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Further questions/comments: 

Work from O. G-M, 2609.07847
and O. G-M, 2610.XXXXX (in preparation)

And partially based on various works *in collaboration with*
H. Elfner, J. Zhu and S. Schlichting, including
Phys.Rev.C 109 (2024) 4, 044916,
Phys.Rev.D 111 (2025) 7, 076029,

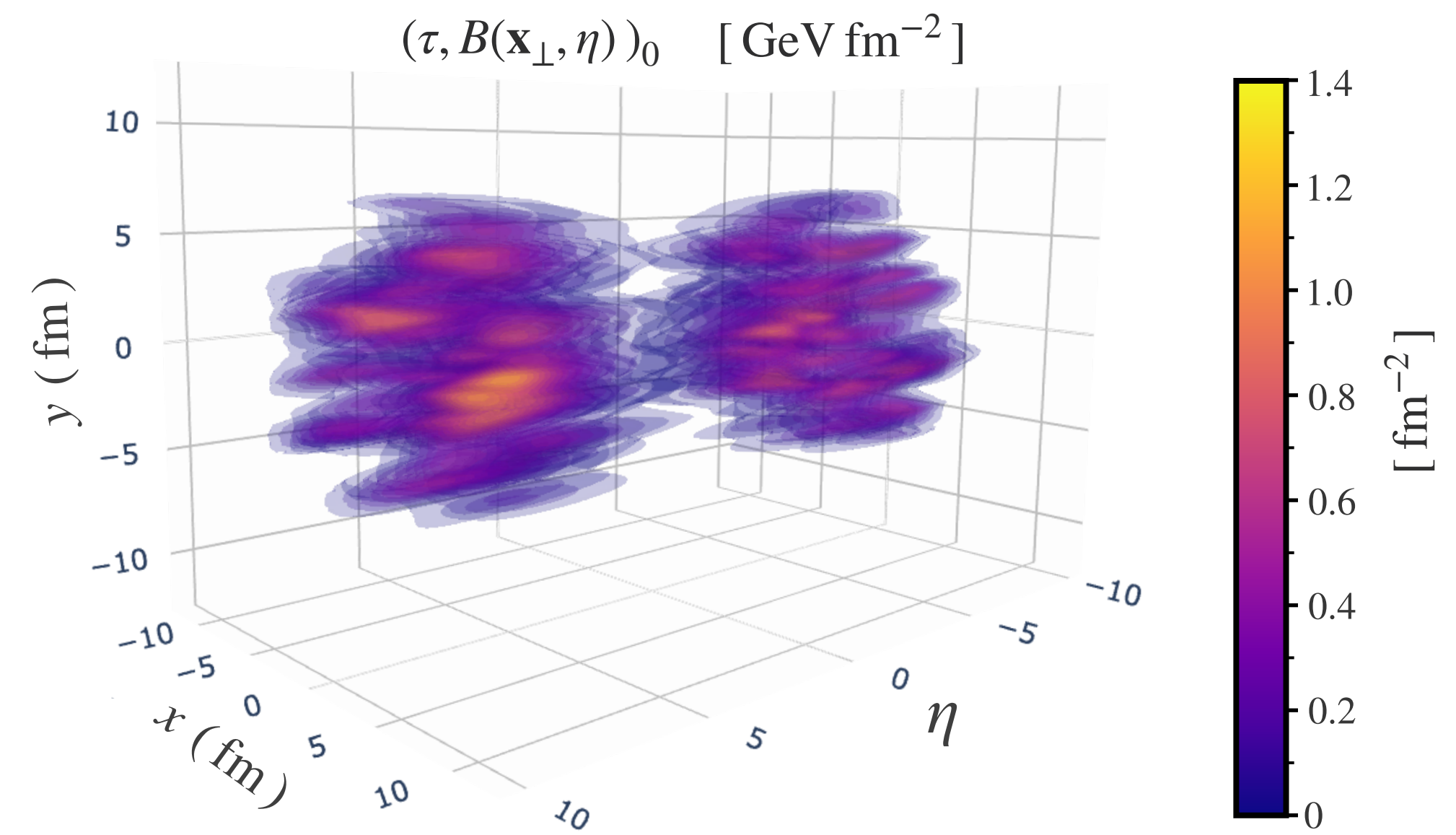
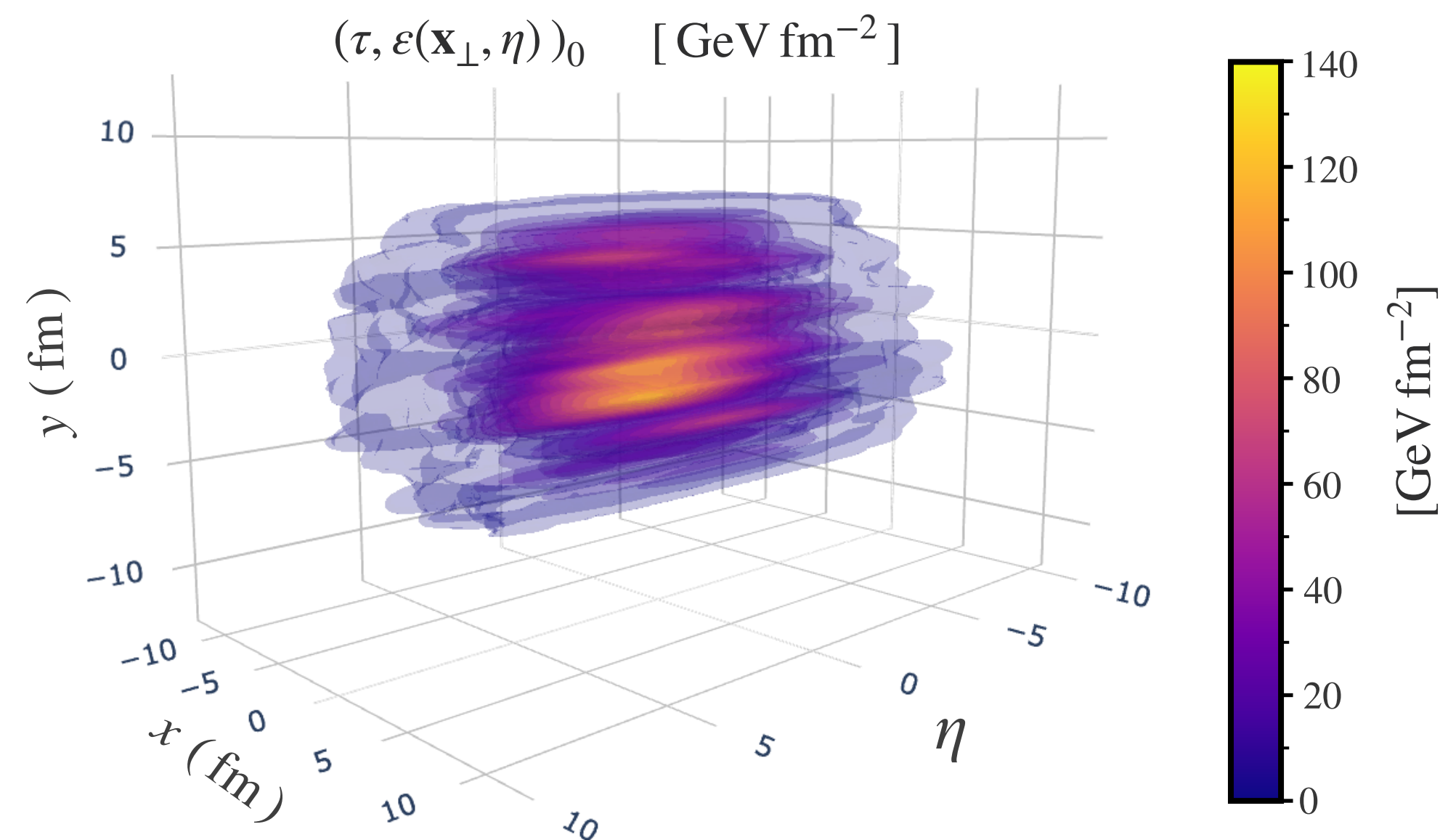


CGC IN 3D: THE MCDIPPER

Monte-Carlo Dipole Parallel Event GeneRator

Framework for comparison of saturation model predictions and creation of IC for HE
Heavy-Ion Collisions

Perturbative realisation of the dilute glasma + Baryon stopping by CGC



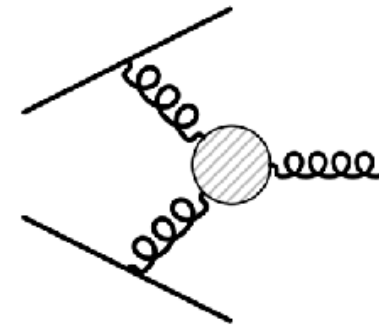
FROM MICRO TO MACRO

CONSERVED CHARGE DEPOSITION FROM THE CGC FORMALISM

- Low- x gluons dominate the midrapidity region

$$\frac{dN_g}{d^2\mathbf{x}d^2\mathbf{p}dy} = \frac{g^2}{8\pi^5 C_F \mathbf{p}^2} \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{d^2\mathbf{k}}{(2\pi)^2} (2\pi)^2 \delta(\mathbf{p} + \mathbf{q} - \mathbf{k})$$

