. Voskresenskaya, S. Typel, Nucl. Phys. A 887 (2012) 42
G. Röpke, N.-U. Bastian, D. Blaschke, T. Klähn, S. Typel, H.H. Wolter, Nucl. Phys. A 897 (2013) 70
S. Typel, H.H. Wolter, G. Röpke, D. Blaschke, acc. for publ. in EPJA, arXiv.org:1309.6934 [nucl-th]

Introduction

Astrophysics and Equation of State

- essential ingredient in astrophysical model calculations:

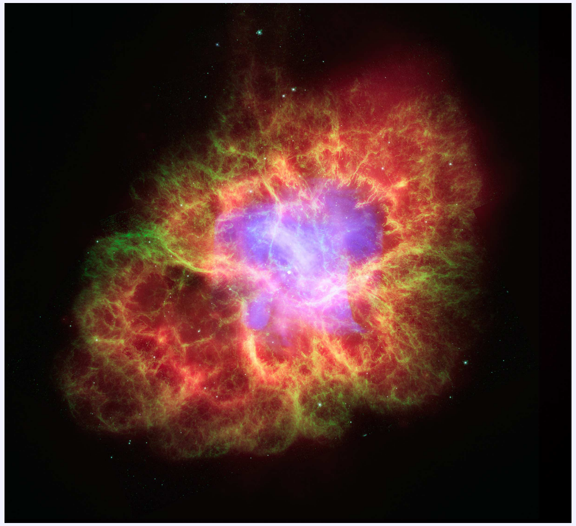
  Equation(s) of State (EoS) of dense matter

  ⇒ dynamical evolution of supernovae

  ⇒ static properties of neutron stars

  ⇒ conditions for nucleosynthesis

  ⇒ energetics, chemical composition, transport properties, . . .



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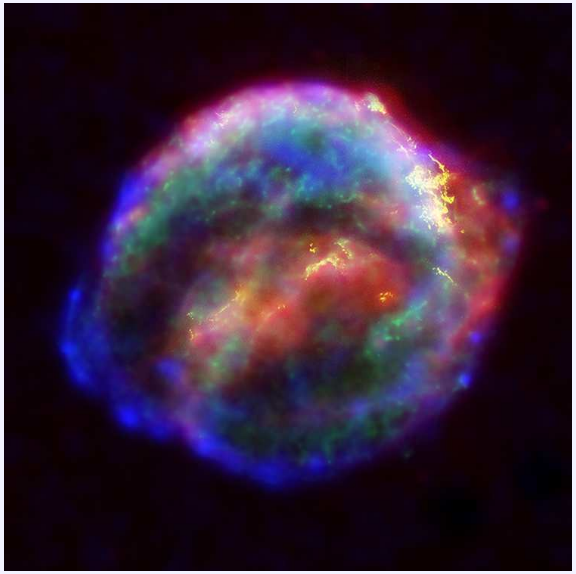

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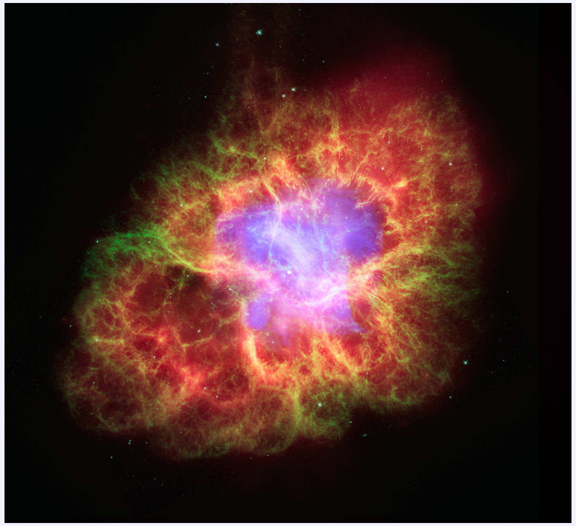
- **timescale of reactions ≪ timescale of system evolution**

  ⇒ equilibrium (thermal, chemical, . . .)

  ⇒ application of **EoS** reasonable



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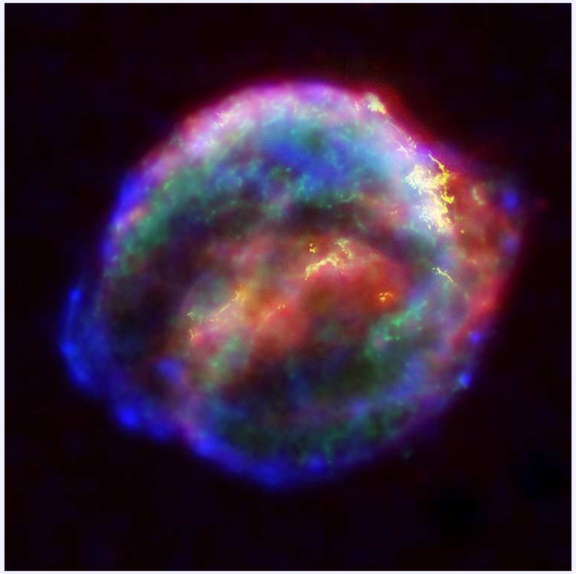

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Typical range of variables:

- density:
  $10^{-9} \lesssim \varrho/\varrho_{\text{sat}} \lesssim 10$
  with nuclear saturation density
  $\varrho_{\text{sat}} \approx 2.5 \cdot 10^{14}$ g/cm^3
  $(n_{\text{sat}} = \varrho_{\text{sat}}/m_n \approx 0.15$ fm$^{-3})$
- temperature:
  0 MeV $\leq k_B T \lesssim 100$ MeV
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- electron fraction:
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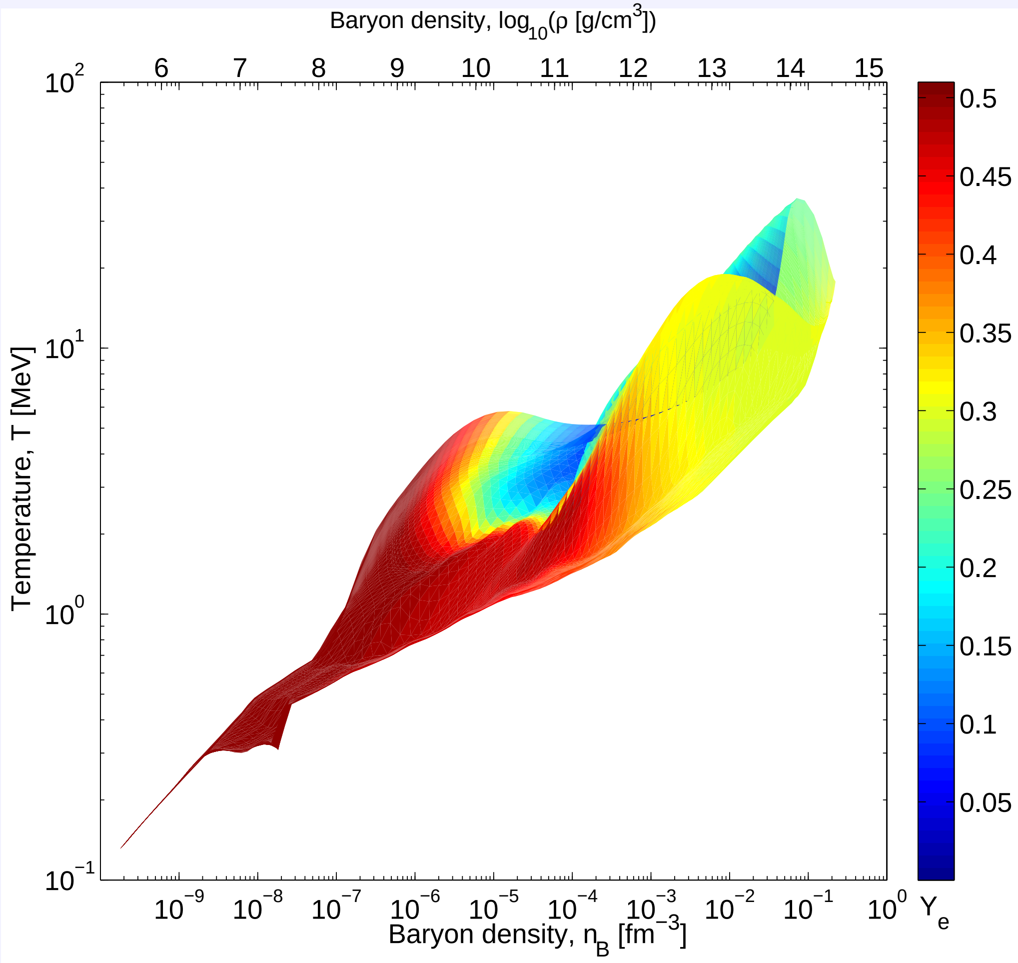
simulation of core-collapse supernova



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global EoS required

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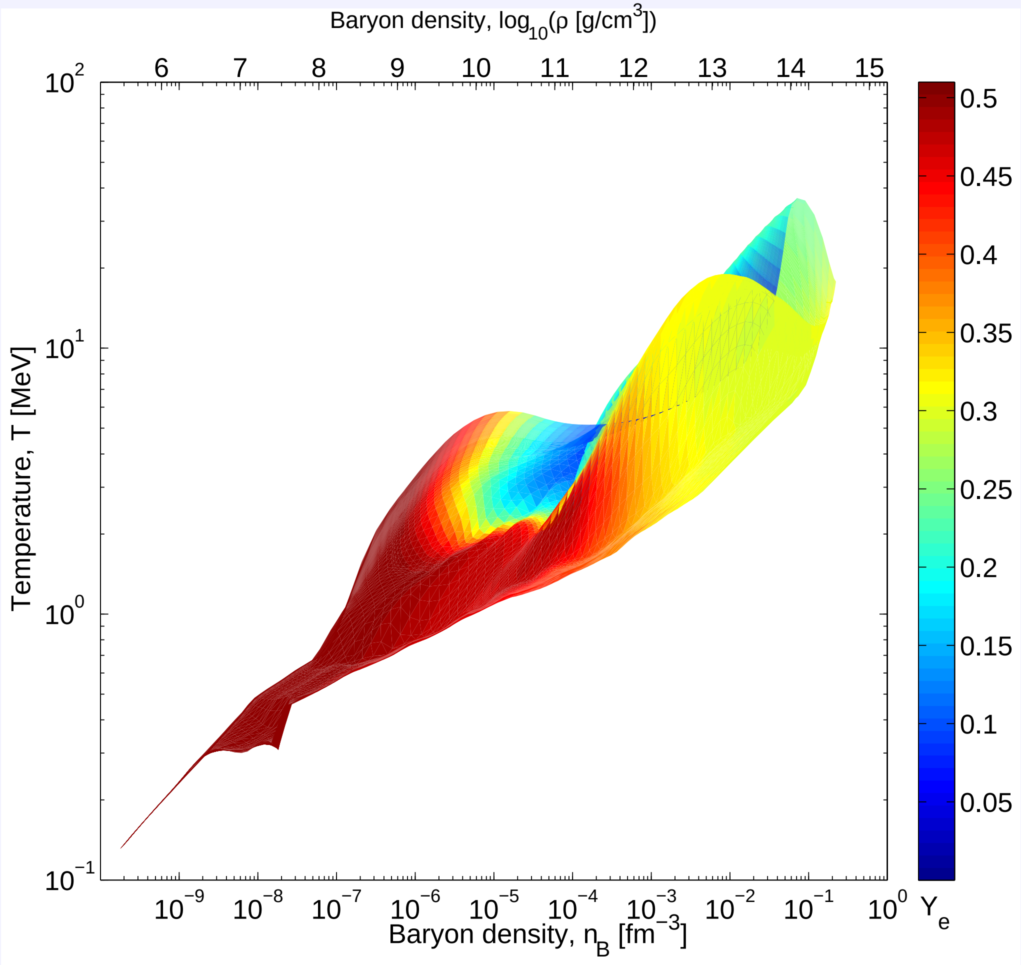

