

Charm Transport in O+O Collisions

Comparing Fokker–Planck to Müller–Israel–Stewart Hydrodynamics

Ruwen Schulz

EMMI Workshop: Light-Ion Collisions at the LHC, Heidelberg

September 21–25, 2026

Based on:

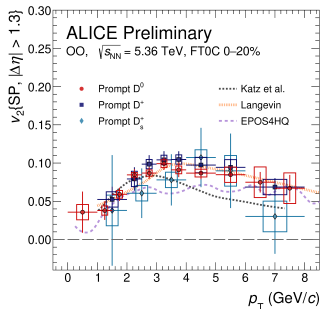
Capellino, Masciocchi, Schulz, in preparation

Boguslavski, Capellino, Schulz, in preparation



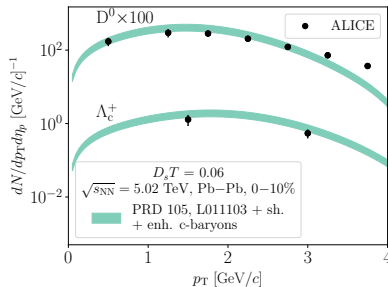
Heavy- and Light-Ion Collisions (Motivation)

O+O, July 2025: ALICE sees
finite D -meson v_2



[ALICE Preliminary, 2608.02161]

Pb+Pb: charm admits
fluid-dynamic description



[Capellino, Facen et al., 2205.07692, 2307.15580]

O+O vs. Pb+Pb: different lifetimes and system sizes.
Is a fluid description for charm valid in O+O?

Kinetic theory — Fokker–Planck

$$\partial_t f + \frac{\mathbf{p}}{E} \cdot \nabla_x f = \nabla_{\mathbf{p}} \cdot \left[\eta_D \mathbf{p} f + \frac{1}{2} \kappa \nabla_{\mathbf{p}} f \right]$$

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Müller–Israel–Stewart Hydrodynamics

$$J^\mu = \int dK k^\mu f = n u^\mu + v^\mu, \quad \nabla_\mu J^\mu = 0$$
$$\tau_n \dot{v}^\mu + v^\mu = -\frac{D_s}{T} \underbrace{\nabla^\mu P_0}_{\text{pressure gradient}} + \underbrace{\mathcal{O}(\nabla u)}_{\text{flow gradients}} + \underbrace{\mathcal{O}(2\text{nd})}_{\text{2nd mom.}}$$

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One input, $D_s T = 0.116$, fixes everything

$$\underbrace{\eta_D = \frac{T^2}{m_c D_s T}}_{\text{drag}} \quad \underbrace{\kappa = 2 m_c T \eta_D}_{\text{mom. diffusion}} \quad \underbrace{\tau_n \propto D_s \frac{m_c}{T}}_{\text{current relaxation}}$$

Kinetic theory — Fokker–Planck

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Müller–Israel–Stewart Hydrodynamics

τ_n
 $J^\mu = \int d^3k \, u^\mu f$

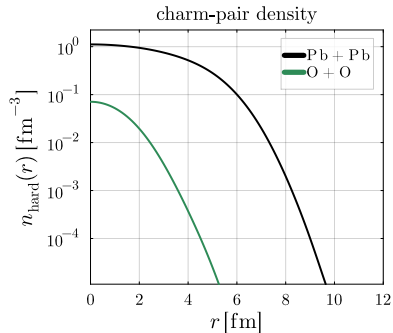
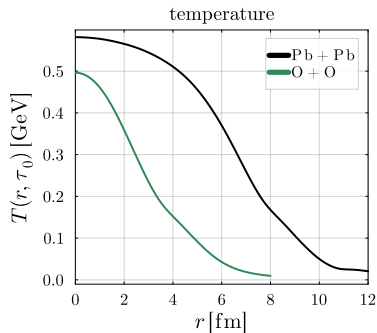
The HQ is evolved on a $T(r, \tau)$, $u(r, \tau)$ BG

pressure gradient flow gradients 2nd mom.