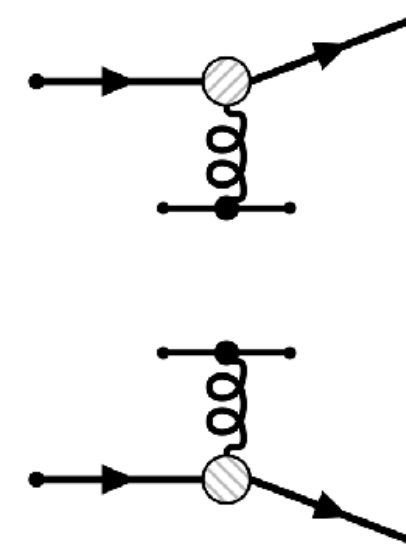
$$\times \Phi_1(x_1, \mathbf{x}, \mathbf{q}) \Phi_2(x_2, \mathbf{x}, \mathbf{k})$$



- At forward/backward rapidities, particle production dominated by baryon stopping

$$\frac{dN_{q_f}}{d^2\mathbf{x}d^2\mathbf{p}dy} = \frac{x_1 q_f^A(x_1, \mathbf{p}^2, \mathbf{x}) D_{\text{fun}}(x_2, \mathbf{x}, \mathbf{p})}{(2\pi)^2}$$

$$+ \frac{x_2 q_f^A(x_2, \mathbf{p}^2, \mathbf{x}) D_{\text{fun}}(x_1, \mathbf{x}, \mathbf{p})}{(2\pi)^2}.$$



THE INPUT

Low- x gluons

uGDFs $\rightarrow \Phi_i(x, \mathbf{r}, \mathbf{q}) \sim q^2 D_{\text{adj}}(x, \mathbf{r}, \mathbf{q})$

Dipoles $\rightarrow D_{\text{adj}}(x, \mathbf{r}, \mathbf{q}), D_{\text{fun}}(x, \mathbf{r}, \mathbf{q})$

└ GBW, IP-Sat, MV...

High- x partons

PDFs $\rightarrow x_i q_f(x_i, \mathbf{p}^2)$

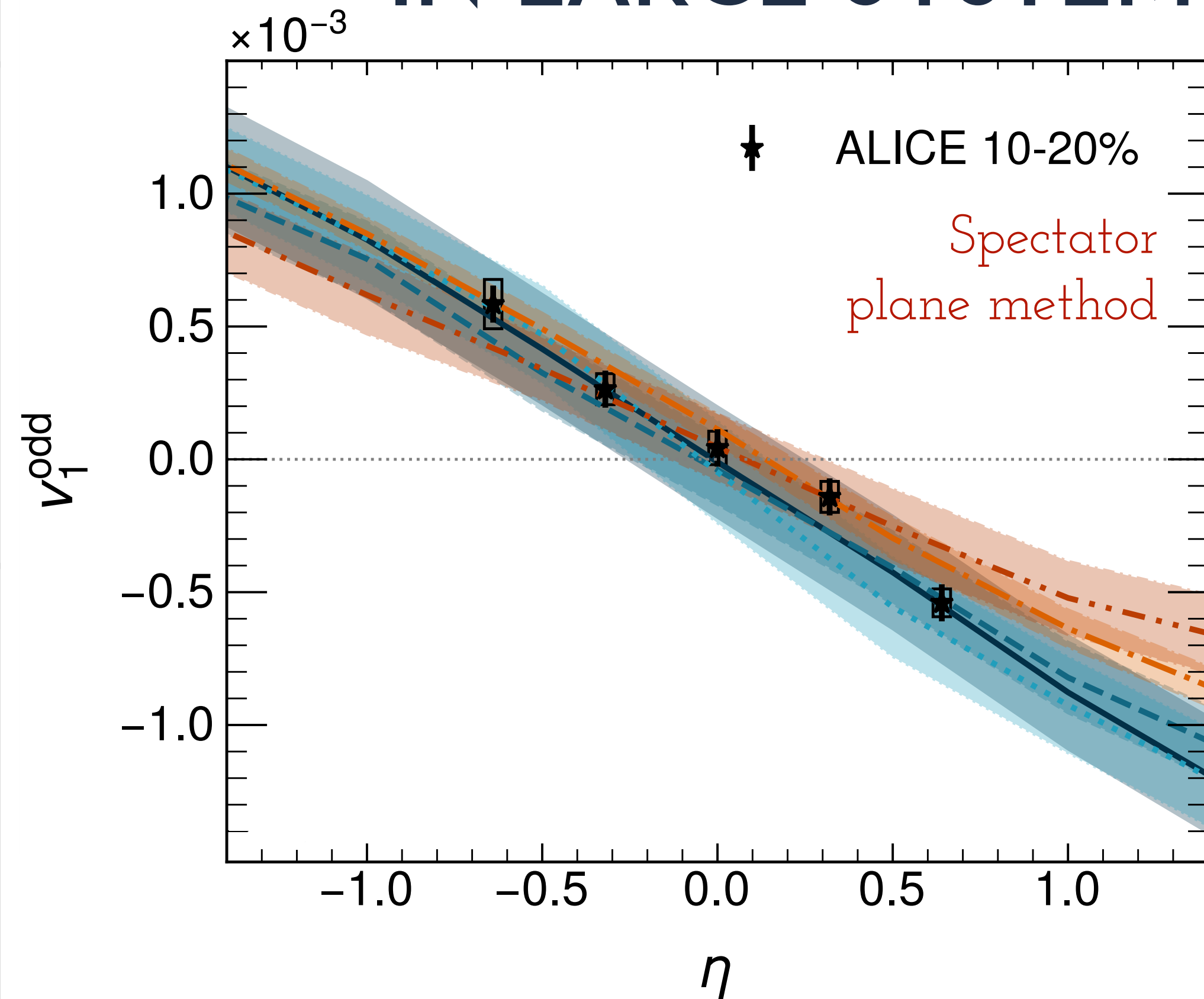
Different PDF sets*.

*Accessible in the MCDIPPER through the LHAPDF library

Systematically Improvable e.g. by including NLO $gg \rightarrow q\bar{q}$ production through gluon fusion

CONSERVED CHARGE DEPOSITION FROM THE CGC FORMALISM

IN LARGE SYSTEMS



THE INPUT

Low- x gluons

uGDFs $\rightarrow \Phi_i(x, r, q) \sim q^2 D_{\text{adj}}(x, r, q)$

Dipoles $\rightarrow D_{\text{adj}}(x, r, q), D_{\text{fun}}(x, r, q)$

└─● GBW, IP-Sat, MV...

High- x partons

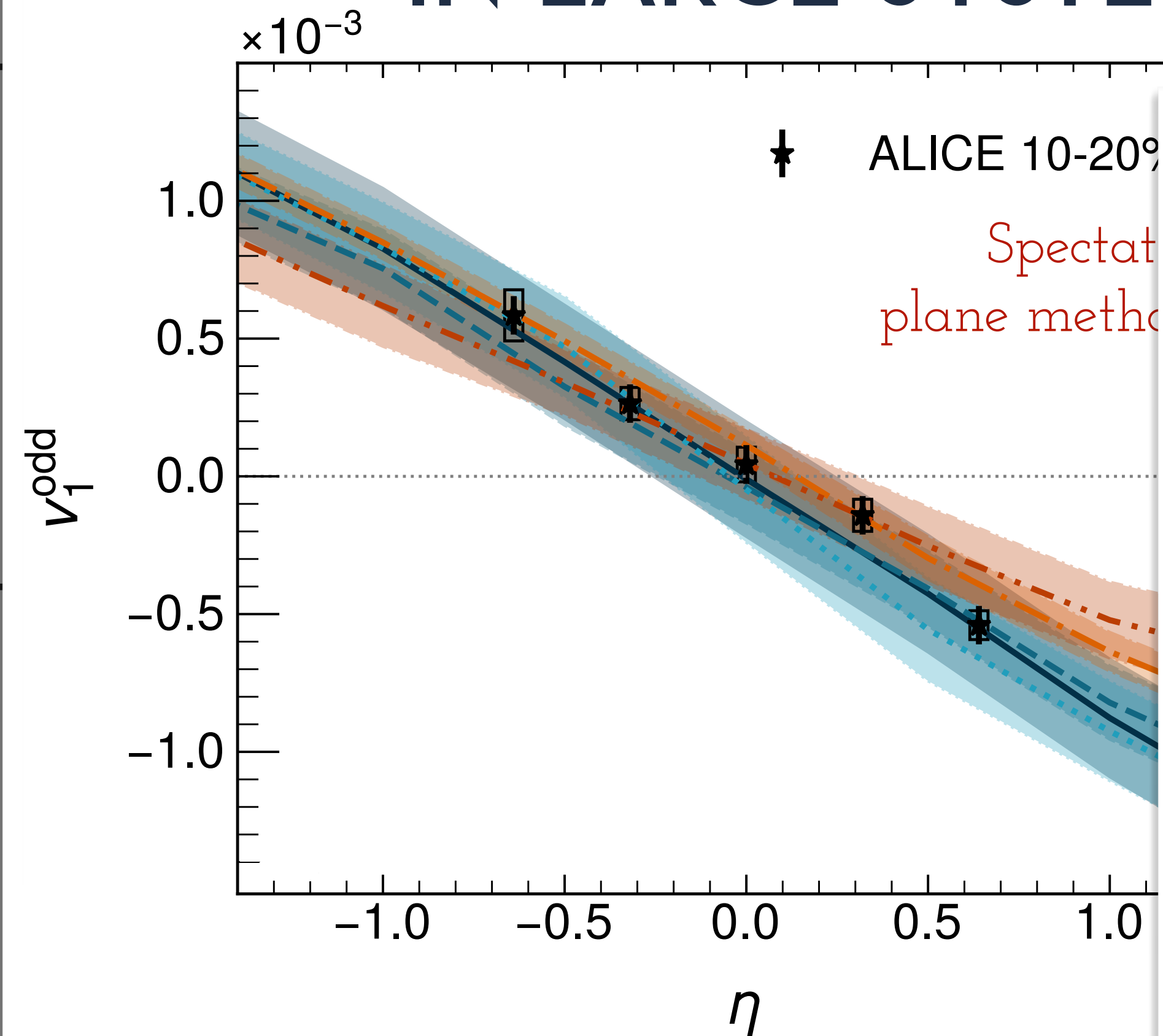
PDFs $\rightarrow x_i q_f(x_i, \mathbf{p}^2)$