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challenge: covering of full range of variables in a unified model

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  $\Rightarrow$ clustering $\Rightarrow$ liquid-gas phase transition in thermodynamic limit
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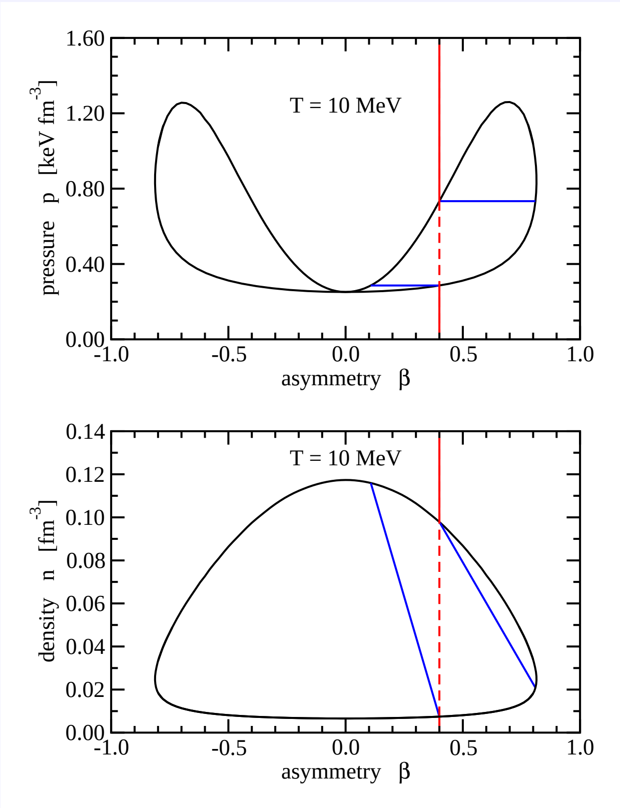

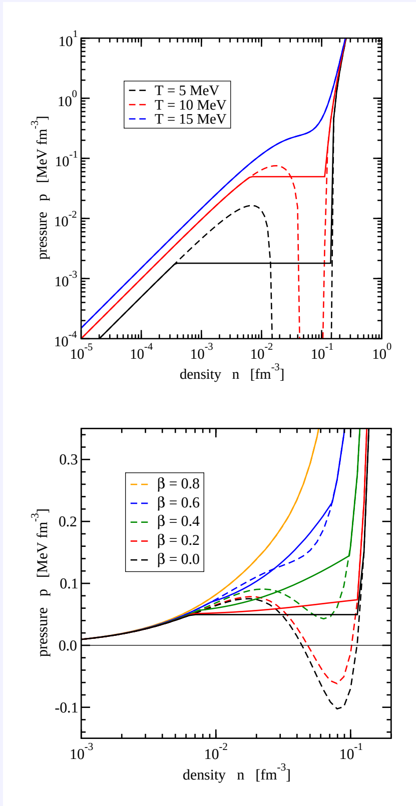

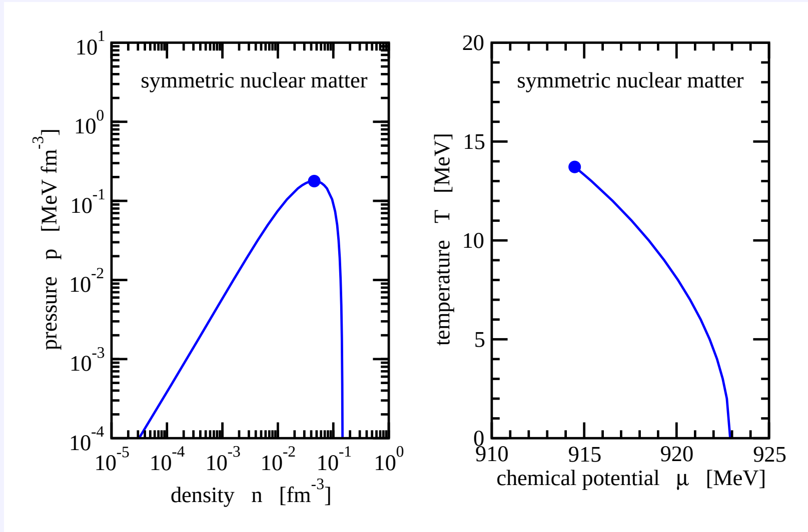

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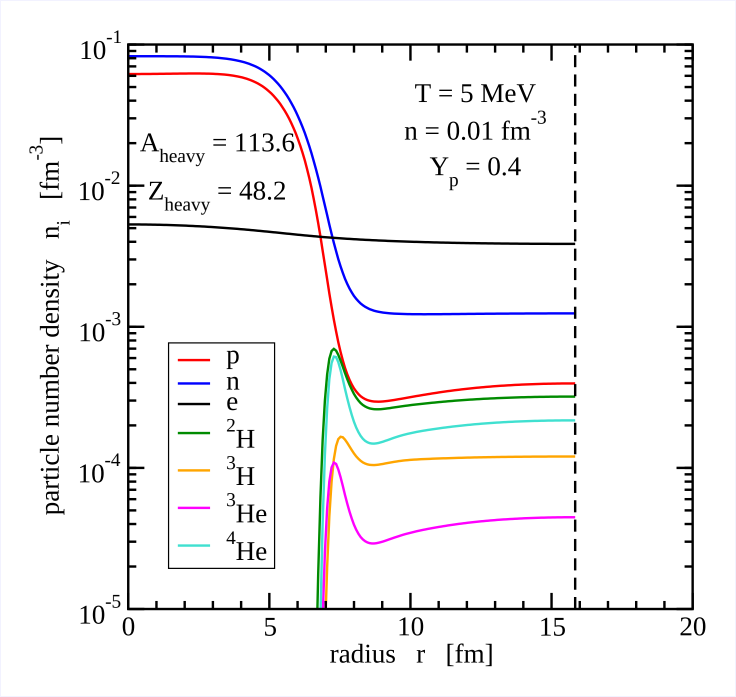

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  - ◦ formation of inhomogeneous matter and finite-size structures
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  - ◦ lattice formation at low temperatures
    ⇒ phase transition: liquid/gas ↔ solid

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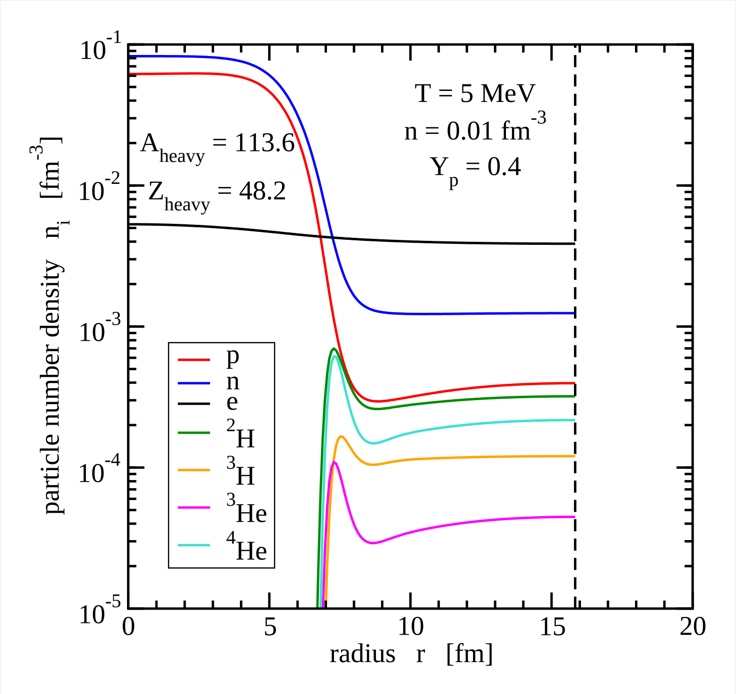