One input, $D_s T = 0.116$, fixes everything

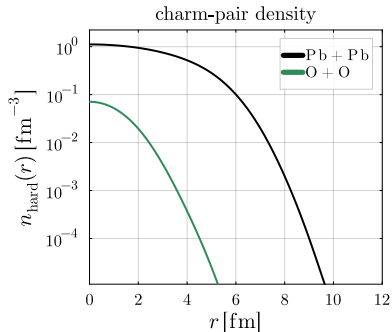
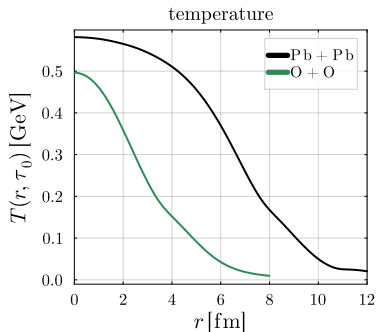
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Initial Conditions and System Size



- 0–5% centrality, $\sqrt{s_{NN}} \approx 5$ TeV

Initial Conditions and System Size



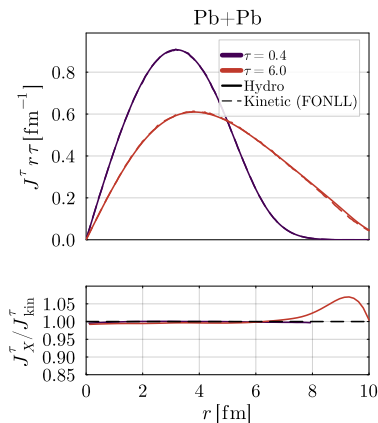
- 0–5% centrality, $\sqrt{s_{NN}} \approx 5$ TeV
- $N_{\text{OO}} \approx \mathbf{0.3}$ charm pairs, $\tau_{\text{life}} \simeq 5.2 \text{ fm}/c \approx \mathbf{5} \tau_n$
- $N_{\text{PbPb}} \approx \mathbf{24}$ charm pairs, $\tau_{\text{life}} \simeq 11.5 \text{ fm}/c \approx \mathbf{13} \tau_n$
- charm momenta are initialized in 3 ways: (FONLL, thermal, narrow Gaussian)

$$\# \tau_n = \int d\tau / \tau_n(T)$$

$\Rightarrow$ less than one pair per event per unit rapidity in O+O

Where the Charge Is

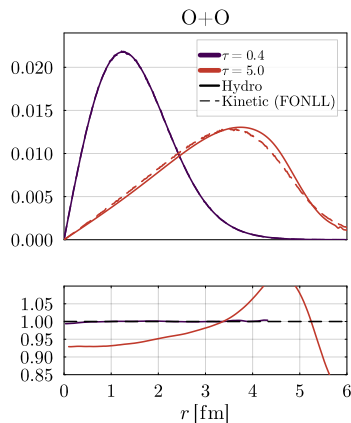
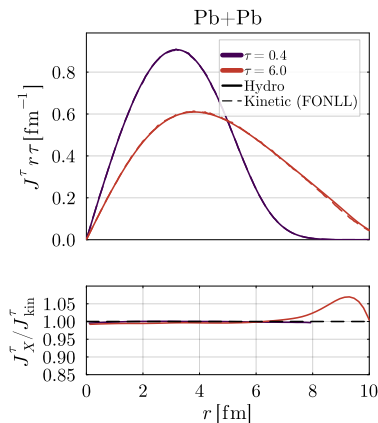
$$J^\mu = nu^\mu + \nu^\mu, \quad \nabla_\mu J^\mu = 0 \quad \Rightarrow \quad N(\tau) = 2\pi\tau \int J^\tau r dr$$



- top: $J^\tau r \tau$, so **area is charge** — solid closure, dashed transport (FONLL, one of the three)
- bottom: $J_{\text{hydro}}^\tau / J_{\text{kin}}^\tau$

Where the Charge Is

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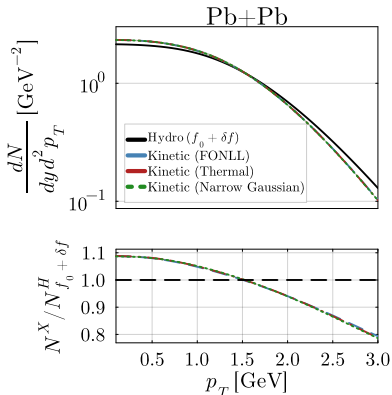
Charm Spectra: The Cooper–Frye Prescription