Different PDF sets*.

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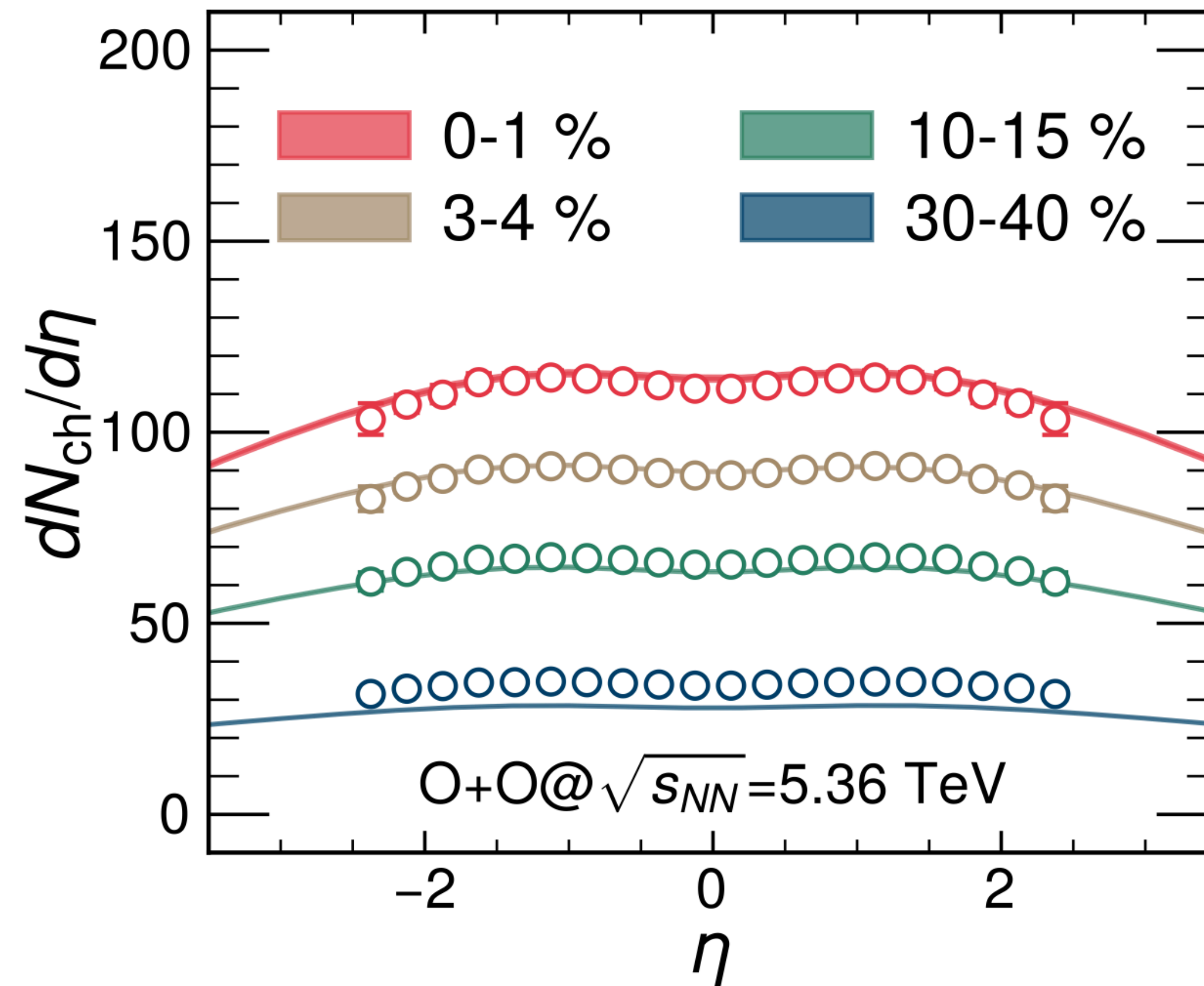
Systematically Improvable e.g. by including NLO $gg \rightarrow q\bar{q}$ production through gluon fusion

IN LARGE SYSTEMS



[See Jie Zhu's talk Wed. 11:45]

IN SMALL SYSTEMS

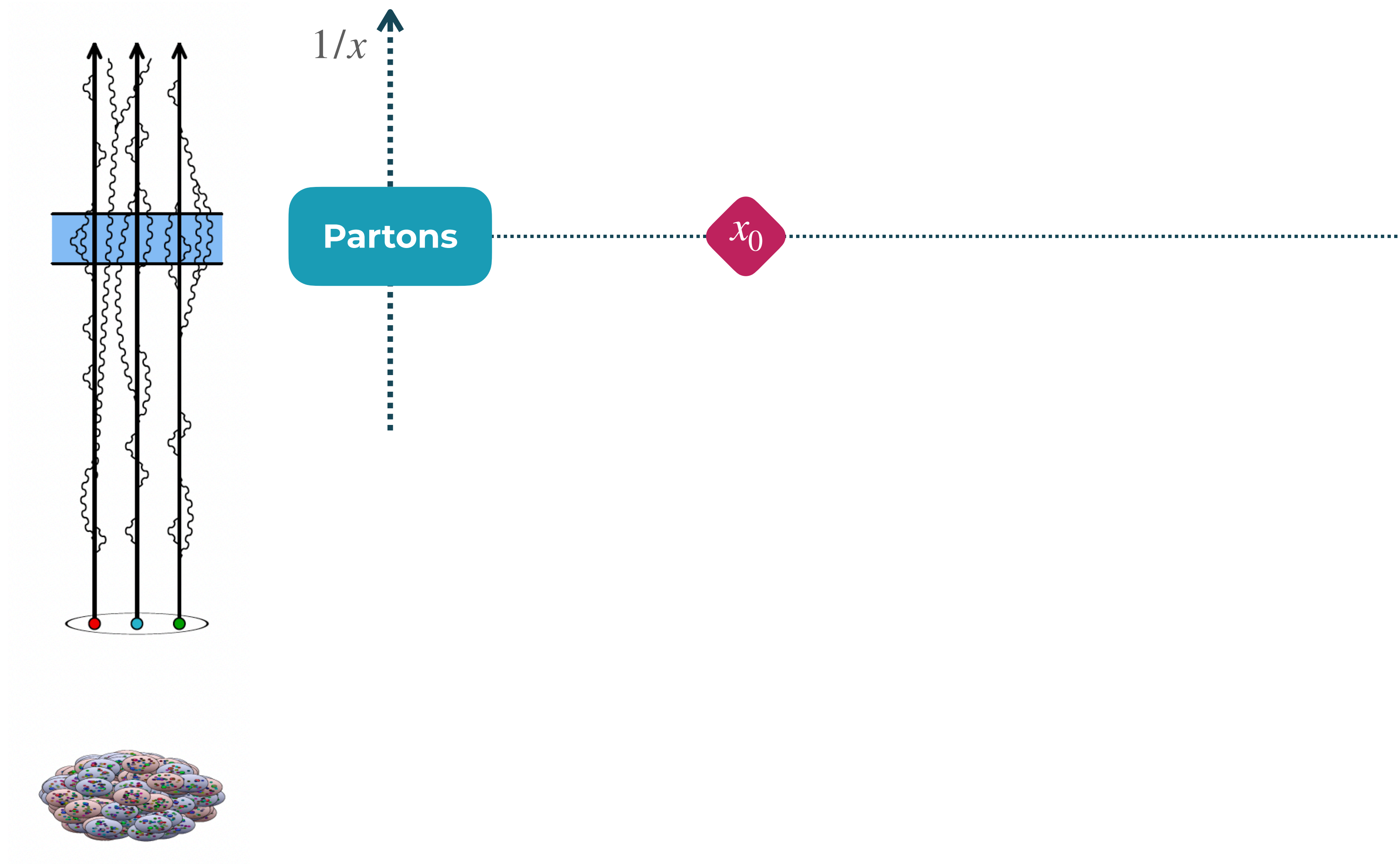


Systematically Improvable e.g. by including NLO $gg \rightarrow q\bar{q}$ production through gluon fusion