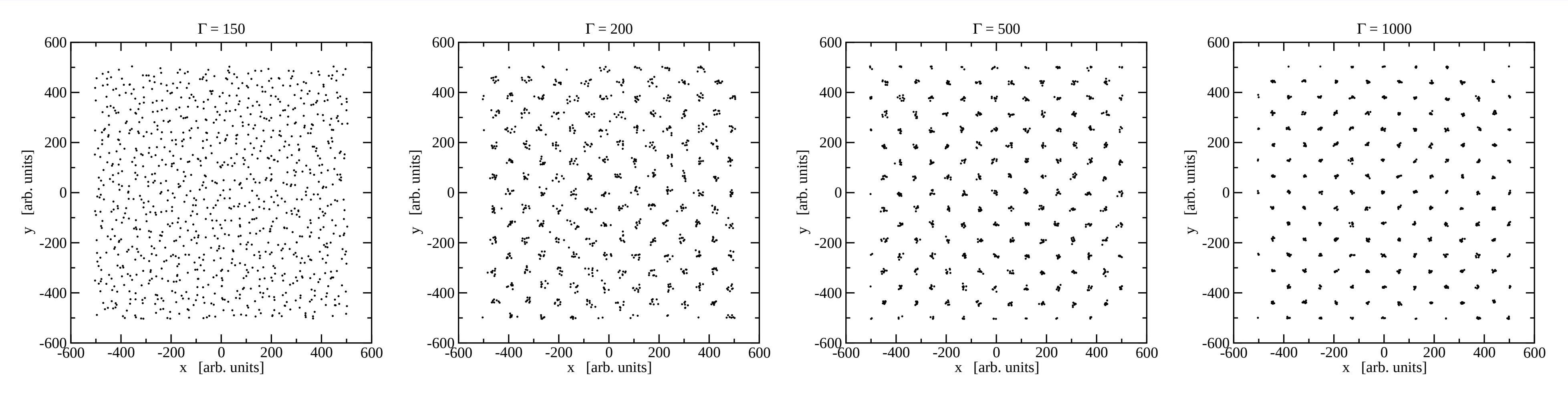

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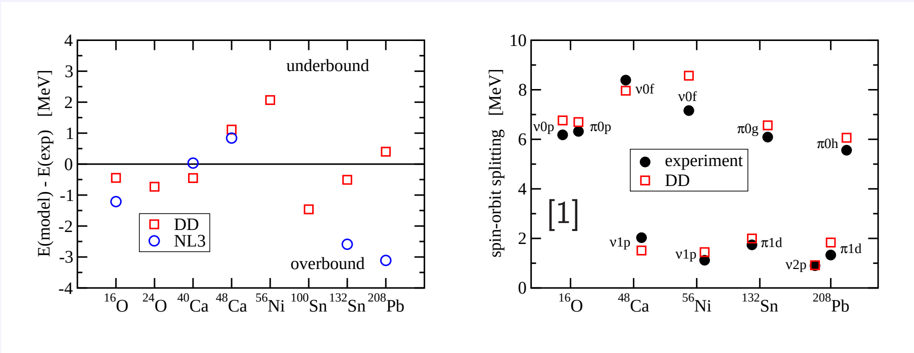

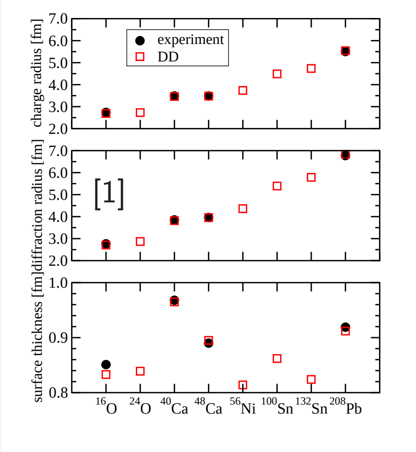

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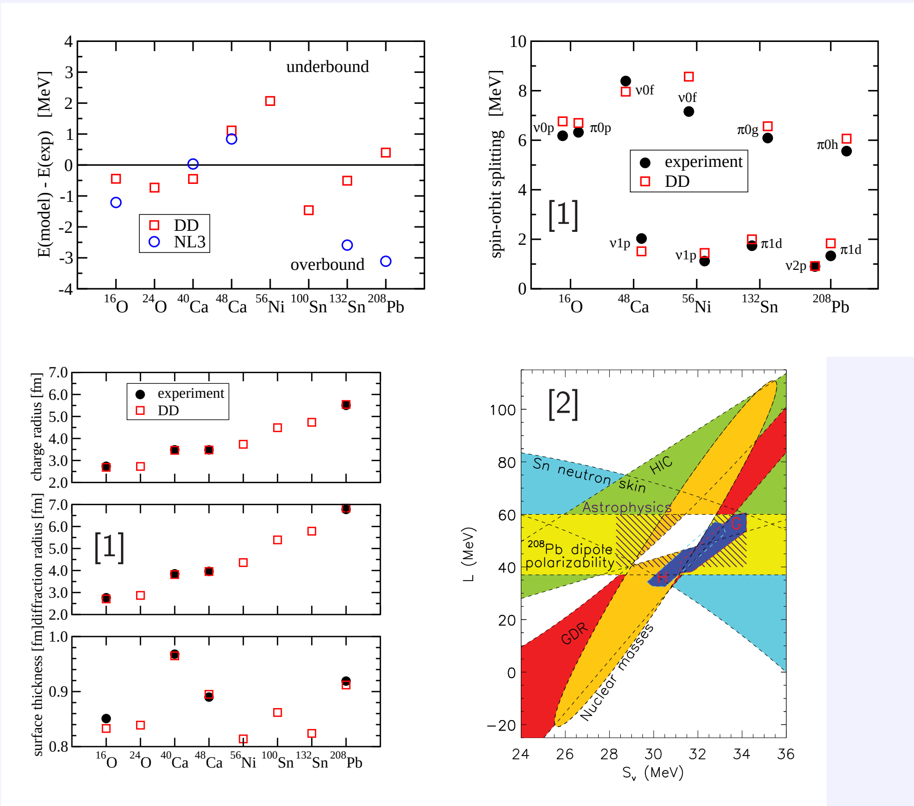

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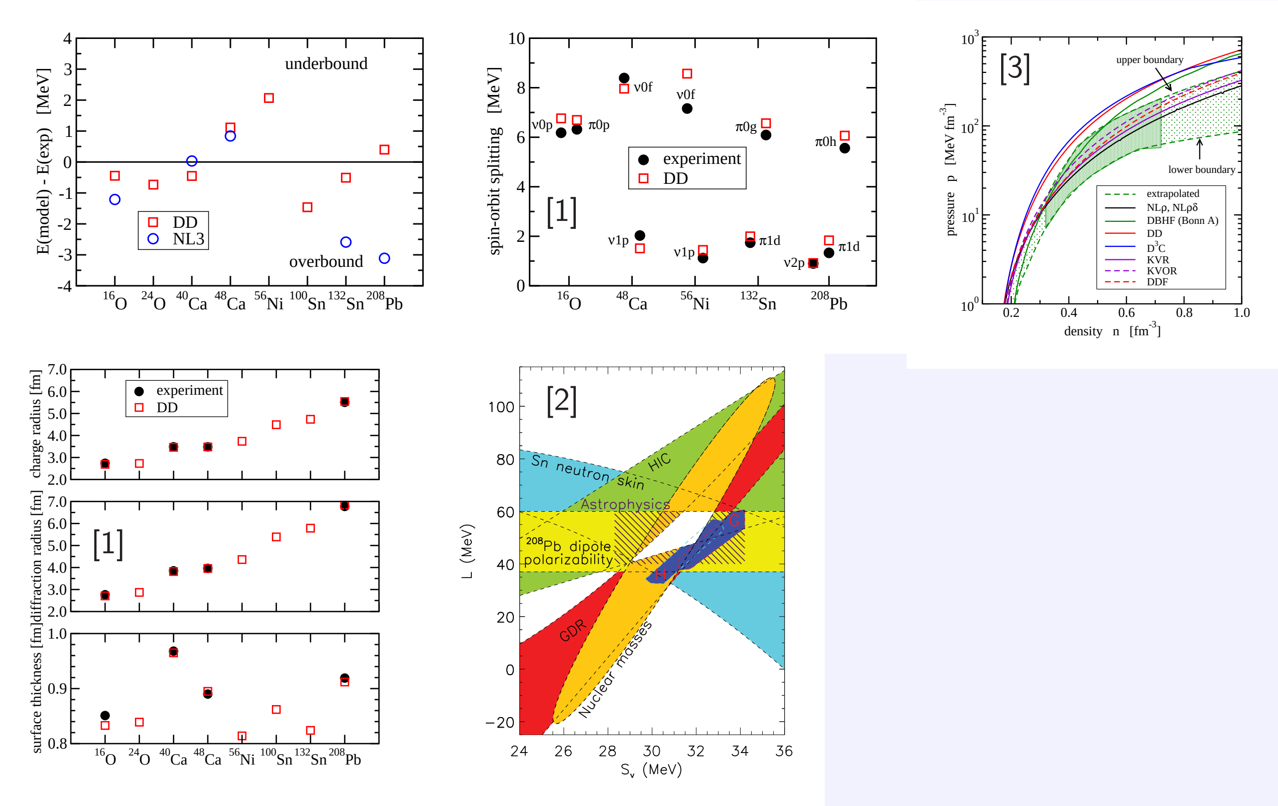

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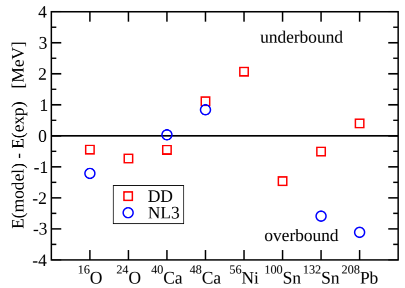
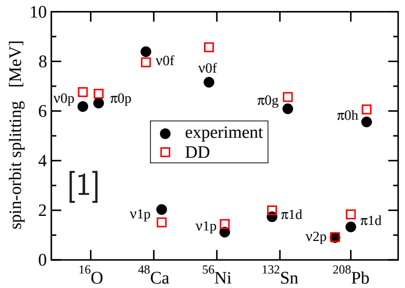
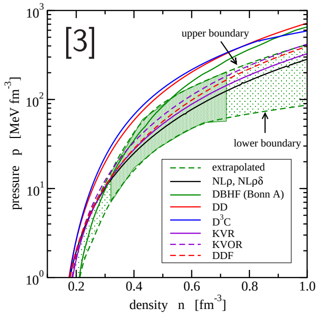
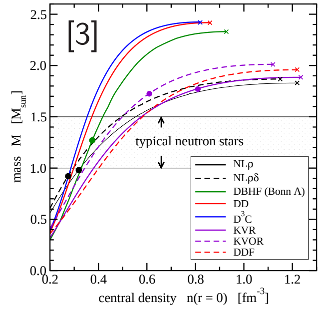
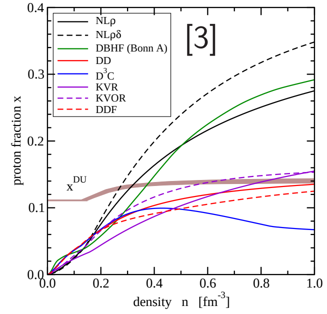
- **astrophysics**
  - compact stars (mass-radius relation, maximum mass, cooling, . . .)