Capellino, Masciocchi, Schulz, in preparation

$$\Sigma \sim (r, \tau) \quad \text{with} \quad T(r, \tau) = 156 \text{ MeV}$$

$$\frac{dN^{\text{hydro}}}{d^2p_T} \propto \int_{\Sigma} [f_0 + \delta f(\nu^\mu)]$$

$$\frac{dN^{\text{kinetic}}}{d^2p_T} \propto \int_{\Sigma} f_{\text{full}}$$

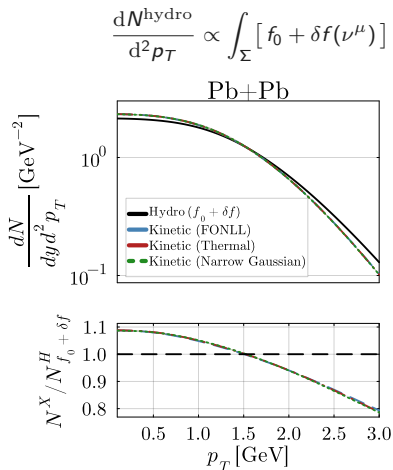


- top: freeze-out spectra, **Kinetic** (3 ICs) vs **Hydro**
- bottom: $N_{\text{kin}}/N_{\text{hydro}}$ — unity is the hydro closure

Charm Spectra: The Cooper–Frye Prescription

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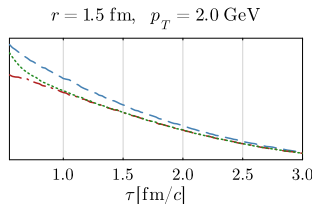
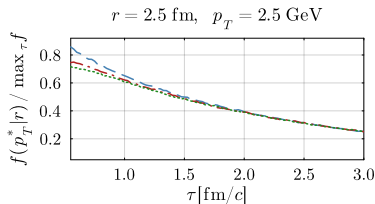
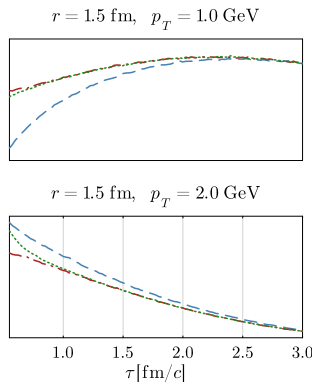
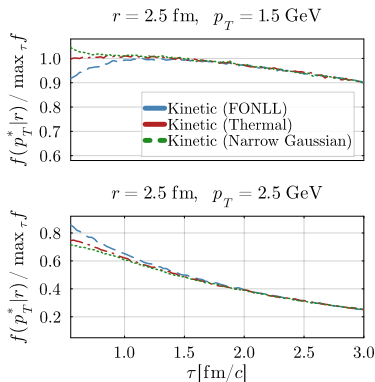
A hydrodynamic description **qualitatively** carries over to O+O

Outlook: Momentum Attractor

Boguslavski, Capellino, Schulz, in preparation

Pb+Pb

O+O



Momentum forgets its initial conditions over time,
Pb+Pb lives long enough to show that in the FO-spectra, O+O not quite.

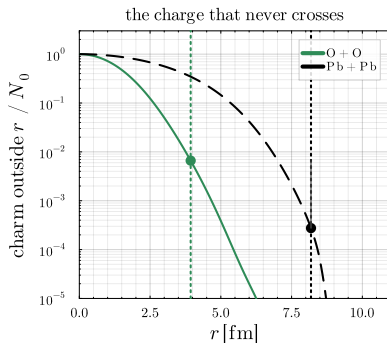
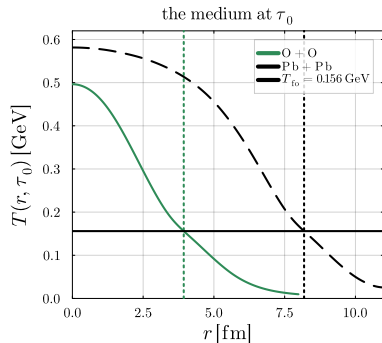
- O+O is **smaller**, ≈ 0.3 in O+O vs. 24 charm pairs in Pb+Pb
- O+O is **shorter lived**, ≈ 5 in O+O vs. ≈ 13 relaxation times in Pb+Pb
- Given that Hydro works "well" in Pb+Pb, quantitatively this could be carried over to O+O
- Outlook: charm attractor

Thank you for your attention.