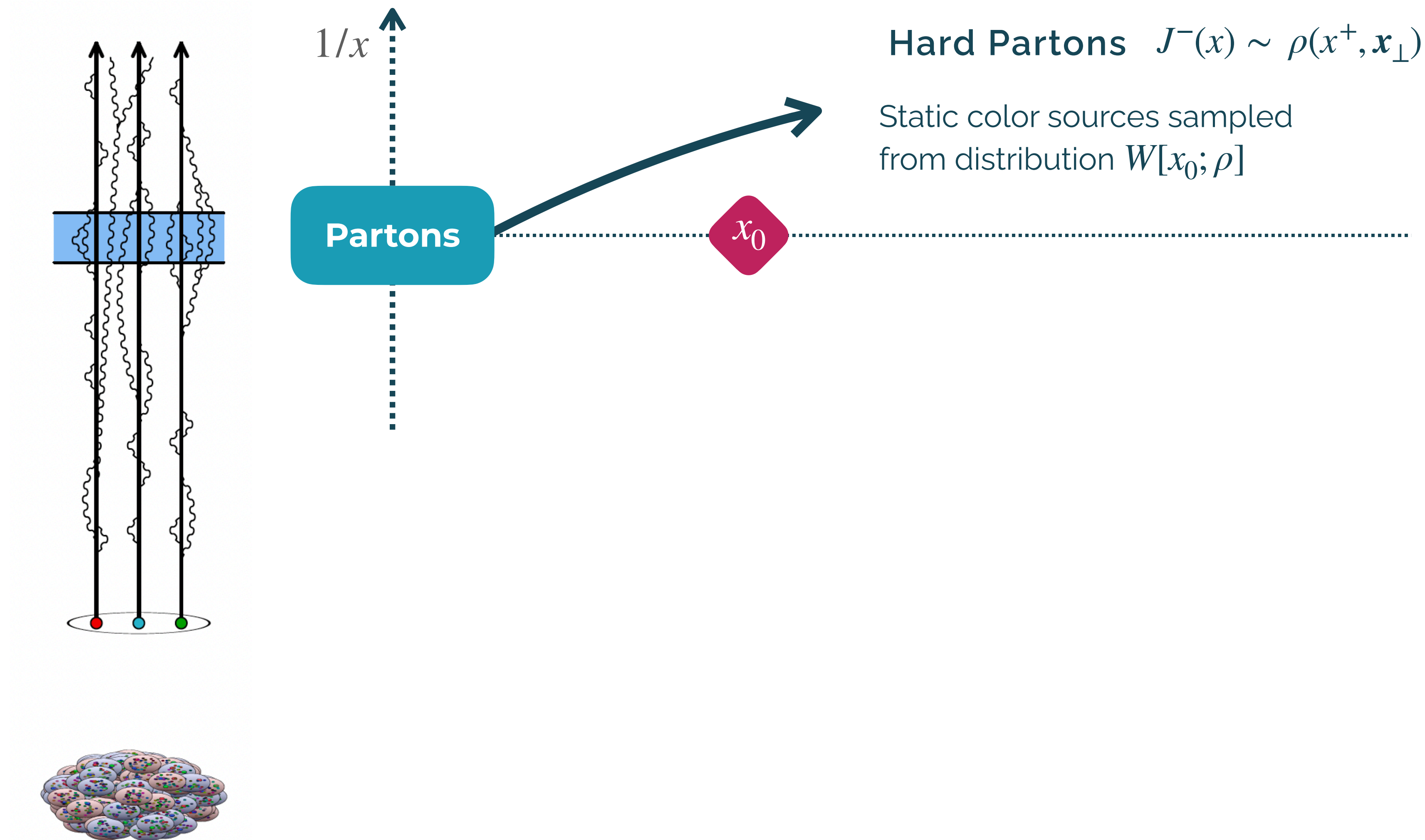
WHY IS THE KT-FACTORISED FORMULA SO... ISOTROPIC?

$$T^{\mu\nu}(\tau_0) = \begin{pmatrix} e & 0 & 0 & 0 \\ 0 & e/2 & 0 & 0 \\ 0 & 0 & e/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