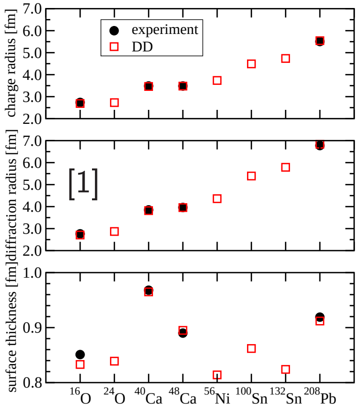


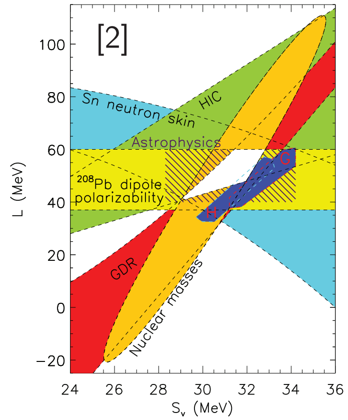


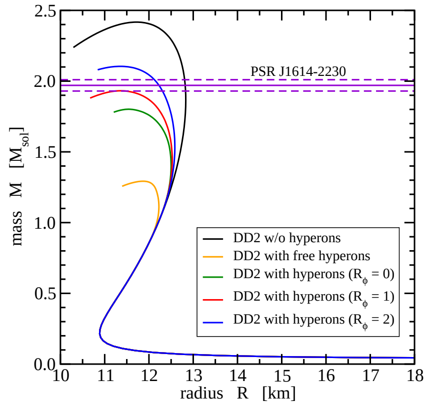












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  - properties of constituents independent of medium
  - interaction between particles ?
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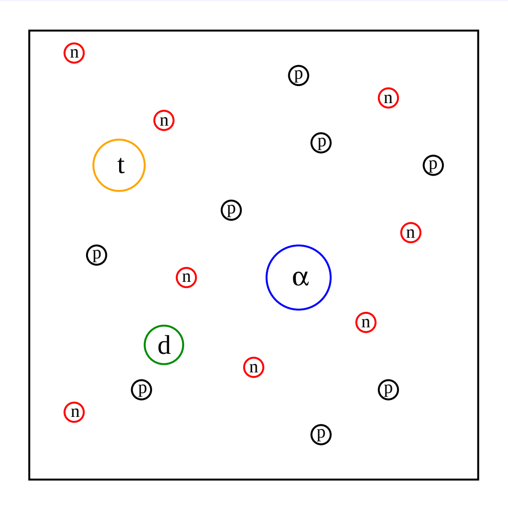

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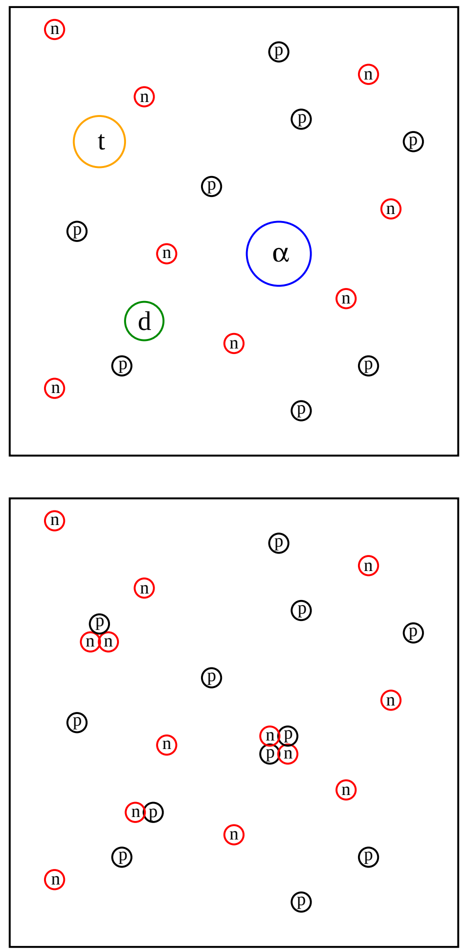
- **physical picture**
  interaction between nucleons ⇒ correlations
  ⇒ formation of bound states/resonances
  - treatment of two-, three-, . . . many-body correlations ?
  - choice of interaction ?

⇒ unified description in a single model ?



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⇒ construction of generalized relativistic density functional with correct limits

Generalized Relativistic Density Functional

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- **constituents of dense matter** (degrees of freedom)
  - baryons (n, p, Λ, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-, ...) $\Rightarrow$ fermions
  - mesons (π^+/π^-, π^0, K^+/K^-, $K^0/\bar{K}^0$, ω, ρ, ...) $\Rightarrow$ bosons
  - light nuclei (^{2}H, ^{3}H, ^{3}He, ^{4}He) $\Rightarrow$ fermions/bosons
  - heavy nuclei ($^{A_i}Z_i$, $A_i > 4$) $\Rightarrow$ classical particles
    - experimental binding energies: AME2012 (M. Wang et al., Chinese Phys. 36 (2012) 1603)
    - extension: DZ10 predictions (J. Duflo and A.P. Zuker, Phys. Rev. C 52 (1995) R23)
  - nucleon-nucleon scattering correlations $\Rightarrow$ classical particles (represented by effective resonances in the continuum)
  - leptons (e^-/e^+, μ^-/μ^+) $\Rightarrow$ fermions
  - photons (γ) $\Rightarrow$ bosons

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  - thermodynamically consistent model
    ($\Rightarrow$ “rearrangement” contributions to vector potential)
  - application to nuclear matter (only hadrons/strong interaction)
    and stellar matter (with leptons/electromagnetic interaction)

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◦ represented by (classical) fields A_m with mass m_m

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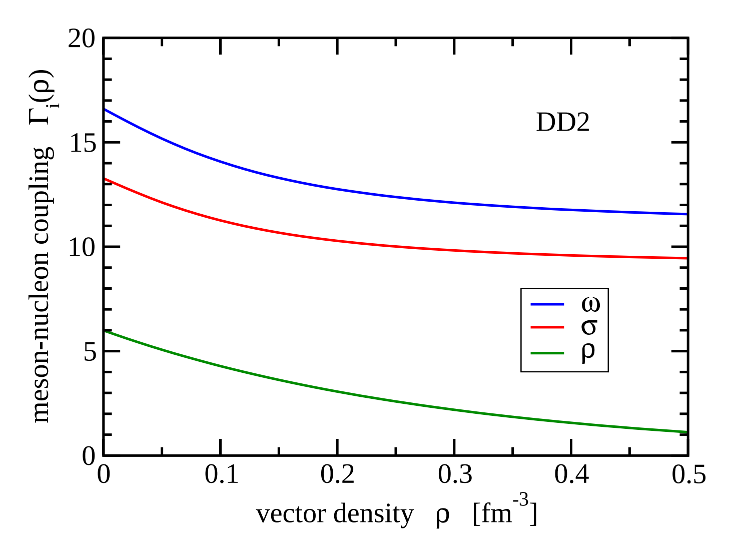
(S. Typel et al., Phys. Rev. C 81 (2010) 015803)



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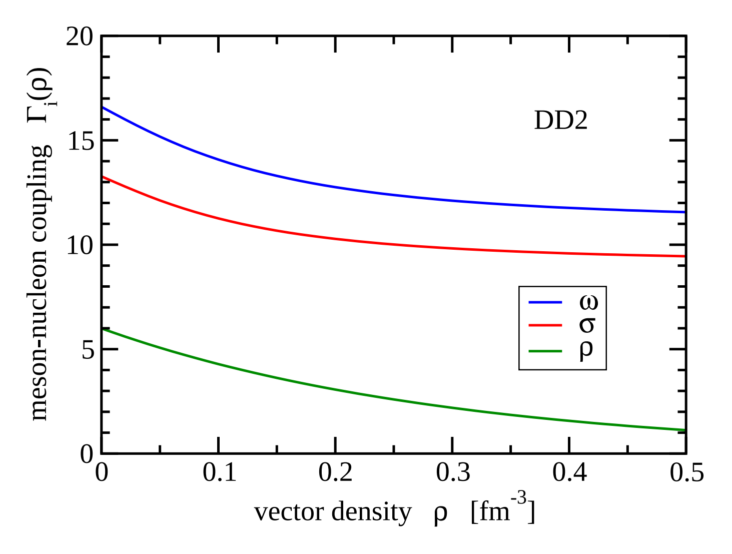

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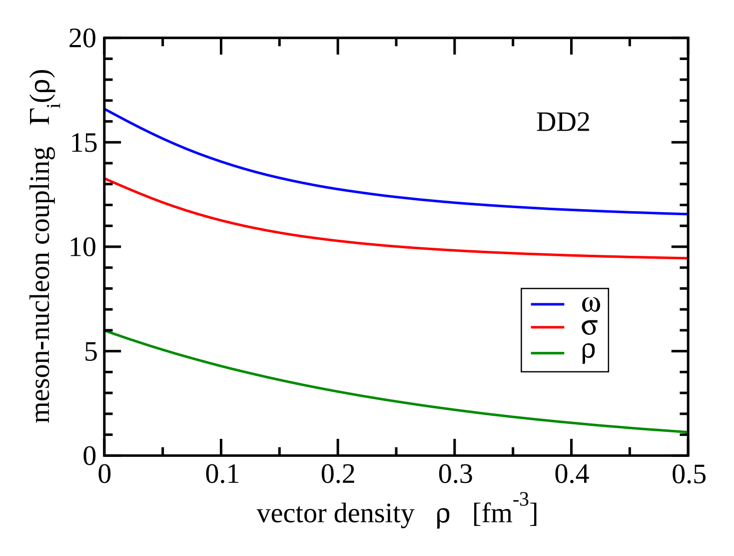