The **code used** is openly available:

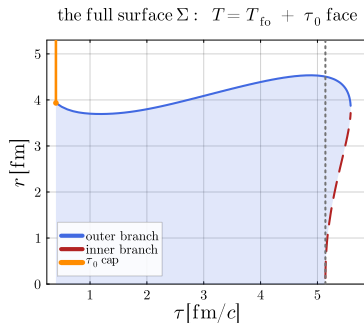
- Fluidum.jl: **finite-difference** viscous hydrodynamics solver
- FiVoHydro.jl: **finite-volume** viscous hydrodynamics solver
- LangevinMedium.jl: relativistic **Langevin** solver

Backup: The τ_0 Cap



- Charm born outside $r_{fo}(\tau_0)$ never crosses the isotherm
- O+O is hot only to 3.93 fm (Pb+Pb 8.19 fm) $\Rightarrow$ cap 0.66% in O+O vs. 0.028% in Pb+Pb.

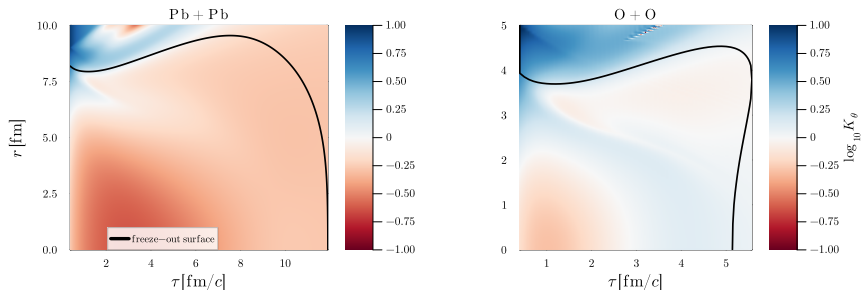
Backup: The Cooper–Frye Prescription



An equilibrium, an out-of-equilibrium, and a cap contribution

$$\frac{dN}{d^2p_T} = \frac{dN^{(0)}}{d^2p_T} + \frac{dN^{(\delta)}}{d^2p_T} + \frac{dN^{\text{cap}}}{d^2p_T}, \quad \frac{dN^{(X)}}{d^2p_T} = \frac{g}{(2\pi)^3} \int_{\Sigma_X} f_X p^\mu d\Sigma_\mu$$

Backup: Knudsen Number

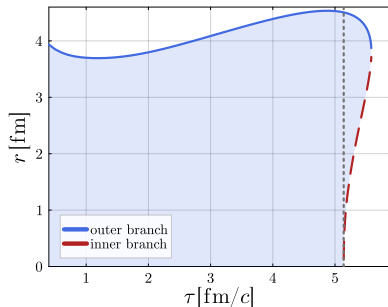


- $K_\theta = \tau_n \theta$ compares the current's relaxation time with the expansion rate. White is $K_\theta = 1$; blue is where the medium expands faster than the current can relax.
- O+O emits 65.9% of its charm through $K_\theta > 1$, Pb+Pb 5.9%; on $\Sigma \langle K_\theta \rangle = 1.07$ in O+O.

Backup: A Two-Branch Freeze-out Surface

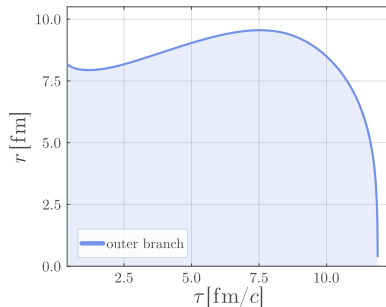
O+O

the freeze-out surface $T = T_{fo}$



Pb+Pb

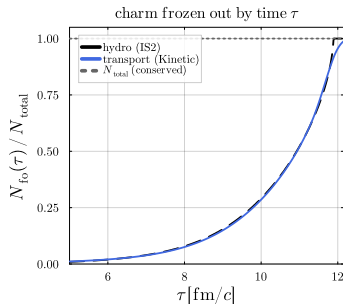
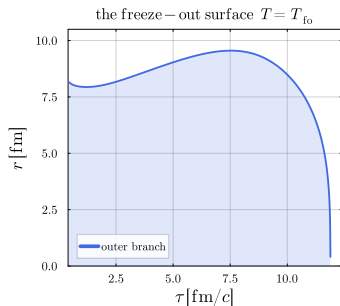
the freeze-out surface $T = T_{fo}$