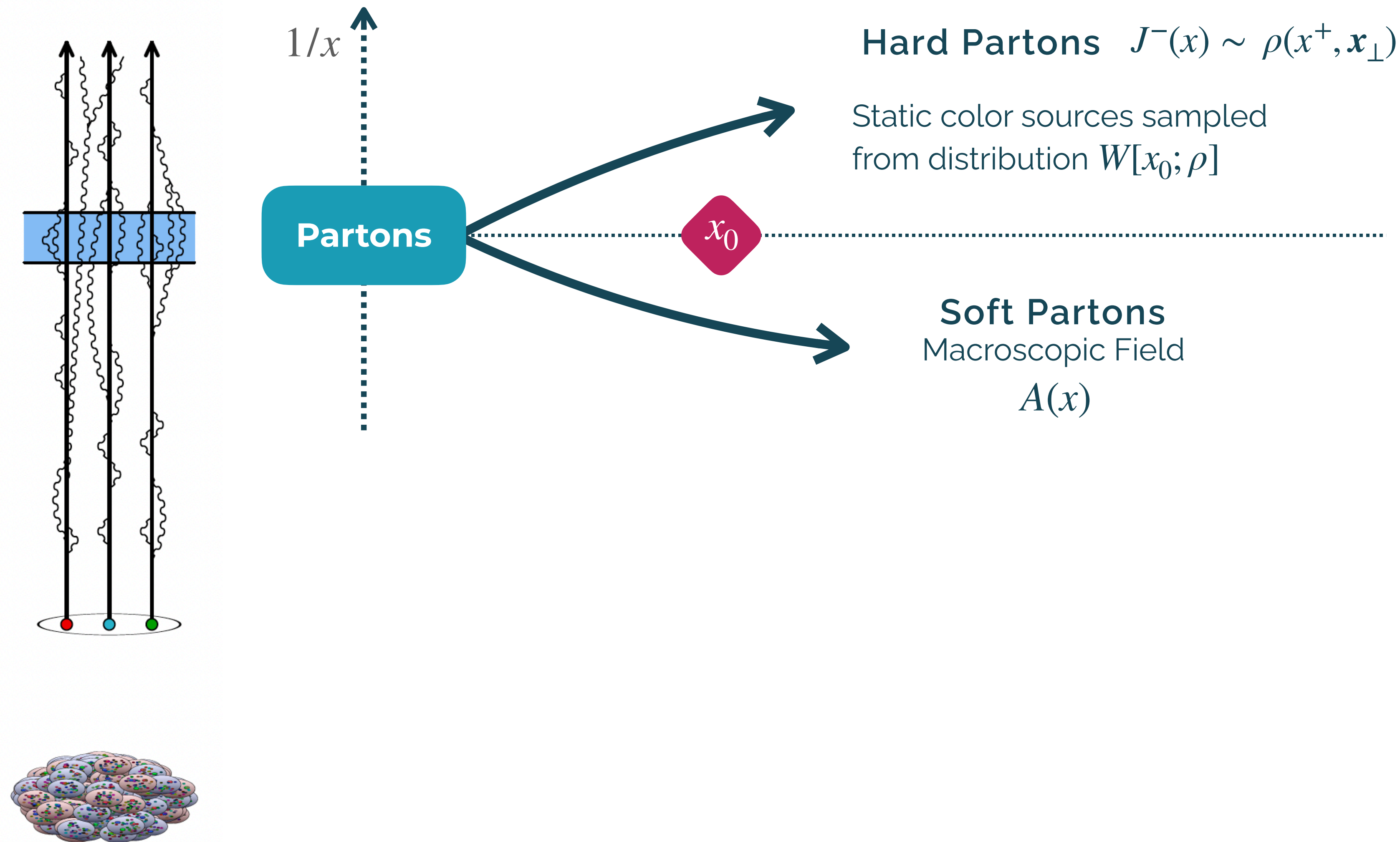
THE COLOR GLASS CONDENSATE



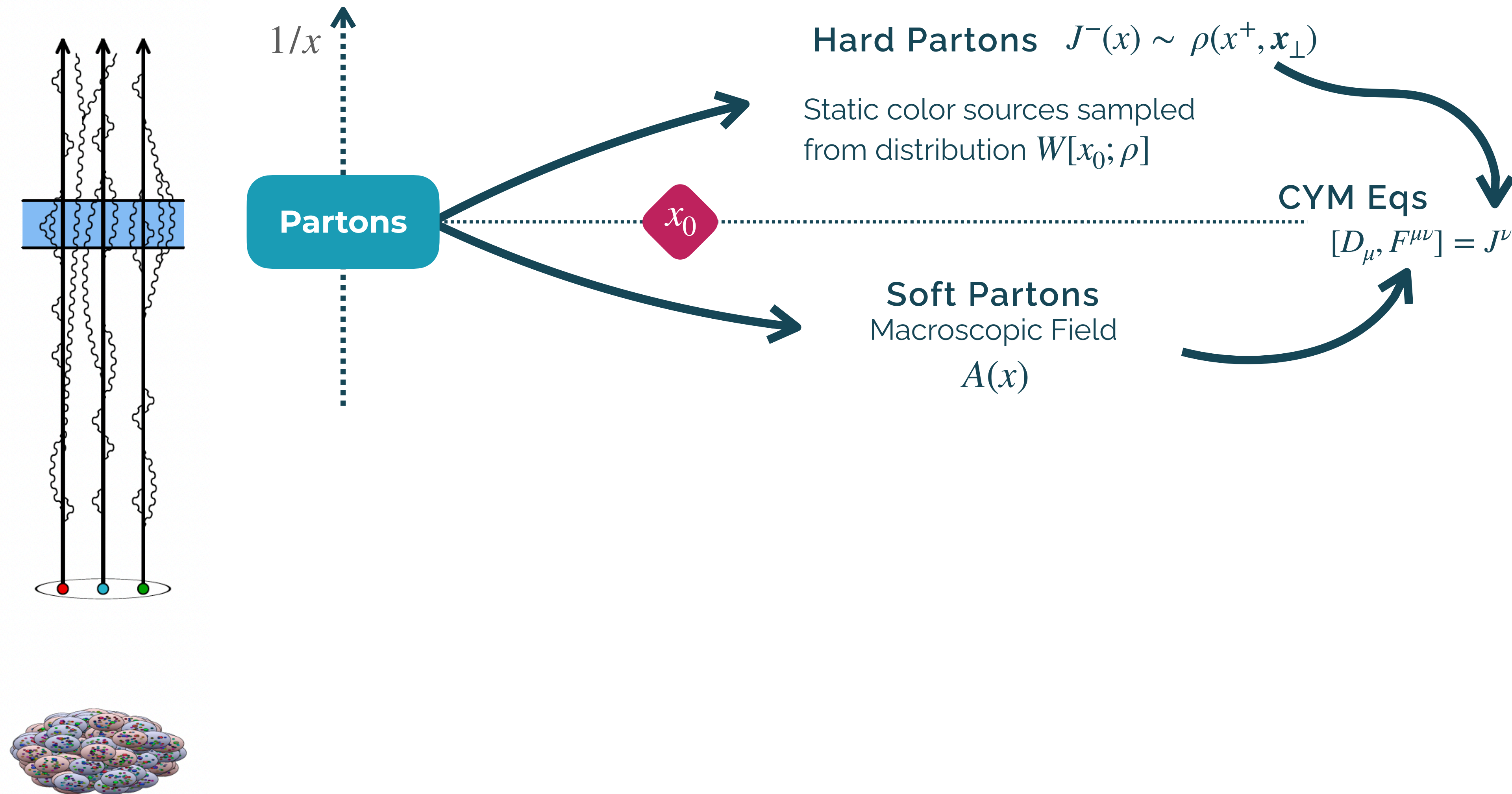
THE COLOR GLASS CONDENSATE



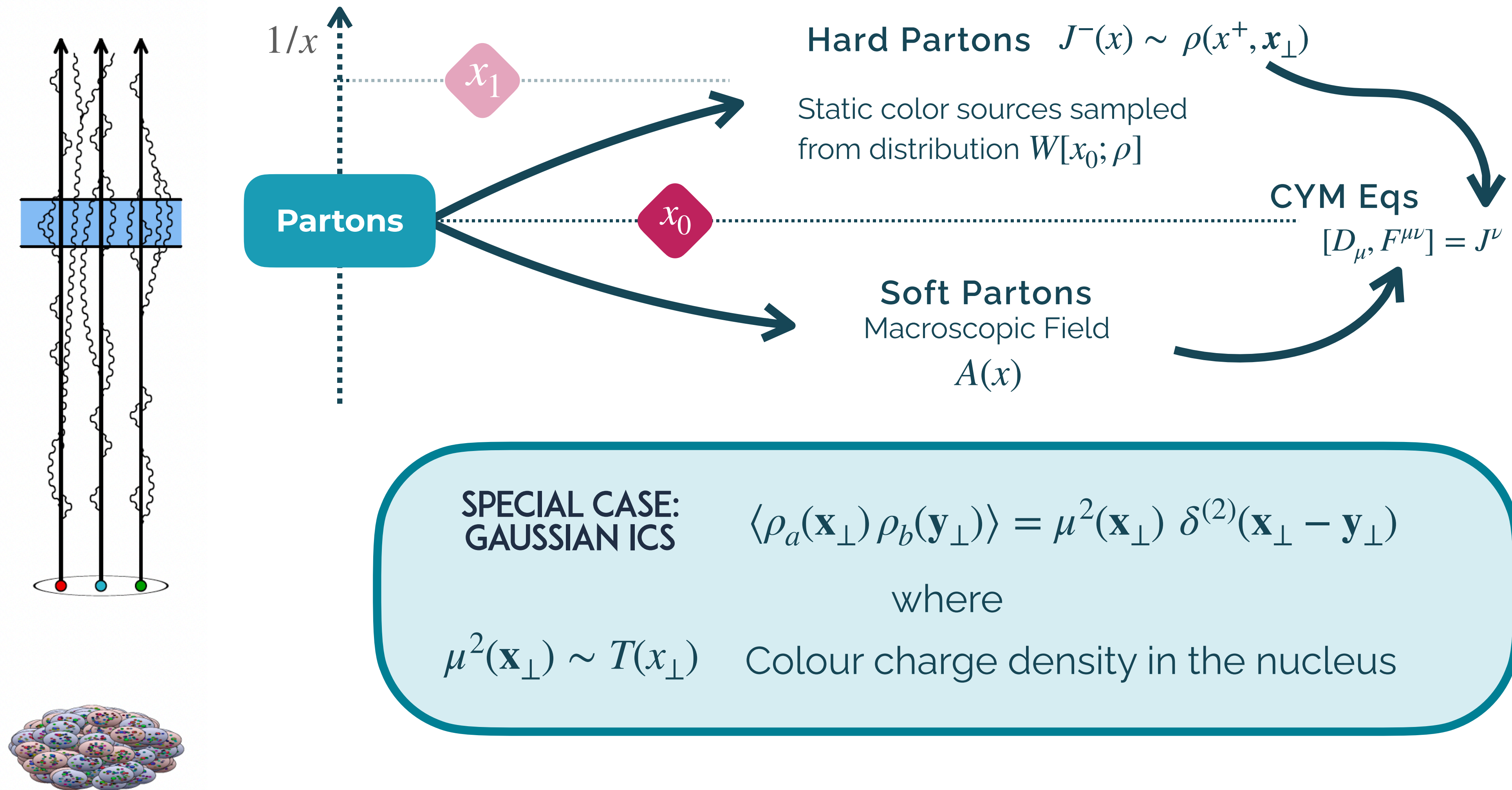
THE COLOR GLASS CONDENSATE



THE COLOR GLASS CONDENSATE

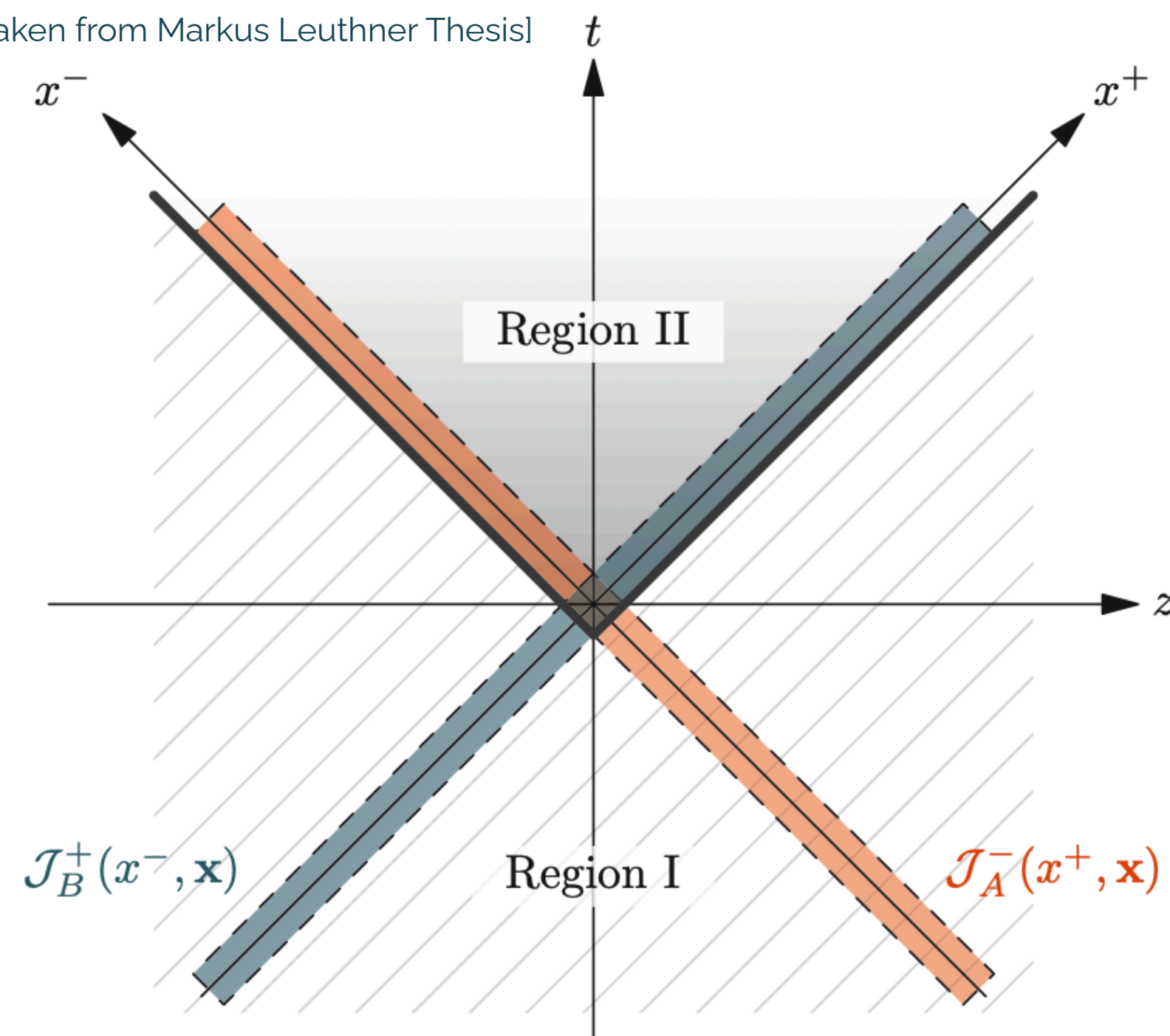


THE COLOR GLASS CONDENSATE



THE DILUTE GLASMA

[Taken from Markus Leuthner Thesis]



Single nucleus source terms for
 "background fields"

$$J_{A/B}^\mu(x^\pm, \mathbf{x}) = \delta_{\mp}^\mu \rho_{A/B}(x^\pm, \mathbf{x})$$

Solve the full CYM equations in the **future light cone (Region II)**
 and presence of both currents in the same limits as the
 McDIPPER

$$D_\mu F^{\mu\nu} = J^\mu \quad \text{With} \quad A^\mu = A_A^\mu + A_B^\mu + a^\mu$$

$$J^\mu = J_A^\mu + J_B^\mu + j^\mu$$

DILUTE-DILUTE

$gA_{1,2} \ll 1$, so that LO:
 $A \sim \mathcal{O}(\rho_1 \rho_2)$

EIKONAL LIMIT

Sources have vanishing support on the
 light-cone (ions infinitely contracted)
 $\rho(x^\pm, \mathbf{x}_\perp) = \delta(x^\pm) \rho(\mathbf{x})$

$$A^\mu = A_A^\mu[\rho_A] + A_B^\mu[\rho_B] + a^\mu[\rho_A \rho_B]$$

$$J^\mu = J_A^\mu[\rho_A] + J_B^\mu[\rho_B] + j^\mu[\rho_A \rho_B]$$

Where the solution can be found by truncating at this order.

INGREDIENTS FOR GLUON $T^{\mu\nu}$

We can find the Energy Momentum Tensor of the produced medium in the forward light-cone via

$$T^{\mu\nu} = 2 \operatorname{Tr} \left[f^{\mu\rho} f_{\rho}^{\nu} + \frac{1}{4} g^{\mu\nu} f^{\rho\sigma} f_{\rho\sigma} \right] \qquad f^{\mu\nu} = \partial^{\mu} a^{\nu} - \partial^{\nu} a^{\mu}$$

The “perturbation field strength”

From which we get, for example.

$$\mathcal{F}^{+-}(x) = g f_{abc} t^c \int_{k_1, k_2} \tilde{A}_{A,a}^{-}(k_1) \tilde{A}_{B,b}^{+}(k_2) (k_1 \cdot k_2) J_0\left(\left|k_1 + k_2\right| \tau\right) e^{-i(k_1+k_2) \cdot x}$$

In the dilute limit we have $Q_S \leq k_1, k_2$ which if $\tau > 1/Q_S \implies$ Plane wave description $J_0(x) \sim \zeta_0 e^{ix} / \sqrt{2\pi x}$

$$\mathcal{F}^{\mu\nu} = \frac{g}{\sqrt{2\pi}} f_{abc} t^c \zeta_0 \int_{k_1, k_2} \tilde{\mathcal{A}}_{A,a}^{-}(k_1) \tilde{\mathcal{A}}_{B,b}^{+}(k_2) \tilde{f}^{\mu\nu}(v, k_1, k_2) e^{i|k_1+k_2| \tau'} e^{-i(k_1+k_2) \cdot x} .$$

GLUON $T^{\mu\nu}$

Then we can find the energy momentum tensor