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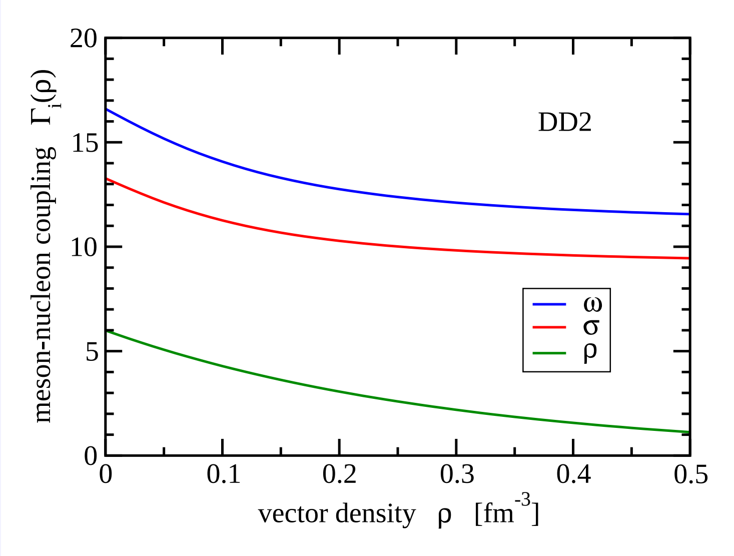

nuclear matter parameters

$n_{\rm sat} = 0.149$ fm^{-3}

$a_V = 16.02$ MeV

$K = 242.7$ MeV

$J = 31.67$ MeV

$L = 55.04$ MeV

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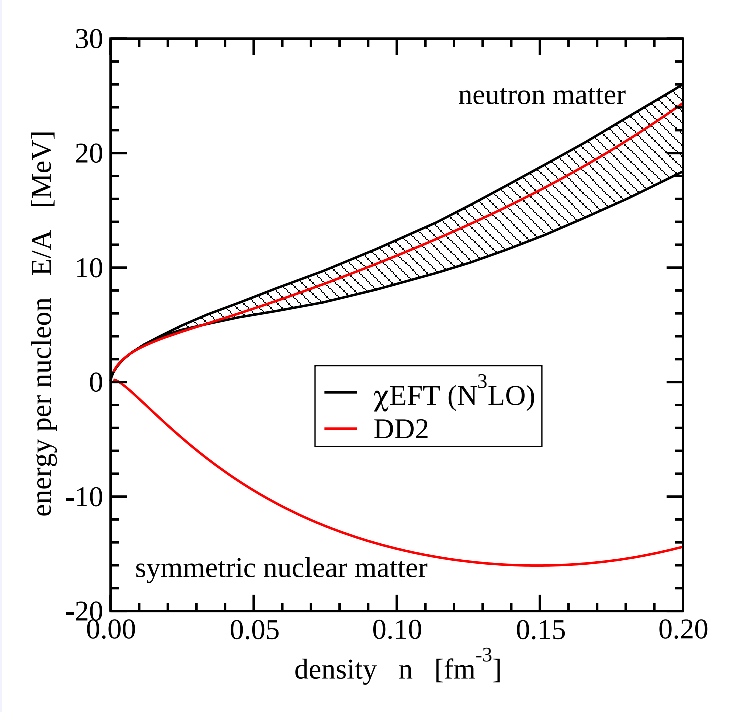

χEFT(N^{3}LO):
I. Tews et al., Phys. Rev. Lett 110 (2013) 032504
T. Krüger et al., Phys. Rev. C 88 (2013) 025802

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$$g_i(T) = g_i^{(gs)} + \int_0^{E_{\max}} d\varepsilon \, \varrho_i(\varepsilon) \exp\left(-\varepsilon/T\right)$$

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  - to be investigated: binomial distribution of states
    (A.P. Zuker, Phys. Rev. C 64 (2001) 021303)

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- **electromagnetic shift** $\Delta E_i^{(\text{Coul})}$ (in stellar matter)
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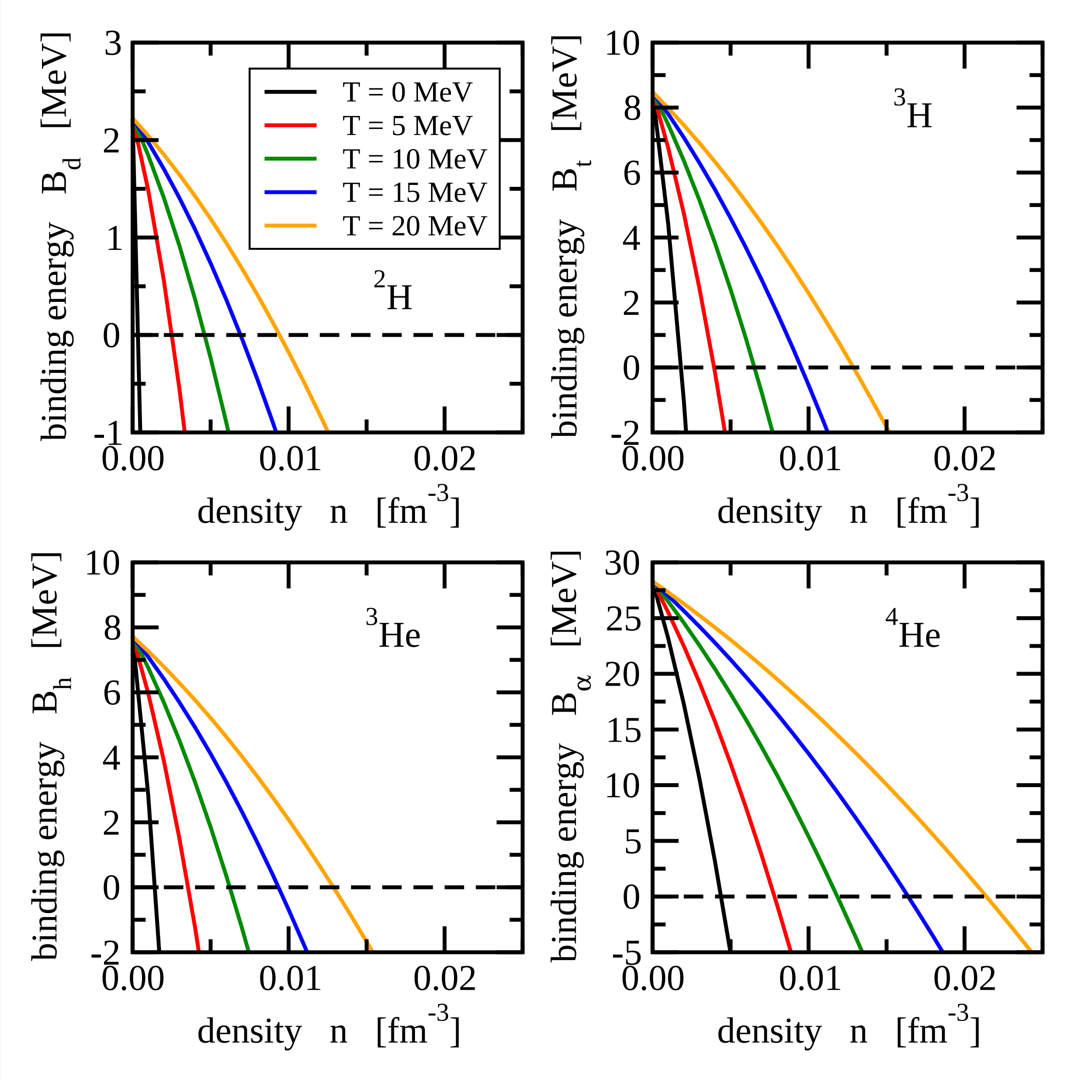
- example: symmetric nuclear matter, nuclei at rest in medium
- nuclei become unbound $(B_i < 0)$ with increasing density of medium
  $\Rightarrow$ dissolution of nuclei



Mass Shifts III

heavy nuclei $(A > 4)$

- spherical Wigner-Seitz cell calculation
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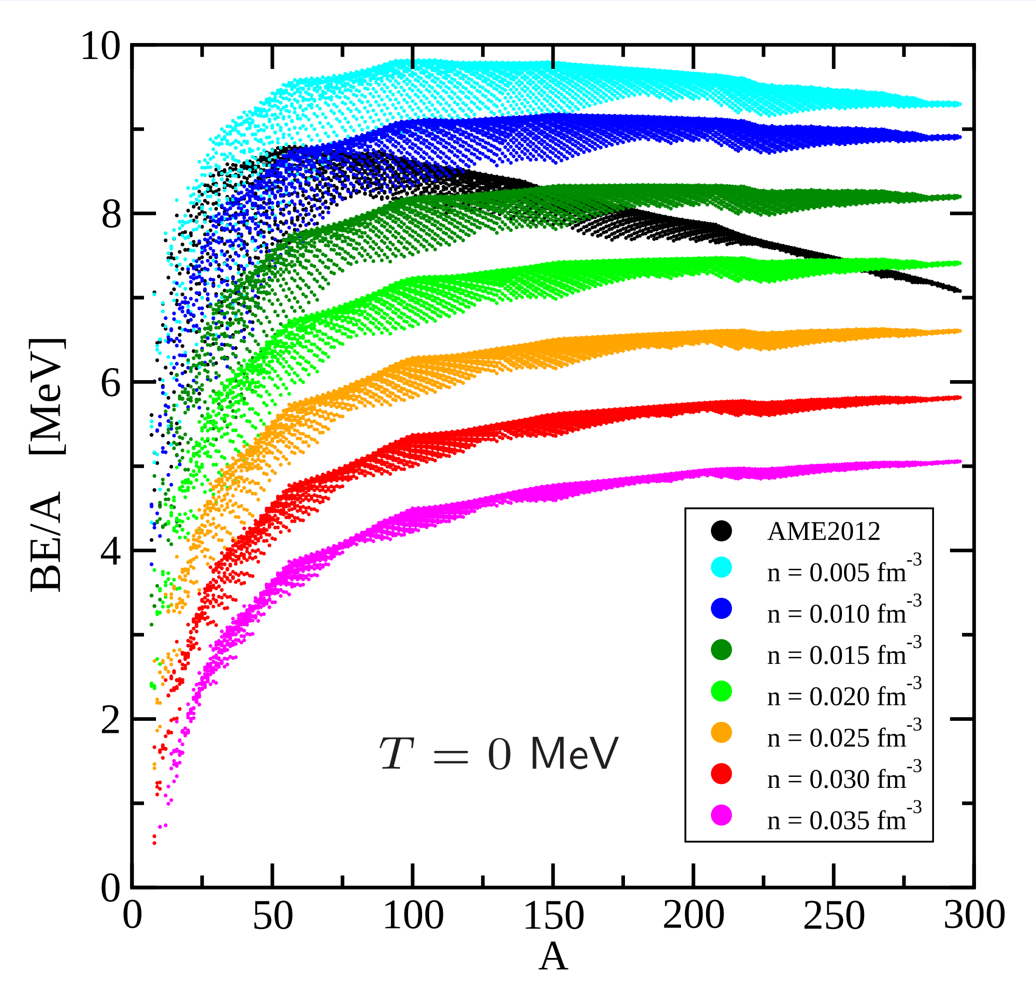