$$T_{fo} = 156 \text{ MeV}$$

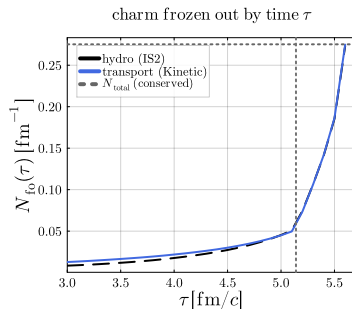
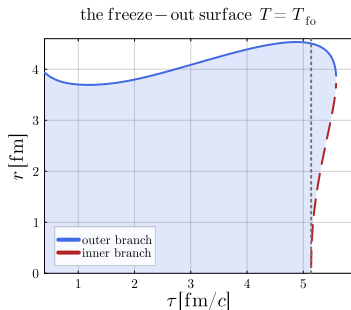
In O+O the hot region turns **disk** $\rightarrow$ **ring** and the isotherm has **two radial branches** at the same τ . In Pb+Pb it is **single-valued**.

Backup: The Pb+Pb Freeze-out Surface



In Pb+Pb $r_{fo}(\tau)$ is **single-valued**

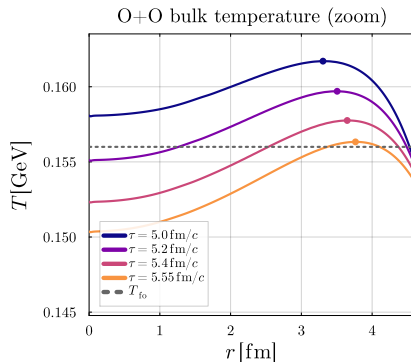
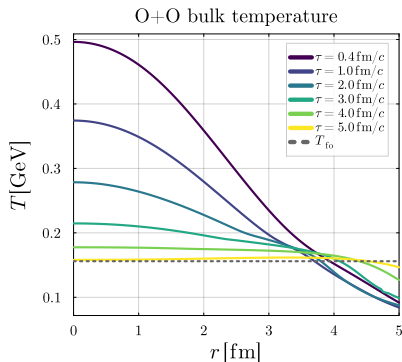
Backup: A Two-Branch Freeze-out Surface



$$N_{fo}(\tau) = \int_{\Sigma, \tau' \leq \tau} d\Sigma_{\mu} J^{\mu} = \int_{\Sigma, \tau' \leq \tau} 2\pi \tau' r [-J^{\tau} dr + J^r d\tau'], \quad J^{\mu} = nu^{\mu} + \nu^{\mu}$$

Both descriptions emit the same conserved charge, the majority freezes out late

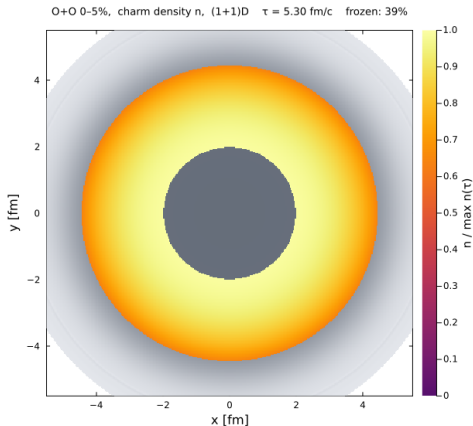
Backup: Background Temperature



$T_{fo} = 156$ MeV (Gray dashed line)

The centre freezes first, at $\tau_c = 5.2$ fm/c.

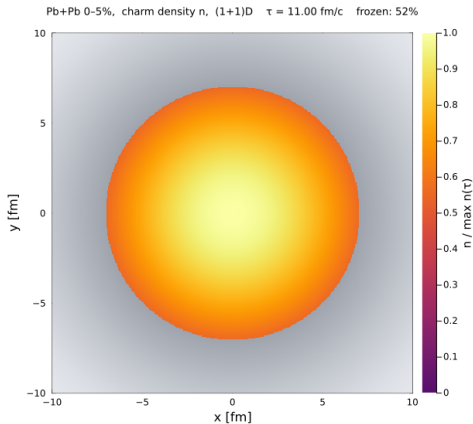
Backup: O+O Charm Density at Freeze-out, Animated



gray is charm in fluid below T_{f0} , on the same colour scale.

The charm freezes out from outside in and inside out.

Backup: Pb+Pb Charm Density at Freeze-out, Animated



gray is charm in fluid below T_{f0} , on the same colour scale.

The charm freezes out outside in.