$$\langle T^{\mu\nu}(x) \rangle = \frac{\alpha_S}{C_F} \int_{k_1, k_2} \int_{\bar{k}_1, \bar{k}_2} \left\langle \tilde{\mathcal{A}}_{A,a}^{-,\dagger}(k_1) \tilde{\mathcal{A}}_{A,a}^-(\bar{k}_1) \right\rangle \left\langle \tilde{\mathcal{A}}_{B,b}^{+,\dagger}(k_2) \tilde{\mathcal{A}}_{B,b}^+(\bar{k}_2) \right\rangle \\ \times t^{\mu\nu}(x, k_1, k_2; x, \bar{k}_1, \bar{k}_2) e^{-i|k_1+k_2|\tau} e^{i(k_1+k_2)\cdot x} e^{i|\bar{k}_1+\bar{k}_2|\tau} e^{-i(\bar{k}_1+\bar{k}_2)\cdot x}$$

$$\left\langle \tilde{\mathcal{A}}_{A,a}^{-,\dagger}(k_1) \tilde{\mathcal{A}}_{A,a}^-(\bar{k}_1) \right\rangle = \frac{1}{\pi} \frac{1}{k_1 \cdot \bar{k}_1} \int_x e^{-ix \cdot (k_1 - \bar{k}_1)} \varphi_A \left(\frac{k_1 + \bar{k}_1}{2}, x \right)$$

We can express the single nucleus correlations in terms of the **unintegrated gluon distributions in the nuclei**

Fourier-conjugate of spatial dependence of distribution

Momentum of the sampled gluon from source

And changing variables to

$$q \equiv \frac{k_1 + \bar{k}_1}{2}, \quad k \equiv \frac{k_2 + \bar{k}_2}{2}, \quad \tilde{q} \equiv k_1 - \bar{k}_1, \quad \tilde{k} \equiv k_2 - \bar{k}_2,$$

GLUON $T^{\mu\nu}$

Then we have the new version

$$\langle T^{\mu\nu}(x) \rangle = \frac{\alpha_S}{\pi^2 C_F} \int_{q,k,\tilde{q},\tilde{k}} \int_{z,y} \mathcal{H}^{\mu\nu} \left(x; \mathbf{q} + \frac{\tilde{\mathbf{q}}}{2}, \mathbf{k} + \frac{\tilde{\mathbf{k}}}{2}; \mathbf{q} - \frac{\tilde{\mathbf{q}}}{2}, \mathbf{k} - \frac{\tilde{\mathbf{k}}}{2} \right) \varphi_A(\mathbf{q}, \mathbf{z}) \varphi_B(\mathbf{k}, \mathbf{y}) e^{-i(\mathbf{z}-\mathbf{x})\cdot\tilde{\mathbf{q}}} e^{-i(\mathbf{y}-\mathbf{x})\cdot\tilde{\mathbf{k}}}$$

Where we have defined

$$\mathcal{H}^{\mu\nu}(x; \mathbf{k}_1, \mathbf{k}_2; \bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2) = \frac{t^{\mu\nu}(x, \mathbf{k}_1, \mathbf{k}_2; x, \bar{\mathbf{k}}_1, \bar{\mathbf{k}}_2)}{(\mathbf{k}_1 \cdot \bar{\mathbf{k}}_1) (\mathbf{k}_2 \cdot \bar{\mathbf{k}}_2)} e^{-i\tau (|\mathbf{k}_1 + \mathbf{k}_2| - |\bar{\mathbf{k}}_1 + \bar{\mathbf{k}}_2|)}$$

INTERMEZZO: THE SCALES OF THIS SYSTEM

The "radius of variation" of a nuclear source is expected to be $\frac{1}{R} \sim |\tilde{\mathbf{q}}|, |\tilde{\mathbf{k}}| < Q_s$