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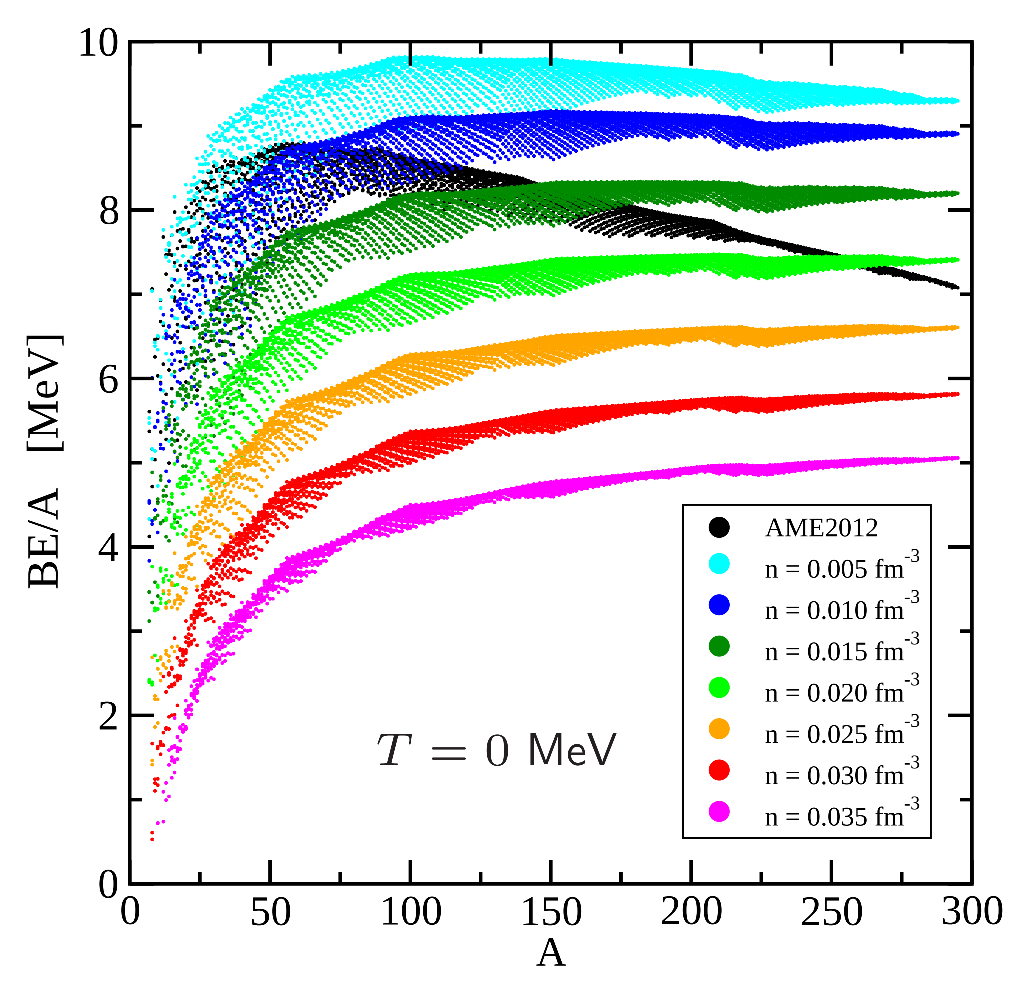
- preliminary parametrization

$$\Delta E_i^{(\text{strong})}(n_i^{(\text{eff})}) = \frac{B^{(\text{vac})}(N_i, Z_i)}{1 - n_i^{(\text{eff})}/n_i^{(0)}}$$

with $n_i^{(0)} = n_{\text{sat}}/(1 + 76/A_i)$



Particle Fractions

- mass fractions

$$X_i = A_i \frac{n_i}{n_B} \qquad n_B = \sum_i A_i n_i$$

- low densities:
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- high densities:
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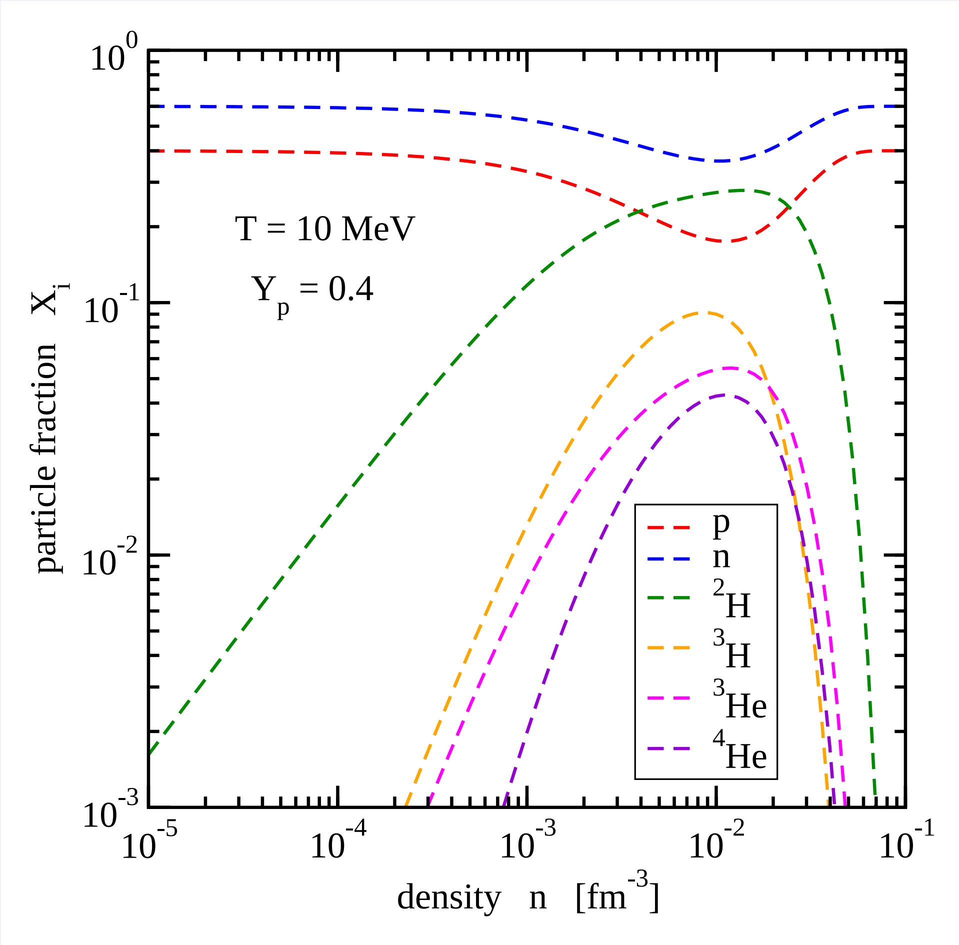
generalized relativistic density functional



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- low densities:
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- high densities:
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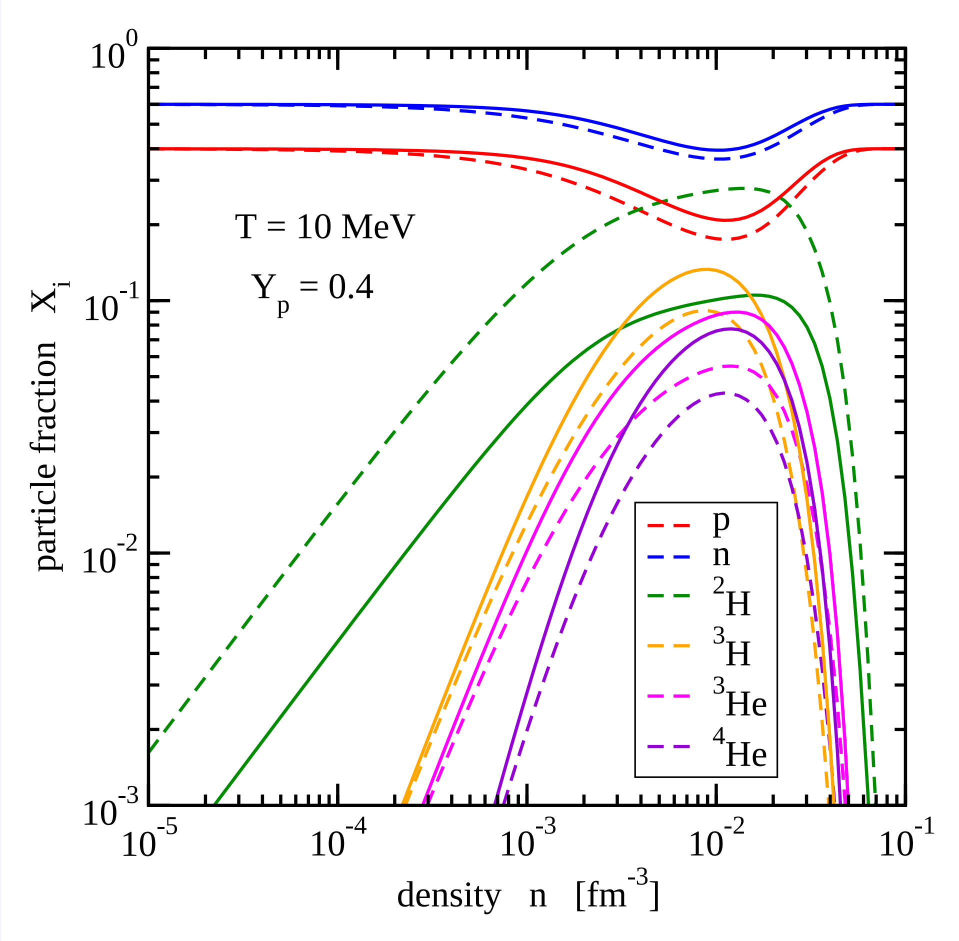
  $\Rightarrow$ Mott effect
- effect of NN continuum correlations
  ◦ dashed lines: without continuum
  ◦ solid lines: with continuum
  $\Rightarrow$ reduction of deuteron fraction,
  redistribution of other particles
- correct limits with extended
  relativistic density functional

generalized relativistic density functional



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  $\Rightarrow$ effective degeneracy factors $g_{ij}^{(\mathrm{eff})}(T)$
  (cf. treatment of excited states of nuclei)

  $\Rightarrow$ relativistic corrections

Low-Density Limit II

- **zero temperature limit of consistency relations without scattering correlations**
  - $C_{\omega} - C_{\sigma} = \frac{\pi}{2m} \left[a_{nn}(^1S_0) + a_{pp}(^1S_0) + a_{np}(^1S_0) + 3a_{np}(^3S_1) \right]$
  - $C_{\rho} - C_{\delta} = \frac{\pi}{2m} \left[a_{nn}(^1S_0) + a_{pp}(^1S_0) - a_{np}(^1S_0) - 3a_{np}(^3S_1) \right]$