Bulk lattice QCD + HRG EoS

$$\frac{\eta}{s} = 0.1, \quad \tau_\pi = \frac{\eta/s}{C_s T}$$

$$\frac{\zeta}{s}(T) = \frac{0.1}{1 + \left[\frac{T - 175 \text{ MeV}}{24 \text{ MeV}} \right]^2}$$

$$\tau_\Pi = \frac{\zeta/s}{C_\zeta T \left(\frac{1}{3} - c_s^2 \right)^2}$$

$$C_s = 0.2, \quad C_\zeta = 15$$

Charm $m_c = 1.5 \text{ GeV}$, $z = m_c/T$

$$D_s T = 0.116 \quad (2\pi T D_s = 0.73)$$

$$\eta_D = \frac{T^2}{m_c D_s T}, \quad \kappa = 2m_c T \eta_D$$

$$\tau_n = D_s z \frac{K_3(z)}{K_2(z)}$$

$$D_s = 0.147 \text{ fm}, \quad \tau_n = 1.81 \text{ fm}/c \quad \text{at } T_{\text{fo}}$$

Transverse: three $f(p_T)$ at τ_0 , all azimuthally symmetric

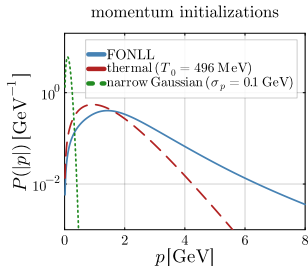
FONLL pQCD charm production spectrum (default)

thermal local Jüttner at $T(r, \tau_0)$

delta narrow Gaussian

Longitudinal: one choice, for every curve shown

$p_z^* = 0$ comoving with the Bjorken fluid at τ_0



Backup: Building the Initial Conditions

① **Glauber** 50,000 min-bias events into $N_{\text{coll}}(r) \propto T_A T_B$

② **Charm** from binary-collisions

$$n_{\text{hard}}(r) = N_{\text{coll}}(r) \frac{d\sigma_{c\bar{c}}/dy}{\sigma_{NN} \tau_0}$$

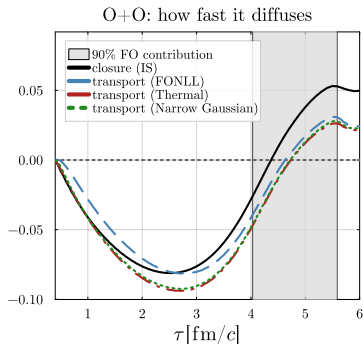
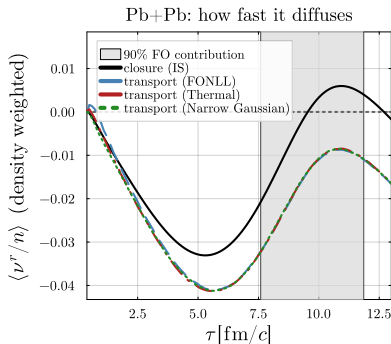
③ **Bulk** $s(r) = \mathcal{N} s_{\text{shape}}(r)$, $s_{\text{shape}} \propto \sqrt{T_A T_B}$, via lattice+HRG EoS to $T(r, \tau_0)$.

④ **Run** the (1+1)D bulk to freeze-out, Cooper–Frye the pions, $\Rightarrow N_\pi$.

⑤ **Iterate** $\mathcal{N}$ steps 3–4 until N_π meets ALICE yield.

	O+O	Pb+Pb
$\sqrt{s_{NN}}$	5.36 TeV	5.02 TeV
$\langle N_{\text{coll}} \rangle_{0-5\%}$	41.6	1806
σ_{NN} (pp inel.)	70 mb	70 mb
$d\sigma_{c\bar{c}}/dy$	463 μb	932 μb
$T(0, \tau_0)$	0.496 GeV	0.581 GeV
$N_{c\bar{c}}$ pairs	0.275	24.0

Backup: The Diffusion Current per Particle



The plotted quantity: the diffusion current per particle

$$n = -u_\mu J^\mu, \quad \nu^\mu = \Delta^\mu_\nu J^\nu, \quad \hat{\nu} \equiv \frac{\nu^r}{n}, \quad \langle \hat{\nu} \rangle(\tau) = \frac{\int dr \, r \, n \, \hat{\nu}}{\int dr \, r \, n}$$

The fluid's current runs ahead of the transport's, in both systems.