In the dilute limit, we have $Q_s \leq |\mathbf{q}|, |\mathbf{k}|$

$\implies$ Scale hierarchy $\frac{1}{R} \sim |\tilde{\mathbf{q}}|, |\tilde{\mathbf{k}}| < Q_s \leq |\mathbf{q}|, |\mathbf{k}|$

GRADIENT EXPANSION

Two-point process: Expand around $\tilde{\mathbf{q}}, \tilde{\mathbf{k}}$, and then use the plane wave to get gradients

$$\langle T^{\mu\nu}(x) \rangle = \frac{\alpha_S}{\pi^2 C_F} \sum_{l=0}^{\infty} \int_{\mathbf{q}, \mathbf{k}} \left[\mathcal{H}_l^{\mu\nu} \left(x, \mathbf{q}, \mathbf{k}; -i \nabla_z, -i \nabla_y \right) \varphi_A(\mathbf{q}, z) \varphi_B(\mathbf{k}, y) \right]_{y=z=x} = \sum_{l=0}^{\infty} \langle T_{(l)}^{\mu\nu} \rangle.$$

Where the gradient functional is short hand for a more complex sum

$$\mathcal{H}_l^{\mu\nu} \left(x, \mathbf{q}, \mathbf{k}; \tilde{\mathbf{q}}, \tilde{\mathbf{k}} \right) = \sum_{n,m=0}^{\infty} \frac{\delta_{l,n+m}}{n! m!} \tilde{q}^{i_1} \dots \tilde{q}^{i_n} \tilde{k}^{j_1} \dots \tilde{k}^{j_m} \mathcal{H}_{(n,m)}^{\mu\nu; i_1, \dots, i_n; j_1, \dots, j_m}(\mathbf{q}, \mathbf{k})$$

We get a ladder of corrections, which are effectively moments of the gluon momentum distributions, and operated up by transverse gradients!

Hierarchy of operators, which in terms of the scale separation are suppressed by powers of

$$1/(Q_s R) \longrightarrow 0.1 - 0.2 \quad \text{Correction for nucleonic EbE in LHC}$$

ZEROth ORDER

The zeroth order term corresponds to the $k_{\perp}$ factorized formula, **the basis of the McDIPPER**.

$$\tau T^{\mu\nu} = \int d^2\mathbf{p} \, dy \, \frac{p^{\mu} p^{\nu}}{p^{\tau}} \frac{dN^0}{d^2\mathbf{x} \, d^2\mathbf{p} \, dy} \delta(\eta - y)$$

This is the EMT of a set of on-shell particles,
and fully isotropic

With the zeroth order particle
yield

$$E_p \frac{dN_g^{(0)}}{d^3p \, d^2\mathbf{x}} = \frac{\alpha_S}{2\pi^4 \mathbf{p}^2 C_F} \int_{qk} (2\pi)^2 \delta^{(2)}(\mathbf{p} - \mathbf{q} - \mathbf{k}) \varphi_A(x_A, \mathbf{q}, \mathbf{x}) \varphi_B(x_B, \mathbf{k}, \mathbf{x})$$

FIRST ORDER

The first order correction arises from the phase in the kernel, and corresponds to a response to the zeroth order yield

$$\left\langle T_{(1)}^{\mu\nu} \right\rangle = - \int d^2\mathbf{p} \left[\frac{p^{\mu} p^{\nu}}{p^{\tau}} \hat{\mathbf{p}} \cdot \nabla_x \left(\frac{dN^{(0)}}{d^2\mathbf{x} \, d^2\mathbf{p} \, dy} \right) \right]_{\eta=y} \sim - \nabla_x (\tau P_0)$$

This creates a **non-zero transverse Poynting**, resulting in a **non-vanishing transverse velocity**

SECOND ORDER

Elliptical response

The second order is a mess, but can be split into

$$\left\langle T_{(2)}^{\mu\nu} \right\rangle = \left\langle T_{(2,os)}^{\mu\nu} \right\rangle + \left\langle T_{(2,\epsilon)}^{\mu\nu} \right\rangle + \left\langle \delta T_{(2)}^{\mu\nu} \right\rangle$$

Yield correction "rest"

The yield correction

$$\frac{dN_g}{d^2\mathbf{p} d^2\mathbf{x} dy} = \frac{\alpha_S}{2\pi^4 \mathbf{p}^2 C_F} \int_{qk} (2\pi)^2 \delta^{(2)}(\mathbf{p} - \mathbf{q} - \mathbf{k}) \frac{P_{qk}^{ij}}{k^2 q^2} \left[\nabla_x^j \varphi_A(x_A, \mathbf{q}, \mathbf{x}) \nabla_y^i \varphi_B(x_B, \mathbf{k}, \mathbf{y}) \right]_{y=x}$$

Suppressed by powers of

$$|\mathbf{p}| = |\mathbf{q} + \mathbf{k}| \gg Q_S$$

Elliptical response

$$\tau \left\langle T_{(2,\epsilon)}^{\mu\nu} \right\rangle = -\frac{1}{8} \int d^2\mathbf{p} \left[\frac{p^\mu p^\nu}{p^\tau} \frac{1}{|\mathbf{p}|^6} (2p^i p^j - \delta^{ij} \mathbf{p}^2) \nabla_x^i \nabla_x^j \left(\frac{dN^{(0)}}{d^2\mathbf{x} d^2\mathbf{p} dy} \right) \right]_{\eta=y}$$

More IR

"rest"
Is complicated, e.g.

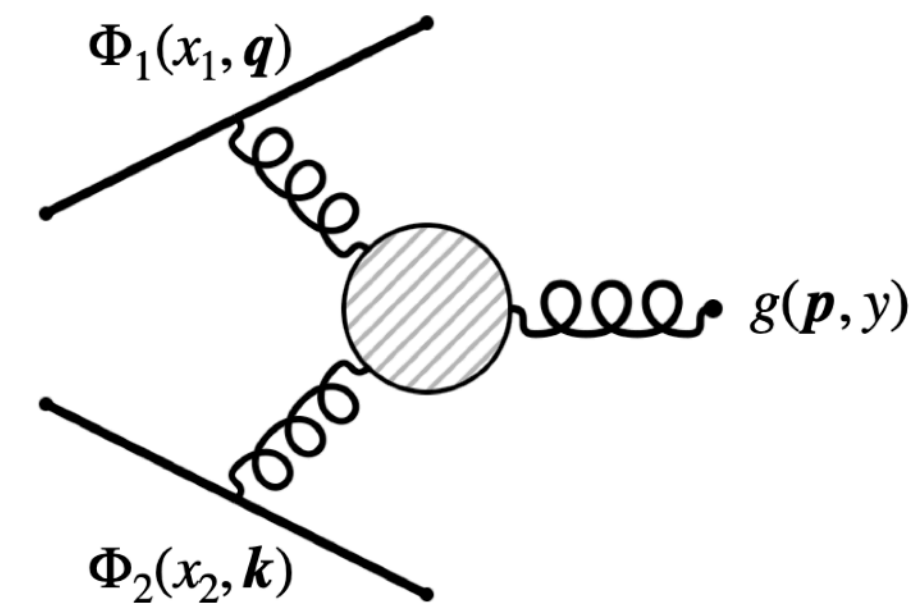
$$\tau \left\langle \delta T_{(2)}^{\tau\tau} \right\rangle = \frac{\alpha_S}{2\pi^2 C_F} \int_{p,q,k} (2\pi)^2 \delta^{(2)}(\mathbf{p} - \mathbf{q} - \mathbf{k}) \times \left[\frac{1}{|\mathbf{p}|^5} \left(\mathbf{p} \times (\nabla_x + \nabla_y) \right)^2 \varphi_A(x_A, \mathbf{q}, \mathbf{x}) \varphi_B(x_B, \mathbf{k}, \mathbf{y}) \right]_{y=x}$$