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- **comparison of experiment with RMF parametrizations**

	exp.	DD2 [1] (ω, σ, ρ)	DD-MEδ [2] $(\omega, \sigma, \rho, \delta)$
$C_\omega - C_\sigma$ [fm^2]	-14.15	-5.39	-4.90
$C_\rho - C_\delta$ [fm^2]	-9.61	2.48	2.55

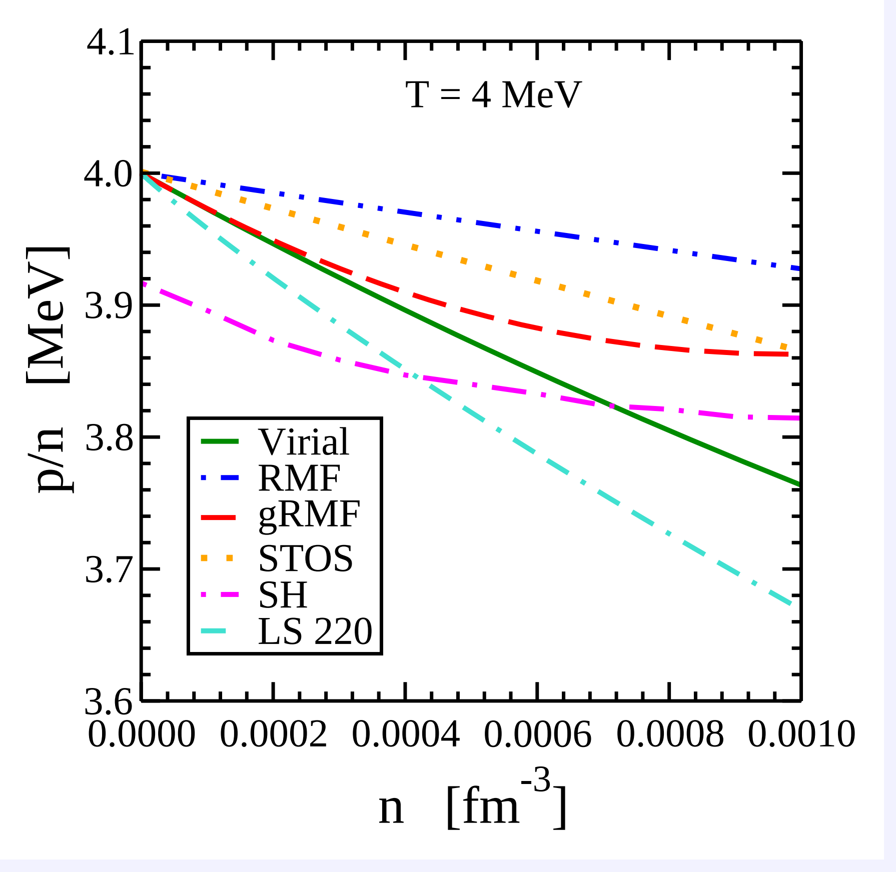
[1] S. Typel et al., Phys. Rev. C 81 (2010) 015803, [2] X. Roca-Maza et al., Phys. Rev. C 84 (2011) 054309

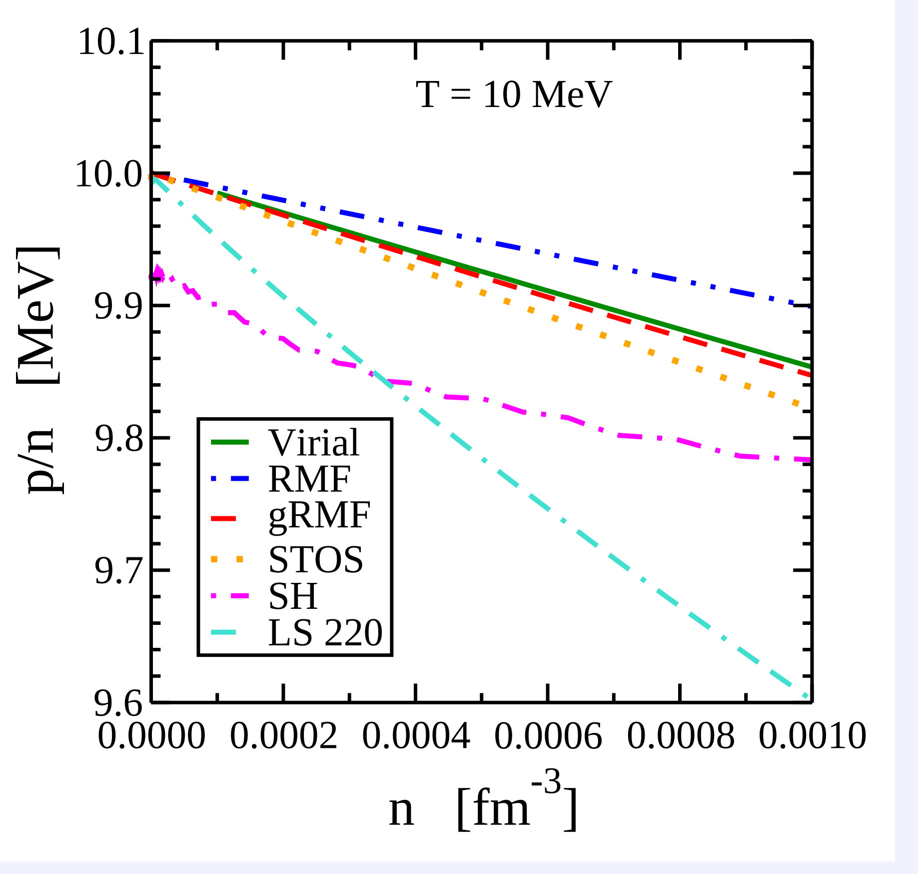
⇒ conventional mean-field models don't reproduce effect of correlations at very low densities

⇒ explicit scattering correlations needed

Neutron Matter at Low Densities

comparison: p/n in different models (ideal gas: $p/n = T$)





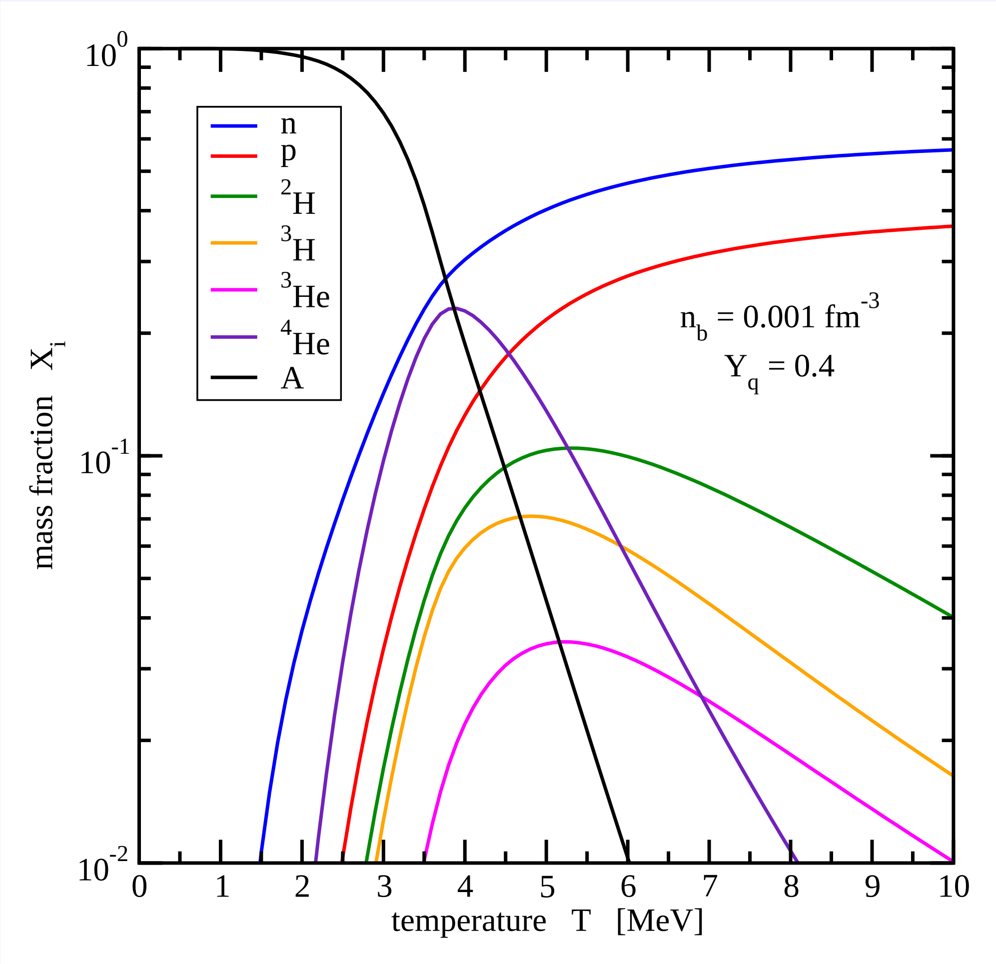
STOS: H. Shen et al., Nucl. Phys. A 637 (1998) 435 (TM1)