Backup: The Full First-Moment Relaxation Equation

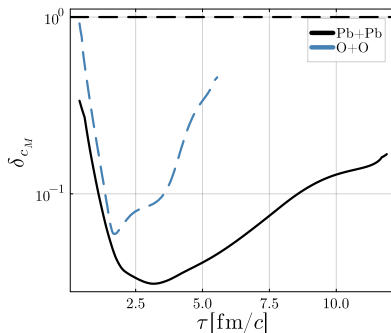
Parent form

$$\begin{aligned} \frac{D_s}{T} \Delta^\mu_\lambda \nabla_\nu T_Q^{\nu\lambda} + \nu^\mu &= 0 \\ T_Q^{\mu\nu} &= \varepsilon_Q u^\mu u^\nu + P_0 \Delta^{\mu\nu} + h(u^\mu \nu^\nu + \nu^\mu u^\nu) \\ P_0 &= nT, \quad \varepsilon_Q + P_0 = nh, \quad h = \tau_n T / D_s \text{ the enthalpy per particle} \end{aligned}$$

Expanded, in the form the code evolves

$$\begin{aligned} \tau_n \Delta^\mu_\nu u^\lambda \partial_\lambda \nu^\nu + \nu^\mu &= -\frac{D_s}{T} \left(nT \nabla^\mu \alpha + (n + T \frac{\partial n}{\partial T}) \nabla^\mu T \right) \\ -\tau_n n a^\mu - \tau_n (\nu \cdot \nabla u)^\mu - \tau_n \nu^\mu (\theta + D \ln h) - \tau_n \nu^\mu \left(\frac{u^T}{\tau} + \frac{u^r}{r} \right) &+ \mathcal{O}(2nd) \\ \mathcal{O}(2nd) &= -c_M \Delta^\mu_\lambda \nabla_\nu \delta M^{\nu\lambda}, \quad c_M = D_s / T \end{aligned}$$

Backup: How Big Is the Term the Closure Drops?



The second moment's feedback, against the first-order drive

$$O_{1\text{st mom}} \sim \nabla P_0, \quad O_{2\text{nd mom}} \sim \nabla \cdot \delta M, \quad \delta c_M(\tau) = \frac{\langle |O_{2\text{nd mom}}| \rangle_r}{\langle |O_{1\text{st mom}}| \rangle_r}$$

Small everywhere, and about twice as large in O+O: median 0.07 against 0.12.

- Charm is made at $\tau = 0$. $M \gg T$ suppresses thermal production and annihilation, so the total number is fixed and the net charm carries nothing:

$$N_{\text{total}} = N_c + N_{\bar{c}} = \text{const.}, \quad N_c - N_{\bar{c}} = 0$$

- The distribution splits into an equilibrium part and a dissipative correction, with *one* fugacity conjugate to that sum:

$$f = f_{\text{eq}} + \delta f, \quad f_{\text{eq}}(\mathbf{k}) = \exp\left(\alpha - \frac{k^0}{T}\right), \quad \alpha = \frac{\mu}{T}, \quad \mu = \mu_c + \mu_{\bar{c}}$$

- The charm current is its first moment, decomposed on the fluid velocity:

$$J^\mu = \int dK k^\mu f(x, k) = n u^\mu + \nu^\mu, \quad dK = \frac{d^3 k}{(2\pi)^3 k^0}, \quad k^0 = \sqrt{M^2 + \mathbf{k}^2}, \quad \nu^\mu u_\mu = 0$$

- Quark and antiquark never appear separately. One current, one conservation law:

$$\nabla_\mu J^\mu = 0$$

Backup: Fokker–Planck and Langevin Are the Same Equation

Fokker–Planck, for the phase-space density $f(t, \mathbf{x}, \mathbf{p})$

$$\partial_t f + \frac{\mathbf{p}}{E} \cdot \nabla_{\mathbf{x}} f = \nabla_{\mathbf{p}} \cdot \left[\eta_D \mathbf{p} f + \frac{1}{2} \kappa \nabla_{\mathbf{p}} f \right]$$

Langevin, for one trajectory $\mathbf{p}^*(t)$ in the fluid rest frame

$$d\mathbf{p}^* = -\eta_D \mathbf{p}^* dt + \sqrt{\kappa} d\mathbf{W}, \quad d\mathbf{x} = \frac{\mathbf{p}}{E} dt$$