Interpolation between "particle" and "field" dynamics



A SIMPLE CASE: A TEASER OF MY WIP

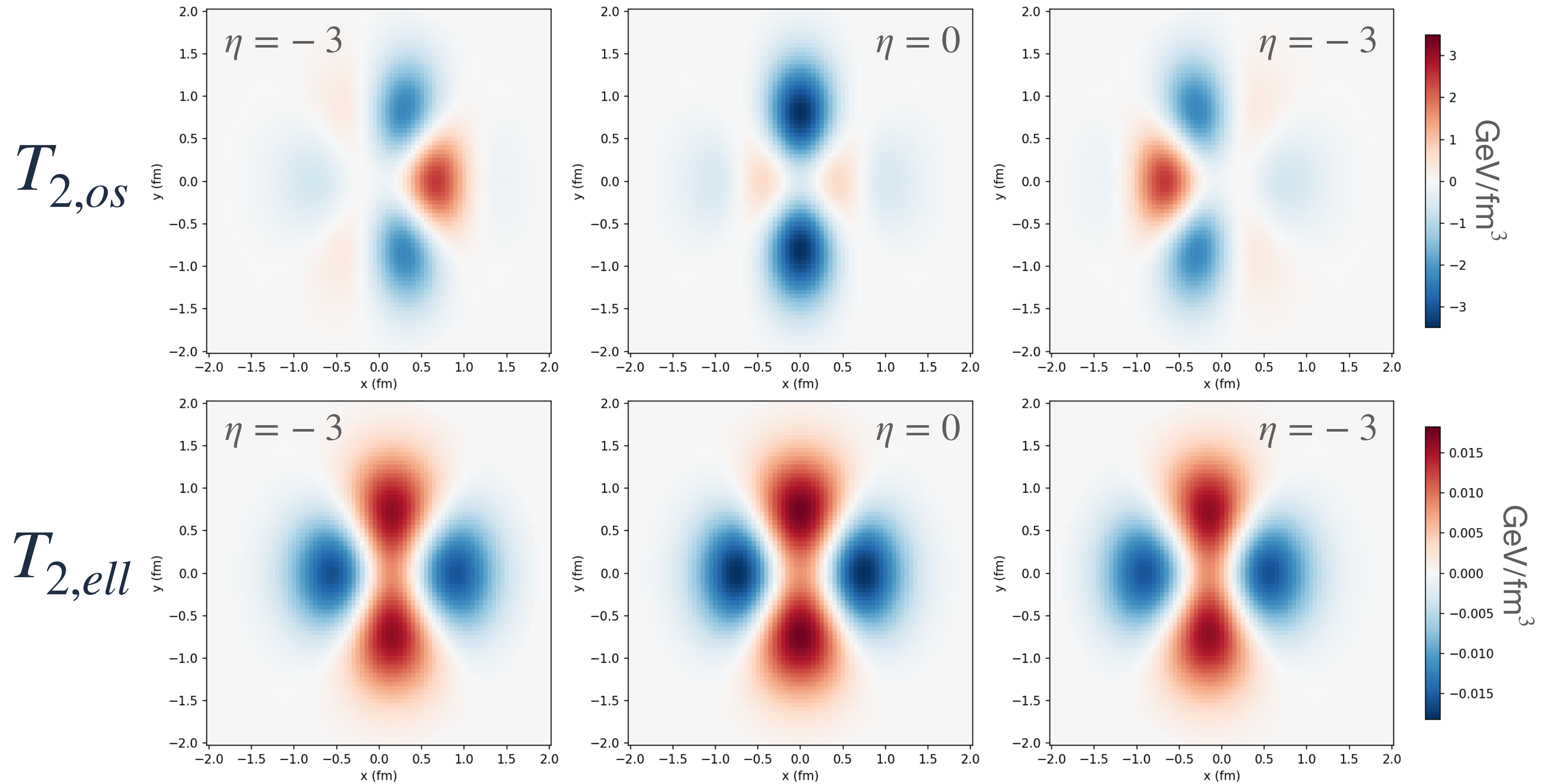
- GBW model (gaussian gluon distributions) $D(x, \mathbf{k}_\perp, \mathbf{x}_\perp) \sim \exp\left(-\frac{\mathbf{k}_\perp^2}{Q_A^2(x, \mathbf{x}_\perp)}\right)$ $Q_A^2(x, \mathbf{x}_\perp) \sim x^{-\lambda} T(\mathbf{x}_\perp)$
- Simple kinematics
Fixed by "measured" gluon $x_1 = \frac{p_\perp e^y}{\sqrt{s_{NN}}}$ $x_2 = \frac{p_\perp e^{-y}}{\sqrt{s_{NN}}}$
- Gluon distribution dominated by saturation scale $x_{1,2} = \frac{Q_{1,2}^2(x, T) e^{\pm y}}{\sqrt{s_{NN}}}$
- Analytical solution to energy $e\tau_0 \sim \frac{Q_A^2 Q_B^2}{(Q_A^2 + Q_B^2)^{5/2}} (2Q_A^4 + 2Q_B^4 + 7Q_A^2 Q_B^2)$ Complex function $e\tau_0[T_1, T_2]$
- Something like T_RENTO $e\tau_0 \sim \left[\frac{T_A^p + T_B^p}{2}\right]^{1/p}$



WHY IS THIS IMPORTANT?

Particle correlations take $\langle \cdot \rangle$ of powers of thermo. quantities.
Even when subleading, extra-powers may induce error.

PRESSURE ANISOTROPY $T^{xx} - T^{yy}$: SINGLE COLLISION CASE



SUMMARY AND CONCLUSIONS



Gradient expansion in the gluon production amplitude yields a hierarchy of a **new scalar and tensor structures**, built from local dipole amplitudes and their transverse derivatives,



Worked an **extended $k_{\perp}$ -factorized ansatz** at first non-trivial gradient order: keeps the compact factorized form while presenting new positional dependence.



The effect should be most pronounced in fluctuation-dominated systems — O+O, Ne+Ne, as the gradients $|\nabla_{\perp} Q_s^2|$ are more important.



What is left to do, is to implement this into phenomenologically successful models like the IP-Sat.



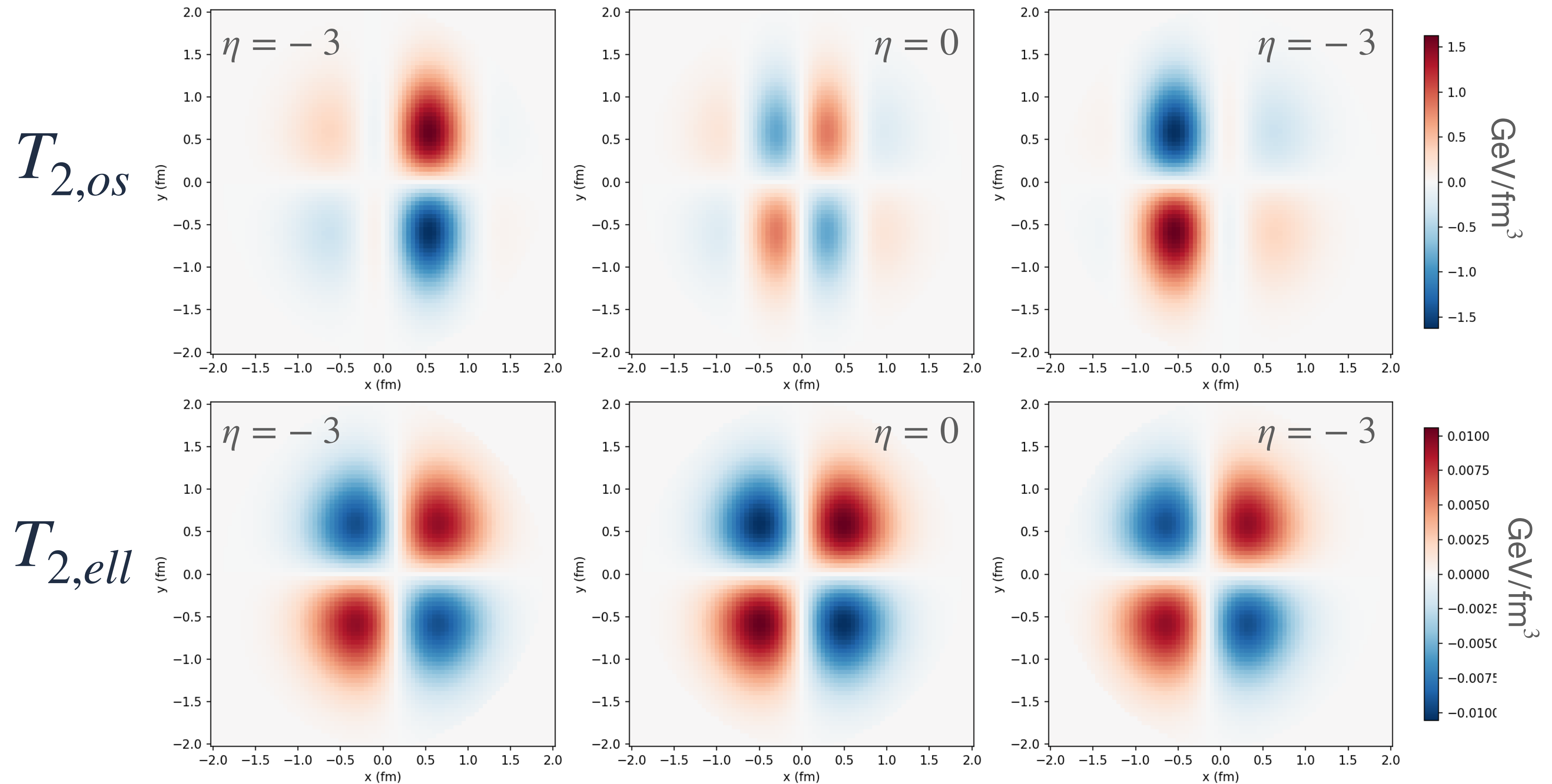
Constraining the first instants of the Quark Gluon Plasma

September 20-24, 2027

Organizers: Oscar Garcia Montero, Pablo Guerrero, Dana Avramescu,
Carlota Andrés, Jacquelyn Noronha-Hostler

BACKUP

PRESSURE ANISOTROPY T^{xy} : SINGLE COLLISION CASE (PP)



PRESSURE ANISOTROPY T^{xy} : SINGLE EVENT CASE (XE-XE)