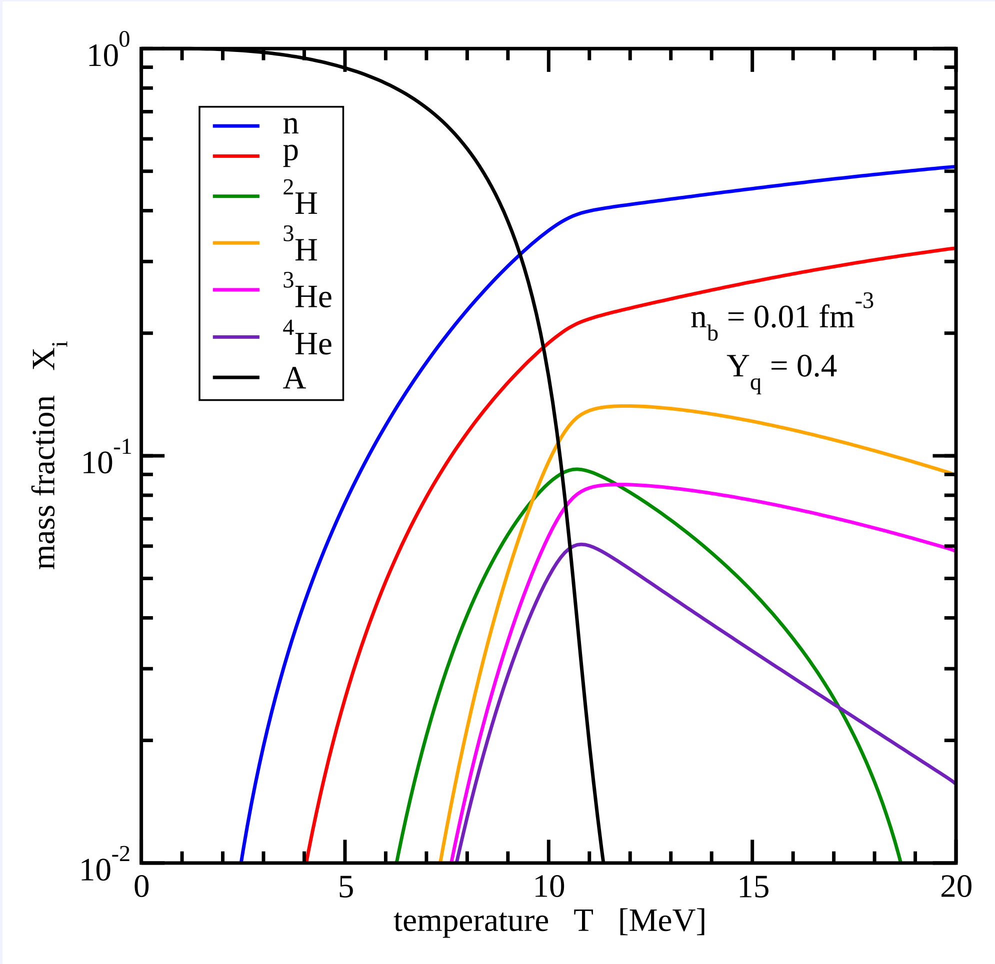
SH: G. Shen et al., Phys. Rev. C 83 (2011) 065808 (FSUGold)

LS 220: J.M. Lattimer et al., Nucl. Phys. A 535 (1991) 331 ($K = 220$ MeV)

Chemical Composition of Stellar Matter I

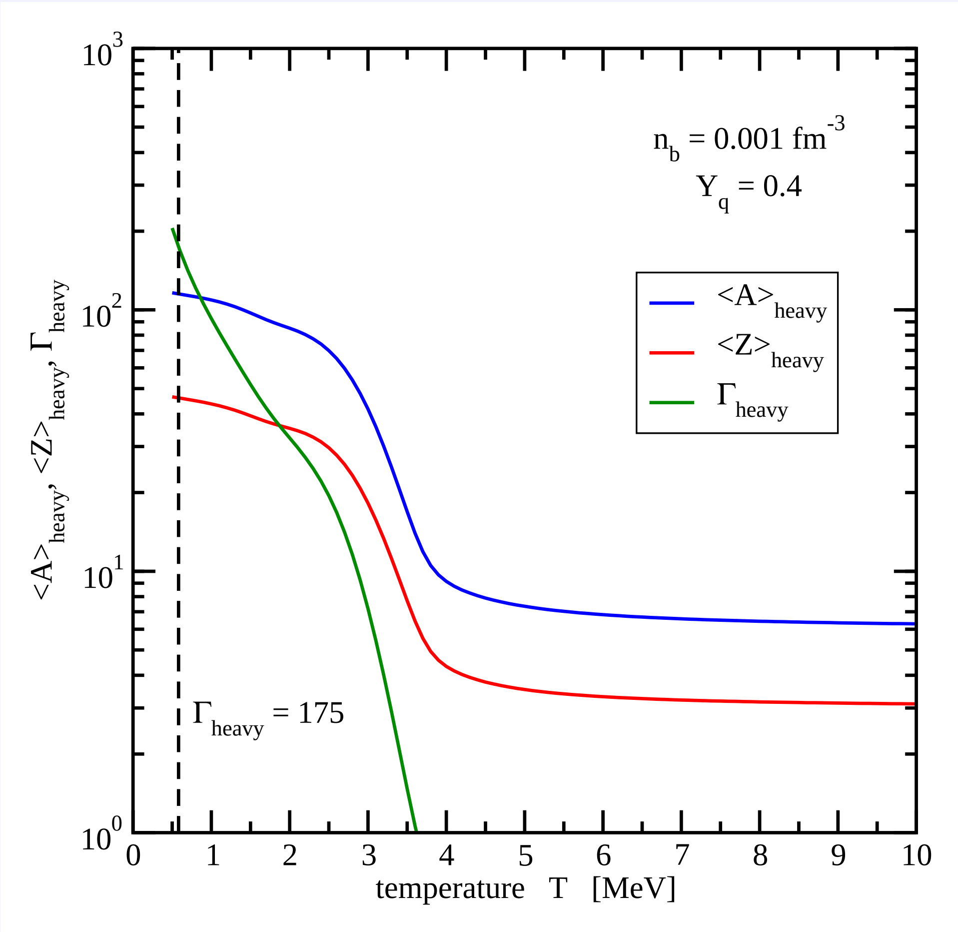
- full calculation in gRDF approach
- mass fractions of nucleons, light and heavy nuclei

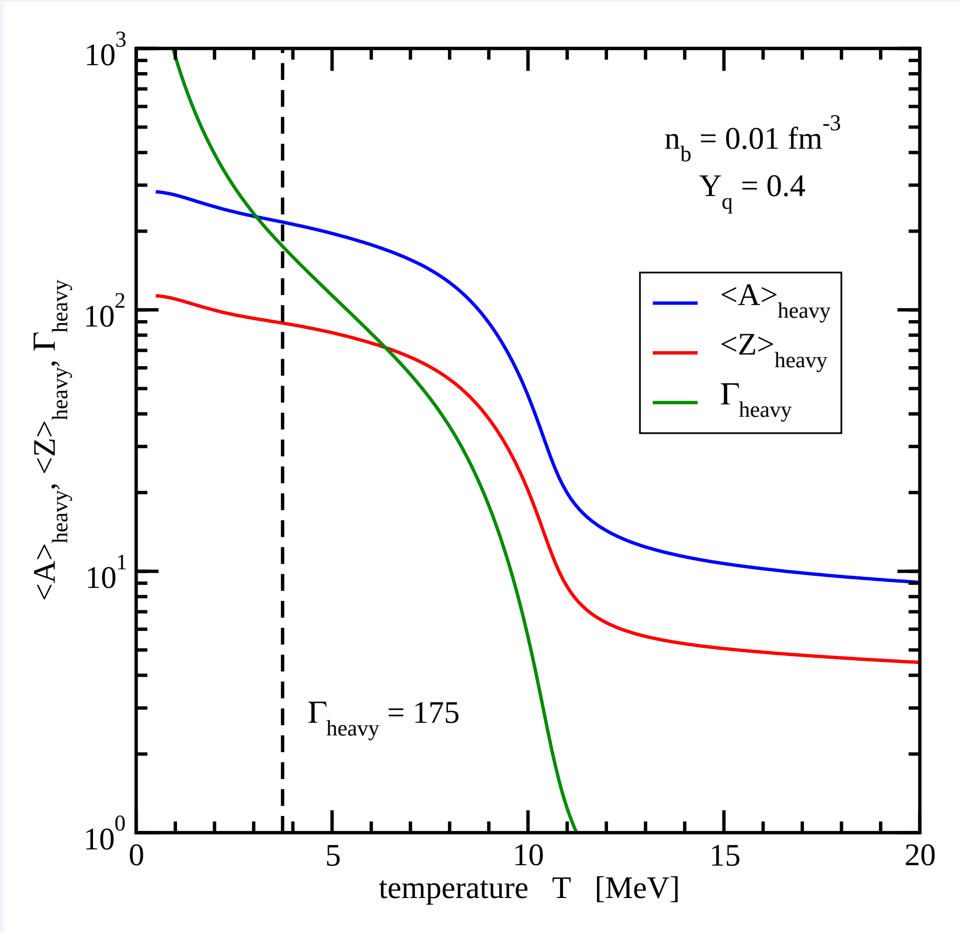




Chemical Composition of Stellar Matter II

- average mass number of heavy nuclei $\langle A\rangle_{\text{heavy}} = \sum_{i,A_i>4} A_i n_i / \sum_{i,A_i>4} n_i$
- average charge number of heavy nuclei $\langle Z\rangle_{\text{heavy}} = \sum_{i,A_i>4} Z_i n_i / \sum_{i,A_i>4} n_i$
- plasma parameter $\Gamma_{\text{heavy}} = \langle Z\rangle_{\text{heavy}}^{5/3} e^2/(a_q T)$ $\quad a_q = [3/(4\pi Y_q n_b)]^{1/3}$





Conclusions

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- nuclear/stellar matter: correlations in many-body system essential
  ⇒ modification of chemical composition and thermodynamic properties
- generalized relativistic density functional for dense matter
  - density-dependent couplings, well-constrained parameters
  - extended set of constituents: explicit cluster degrees of freedom, quasiparticle description
  - medium-dependent properties (mass shifts!) of composite particles
    ⇒ formation and dissolution of clusters, correct limits
  - Coulomb correlations considered
  - thermodynamic consistency ⇒ rearrangement contributions
- application: equation of state of stellar matter
  ⇒ astrophysical simulations
- remaining tasks:
  - implementation of solid phase calculation in code
  - full treatment of phase transitions
  - minor improvements (degeneracy factors of nuclei, extension of mass table, parametrisation of mass shifts, . . .)
  - preparation of global EoS table

Thanks

- **to my collaborators**

  Gerd Röpke, Niels-Uwe Bastian (Universität Rostock)

  David Blaschke, Thomas Klähn (Uniwersytet Wrocławski)

  Hermann Wolter (Ludwig Maximilians-Universität München)

  Maria Voskresenskaya (TU Darmstadt)

  Sofija Antić (GSI Darmstadt)

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  - CompStar - Research Networking Program of the European Science Foundation (ESF)
  - European Nuclear Science and Application Research Joint Research Activity THEXO
  - ExtreMe Matter Institute EMMI

- **to you, the audience**

  for your attention and patience

CompOSE
CompStar Online Supernovae Equations of State

Micaela Oertel (LUTH Meudon)
Thomas Klähn (Uniwersytet Wrocławski)
Stefan Typel (GSI Darmstadt)
and the CompOSE core team

- **features**
  - repository of equations of state (data tables and additional information)
  - tools for extracting, interpolating and generating EoS tables according to the needs of the user
  - flexible data format for storage of EoS tables, supports ASCII and HDF5 data formats in output
- **access & information**
  - website: `compose.obspm.fr`
  - manual (≈ 70 pages): available from website, or arXiv:1307.5715 [astro-ph.SR]

please contribute your favorite EoS! (see manual for details)